Direct Extraction of QCD $\Lambda_{\overline{MS}}$ from Moments of DIS Structure Functions.

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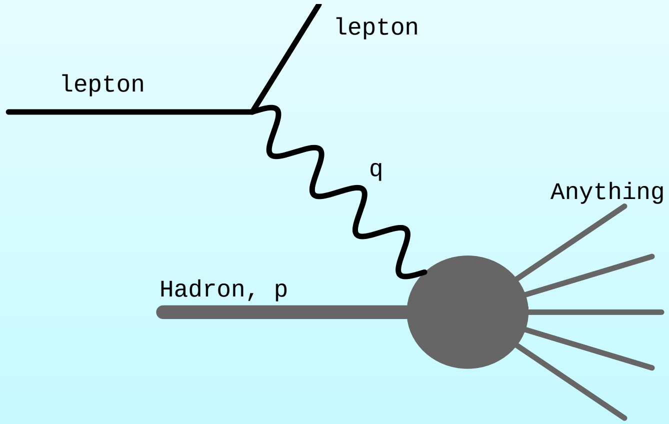
Outline.

- What challenges are we faced with when attempting to compare data with theoretical predictions for moments of structure functions?
- How can we overcome these challenges or at least quantify the uncertainties involved?
- Extracting a value for QCD $\Lambda_{\overline{MS}}$ by fitting theory predictions to data.

Deep Inelastic Scattering

Structure functions can be used to describe DIS processes



$$W_{\mu\nu} = e_{\mu\nu}\frac{F_L}{2x} + d_{\mu\nu}\frac{F_2}{2x} - i\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{\nu}F_3$$

We wish to compare experimental data for F_2 and F_3 with QCD predictions. To compare with theory we compute the moments of the structure functions.

$$\mathcal{M}_n = \int_0^1 x^{n-2} F_3(x, Q^2)\,dx$$

Challenges.

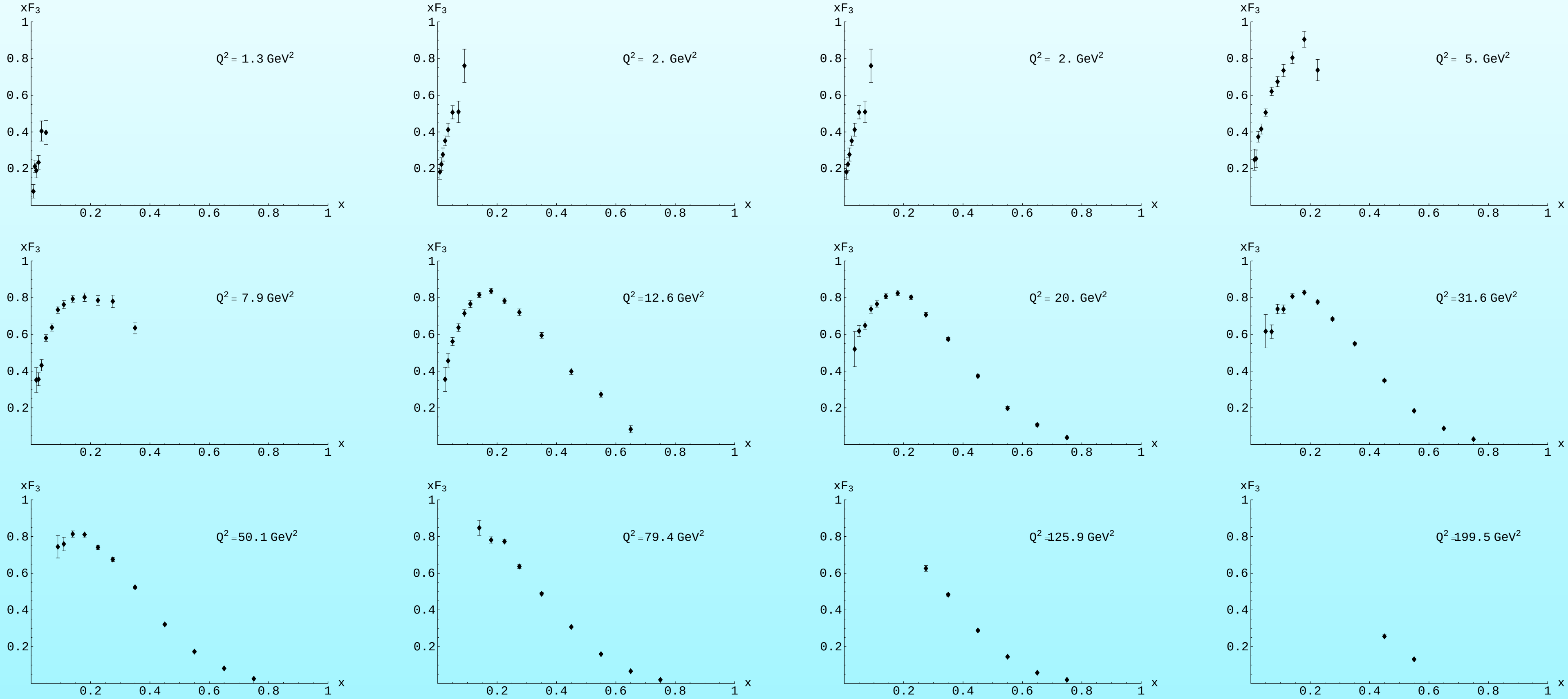
Limitations of the data:

