Superconductive cavity driving with FPGA controller

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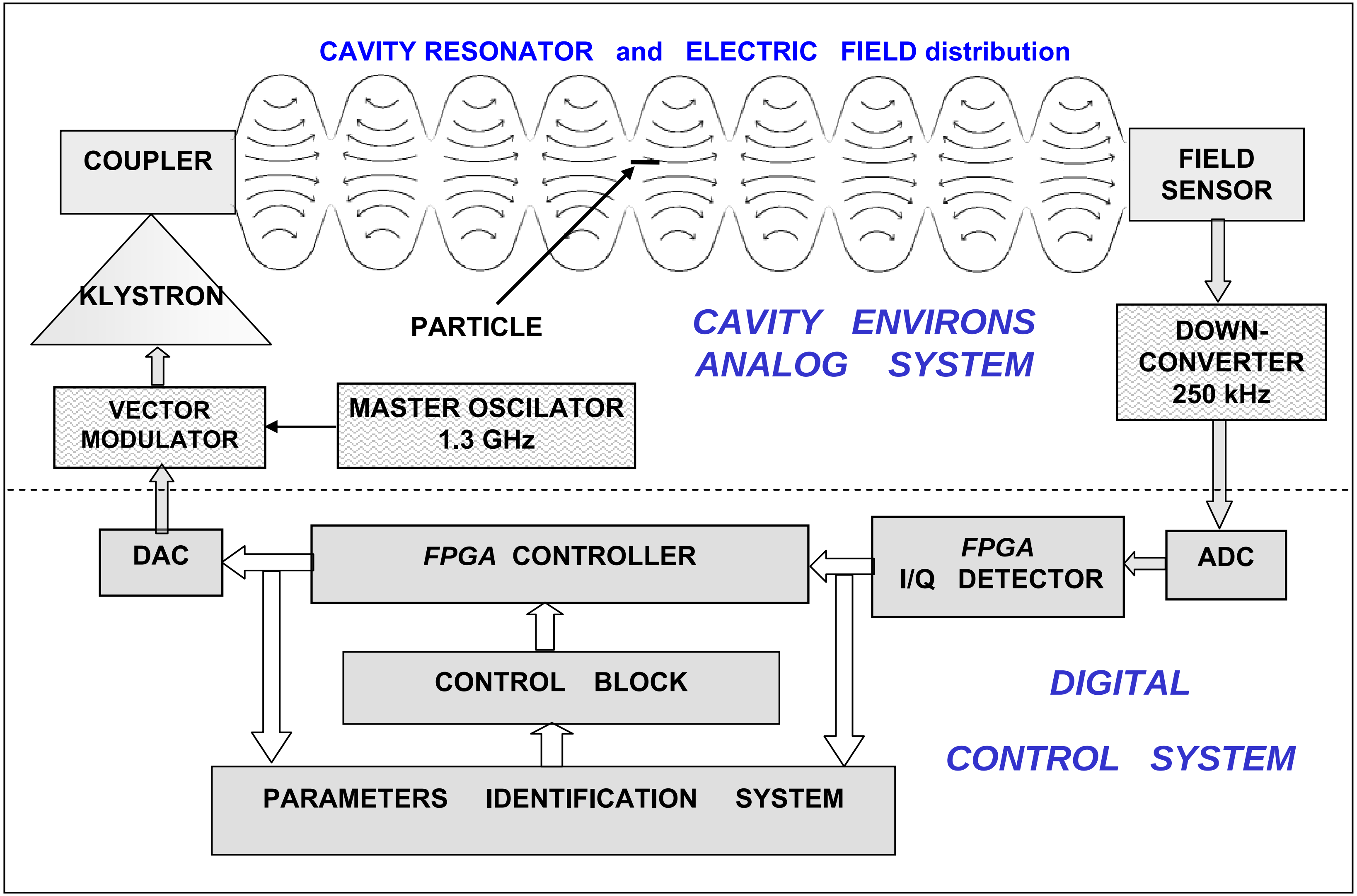
Main topics

- Low Level RF control system introduction
- Superconducting cavity modeling
- Cavity features
- Recognition of real cavity system
- Cavity parameters identification
- Control system algorithm
- Experimental results

