

Recent Progress in Kaon Physics

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many thanks to the organizers

IFAE, Catania March 30 – April 2

A REMARKABLE HISTORY

1947

Rochester and Butler

Observation of K^0 and K^+ particles

1964

Cronin et al., $K \rightarrow \pi\pi$ CP VIOLATION

1963

Cabibbo – V_{us}

Universality of Weak Interactions

1970

Glashow, Iliopoulos, Maiani – GIM
Suppression of $K \rightarrow \mu^+\mu^-$, $K \rightarrow \pi l^+l^-$

Kobayashi, Maskawa – CKM Matrix

1972

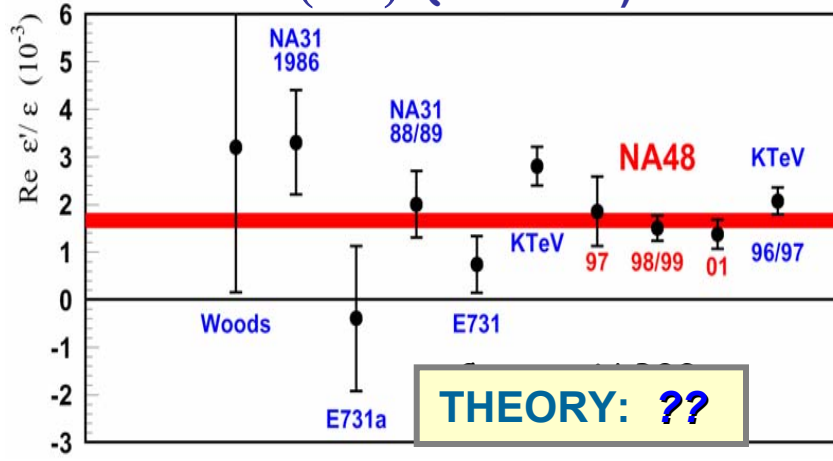
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

For the Future, Kaon physics will help looking for inconsistencies in SM using independent observables affected by **small theoretical uncertainties** and different sensitivity to new physics.

Recently

KTeV-NA48

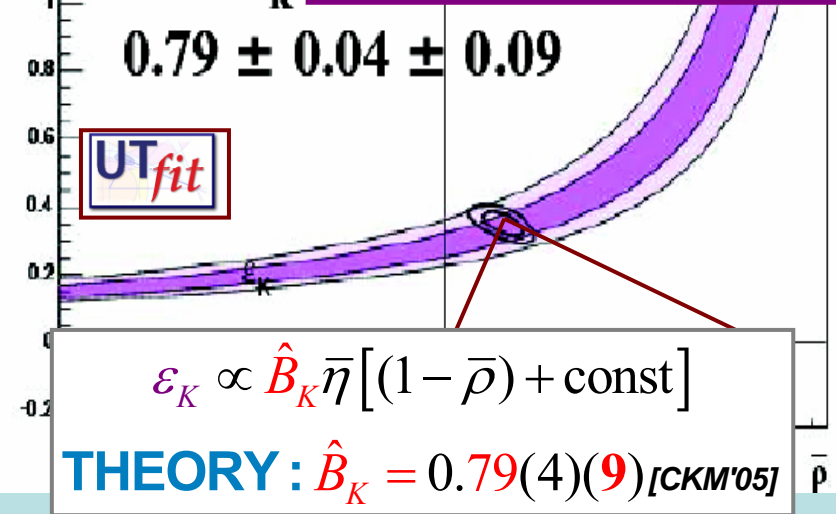
$$\text{Re}(\varepsilon'/\varepsilon) = (16.6 \pm 1.6) \times 10^{-4}$$