We have no data for high x at low Q^2 and for low x at high Q^2.

Theoretical Uncertainty:

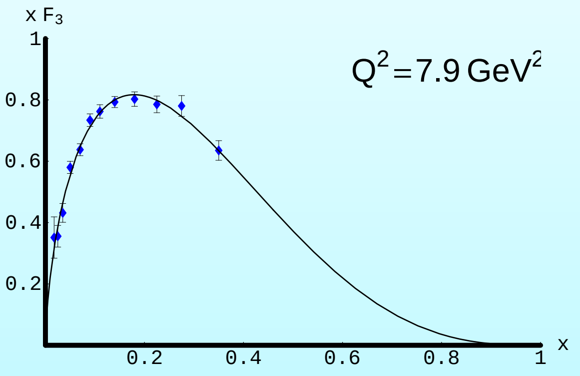
The Theoretical prediction depends on our choice of Factorization and Renormalization Schemes (FRS).

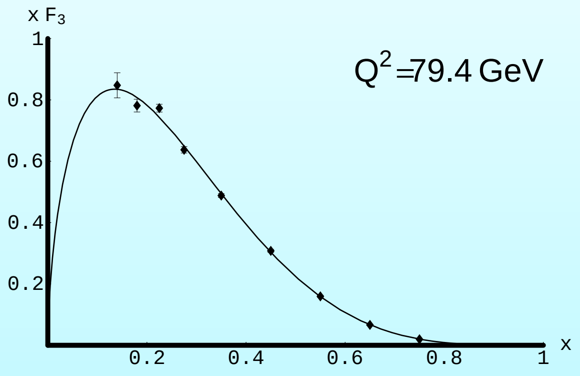
Data for xF_3 from CCFR



Modelling the Structure Functions.

We choose some fitting method e.g. $xF_3 = Ax^B(1-x)^C$.





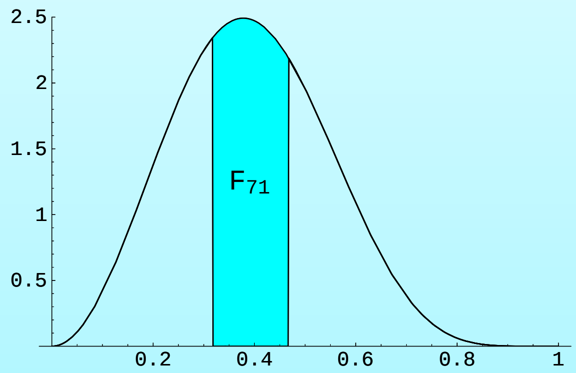
Large uncertainty due to gaps in data.

