Neutrino Masses in the Lepton number violating Minimal Supersymmetric Standard Model

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Superpotential

The Lagrangian contains terms of the form

Yukawa terms: $$-\frac{1}{2}\frac{\partial^2\mathcal{W}}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\mathcal{W}^\dagger}{\partial\varphi_i^*\partial\varphi_j^*}\bar{\psi}_i\bar{\psi}_j$$

F-terms: $$-\sum_i \left|\frac{\partial\mathcal{W}}{\partial\varphi_i}\right|^2$$

$$\mathcal{W} = (Y_E)\ \ LH_1E^c + (Y_D)\ \ QH_1D^c + (Y_U)\ \ QH_2U^c - \mu H_1H_2$$

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$$+\frac{1}{2}\lambda''\ \ U^cD^cD^c$$

Mixing

Fields which transform the same way under all unbroken symmetries will mix.

If conservation of R-parity is not imposed mixing occurs between

- neutral gaugino, neutral higgsinos, neutrinos
- charged gauginos, charged higgsinos, charged leptons
- one of the higgs doublets, selectron-sneutrino doublet
- . . .

In fact, even before symmetry breaking there are no quantum numbers which differentiate between one of the higgs doublets and the lepton doublet. The lepton doublet will play the same role as the higgs doublet and vice versa.

Neutrino-neutralino mass matrix

Notation

$$\nu_{L\alpha} = (\tilde{h}_1^0, \nu_{Li}) \qquad \tilde{\nu}_{L\alpha} = (h_1^0, \tilde{\nu}_{Li})$$

$$\langle h_2^0 \rangle = v_u \qquad \langle \tilde{\nu}_{L\alpha} \rangle = v_\alpha = (v_d, v_i)$$

$$\mathcal{L} \subset -\frac{1}{2} \begin{pmatrix} -i\widetilde{B} & -i\widetilde{W}^0 & \tilde{h}_2^0 & \tilde{h}_1^0 & \nu_{Li} \end{pmatrix} \mathcal{M}_N \begin{pmatrix} -i\widetilde{B} \\ -i\widetilde{W}^0 \\ \tilde{h}_2^0 \\ \tilde{h}_1^0 \\ \nu_{Lj} \end{pmatrix}$$

Neutrino-neutralino mass matrix

$$\mathcal{M}_N =$$

$$\begin{pmatrix} M_1 & 0 & gv_u/\sqrt{2} & -gv_d/\sqrt{2} & -gv_j/\sqrt{2} \\ 0 & M_2 & -g_2v_u/\sqrt{2} & g_2v_d/\sqrt{2} & g_2v_j/\sqrt{2} \\ gv_u/\sqrt{2} & -g_2v_u/\sqrt{2} & 0 & -\mu & -\kappa_j \\ gv_d/\sqrt{2} & g_2v_d/\sqrt{2} & -\mu & 0 & 0 \\ gv_i/\sqrt{2} & g_2v_i/\sqrt{2} & -\kappa_i & 0 & 0_{ij} \end{pmatrix}$$

Neutrino-neutralino mass matrix

$\mathcal{M}_N =$

$$\left(\begin{array}{cccc|c} M_1 & 0 & gv_u/\sqrt{2} & -gv_d/\sqrt{2} & -gv_j/\sqrt{2} \\ 0 & M_2 & -g_2v_u/\sqrt{2} & g_2v_d/\sqrt{2} & g_2v_j/\sqrt{2} \\ gv_u/\sqrt{2} & -g_2v_u/\sqrt{2} & 0 & -\mu & -\kappa_j \\ gv_d/\sqrt{2} & g_2v_d/\sqrt{2} & -\mu & 0 & 0 \\ \hline gv_i/\sqrt{2} & g_2v_i/\sqrt{2} & -\kappa_i & 0 & 0_{ij} \end{array}\right)$$

This structure is suggestive of the seesaw formula, with

$$\mathcal{M}_N = \begin{pmatrix} M_{\tilde{\chi}} & m \\ m^T & 0 \end{pmatrix}$$

4 eigenvalues $\sim M_{\tilde{\chi}}$ $\qquad$ 3 eigenvalues $\sim \frac{mm^T}{M_{\tilde{\chi}}}$

Neutrino-neutralino mass matrix

$\mathcal{M}_N =$

$$\left(\begin{array}{ccc|cc} M_1 & 0 & gv_u/\sqrt{2} & -gv_d/\sqrt{2} & -gv_j/\sqrt{2} \\ 0 & M_2 & -g_2v_u/\sqrt{2} & g_2v_d/\sqrt{2} & g_2v_j/\sqrt{2} \\ gv_u/\sqrt{2} & -g_2v_u/\sqrt{2} & 0 & -\mu & -\kappa_j \\ gv_d/\sqrt{2} & g_2v_d/\sqrt{2} & -\mu & 0 & 0 \\ \hline gv_i/\sqrt{2} & g_2v_i/\sqrt{2} & -\kappa_i & 0 & 0_{ij} \end{array}\right)$$

$$\mathcal{M}_N = \begin{pmatrix} N_{4\times3} & n_{4\times4} \\ n'_{3\times3} & 0_{3\times4} \end{pmatrix}$$

The masses squared are given by the eigenvalues of $\mathcal{M}_N^\dagger \mathcal{M}_N$