new B_K $|\varepsilon_K| = 2.280(13) \times 10^{-3}$

$$0.79 \pm 0.04 \pm 0.09$$

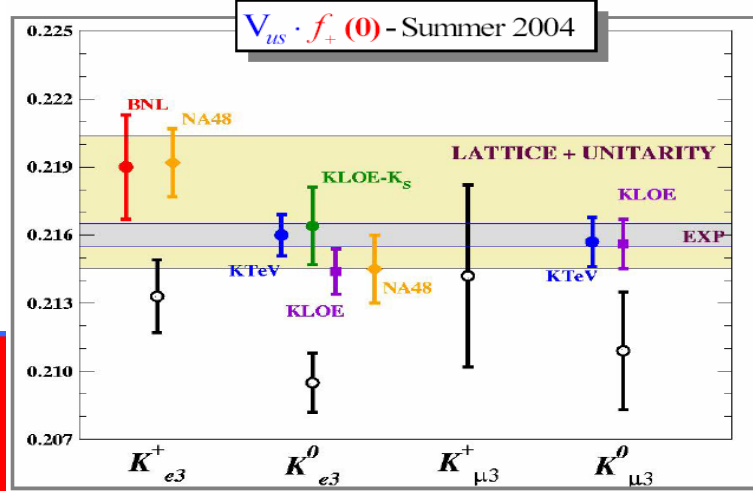
UT fit



2004

BNL-KTeV-NA48-KLOE

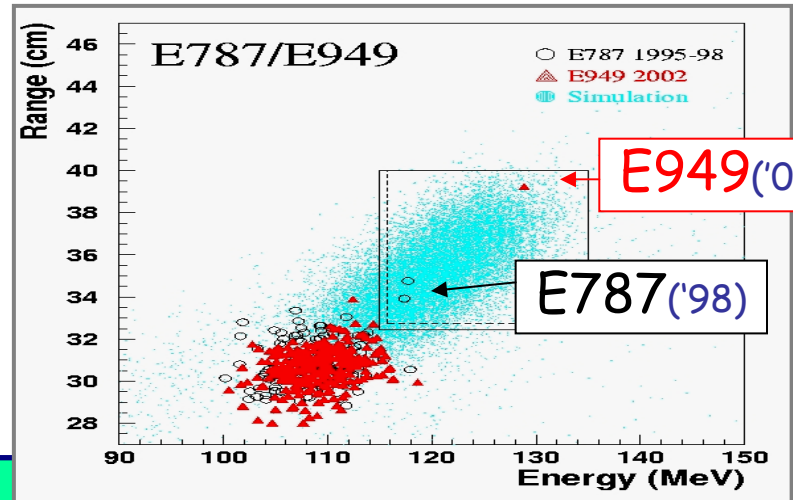
$$|V_{us}|^2 + |V_{ud}|^2 + |V_{ub}|^2 = 0.999(1)$$



THEORY: *Becirevic et al, Nucl.Phys.B705:339,2005*

a look to THE FUTURE

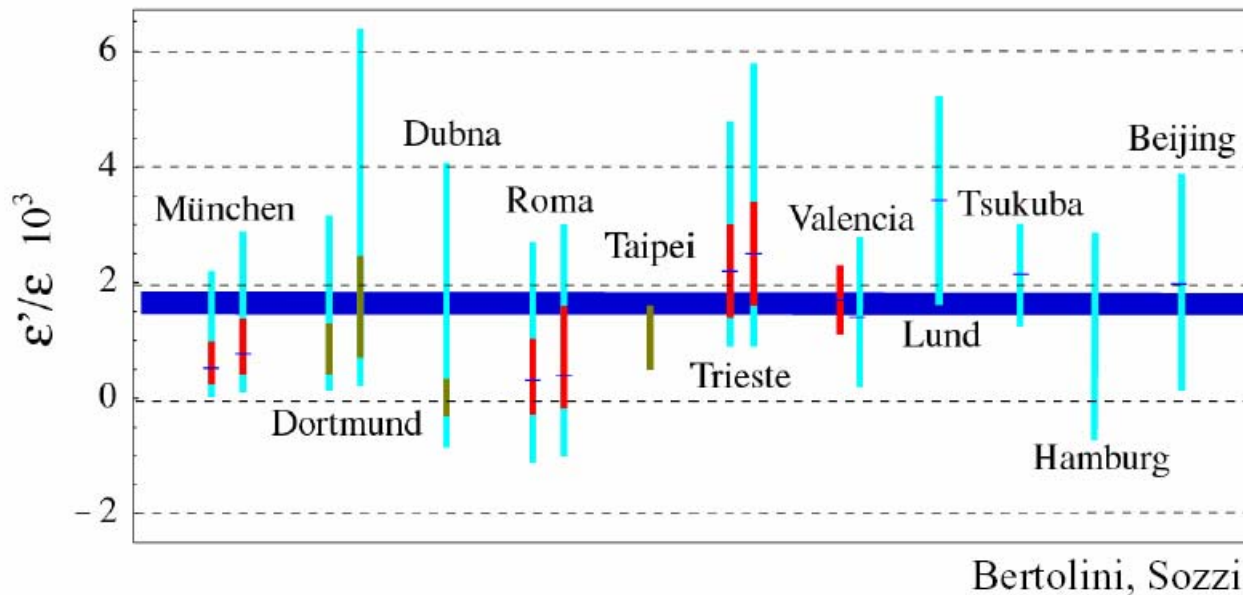
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$: the signal (?)