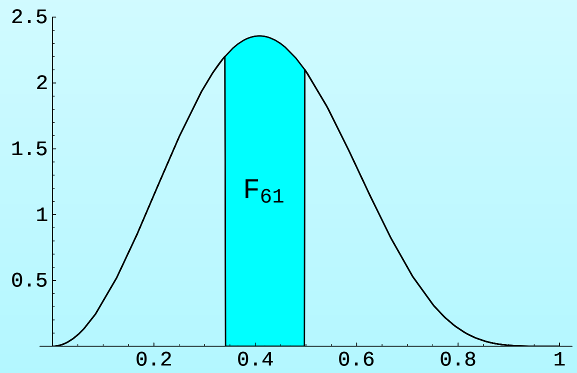
$\Rightarrow$ Result is dependent on method of fit.

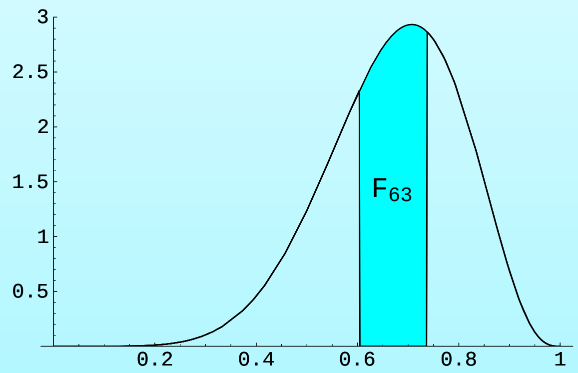
Bernstein Averages

Bernstein Polynomials are polynomials in x^2.

$$p_{nk}(x) = \frac{2\Gamma(n+\frac{3}{2})}{\Gamma(k+\frac{1}{2})\Gamma(n-k+1)} x^{2k}(1-x^2)^{n-k}$$



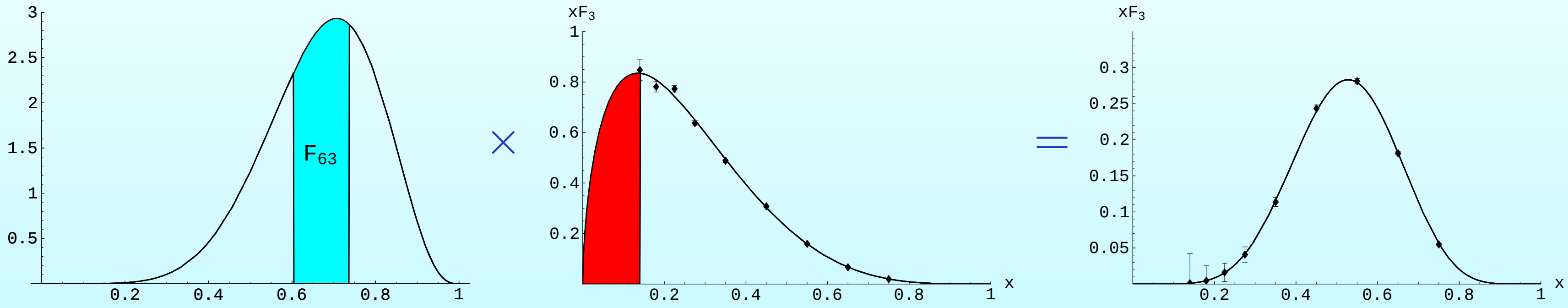




Constructed to be 0 at $x = 0, 1$ and normalized to unity.

Bernstein Averages

If we multiply the data for xF_3 by these polynomials.