Functional block diagram of Low Level Radio Frequency Cavity Control System

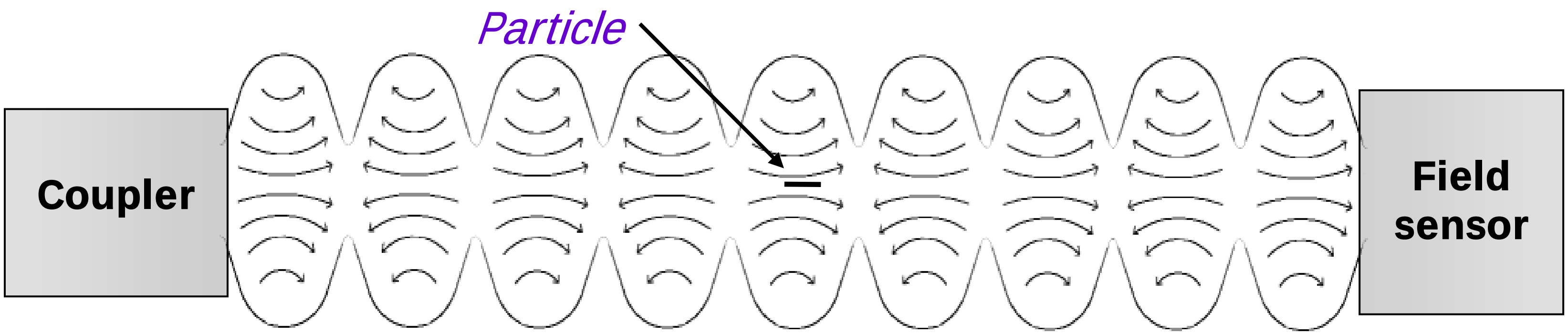


Superconductive cavity modeling

RF current	CAVITY transfer function	*RF voltage*
$\mathbf{I}(\mathbf{s}-\boldsymbol{i}\omega_g)$	$Z(\mathbf{s}) = (1/R+\mathbf{s}C+1/\mathbf{s}L)^{-1}$	$Z(\mathbf{s})\cdot\mathbf{I}(\mathbf{s}-\boldsymbol{i}\omega_g)$
Modulation $\exp(\boldsymbol{i}\omega_g t)$	*Analytic signal:* $a(t)\cdot\exp[\boldsymbol{i}(\omega_g t + \varphi(t))]$ *Complex envelope:* $a(t)\cdot\exp[\boldsymbol{i}\varphi(t)] = [I,Q]$	*Demodulation* $\exp(-\boldsymbol{i}\omega_g t)$
Input signal $\mathbf{I}(\mathbf{s})\leftrightarrow\mathbf{i}(t)$	Low pass transformation $Z(\mathbf{s}+\boldsymbol{i}\omega_g) \approx \omega_{1/2}\cdot R/(\mathbf{s}+\omega_{1/2}-\boldsymbol{i}\Delta\omega)$ *half-bandwidth* $= \omega_{1/2}$ *detuning* $= \Delta\omega$	*Output signal* $Z(\mathbf{s}+\boldsymbol{i}\omega_g)\cdot\mathbf{I}(\mathbf{s})\leftrightarrow\mathbf{v}(t)$

State space – continuous and discrete model	
$d\mathbf{v}/dt = \mathbf{A}\cdot\mathbf{v} + \omega_{1/2}\cdot R\cdot\mathbf{i}$ $\mathbf{A} = -\omega_{1/2}+\boldsymbol{i}\Delta\omega$	$\mathbf{v}_n = \mathbf{E}\cdot\mathbf{v}_{n-1} + T\cdot\omega_{1/2}\cdot R\cdot\mathbf{i}_{n-1}$ $\mathbf{E} = (1-\omega_{1/2}\cdot T) + \boldsymbol{i}\Delta\omega\cdot T$