THEORY: *Isidori, F.M., Smith hep-ph/0503107*

Direct CP-Violation - $\text{Re}(\epsilon'/\epsilon)$

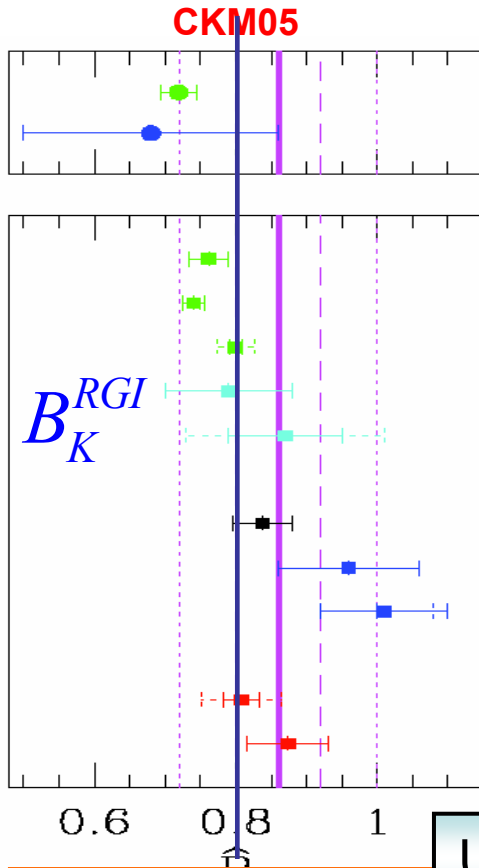
Calculations of $\text{Re}(\epsilon'/\epsilon)$



$\text{Re}(\epsilon'/\epsilon) \propto \eta$, but large hadronic uncertainties currently make it impossible to convert measurement of $\text{Re}(\epsilon'/\epsilon)$ into a meaningful CKM constraint (except that $\eta \neq 0$).

Rare Kaon Decays will fill the miss

Current Lattice results for B_K



RBC 2004, $N_f=2$
 UKQCD 2004, $N_f=2$

RBC 2004
 RBC 2002
 CP-PACS 2001
 MILC 2003
 BosMar 2003

ALPHA 2005
 SPQ_{cd}R 2004
 SPQ_{cd}R 2000

Lee et al. 04
 JLQCD 1997

Quenched Simulations

- ✓ High level of accuracy:
- ✓ Newer values by chirally-improved fermions lower than reference JLQCD 97 value of **0.86(6)**
- ✓ Estimate of quenching error by $N_f=2$