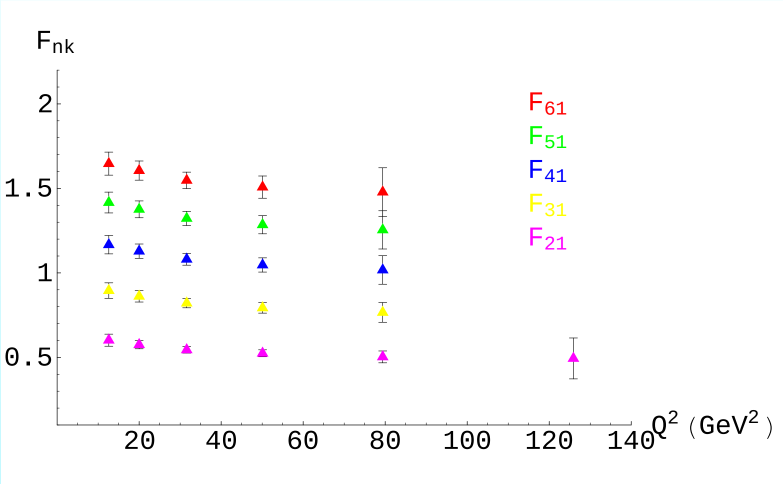


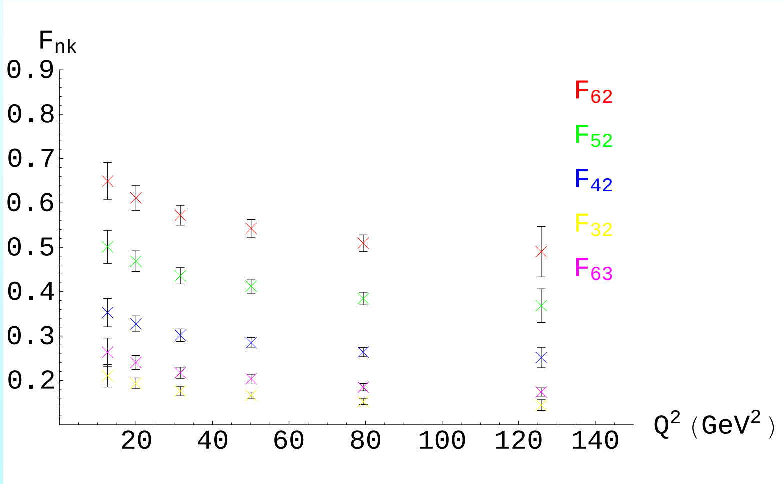
This defines the Bernstein average of F_3

$$F_{nk}(Q^2) = \int_0^1 p_{nk}(x) F_3(x, Q^2) dx$$

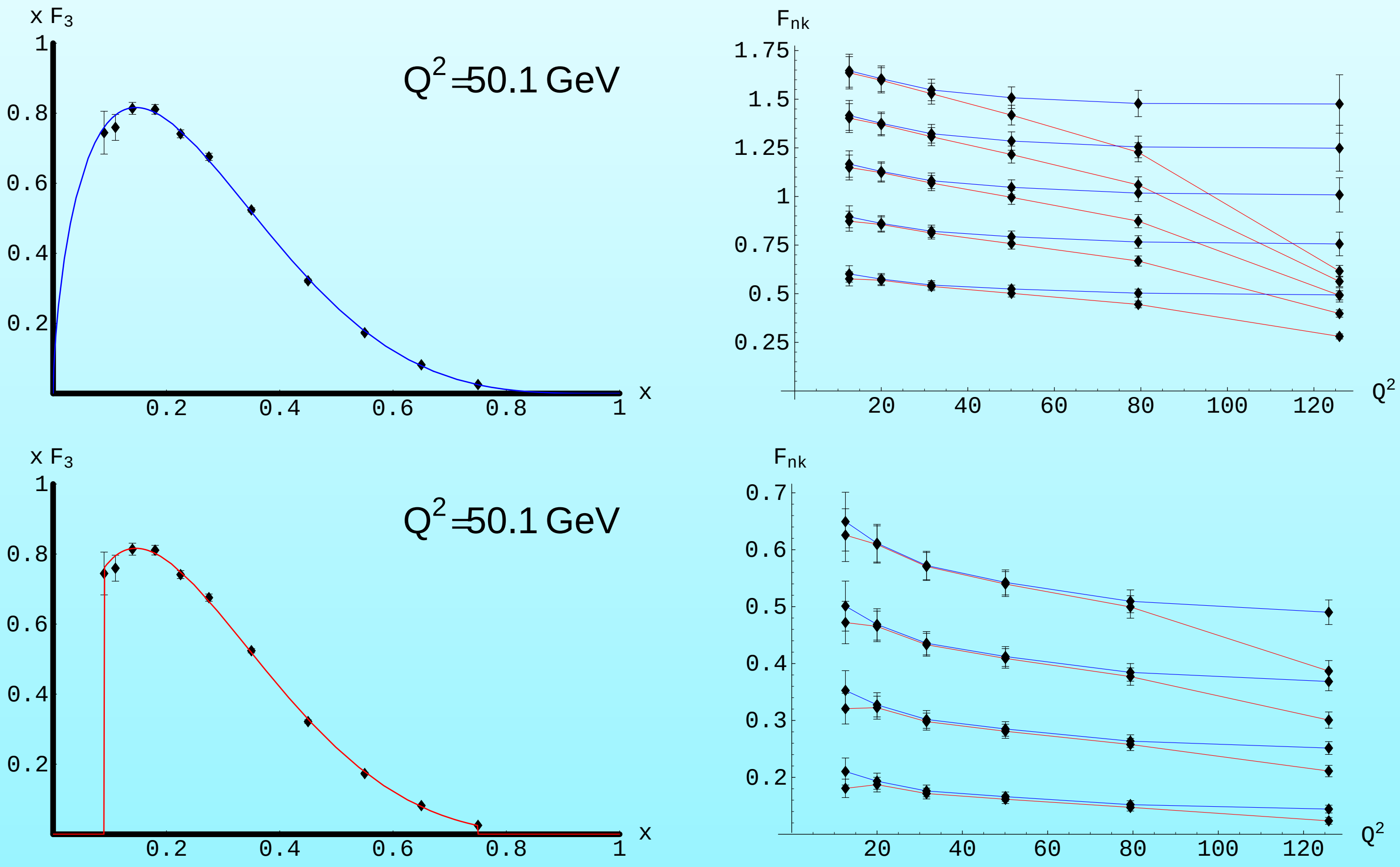
$$\mathcal{M}_n = \int_0^1 x^{n-2} F_3(x, Q^2) dx$$