Cavity features

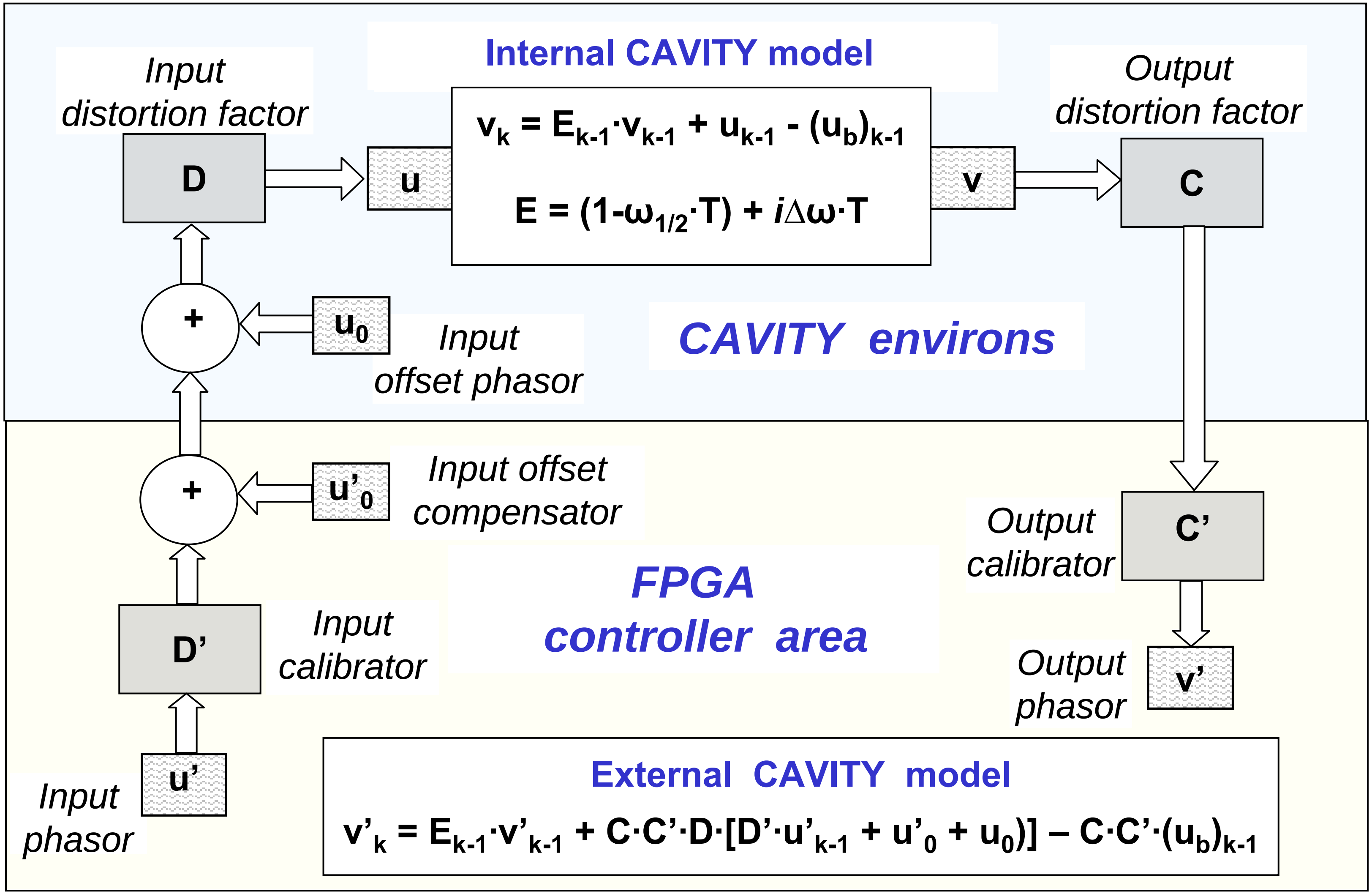


Carrier frequency	Pulse time	Repetition time	Field gradient	Beam pulse repetition	Average beam current
1.3 GHz	1.3 ms	100 ms	25 MV/m	1μs	8 mA

Cavity parameters: *resonance circuit parameters:* R L C

Resonance frequency	Characteristic resistance	Quality factor	Half-bandwidth	Detuning
$\omega_0 = (LC)^{-½}$ $2\pi \cdot 1.3$ GHz	$\rho = (L/C)^{½}$ 520 Ω	$Q = R/\rho$ $3 \cdot 10^6$	$\omega_{1/2} = 1/2RC$ $2\pi \cdot 215$ Hz	$\Delta\omega = \omega_0 - \omega_g$ $\sim 2\pi \cdot 600$ Hz

Algebraic model of cavity environment system



Parameters identification of cavity system in noisy and no stationary condition

External CAVITY model

$$\mathbf{v}_{k+1} = \mathbf{E}_k \cdot \mathbf{v}_k + \mathbf{F}_\mathbf{k} \cdot \mathbf{u}_k \qquad \mathbf{E}_\mathbf{k} = (1 - \omega_{1/2} \cdot T) + \boldsymbol{i}\Delta\omega_k \cdot T$$

Linear decomposition of no stationary parameter **Y**:

$$\mathbf{Y} = \mathbf{W} * \mathbf{x}$$

W – matrix of base functions: *polynomial* or *cubic B-spline set*

x – unknown vector of series coefficients

Over-determined matrix equation for measurement range**:**

$$\mathbf{V} = \mathbf{Z} * \mathbf{z}$$

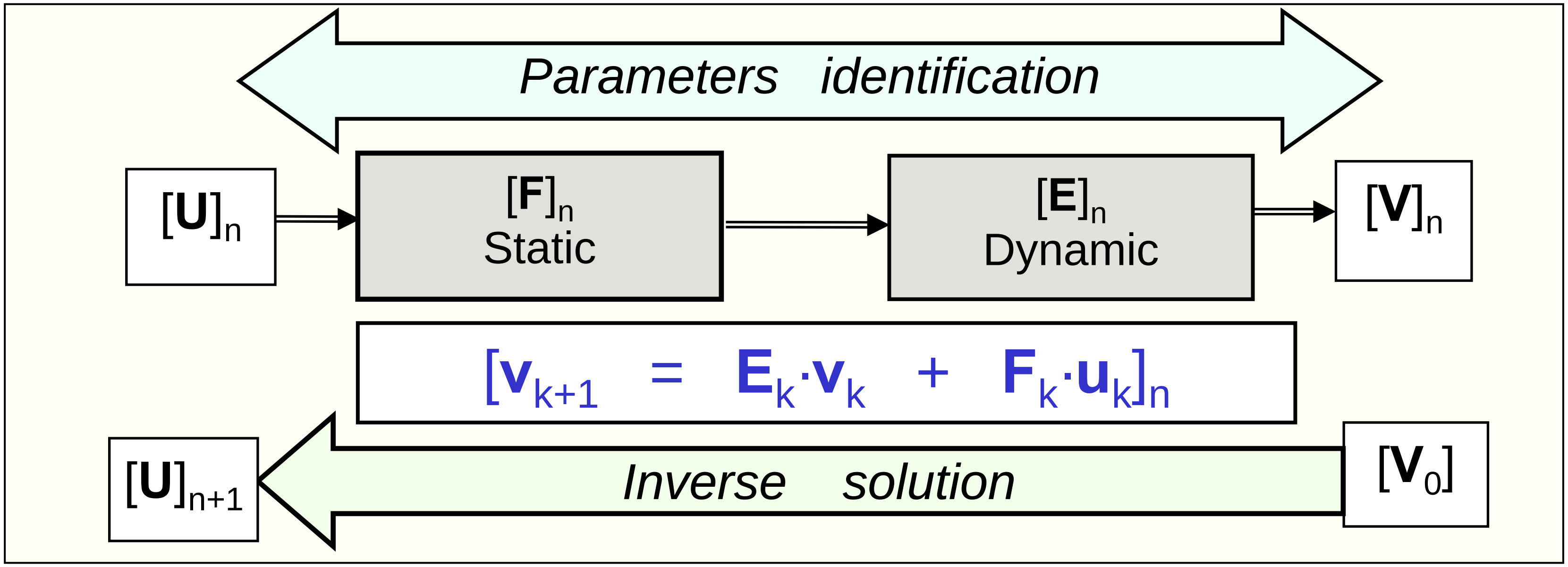
V – total output vector, **Z** – total structure matrix

