

Michele Treccani



*Dipartimento di Fisica Nucleare e Teorica  
Università degli Studi Pavia*

# Correzioni multifotoniche alla produzione di W & Z ai collider adronici

In collaborazione con:  
C.M. Carloni Calame,  
G. Montagna,  
O. Nicrosini.

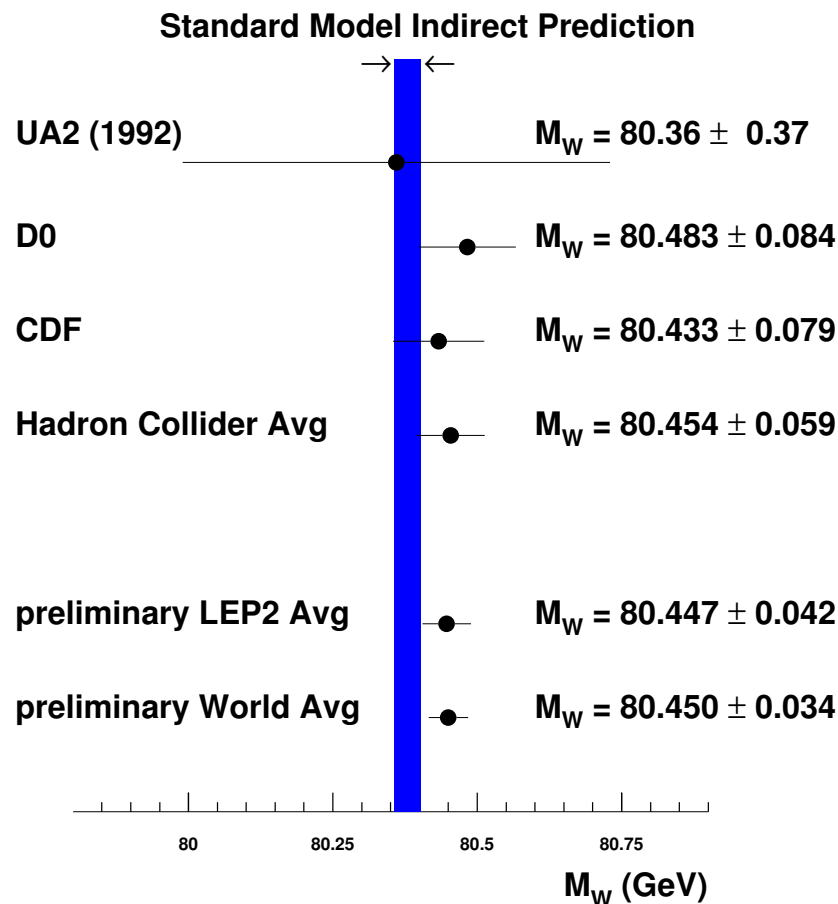
*Incontri sulla Fisica delle Alte Energie  
Catania, 30 Marzo-1 Aprile 2005*

# Outline

- ★ Motivations for RC to  $W/Z$  production
- ★ Theoretical calculation & tools
  - ✓ QCD corrections (briefly)
  - ✓ Electroweak corrections
- ★ The event generator **HORACE** for higher order QED corrections
  - ✓ tuned comparisons with **WINHAC**
  - ✓ estimating h.o. QED shifts on  $M_W$  and  $M_Z$  and their effects on distributions
- ★ Conclusions

# Motivations

- Precise measurements require **precise calculations**
- The precise  $M_W$  (and  $M_{top}$ ) measurement will highly improve the indirect bound on  $M_H$  via RC ( $\Delta M_H \sim 35\%$  if  $\Delta M_W = 30 \text{ MeV}$  and  $\Delta M_{top} = 2 \text{ GeV}$ )
- Present experimental status and future goal:



Target precision for  $\Delta M_W$

★ Tevatron RunII  $\lesssim 30 \text{ MeV}$

★ LHC  $\Rightarrow 15 \text{ MeV}$

$\Rightarrow \Delta M_W / M_W = \mathcal{O}(1 \times 10^{-4})$

$\Rightarrow M_W$  measured preferably  
from  $M_T^W$  distribution

## Motivations (II)

- $\sigma_W$  (and  $\sigma_Z$ ) can be used to monitor **hadron and parton luminosities and/or determine PDF**

Dittmar, Pauss, Zurcher, Phys. Rev. **D56** (1997) 7284; Khoze *et al.*, Eur. Phys. J. **C19** (2001) 313;

Giele, Keller hep-ph/0104053; Frixione, Mangano, JHEP **0405** (2004) 056

$$\sigma^{theory}(W) = \sum_{ij} \mathcal{P}_{ij} \otimes \hat{\sigma}_{ij} = \sigma^{exp}(W) = \frac{N_W}{BR(W \rightarrow \ell\nu) \int \mathcal{L} dt A_W}$$

- measure  $\Gamma_W$  with  $\Delta\Gamma_W \simeq 50$  MeV (from  $W/Z$  cross section ratio)
- $Z$  data are used to calibrate detectors (**lepton scale and resolution**): a precise simulation is mandatory for accurate  $M_W$  measurement
- $p_{\perp}^W$  can be simulated **measuring  $p_{\perp}^Z$**
- $\sin^2 \theta_{eff}^{lept.}$  can be measured from  $A_{FB}$  at the  $Z$  pole
- the  $M_T^W / M_T^Z$  **ratio** can be used to measure  $M_W$ . This method is competitive at high luminosities

# QCD calculations & tools

- resummation of leading and next-to-leading  $p_T^W / M_W$  logarithms, implemented in **RESBOS**  
C. Balazs and C.P. Yuan, Phys. Rev. **D56** (1997) 5558
- NLO corrections merged with the HERWIG Parton Shower (**MC@NLO**)  
S. Frixione and B.R. Webber, JHEP **0206** (2002) 029 and hep-ph/0402116
- NNLO corrections to  $W/Z$  production **total cross sections**  
R. Hamberg, W.L. van Neerven, T. Matsuura, Nucl. Phys. **B359** (1991) 343  
W.L. van Neerven and E.B. Zijlstra, Nucl. Phys. **B382** (1992) 11  
R.V. Harlander and W.B. Kilgore, Phys. Rev. Lett. **88** (2002) 201801
- NNLO corrections to  $W/Z$  **rapidity distribution**  
C. Anastasiou *et al.* , Phys. Rev. **D69** (2004) 094008

→ notice that

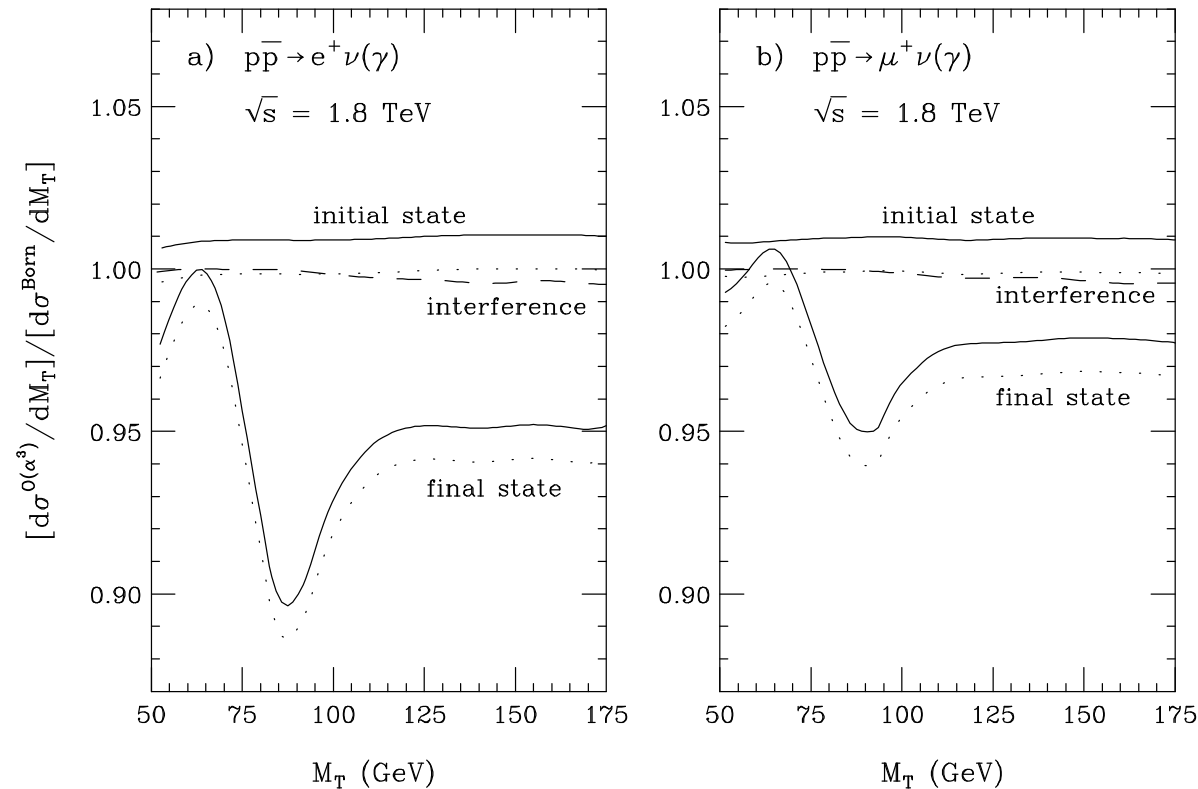
$$\frac{\alpha_s^2}{\pi^2} \sim \frac{\alpha_{qed}}{\pi}$$

# Status of $\mathcal{O}(\alpha)$ electroweak calculations & tools

- QED corrections to  $W$  and  $Z$  boson decay
  - Berends, Kleiss, Revol, Vialle, Z. Phys. **C27** (1985) 155; Berends, Kleiss, *ibidem* 365
- Electroweak corrections to  $W$  production and decay
  - ★ Pole approximation
    - Wackerth and Hollik, Phys. Rev. **D55** (1997) 6788
    - Baur, Keller, Wackerth, Phys. Rev. **D59** (1999) 013002 → WGRAD
  - ★ Complete  $\mathcal{O}(\alpha)$  corrections
    - Dittmaier and Krämer, Phys. Rev. **D65** (2002) 073007
    - Baur and Wackerth, Phys.Rev. **D70** (2004) 073015 → WGRAD2
- Electroweak corrections to  $Z$  production and decay
  - ★  $\mathcal{O}(\alpha)$  photonic corrections
    - Baur, Keller, Sakumoto, Phys. Rev. **D57** (1998) 199 → ZGRAD
  - ★ Complete  $\mathcal{O}(\alpha)$  corrections → ZGRAD2
    - Baur, Brein, Hollik, Schappacher, Wackerth, Phys. Rev. **D65** (2002) 033007

# Initial vs final state photonic corrections

U. Baur, S. Keller, D. Wackerath, Phys. Rev. **D59** (1999) 013002



- **initial-state radiation** and **initial-final-state interference** do not modify the shape of the distributions and are at the 1% level (or below)
- **final-state radiation** does modify the shape of the distributions and is important because it contains mass logarithms of the form  $\log(\hat{s}/m_\ell^2)$

# Higher order final-state QED corrections

- Why worrying about FS **higher-order QED** corrections?
  - $\mathcal{O}(\alpha)$  FS QED radiation changes by **several %** the shape of the relevant distributions
  - single photon emission induces  $W/Z$  mass shift **at the 0.1% level** ( $\sim 100$  MeV)
  - a shift due to multiple photons **at the 0.01% level** can be expected
  - presently, it is a theoretical systematic error
- Recent literature
  - ★ double real bremsstrahlung  $q\bar{q}' \rightarrow \ell^\pm \nu \gamma \gamma$  and  $q\bar{q} \rightarrow \ell^+ \ell^- \gamma \gamma$  for  $W$  &  $Z$   
Baur, Stelzer, Phys. Rev. **D61** (2000) 073007
  - ★ Higher-order (real+virtual) QED corrections to the leptonic boson decay
    - **HORACE** MC (for  $W$  &  $Z$ ): **QED Parton Shower**  
C.M. Carloni Calame *et al.*, Phys. Rev. **D69** (2004) 037301
    - **WINHAC** (for  $W$ ): **YFS exponentiation** of order  $\alpha$  matrix element  
Jadach, Płaczek, Eur. Phys. J. **C29** (2003) 325
  - ★ Tuned comparison of **WINHAC** and **HORACE** for  $W$   
C.M. Carloni Calame *et al.*, Acta Phys. Pol. **B35** (2004) 1643



# Our Parton Shower approach

- Multiphoton corrections are computed by means of the QED Structure Function  $D(x, Q^2)$ , solution of DGLAP equation in QED.