Bernstein Averages for xF_3





Testing the Method



Theoretical Prediction

The perturbative contributions to the F_3 moments are given by

$$\mathcal{M}_n =< \mathcal{O}(M) > \mathcal{C}(\mu, M)$$

Where

$$\begin{aligned} \frac{M}{< \mathcal{O} >} \frac{\partial < \mathcal{O} >}{\partial M} &= -da - d_1 a^2 - d_2 a^3 - \cdots \\ M \frac{\partial a}{\partial \ln M} &= \beta(a) \end{aligned}$$

and $\mathcal{C}$ is the coefficient function.

$$\mathcal{C}(\mu, M) = 1 + r_1 \tilde{a} + r_2 \tilde{a}^2 + \cdots$$

Scheme Dependence of the Moments.

Integrating the equations for $< \mathcal{O} >$ gives us

$$\mathcal{M} = A(n) \left(\frac{ca}{1+ca} \right)^{d/b} \exp(\mathcal{I}(a))(1 + r_1 \tilde{a} + r_2 \tilde{a}^2 \cdots)$$

Where $a = a(M)$ and $\tilde{a} = a(M = \mu)$. $\mathcal{I}(a)$ is a polynomial in a with coefficients related to the beta function and anomalous dimension coefficients.

The standard approach is choose a physical scale, setting $M = \mu = Q$.

$$\mathcal{M} = A(n) \left(\frac{ca(Q)}{1+ca(Q)} \right)^{d/b} (1 + R_1 a(Q) + R_2 a(Q)^2 \cdots)$$

But there is no concrete physical motivation for this.

FRS Invariant Quantities.

We can separate the coefficients r_n into FRS dependent and FRS invariant pieces.

$$
\begin{aligned}
r_1 &= r_1^{FRS} - X_1(Q) \\
&= \underbrace{d \log\left[\frac{M}{\Lambda_{\mathcal{M}}}\right] - \frac{d_1}{b}}_{\text{FRS dependent}} - X_1(Q)
\end{aligned}
$$

$$
\begin{aligned}
r_2 &= r_2^{FRS} + X_2 \\
&= \underbrace{\frac{d_1^2}{2db} + \frac{cd_1}{2b} - \frac{c_2 d}{2b} - \frac{d_2}{2b} + \frac{br_1^2}{2d} + \frac{r_1^2}{2} + \frac{r_1 d_1}{d}}_{\text{FRS dependent}} + X_2
\end{aligned}
$$

X_i are FRS invariant quantities.

FRS Invariant Quantities.