z – total vector of unknown values

***Least square (LS)* solution**:

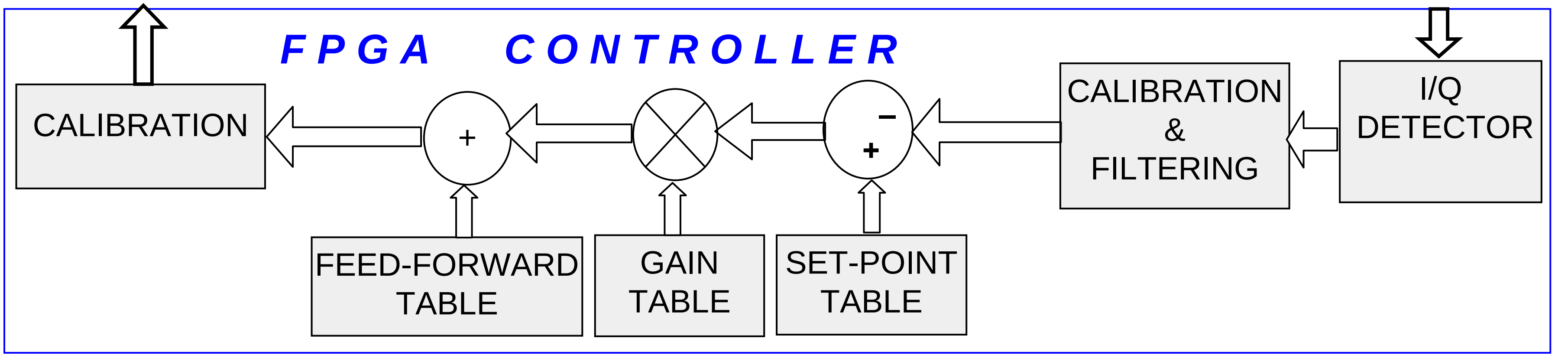
$$\mathbf{z} = (\mathbf{Z}^T * \mathbf{Z})^{-1} * \mathbf{Z}^T * \mathbf{V}$$

Implementation of parameters estimation for adaptive control of pulsed operated cavity