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} P_+(y) D\left(\frac{x}{y}, Q^2\right)$$

$$P_+(x) = \frac{1+x^2}{1-x} - \delta(1-x) \int_0^1 dt P(t)$$

- Exactly solved by means of a Monte Carlo **Parton Shower** algorithm
- PS is “attached” to parton level process for FS radiation:
  - $\gamma$ 's multiplicity and energies given by the **PS algorithm**
  - $\gamma$ 's generated according to the factorized part of the  $W/Z$  radiative decay matrix element (eikonal current → gauge invariance in the approximate ME)

$$\cos \theta_\gamma \sim - \left( \frac{p_\lambda^\ell}{p^\ell \cdot k} - \frac{Q_\lambda^W}{Q^W \cdot k} \right) \cdot \left( \frac{p^{\ell\lambda}}{p^\ell \cdot k} - \frac{Q^{W\lambda}}{Q^W \cdot k} \right) \quad (\text{for } W)$$

$$\cos \theta_\gamma \sim - \left( \frac{p_\lambda^\ell}{p^\ell \cdot k} - \frac{p_\lambda^\ell}{p^\ell \cdot k} \right) \cdot \left( \frac{p^{\ell\lambda}}{p^\ell \cdot k} - \frac{p^{\ell\lambda}}{p^\ell \cdot k} \right) \quad (\text{for } Z)$$

# The event generator HORACE at present

The event generator **HORACE** (Higher Order Radiative Corrections) for  $W/Z$  production in Drell-Yan processes was developed

- ✓ parton level processes convoluted with initial state PDFs
- ✓ it generates **unweighted events**
- ✓ QED RC are included by means of the **Parton Shower algorithm**. It can be run in
  - ★  $\mathcal{O}(\alpha)$  mode: only QED **order  $\alpha$**  corrections are included
  - ★ *exp. mode*: QED corrections are included **up to all orders**
- of course, any experimental cut can be applied (for our studies, also a particles' smearing implemented)
- ★ it can run also in parallel mode (using **MPI libraries**)

# Tuned Comparisons with WINHAC

C.M. Carloni Calame *et al.*, Acta Phys. Pol. **B35** (2004) 1643

- Performed during the MC4LHC workshop held at CERN in summer 2003
- Processes (with  $l = e, \mu$ )

$$d\bar{u} \longrightarrow W^- \longrightarrow l^- + \bar{\nu}_l \qquad pp \longrightarrow W^\pm + X \longrightarrow l^\pm + \nu_l + X$$

- Input parameters:

$G_\mu$  scheme and fixed-width scheme, PDFs set: MRS(G) from PDFLIB

$$m_e = 0.511 \times 10^{-3} \text{ GeV}, \quad m_\mu = 0.10565836 \text{ GeV}, \quad m_{\nu_e} = m_{\nu_\mu} = 0,$$

$$m_u = m_d = m_s = m_c = m_b = 0,$$

$$V_{ud} = 0.97483, \quad V_{us} = 0.22290, \quad V_{ub} = 0.00360,$$

$$V_{cd} = -0.22286, \quad V_{cs} = 0.97398, \quad V_{cb} = 0.04120,$$

$$V_{td} = 0.00568, \quad V_{ts} = -0.04097, \quad V_{tb} = 0.99914,$$

$$M_W = 80.423 \text{ GeV}, \quad M_Z = 91.1882 \text{ GeV}$$

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2}, \quad \Gamma_W = \frac{3G_\mu M_W^3}{2\sqrt{2}\pi} \left( 1 + \frac{2\alpha_s}{3\pi} \right),$$

$$\alpha^{-1} = 137.03599976, \quad G_\mu = 1.16639 \times 10^{-5} \text{ GeV}^{-2}, \quad \alpha_s = 0.1185,$$

$$E_{\text{CM}} = \sqrt{s} = 14 \text{ TeV}$$

- Event selection criteria

- the charged lepton transverse momentum:  $p_T^\ell > 25 \text{ GeV}$
- the charged lepton pseudorapidity:  $|\eta_\ell| < 2.4$
- the missing transverse energy:  $E_T^{\text{miss}} > 25 \text{ GeV}$  (we used  $E_T^{\text{miss}} = p_T^\nu$ )
- the size of an electron cluster (for **electron–photon recombination**):  $\Delta\eta_e \times \Delta\phi_e = 0.075 \times 0.175 \text{ rad}$ , where  $\eta_e$  and  $\phi_e$  are the electron pseudorapidity and azimuthal angle
- no photon recombination with muons
- photon transverse momentum:  $p_T^\gamma > 25 \text{ GeV}$  (for exclusive photonic distributions)
- photon pseudorapidity:  $|\eta_\gamma| < 2.4$  (for exclusive photonic distributions)

- Observables

1.  $W$ -boson transverse mass:  $m_T^W$
2. charged lepton transverse momentum:  $p_T^\ell$
3.  $W$ -boson rapidity:  $y_W$
4. charged lepton pseudorapidity:  $\eta_\ell$
5. hardest photon transverse momentum:  $p_T^\gamma$
6. hardest photon pseudorapidity:  $\eta_\gamma$

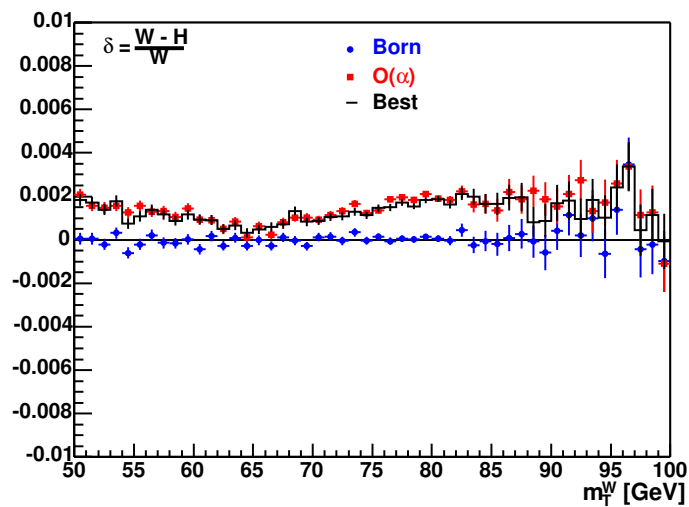
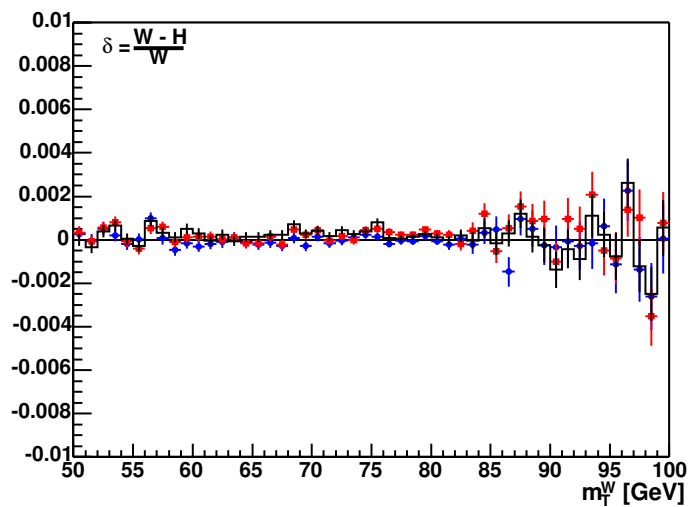
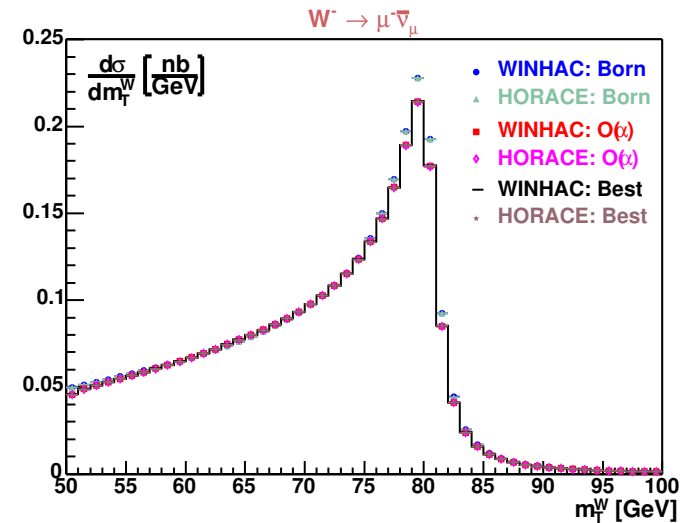
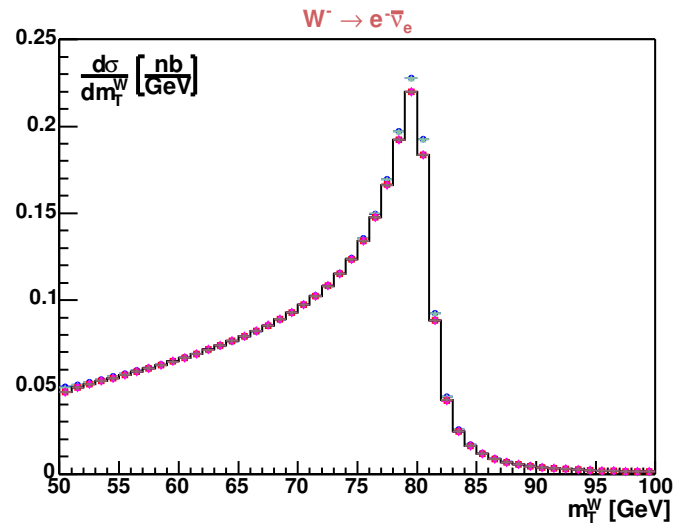
## HORACE and WINHAC: total cross sections

- Total cross sections comparisons, at parton (p.l.) and hadron level (h.l.). Cross sections for  $W^-$  production (in  $nb$ ):