$$B_K^{RGI}(\text{CKM03}) = 0.87 \pm 0.06 \pm 0.14$$

$$B_K^{RGI}(\text{CKM05}) = 0.79 \pm 0.04 \pm 0.09$$

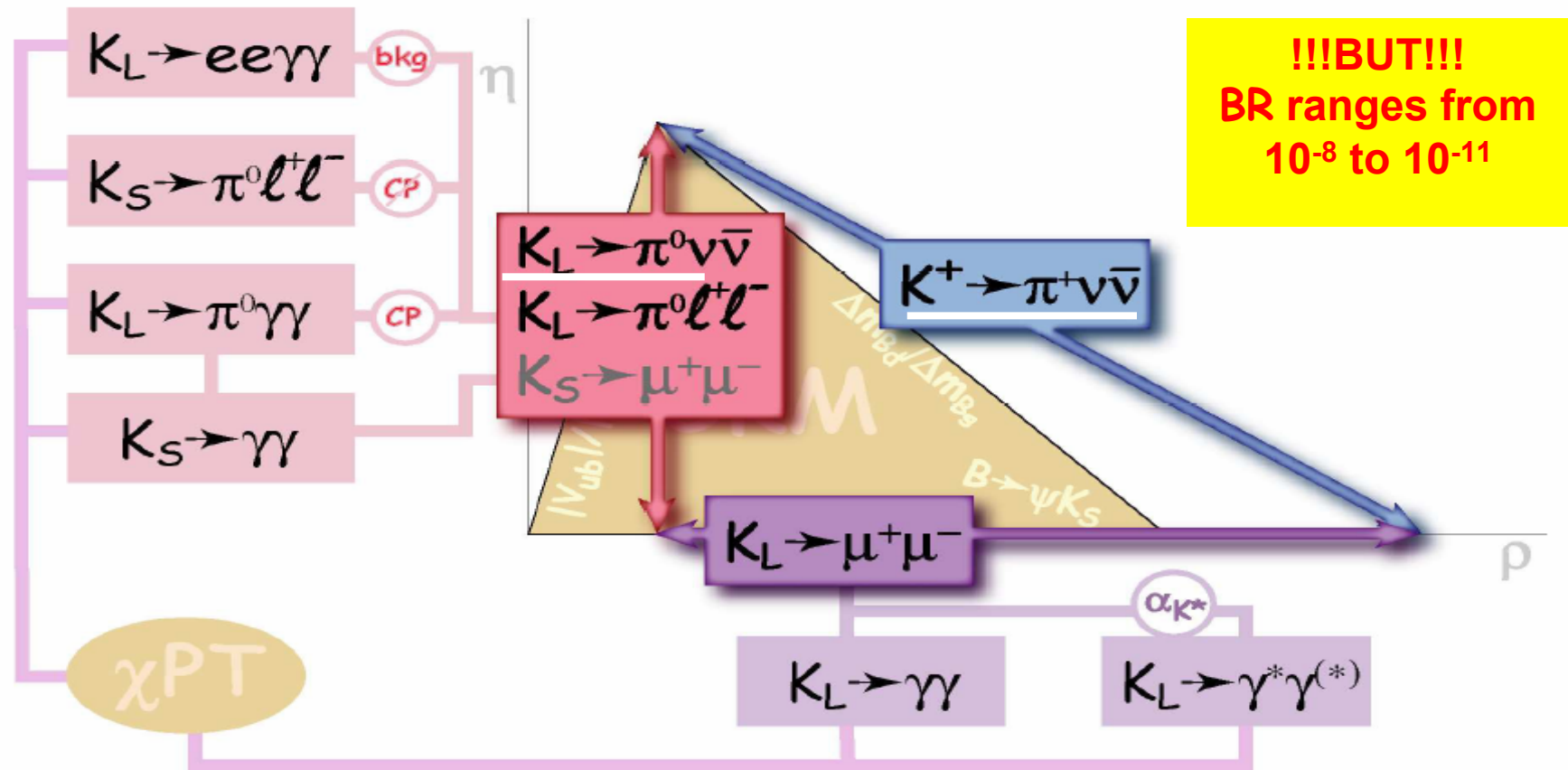
[CKM 2005]

Quenching
 The largest
 uncertainty

Unquenching

- ✓ A few weak attempts (<'99): limited parameter space and no sea quark dependence was possible to observe.
- ✓ Recently, first results: **W. Fermions** [Flynn, F.M., Tariq UKQCD04] and **D.W. Fermions** [Dawson et al/RBC04]: a mild sea quark dependence seen.
- ✓ In conclusion, still much to do.

Rare K Decays and the Unitarity Triangle



by Komatsubara FPCP04

Rare Kaon decays provide quantitative tests of SM independent from B mesons: they are theoretically clean and highly sensitive to NP.

Rare K Decays: Present Status

Golden Modes	Standard Model	Experiment
$K_L \rightarrow \pi^0 \nu \bar{\nu}$	$3.0^{+0.6}_{-0.6} \times 10^{-11}$	$< 5.9 \times 10^{-7}$ KTeV
$K_L \rightarrow \pi^0 e^+ e^-$	$3.7^{+1.1}_{-0.9} \times 10^{-11}$	$< 2.8 \times 10^{-10}$ KTeV
$K_L \rightarrow \pi^0 \mu^+ \mu^-$	$1.5^{+0.3}_{-0.3} \times 10^{-11}$	$< 3.8 \times 10^{-10}$ KTeV
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	$7.8^{+1.2}_{-1.2} \times 10^{-11}$	$14.7^{+13.0}_{-8.9} \times 10^{-11}$ E787 E949