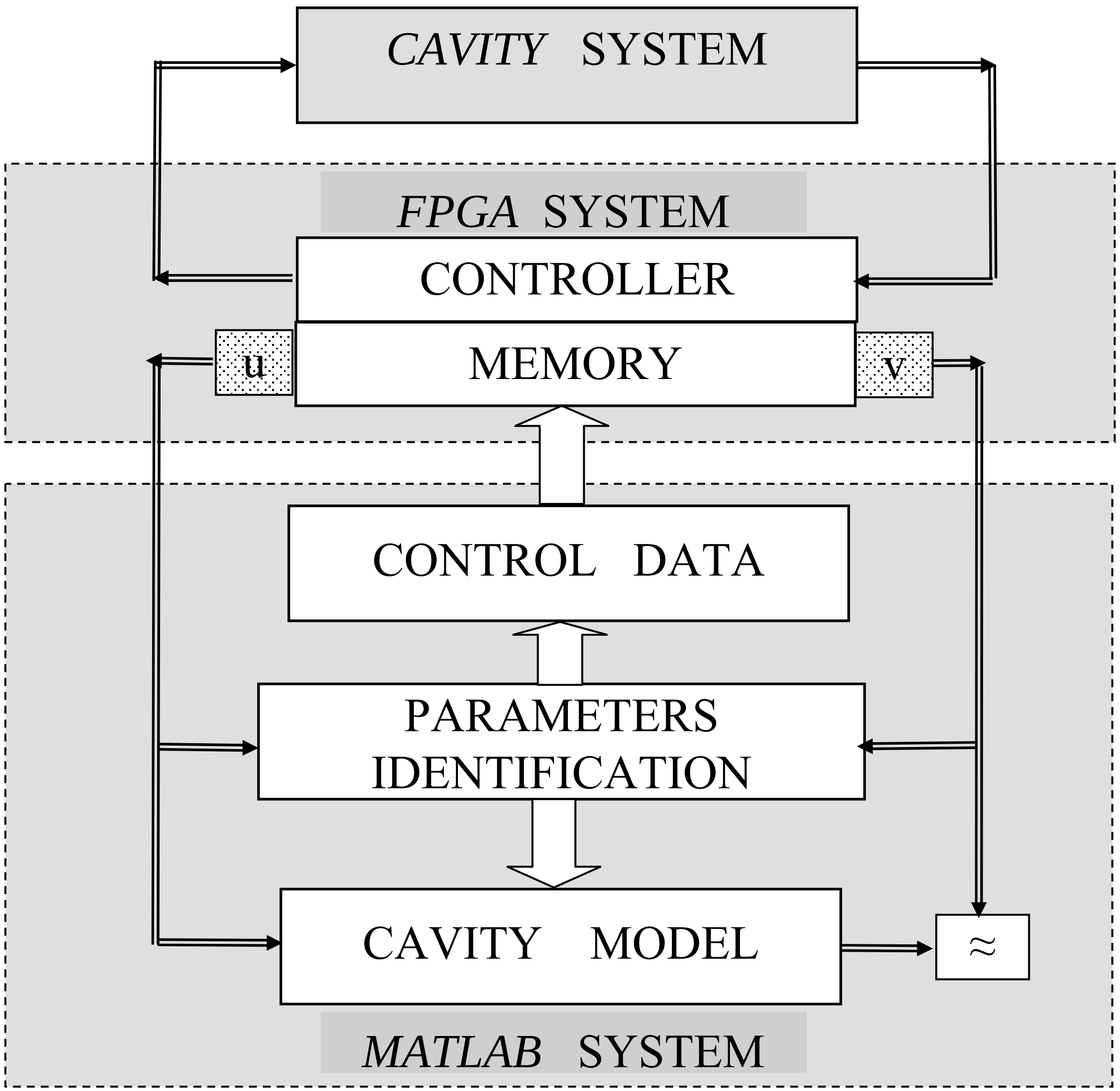
RANGE	SIGNAL CONDITION	ASSUMPTION	ESTIMATED VALUE
DECAY	$\mathbf{u} = 0$, $\mathbf{u}_b = 0$	–	$\omega_{1/2}$ and $[\Delta\omega]$
FILLIG	$\mathbf{u}_b = 0$	$\arg(\mathbf{F}) = 0$, $\omega_{1/2} = \text{const.}$	$[\lvert\mathbf{F}\rvert]$ and $[\Delta\omega]$
FLATTOP	$\mathbf{u}_b = 0$	$[\Delta\omega]$ – linear, $\omega_{1/2} = \text{const.}$	$[\mathbf{F}]$