If we define a_0 as the FRS predictable part of $\mathcal{C}$.

$$
\begin{aligned}
\mathcal{C} &= a + r_1 a^2 + (r_1^2 + c r_1 - c_2 + X_2) a^3 + \cdots \\
a_0 &= a + r_1 a^2 + (r_1^2 + c r_1 - c_2) a^3 + \cdots
\end{aligned}
$$

Defining a scheme for which $r_1 = c_2 = c_3 = \cdots = d_1 = d_2 = \cdots = 0$ allows us to write the moments as

$$
\mathcal{M} = A(n) \left(\frac{c a_0}{1 + c a_0} \right)^{d/b} (1 + X_2 a_0^2 + \cdots)
$$

CORGI QCD

This scheme is known as the ’t Hooft scheme and in it the beta function has a particularly simple form

$$\frac{\partial a_0}{\partial \ln \mu} = -b a_0^2 (1 + c a_0)$$

Which has the following solution

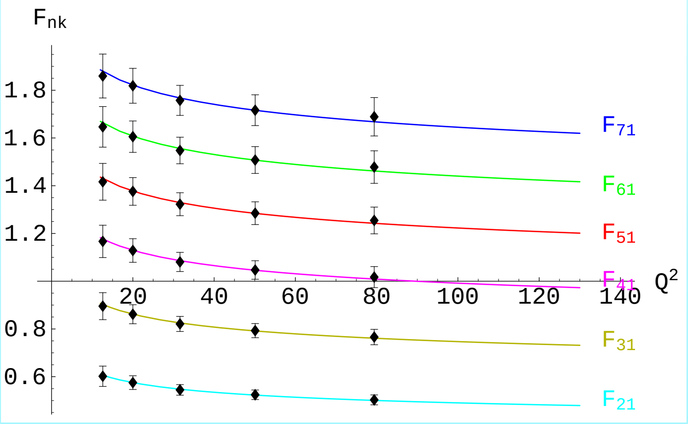
$$a_0 \quad = \quad \frac{1}{c[1 + W(z(Q))]}$$

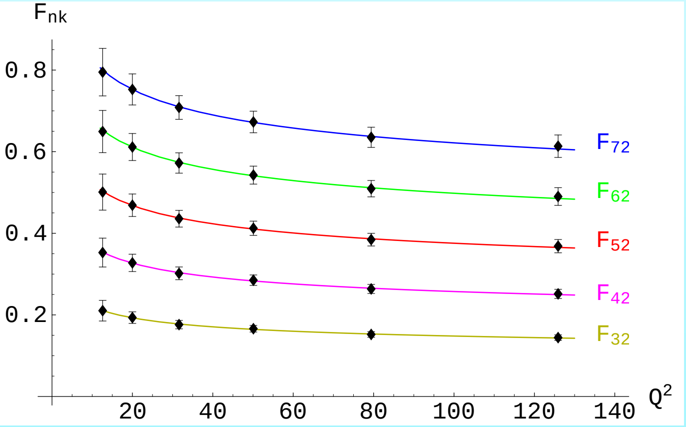
Where W is the Lambert W function defined by $W(z) \exp[W(z)] = z$ and

$$z(Q) \quad \equiv \quad -\frac{1}{e} \left(\frac{Q}{\Lambda_{\mathcal{R}}} \right)^{-b/c}$$

Results for F_3

- We compare the theoretical prediction for the Bernstein Polynomials with the experimental data.
- We have 201 Bernstein averages, the highest being $F_{10,9}$. The highest moment we use is $\mathcal{M}_{22}$.
- The fitting parameters are $\Lambda_{\overline{MS}}$ and the non-perturbative constants, $A(n)$.





With $\Lambda_{\overline{MS}} = 240 \pm 41 MeV$ and $\chi^2/d.o.f = 4.2/(201 - 11)$.

Further Details

- We have also performed an analysis of F_2 moments.
- We now have access to theoretical predictions for odd *and* even moments.
- We have modified the expressions in order to account for quark thresholds.

Summary.

- We can minimize the impact of ‘missing’ data by using Bernstein averages instead of the moments.
- We need to be careful about which Averages we use.
- Factorization and Renormalization scheme dependence can be quantified.
- CORGI QCD is consistent with experimental data.