	Born		$\mathcal{O}(\alpha)$		with higher orders	
	HORACE	WINHAC	HORACE	WINHAC	HORACE	WINHAC
p. l. $e^-$ (no cuts)	8.8872	8.8872(2)	8.8872	8.8855(1)	8.8872	8.8840
p. l. $\mu^-$ (no cuts)	8.8872	8.8872(1)	8.8863	8.8853(1)	8.8863	8.8844
h. l. $e^-$ (no cuts)	7.7331(4)	7.7332(1)	7.7331(4)	7.7317(1)	7.7325(4)	7.7304
h. l. $\mu^-$ (no cuts)	7.7332(4)	7.7332(1)	7.7332(4)	7.7316	7.7328(4)	7.7307
h. l. $e^-$ (with cuts)	3.2363(1)	3.2363(1)	3.1871(1)	3.1878(1)	3.1870(1)	3.1876(1)
h. l. $\mu^-$ (with cuts)	3.2363(1)	3.2363(1)	3.1599(1)	3.1642(1)	3.1601(1)	3.1641(1)

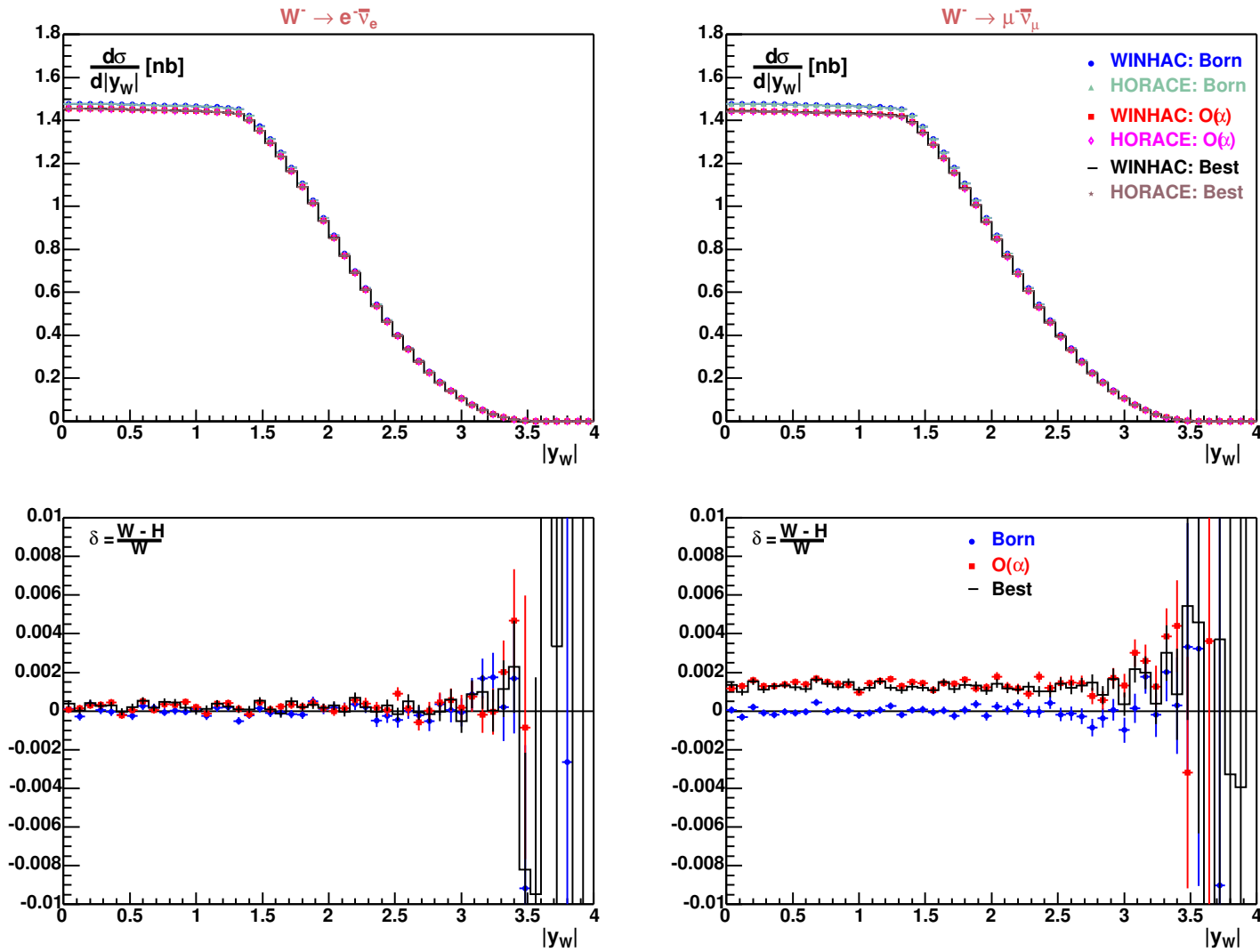
- Differences are at the  $10^{-4} \div 10^{-3}$  level
- ★ Effect of QED RC on integrated cross section with cuts at **1.5% for  $e^-$**  and **2.3% for  $\mu^-$**

# HORACE and WINHAC: differences on $M_T^W$



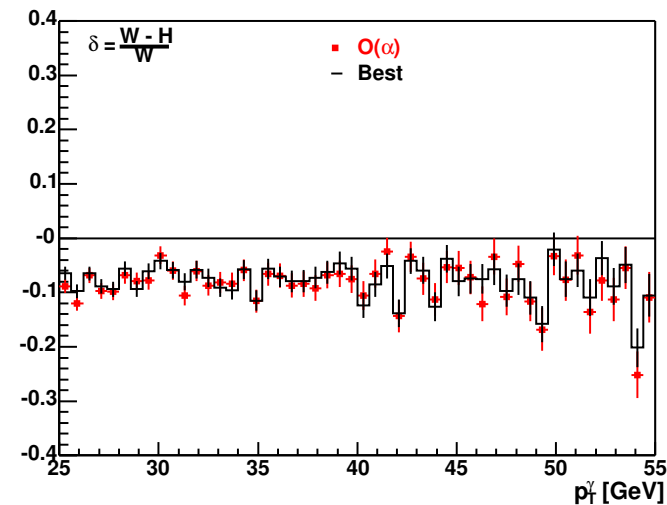
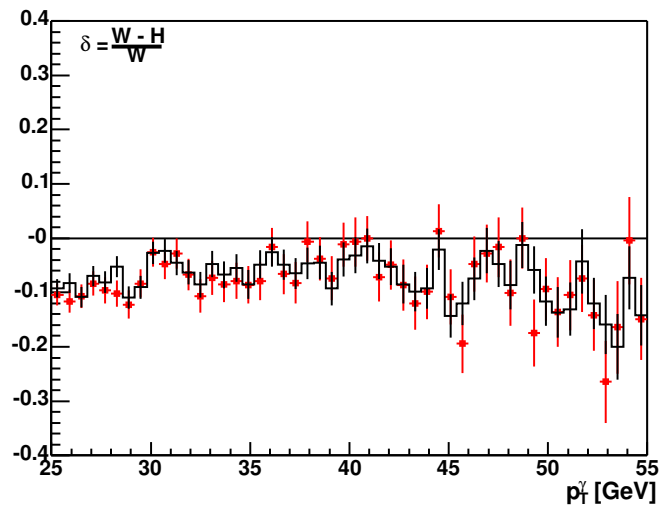
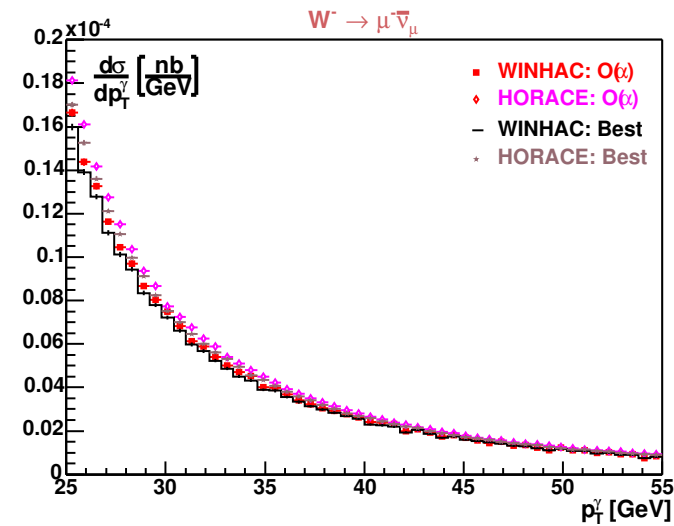
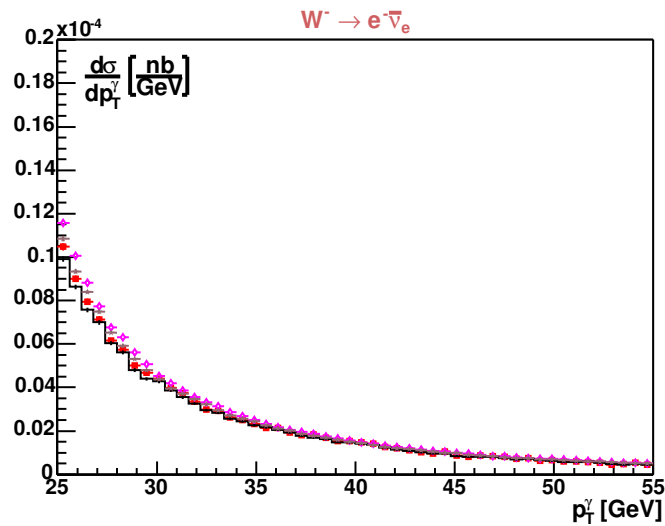
- Flat  $\mathcal{O}(\alpha)$  differences at 0.2% level for  $\mu$ . Same relative effect of QED exponentiation

# HORACE and WINHAC: differences on $W$ rapidity



- Flat  $\mathcal{O}(\alpha)$  differences at 0.2% level for  $\mu$ . Same relative effect of QED exponentiation

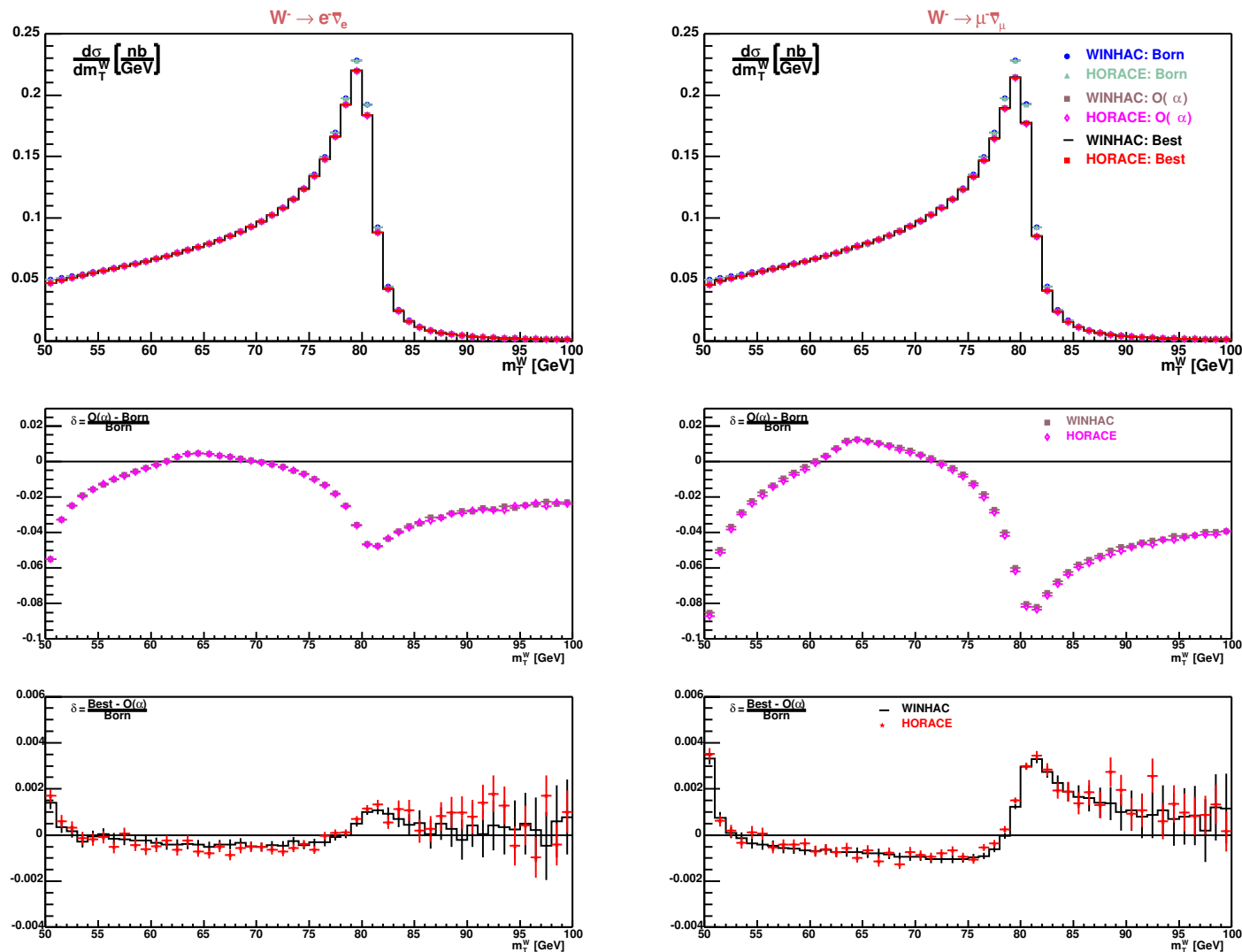
# HORACE and WINHAC: differences on hardest photon $p_T$