Functional black diagram of FPGA controller and control tables determination

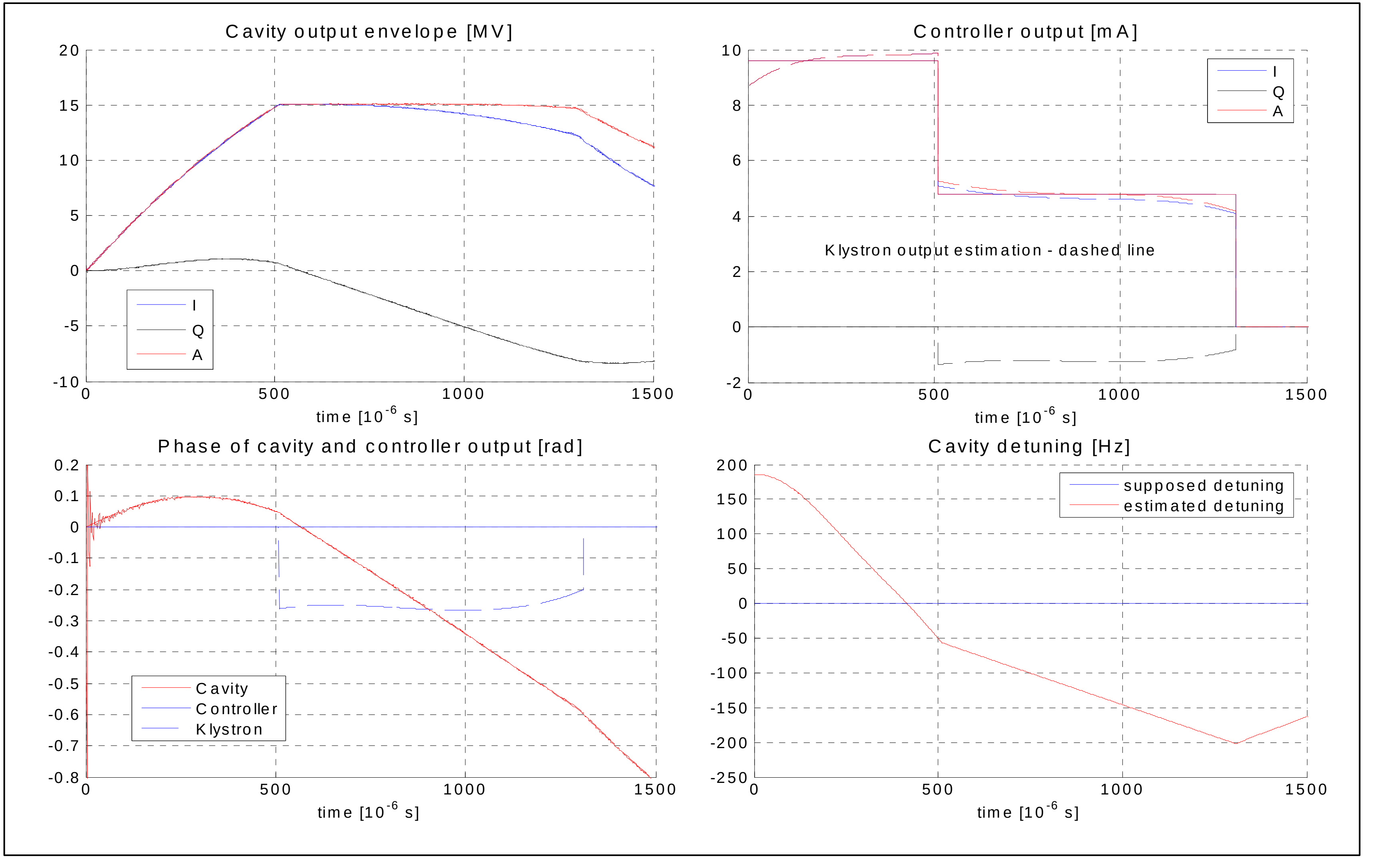


CONTROL	FEED-FORWARD	SET-POINT
Filling Cavity is driven in resonance condition	$\mathbf{i}(t) = Io \cdot \exp(\boldsymbol{i}\varphi(t))$ $d\varphi(t)/dt = \Delta\omega(t)$	$\mathbf{v}(t) = \mathbf{i}(t) \cdot R(1 - \exp(-\omega_{1/2} \cdot t))$
Flattop Envelope of cavity voltage is stable	$\mathbf{i}(t) = \mathbf{V_0} \cdot (1 - \boldsymbol{i}\Delta\omega(t)/\omega_{1/2})/R$	$\mathbf{v}(t) = \mathbf{V_0} = V_0 \cdot \exp(\boldsymbol{i}\Phi_0)$

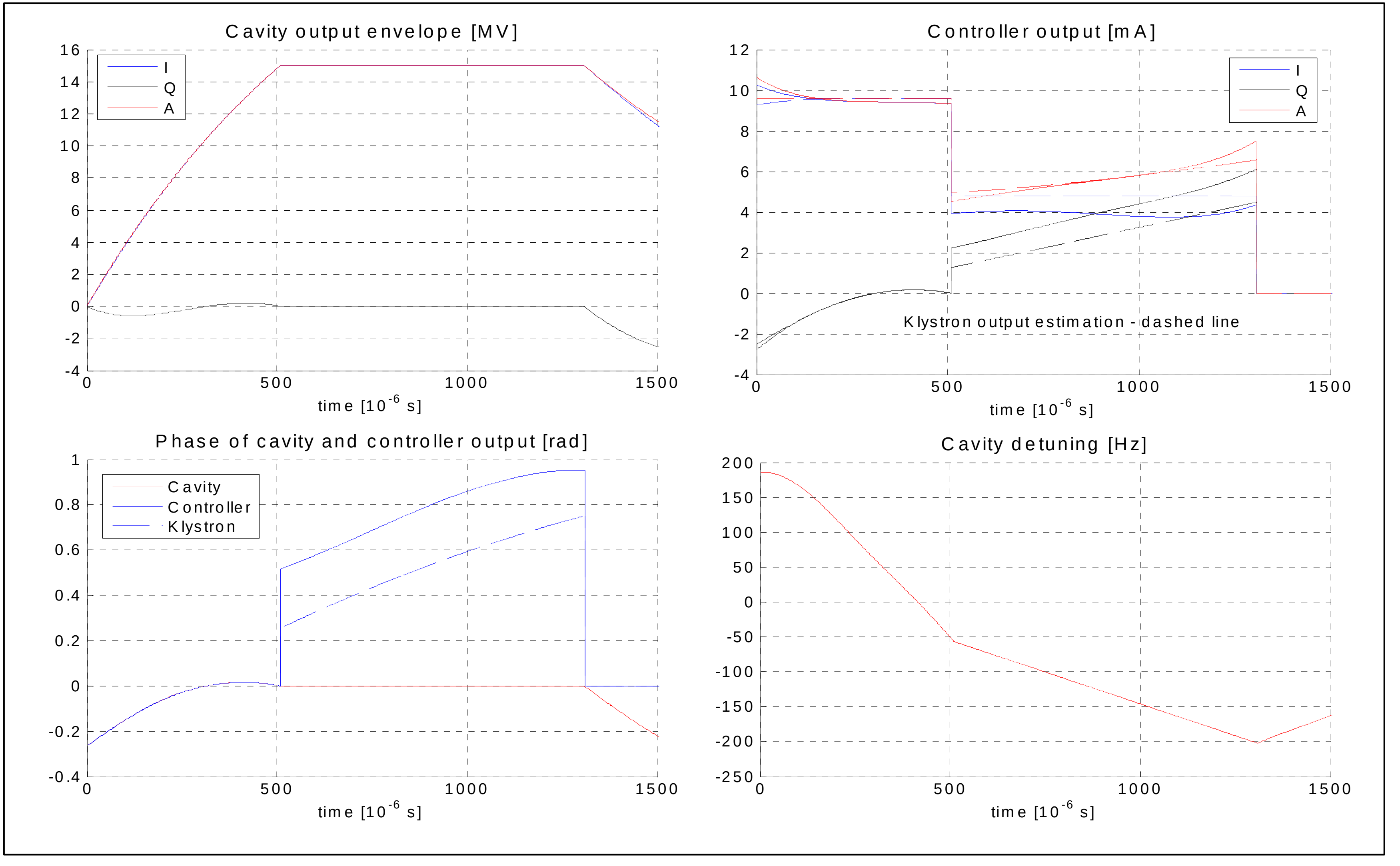
Functional diagram of cavity testing system