Will find that there are two zero eigenvalues

Neutrino-neutralino mass matrix

$$n = \begin{pmatrix} -gv_d/\sqrt{2} & -gv_1/\sqrt{2} & -gv_2/\sqrt{2} & -gv_3/\sqrt{2} \\ g_2 v_d/\sqrt{2} & g_2 v_1/\sqrt{2} & g_2 v_2/\sqrt{2} & g_2 v_3/\sqrt{2} \\ -\mu & -\kappa_1 & -\kappa_2 & -\kappa_3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

n has three rows which are linearly dependant $\longrightarrow$ two zero eigenvalues with corresponding eigenvectors $\vec{v}_{1,2}$.

$$\mathcal{M}_N \vec{q}_i = \begin{pmatrix} N_{4\times 3} & n_{4\times 4} \\ n'_{3\times 4} & 0_{3\times 3} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \vec{v}_i \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ \vdots \\ w_i \vec{v}_i \\ \vdots \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\vec{q}_{1,2}$ are eigenvectors for the full matrix with zero eigenvalues

$$\mathcal{M}_N^\dagger \mathcal{M}_N \vec{q}_{1,2} = 0$$

Rotation matrix

Define U_N such that $U_N^T \mathcal{M}_N U_N$ is diagonal

Consider an approximate form of, U_N, where $\xi = M_{\tilde{\chi}^0}^{-1}$

$$U_N = \begin{pmatrix} 1 & -\xi m \\ m^T \xi^\dagger & 1 \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix} = \begin{pmatrix} U & -\xi m V \\ m^T \xi^\dagger U & V \end{pmatrix}$$

$$U_N^T \mathcal{M}_N U_N = \begin{pmatrix} U^T M_{\tilde{\chi}^0} U & 0 \\ 0 & -V^T m^T M_{\tilde{\chi}^0}^{-1} m V \end{pmatrix}$$

Neutrino sector

The neutrino part of the mass matrix is given approximately by
$m_{\text{eff}} = -m^T M_{\tilde{\chi}^0}^{-1} m$

$$= \frac{M_1 g_2^2 + M_2 g^2}{4\text{Det}[M_{\tilde{\chi}^0}]} \begin{pmatrix} \Lambda_1^2 & \Lambda_1\Lambda_2 & \Lambda_1\Lambda_3 \\ \Lambda_1\Lambda_2 & \Lambda_2^2 & \Lambda_2\Lambda_3 \\ \Lambda_1\Lambda_3 & \Lambda_2\Lambda_3 & \Lambda_3^2 \end{pmatrix}$$

where $\Lambda_i = \mu v_i - v_d \kappa_i$

Define, V, such that $-V^T m^T M_{\tilde{\chi}^0}^{-1} m V$ is diagonal.

Tree-level neutrino mass given by

$$m_{\nu,\text{tree}} = \left| \frac{M_1 g_2^2 + M_2 g^2}{4\text{Det}[M_{\tilde{\chi}^0}]} \right| |\vec{\Lambda}|^2$$

Lepton-Chargino mass matrix

$$\mathcal{L} \subset - \left(\begin{array}{cc} -i\widetilde{W}^- & e_{L\alpha} \end{array} \right) \mathcal{M}_C \left(\begin{array}{c} -i\widetilde{W}^+ \\ \tilde{h}_2^+ \\ e_{Rk} \end{array} \right)$$

where $\mathcal{M}_C$ takes the form

$$\mathcal{M}_C = \left(\begin{array}{ccc} M_2 & g_2 v_u & 0_k \\ g_2 v_\alpha & \mu_\alpha & \lambda_{\beta\alpha k} v_\beta \end{array} \right)$$

Mixing matrices

The Lagrangian contains interactions of fermions with gauge bosons

$$\mathcal{L} \subset \frac{g_2}{\sqrt{2}} \bar{\nu}_{L\alpha} \bar{\sigma}^\mu W^+_\mu e_{L\alpha}$$

$$= \frac{g_2}{\sqrt{2}} \left(\bar{\tilde{\chi}}^0 \right)'^{4+\alpha} \bar{\sigma}^\mu W^+_\mu \left(\tilde{\chi}^- \right)'^{2+\alpha}$$

$$= \frac{g_2}{\sqrt{2}} \left(\bar{\tilde{\chi}}^0 \right)^{4+\alpha} U^\dagger_{Nr(4+\alpha)} \bar{\sigma}^\mu W^+_\mu \left(\tilde{\chi}^- \right)^{2+\alpha} V^\dagger_{C(2+\alpha)}$$

$$= \frac{g_2}{\sqrt{2}} U_{rs} \left(\bar{\tilde{\chi}}^0 \right)^{r} \bar{\sigma}^\mu W^+_\mu \left(\tilde{\chi}^- \right)^{s}$$

where U_{rs} is a 5×7 mixing matrix given by $U_{rs} = U^\dagger_{Nr(4+\alpha)} V^*_{C(2+\alpha)s}$

the PMNS matrix, is a 3×3 submatrix $U_{(4+i)(2+j)}$

Summary

- Within the R-parity violating minimal supersymmetric standard model, one non-zero neutrino mass is generated at tree level
- The lightness of the neutrino mass can be understood in terms of a see-saw mechanism where soft supersymmetry breaking parameters play the role of the mass scale supressing the neutrino mass
- It is also possible to investigate the PMNS mixing matrix within this framework, as a submatrix of a larger neutralino-neutrino/chargino-charged lepton mixing matrix

Future work

- This scenario is well understood at tree-level, but loop effects could give rise to interesting phenomenology
- Need to derive a full set of Feynman rules for the model in question, having rotated all the fields into their mass basis
- Attempt to relate other physical observables, such as lepton flavour violating processes, to neutrino masses and mixing angles, as measured in neutrino oscillation experiments