- Radiative event  $W \rightarrow l\nu\gamma + (n\gamma)$
- Flat differences at some per cent level

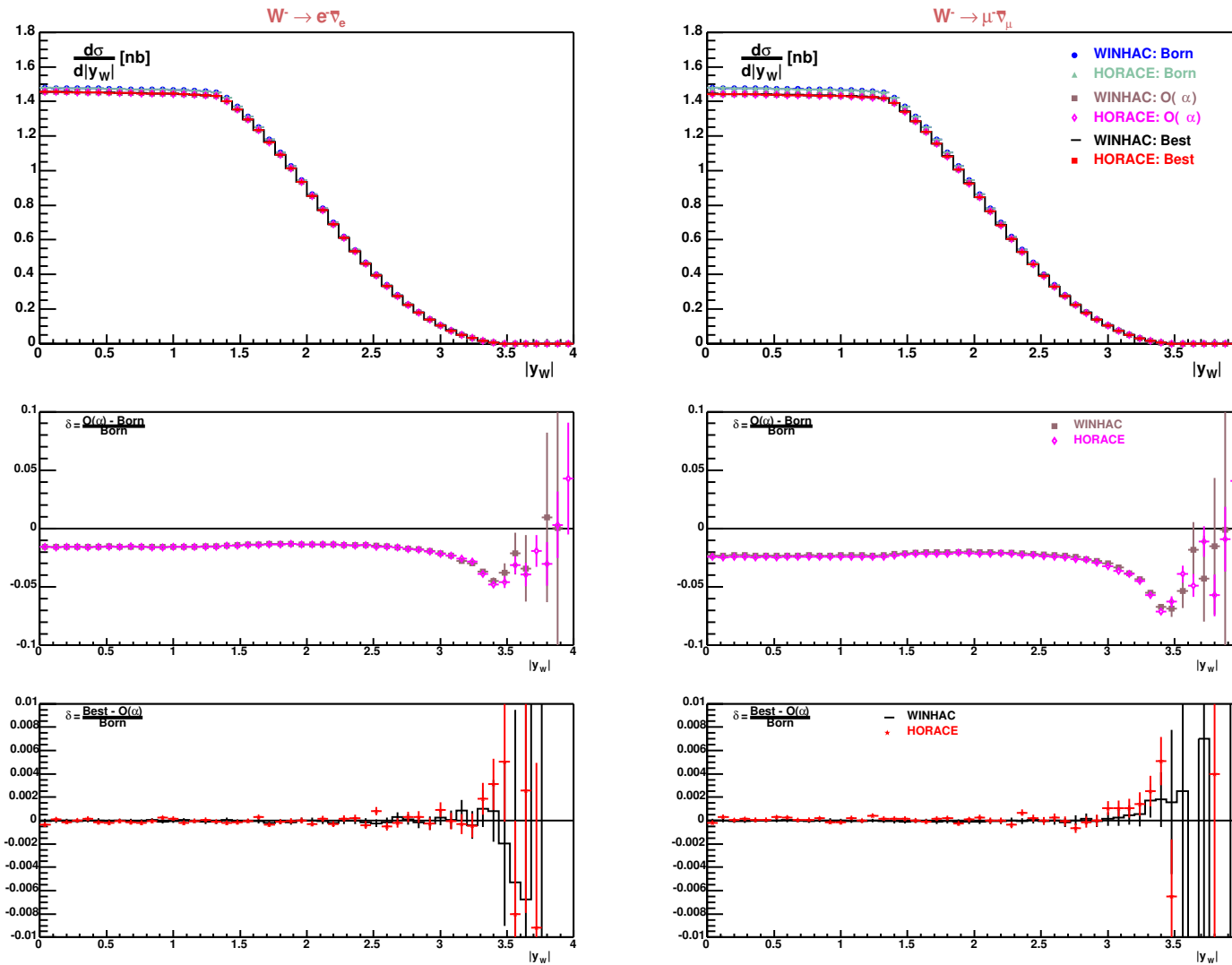


# HORACE and WINHAC: QED effects on $W$ transverse mass



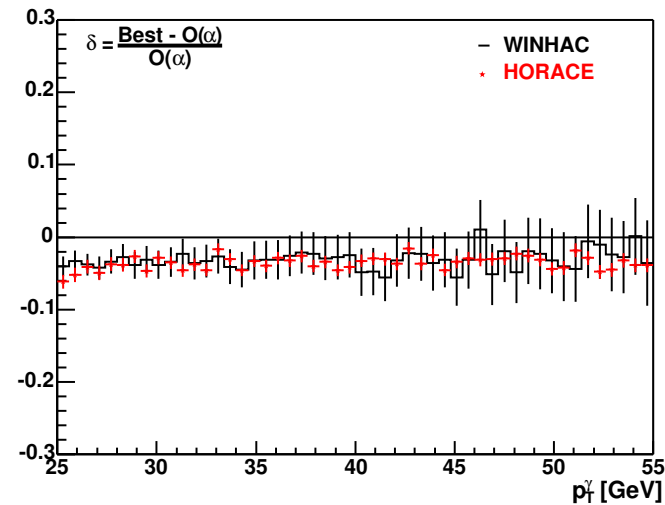
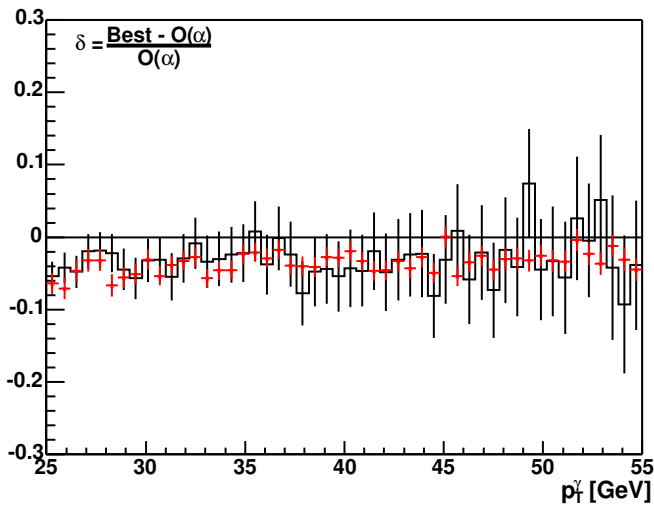
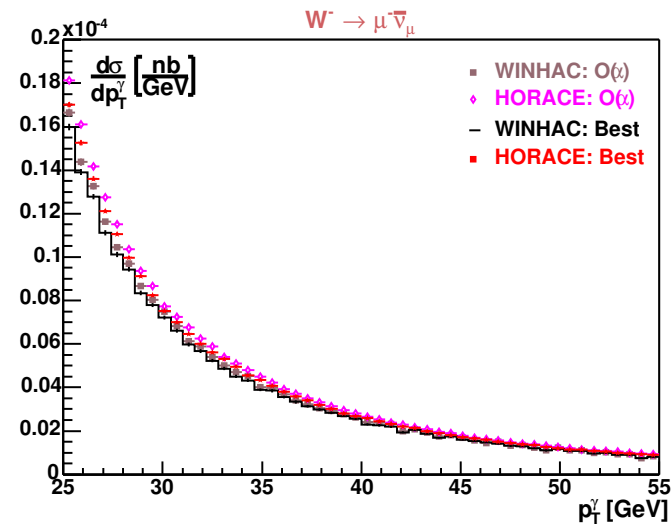
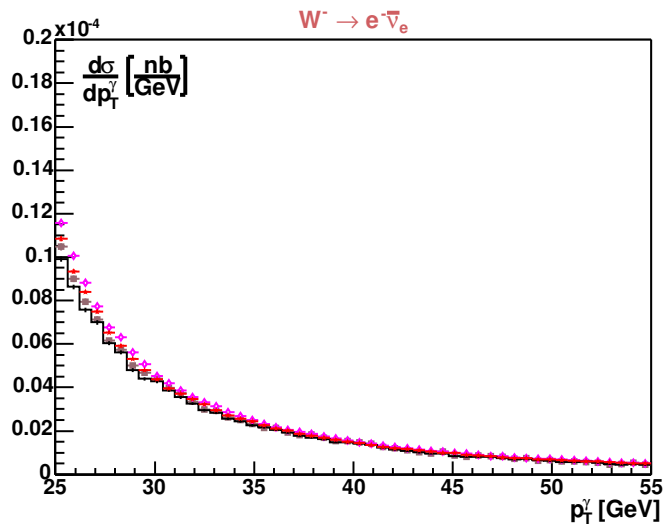
- $\mathcal{O}(\alpha)$  corrections at 5%(10%) level for  $e(\mu)$ , higher-order effects at 0.2%(0.5%) level for  $e(\mu)$  around the  $W$  peak

# HORACE and WINHAC: QED effects on $W$ rapidity



- $\mathcal{O}(\alpha)$  corrections at 2-5% level both for  $e/\mu$ , very small effect of QED exponentiation

# HORACE and WINHAC: QED effects on hardest photon $p_T$



- Radiative event  $W \rightarrow l\nu\gamma + (n\gamma)$
- QED exponentiation at a few % level

# Neutral current case

## Z simulation setup (see hep-ph/0502218)

- Input parameters (**Born**  $\rightarrow$  **EBA**) and cuts

$p\bar{p}$  collisions at  $\sqrt{s} = 2$  TeV. CTEQ6 PDF's at LO

$$p_T(\ell^\pm) > 25 \text{ GeV} \quad |\eta(\ell^\pm)| < 1.2$$

$$m_e = 0.511 \times 10^{-3} \text{ GeV} \quad m_\mu = 0.10565836 \text{ GeV}$$

$$\alpha^{-1} = 137.03599976 \quad G_\mu = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$$

$$M_Z = 91.1876 \text{ GeV} \quad \sin^2 \theta_W^{\text{eff}} = 0.23150 \quad \Gamma_Z = 2.4952 \text{ GeV}$$