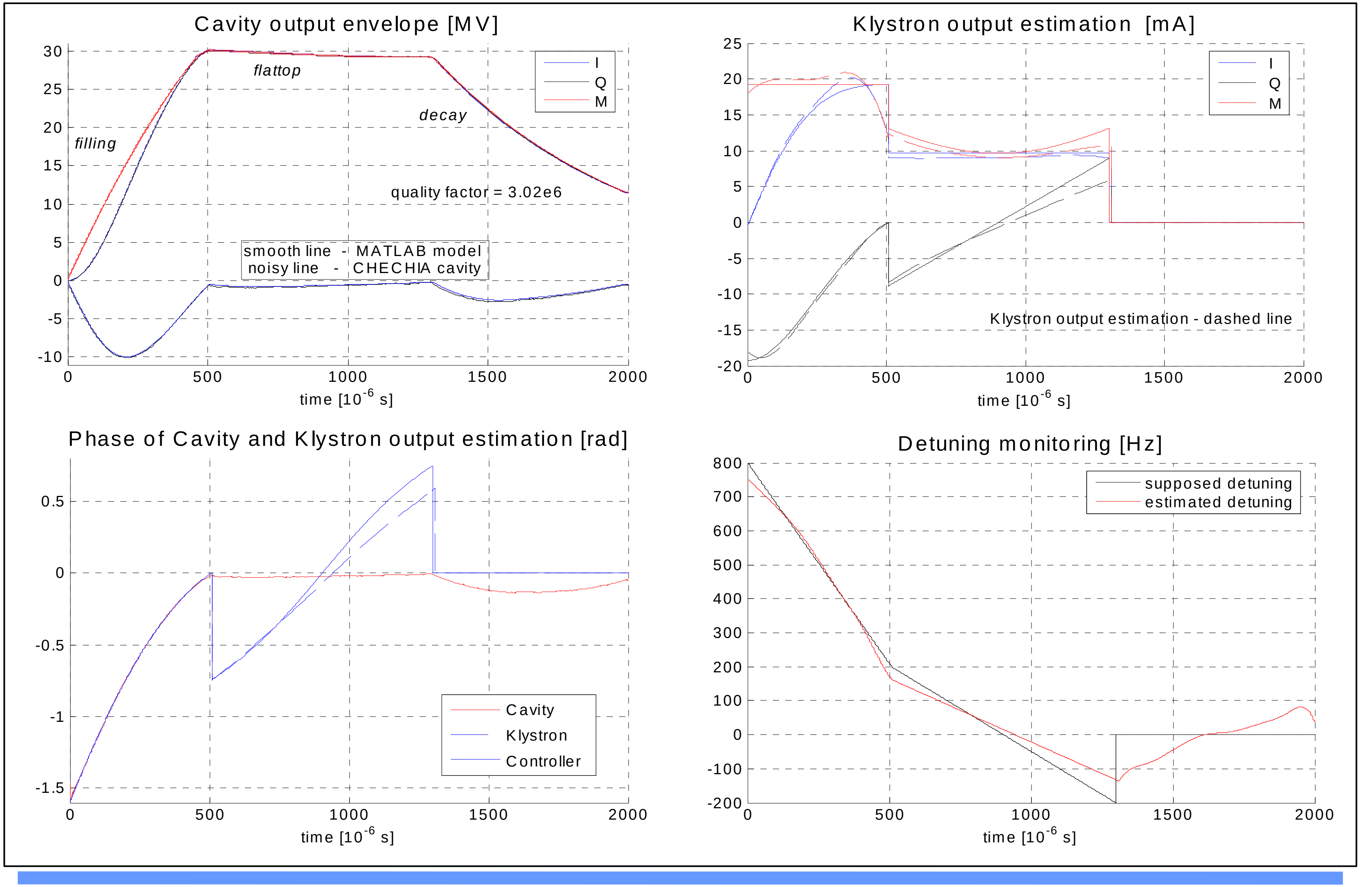
Adaptive feed-forward cavity driving
first step of iterative procedure – real process