New generation of experiments needed
 [KOPIO@BNL]
 [JPARC@KEK]
 [NA48-@CERN]
 2008 2010

by C. Smith Moriond'05

- $K^0_L \rightarrow \pi^0 \nu \nu$
 –Expect results from data collected by E391a (proposed SES~3 10^{-10})

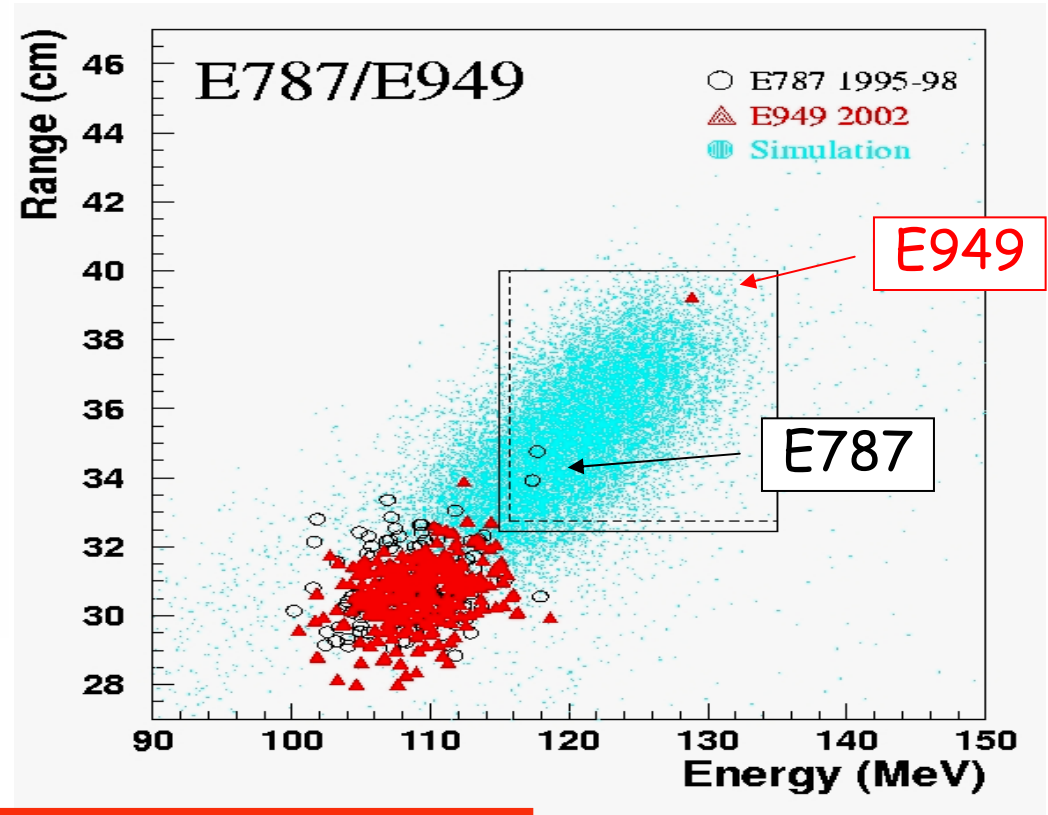
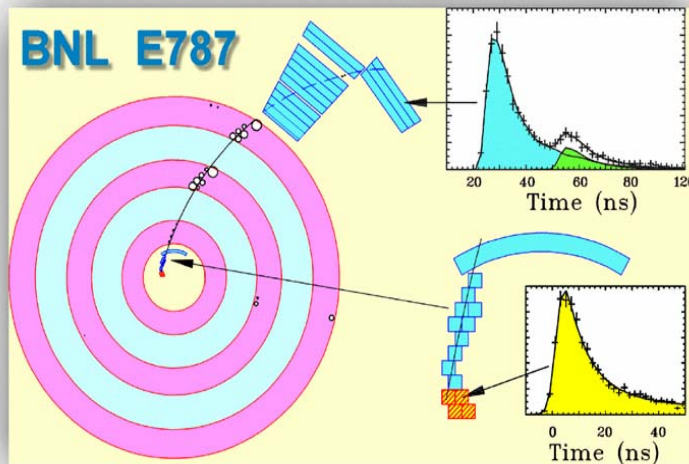
2004-Progress