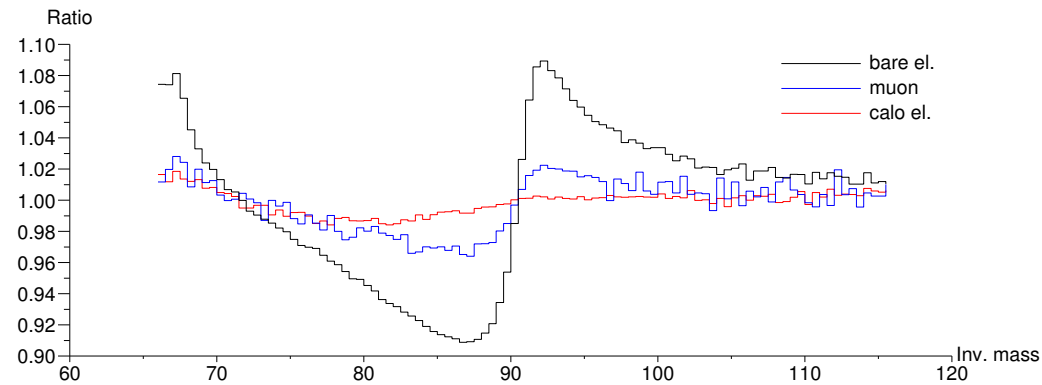
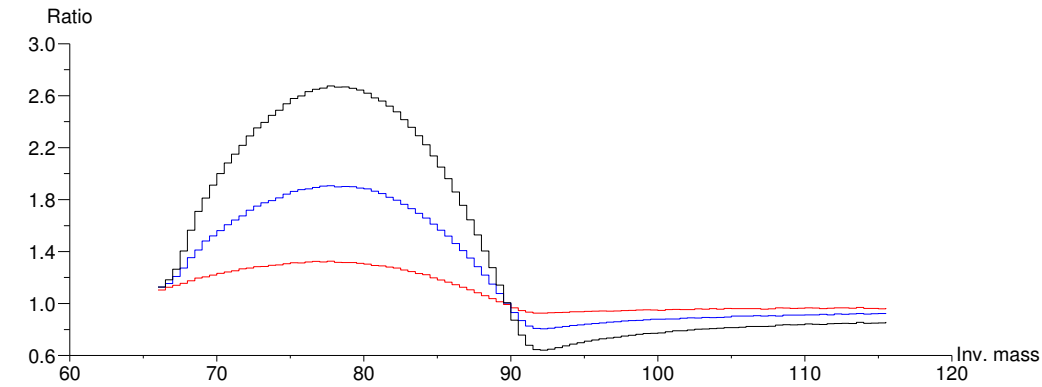
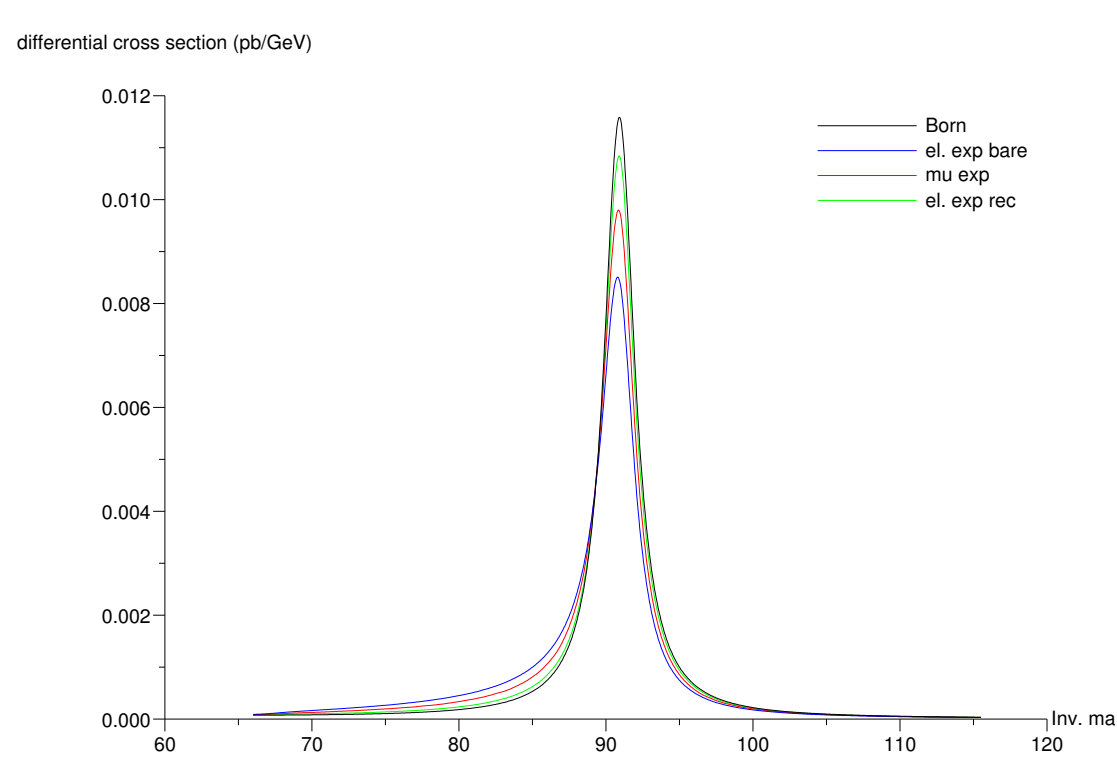
$$\alpha \rightarrow \frac{\alpha}{1 - \Delta\alpha} \quad \Gamma_Z \rightarrow \Gamma_Z \frac{s}{M_Z^2}$$

- Particle's ID requirements ( $\Delta R = \sqrt{\Delta\eta_{\ell\gamma}^2 + \Delta\phi_{\ell\gamma}^2}$ )

	$\ell = \text{electron}$	$\ell = \mu$
recombined	$\Delta R < 0.2$ or $0.2 < \Delta R < 0.3$ if $E_\gamma > 0.15E_e$	
rejected	$0.1 < \Delta R < 0.4$ and $E_\gamma > 0.15E_e$	$\Delta R < 0.2$ if $E_\gamma > 2 \text{ GeV}$ or $0.2 < \Delta R < 0.6$ if $E_\gamma > 6 \text{ GeV}$

- particles' momenta can be smeared **similarly** to RunII DØ detector specifications

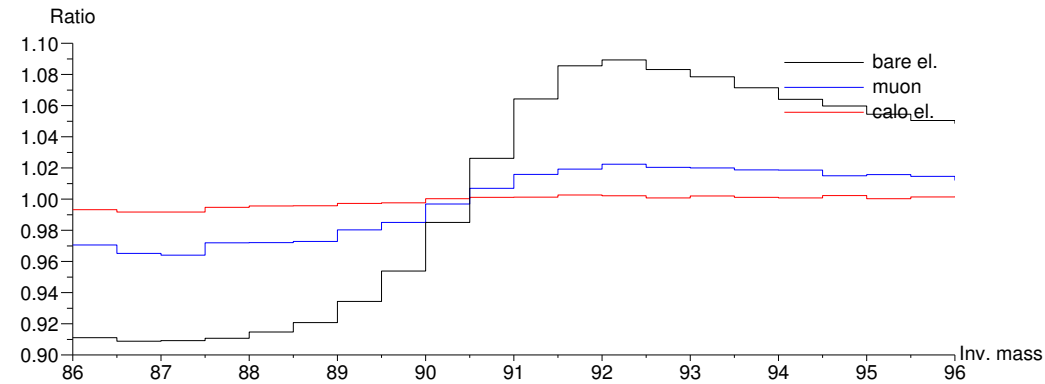
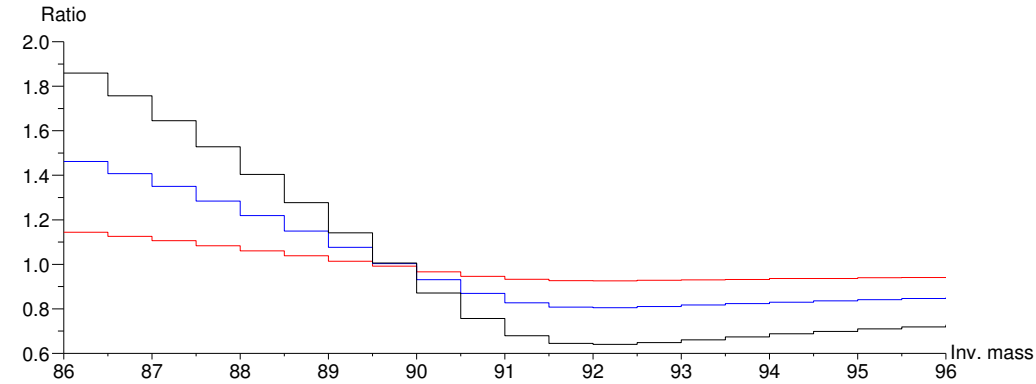
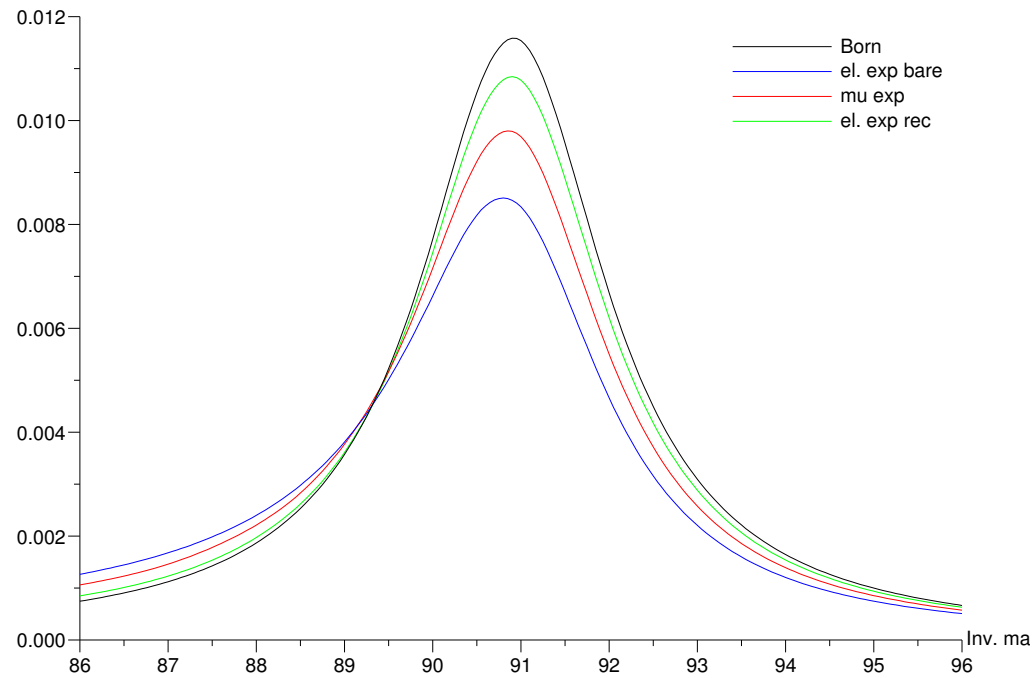
# QED effects on $Z$ line shape