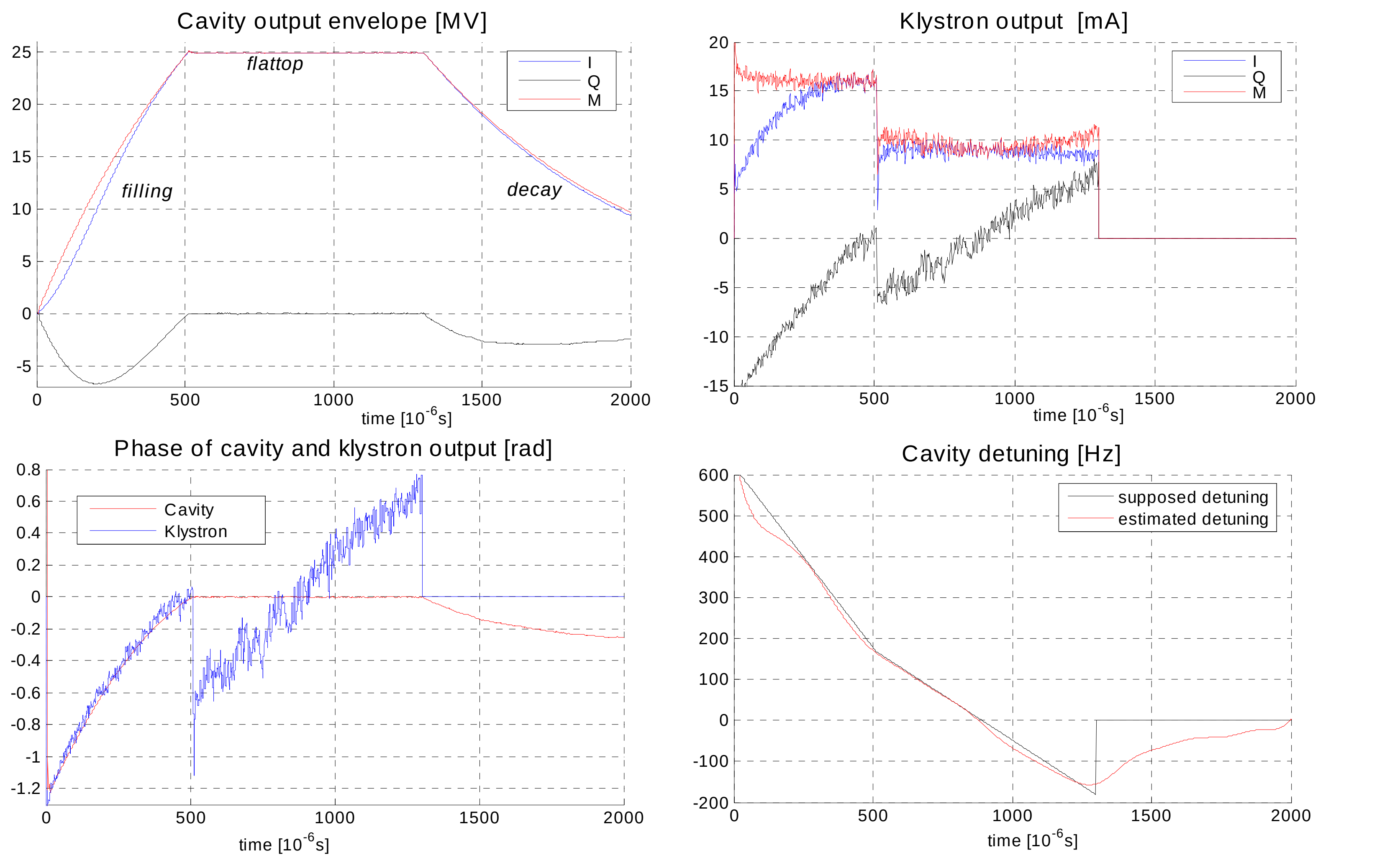
Adaptive feed-forward cavity driving
second step of iterative procedure – MATLAB simulation



Feed-forward cavity driving
CHECHIA cavity and model comparison



Feedback cavity driving
CHECHIA cavity and model comparison



CONCLUSION

- Cavity model has been confirmed according to reality
- Cavity parameters identification has been verified for control purpose

Future plans

- Vector sum control with beam
- Forward and reflected signal application for control purpose
- Normal-conductive cavity control for RF Gun