- Detector effects are switched off
- $\mathcal{O}(\alpha)$  corrections are large in the lower tail
- QED exponentiation at some % in the peak region: 10% for bare  $e$ , 3% for  $\mu$ , 1% for recombined  $e$

# QED effects on $Z$ line shape (zoom)

differential cross section (pb/GeV)

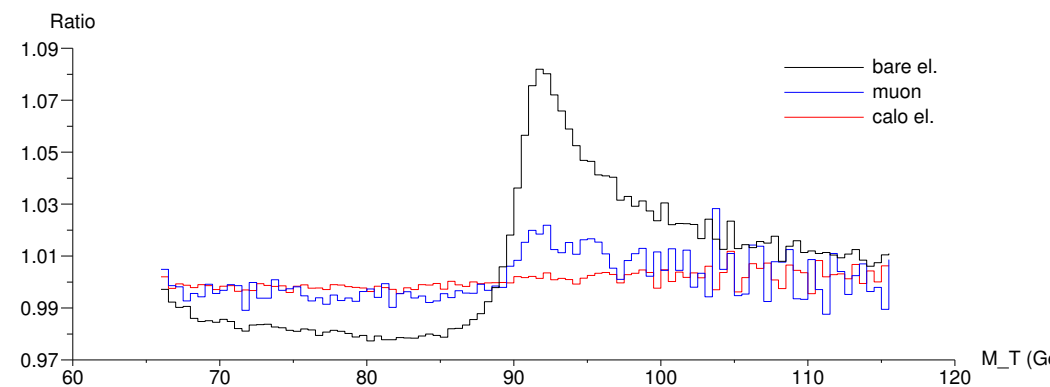
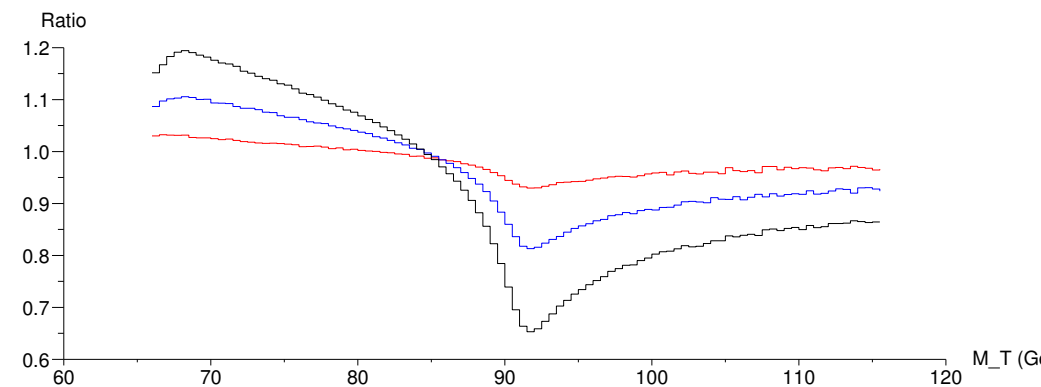
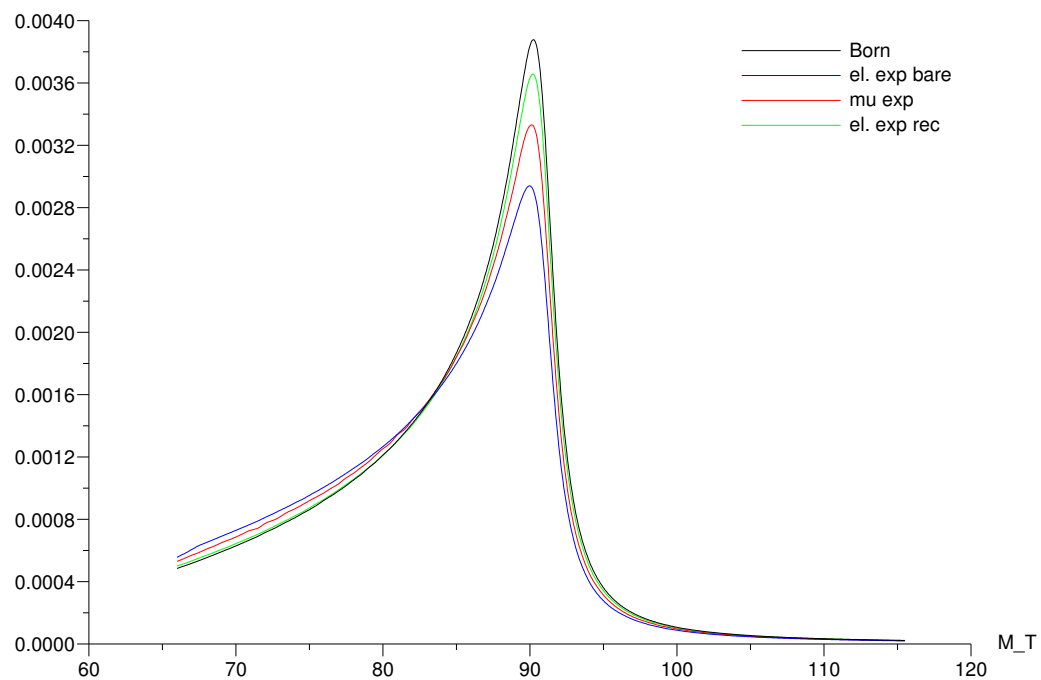


→  $\mathcal{O}(\alpha)$  corrections at some 10% near the peak

→ QED exponentiation at some % in the peak region

# QED effects on $Z$ transverse mass

differential cross section (pb/GeV)

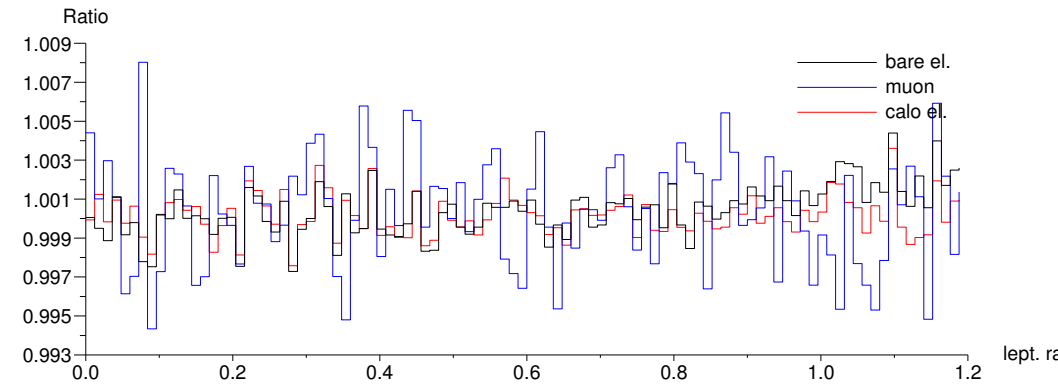
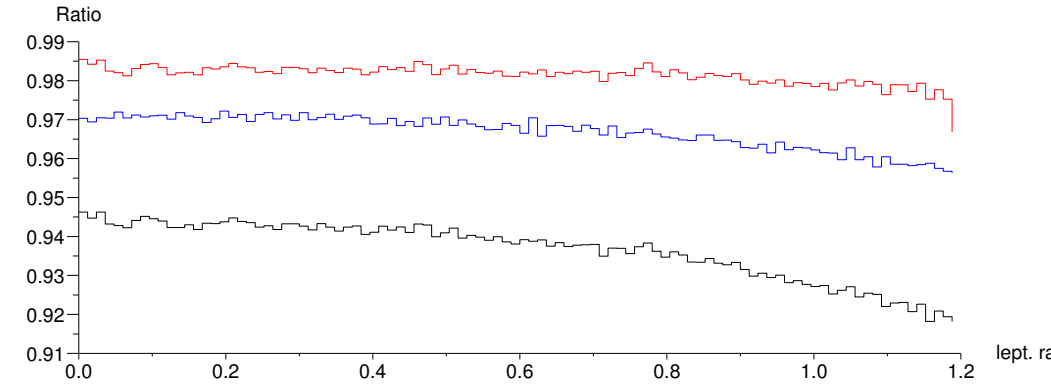
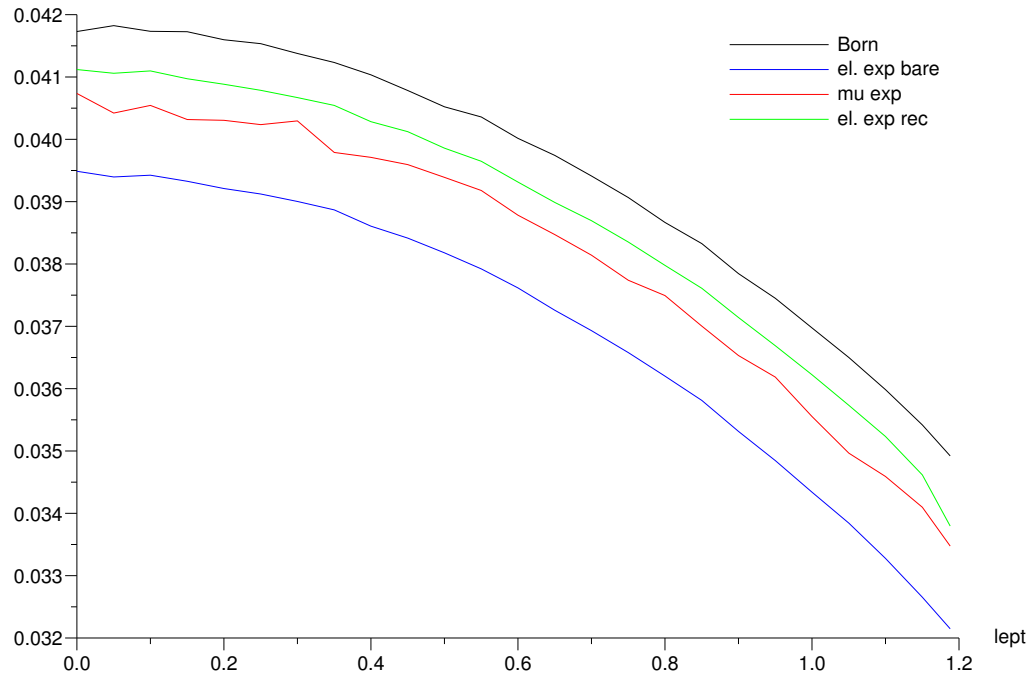


→  $\mathcal{O}(\alpha)$  corrections at a few 10% on the peak

→ QED exponentiation at some % in the peak region

# QED effects on $\eta_\ell$ in $Z$ decay

differential cross section (pb)



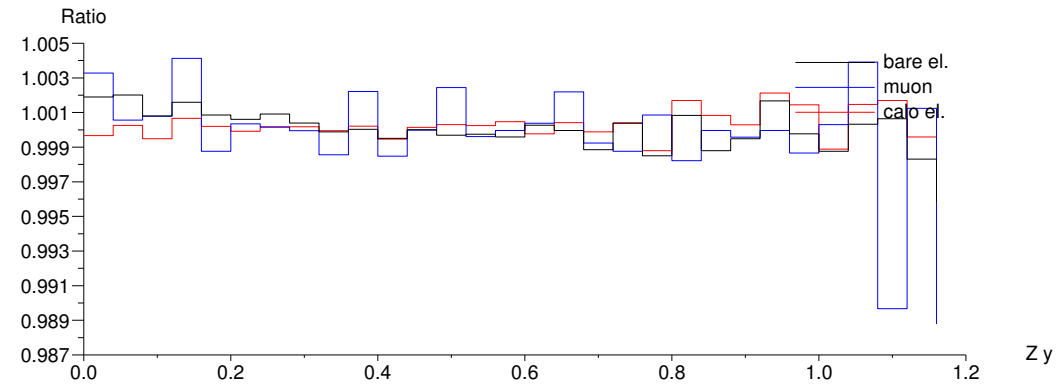
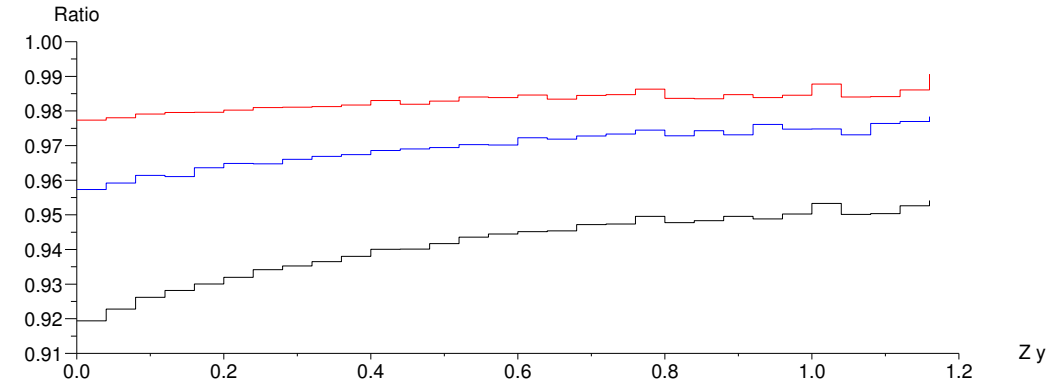
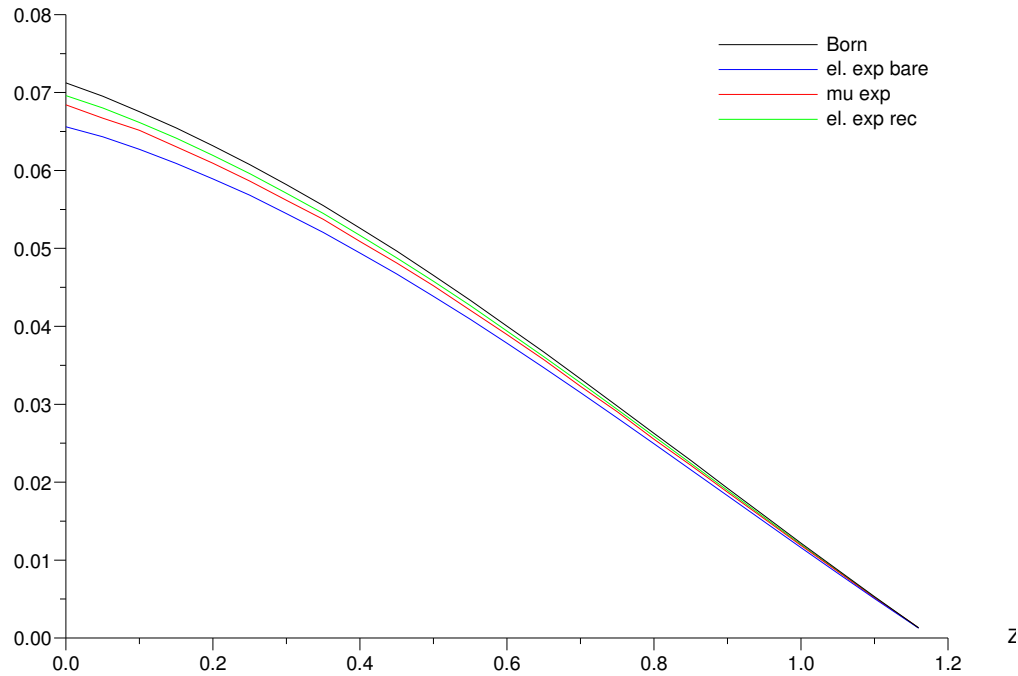
→ flat  $\mathcal{O}(\alpha)$  corrections at a few %

→ QED exponentiation is almost negligible



# QED effects on $Z$ rapidity

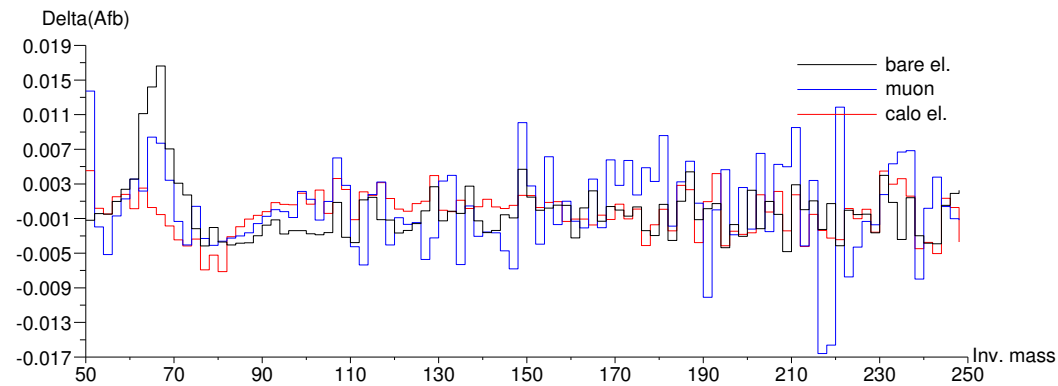
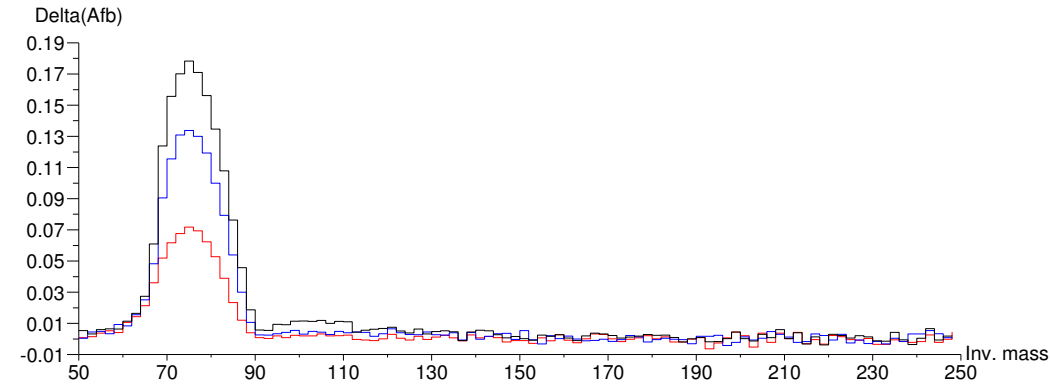
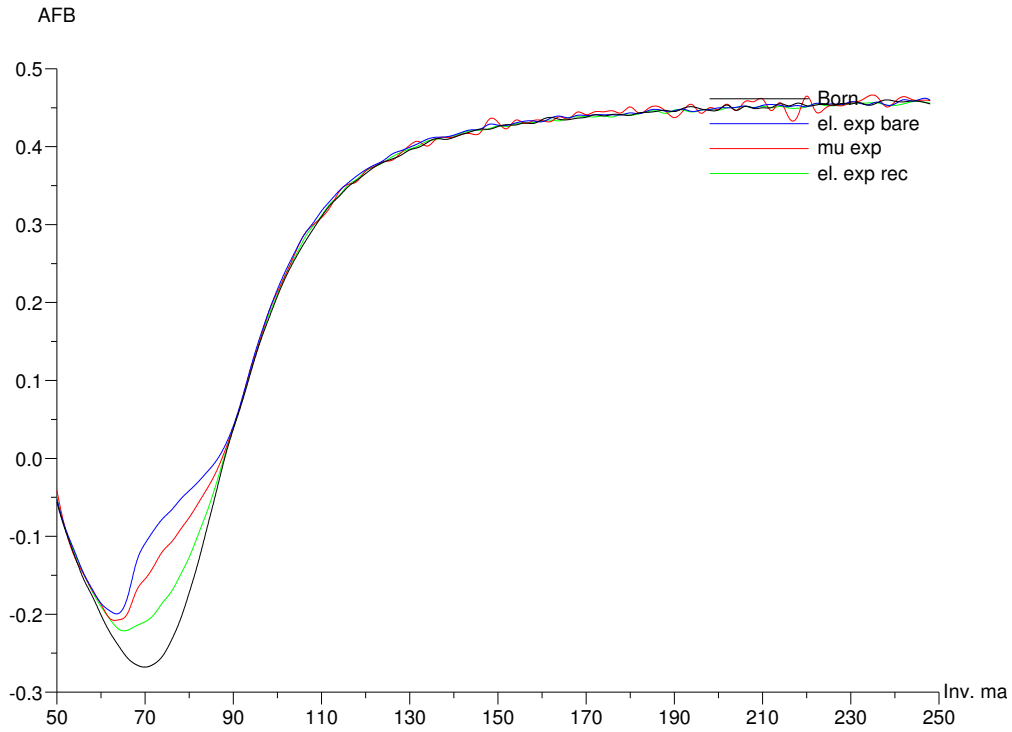
differential cross section (pb)



→  $\mathcal{O}(\alpha)$  corrections at few %

→ QED exponentiation is almost negligible

# QED effects on $A_{FB}$

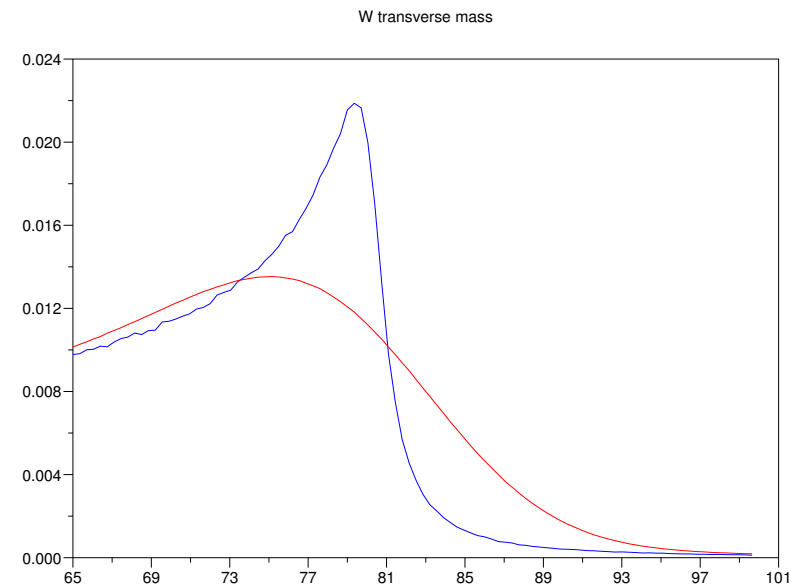
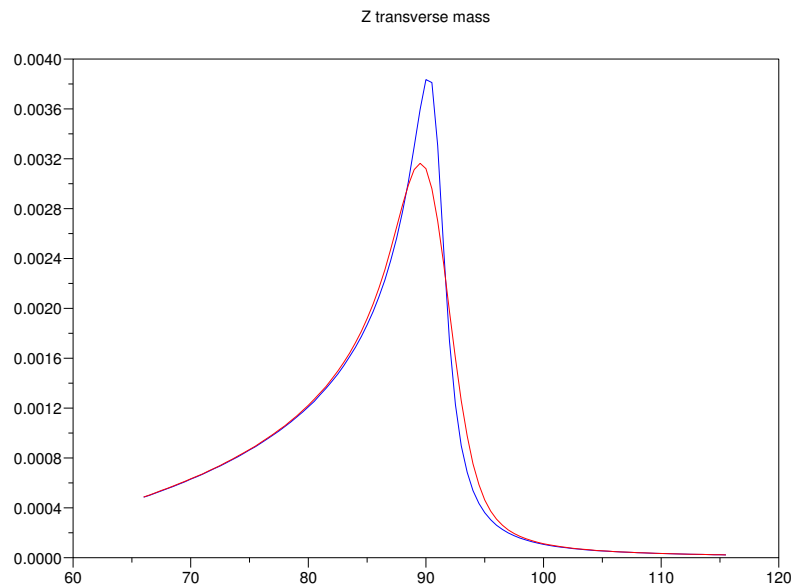


→ small effect of QED exponentiation

# Estimate of RC effects on the fitted $W$ & $Z$ masses

By means of HORACE, we estimated the shifts induced on extracted boson masses by RC

- given a pseudo-data distribution  $(M_T^W, M^Z)$ , we fit it with RC corrected distributions:
  - ★ we generate RC corrected distributions for  $N$  different masses
  - ★ we calculate the  $\chi^2$  for each of them w.r.t. the pseudo-data
  - ★ at the minimum  $\chi^2$  we read the shift
  - ★ the same procedure to quantify  $\mathcal{O}(\alpha)$  and HO shifts
- the exact value of the shift depends on cuts, particles' ID and **smearing!**
  - ★ e.g.,  $M_T$  for  $Z$  &  $W$  with and without smearing



# $W$ & $Z$ mass shifts estimate

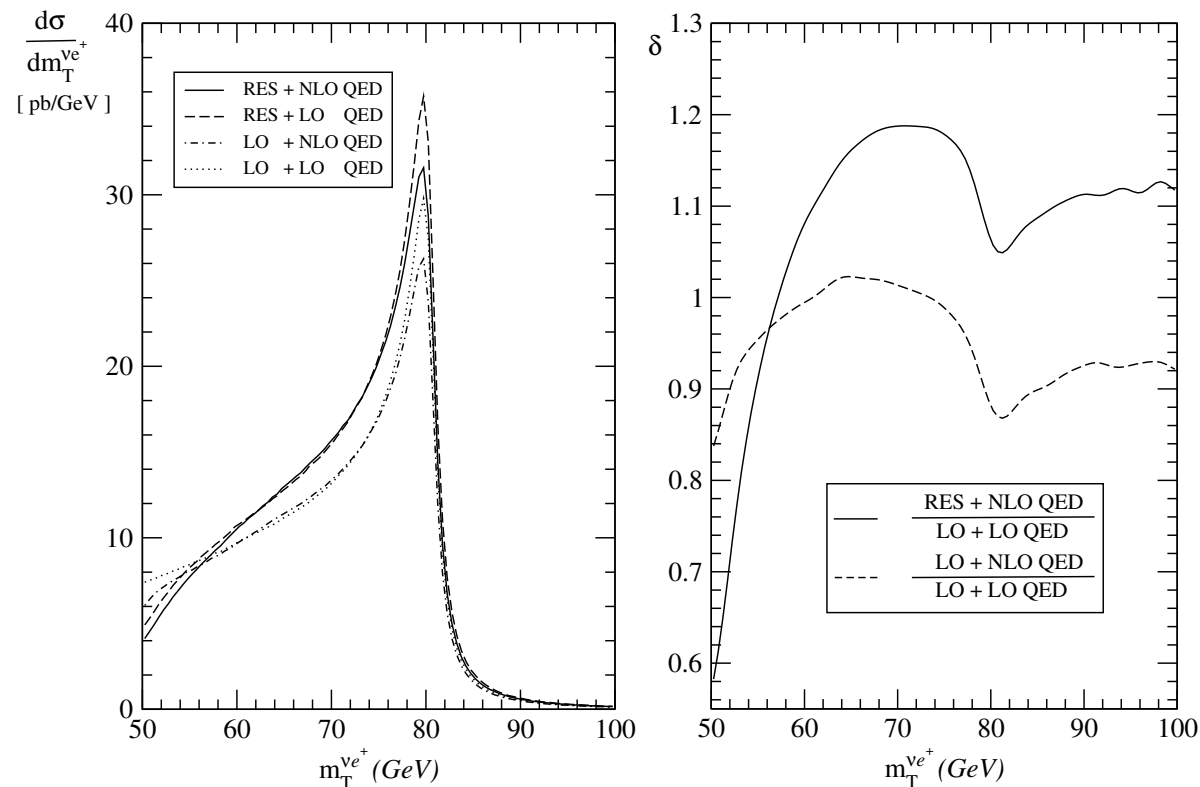
→ the  $\chi^2$  fit shows (with “our” simplified detector!)

Particle	Smearing	Lepton ID	$\Delta M_Z^\alpha$ (MeV)	$\Delta M_Z^{\text{h.o.}}$ (MeV)	$\Delta M_W^\alpha$ (MeV)	$\Delta M_W^{\text{h.o.}}$ (MeV)
$e$	✗	✗	595	-135		
$\mu$	✗	✗	270	-31		
$e$	✗	✓	75	-5		
$\mu$	✗	✓	215	-28		
$e$	✓	✗	780	-159	400	-40
$\mu$	✓	✗	565	-49	220	-10
$e$	✓	✓	105	-6	20	-2
$\mu$	✓	✓	420	-44	110	-10

- ★ detailed simulation performed for the TeVatron. For the LHC, the results are similar
- ★ higher orders reduce the  $\mathcal{O}(\alpha)$  effect
- ★ |higher orders|  $\sim 10\% \times \mathcal{O}(\alpha)$
- ★ the shifts significantly depend on the detector details... ; - )

# FS QED + IS QCD resummation (for $W$ )

- A unified MC including both EW and QCD corrections would be very helpful
- Cao and Yuan (PRL 93 042001 (2004) and hep-ph/0401171) included in **RESBOS** (Balazs and Yuan, PRD 56 5558 1997) the **QED  $\mathcal{O}(\alpha)$  corrections** of **WGRAD**
  - ★ resummation of soft gluons ( $\Rightarrow p_{\perp}^W$  generation)
  - ★ bulk of EW radiative corrections: **final-state QED  $\mathcal{O}(\alpha)$  corrections**



# Conclusions

- The Drell-Yan-like production of **single  $W/Z$  boson** is a process relevant for many aspects of the physics programme of hadron colliders
- Recent big theoretical effort towards **high-precision predictions**, including higher-order QCD and electroweak corrections, to keep under control **theoretical systematics**
- **Full  $\mathcal{O}(\alpha)$  EW corrections** are available and are dominated by FS photonic corrections
- We focused on **multi-photon corrections**:
  - the MC **HORACE** developed (for  **$W&Z$** ). **QED radiation included by means of a PS**
  - for  $W$ , tuned and detailed comparisons with WINHAC
  - $W/Z$ -mass shift due to higher-orders is **about 10%** of that due to  $\mathcal{O}(\alpha)$  (**with simplified detector effects!**)
- **Work in progress**: include the exact  $\mathcal{O}(\alpha)$  electroweak corrections in **HORACE** (in collaboration with **A. Vicini**)
- It's necessary to **combine EW and QCD corrections** in a “unified” generator

## HORACE cross-checks

- for  $W^{(+)}$  cross-checked with WGRAD (Baur, Keller, Wackerath)

(cross sections in pb)	TeVatron		LHC	
	$e$	$\mu$	$e$	$\mu$
WGRAD Born	441.7(1)		1906(1)	
WGRAD	418.3(4)	429.4(3)	1800(2)	1845(2)
WGRAD final-state	419.7(1)	430.0(1)	1808(1)	1854(1)
HORACE Born	441.6(1)		1905(1)	
HORACE $\mathcal{O}(\alpha)$	419.4(1)	429.9(1)	1806(1)	1853(1)
HORACE exponentiated	419.5(1)	430.0(1)	1808(1)	1853(1)

→  $\mathcal{O}(\alpha)$  corrections  $\sim 5\%$

- for  $Z$  checking versus ZGRAD2 (Baur et al.) is in progress

→ preliminary results for  $e^+e^-$  at the TeVatron

(cross sections in pb)	Born	FS $\mathcal{O}(\alpha)$
ZGRAD2	51.625(5)	47.72(4)
HORACE	51.639(5)	47.621(1)

## HORACE versus B&K (Z. Phys. C27 (1985) 365)

- $\mathcal{O}(\alpha)$  Z at parton level,  $\sqrt{s} = 90$  GeV,  $k_0 = E_\gamma/E_{beam}$
- $R = \sigma(k > k_0)/\sigma_0$

$k_0$	$e$		$\mu$	
	HORACE	B&K	HORACE	B&K
0.01	0.416	0.411	0.224	0.220
0.05	0.246	0.242	0.133	0.129
0.10	0.178	0.173	0.096	0.090
0.15	0.139	0.135	0.075	0.071
0.20	0.113	0.109	0.061	0.057
0.30	0.079	0.075	0.042	0.039
0.40	0.056	0.054	0.030	0.028
0.50	0.041	0.038	0.022	0.020
0.60	0.029	0.027	0.016	0.014
0.70	0.020	0.018	0.011	0.009
0.80	0.012	0.011	0.006	0.005
0.90	0.006	0.005	0.003	0.002

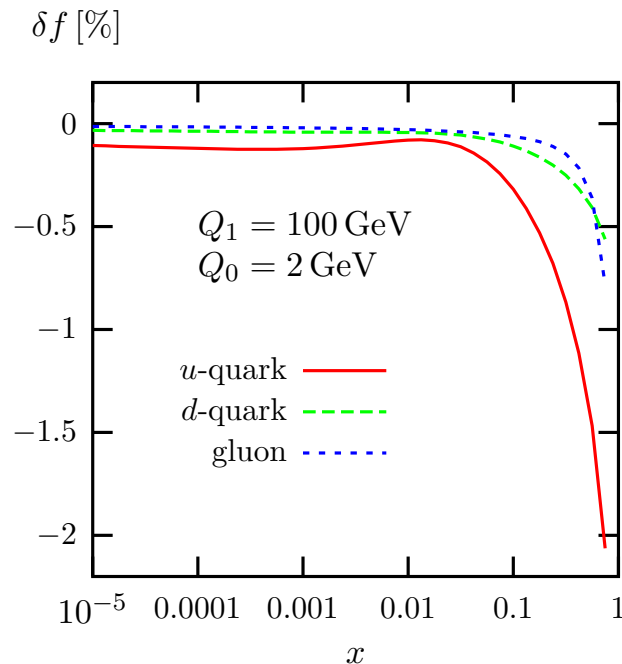


# Initial-state photonic corrections & PDFs

H. Spiesberger, Phys. Rev. **D52** (1995) 4936

M. Roth and S. Weinzierl, Phys. Lett. **B590** (2004) 190

- QED radiation off quarks give rise to **factorizable and universal** mass singularities
- they can be absorbed by a redefinition (*renormalization*) of PDFs, in analogy to QCD, introducing a dependence on a QED factorization scale
- the DGLAP evolution must contain an additional term proportional to  $\alpha_{QED}$  and fits to data should include QED contributions (**now included in MRST**, hep-ph/0411040)



- QED terms modify PDFs at the 0.1% level for  $x < 1 \Rightarrow$  “small” for  $W/Z$  production