$K^0_L \rightarrow \pi^0 ee (\mu\mu)$

- Long distance contributions under control
 [Buchalla, D'Ambrosio, Isidori, '03; Friot, Greynat, De Rafael '04; Isidori, Unterdorfer, Smith '04]
- **NA48** result of $\text{Br}[K^0_S \rightarrow \pi^0 ee]$ and $\text{Br}[K^0_S \rightarrow \pi^0 \mu\mu]$ has allowed SM prediction [D'Ambrosio, Ecker, Isidori, Portolés ('98)]

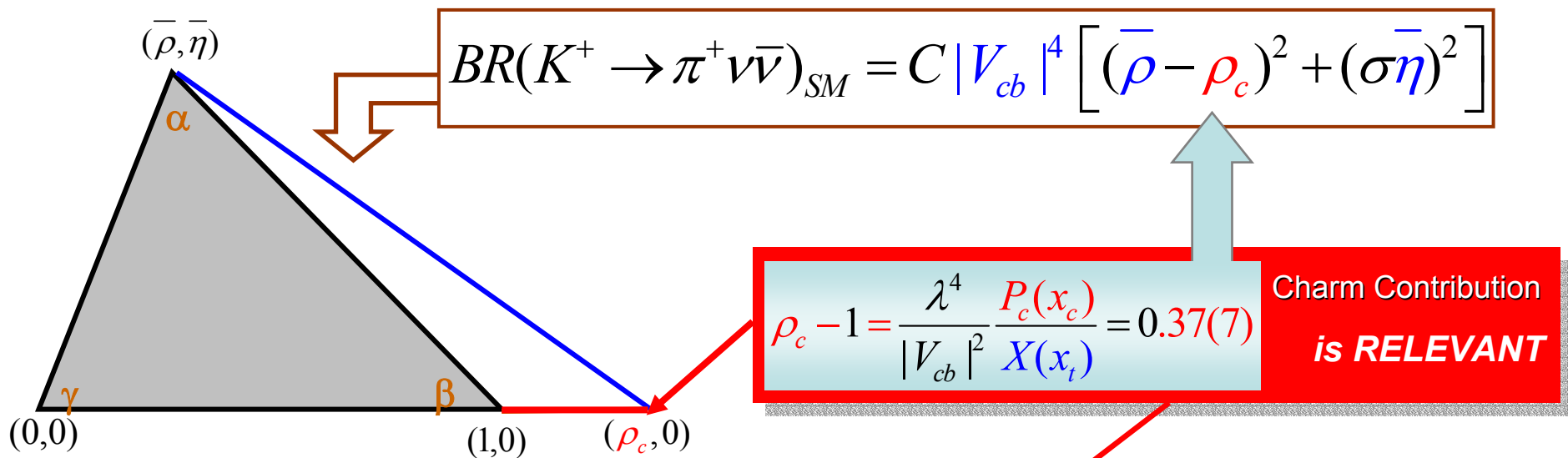
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$: State of the Art

hep-ex/0403036 PRL93 (2004)



$$Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.47_{-0.89}^{+1.30} \times 10^{-10}$$

Twice the SM but compatible within errors



Precision Test: $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{SM} = (7.8 \pm \underline{0.82}_{P_c} \pm 0.91_{CKM, m_t}) \times 10^{-11}$

⊙ **Hadronic** ME from K_{13} via isospin corrections (2%) (Marciano-Parsa 96):

⊙ The **t-quark** loops: $X(x_t) = 1.53 \pm 0.04$ *known to ~3%*

⊙ The **c-quark** loops: $P_c(x_c) = 0.39 \pm 0.07$ *known to ~18% [8% @N²LO Buras soon].*

Br : 15% of uncertainty

10% comes from CHARM Dim=6 Contributions @NLO:

to be reduced to below 4% by the NNLO calculation (Buras)

***Aiming to get a few % th. accuracy on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$,
subleading terms $\propto (1/m_c)^2$ have to be considered:***

1) Subleading *c-quark loops*: Dimension-eight operators

Possible correction to $P_c(x_c)$ of the order of $m_K^2 / m_c^2 \approx 15\%$

Lu and Wise (1994)

2) Residual *u-quark loops*: Genuine $\Delta S=1$ Long-distance effects

Possible correction to $P_c(x_c)$ of the order of $\Lambda_{QCD}^2 / m_c^2 \approx 10\%$

Falk, Lewandowski, Petrov (2000)

**Theoretical Progress
(2005)**

Isidori, F.M., Smith hep-ph/0503107

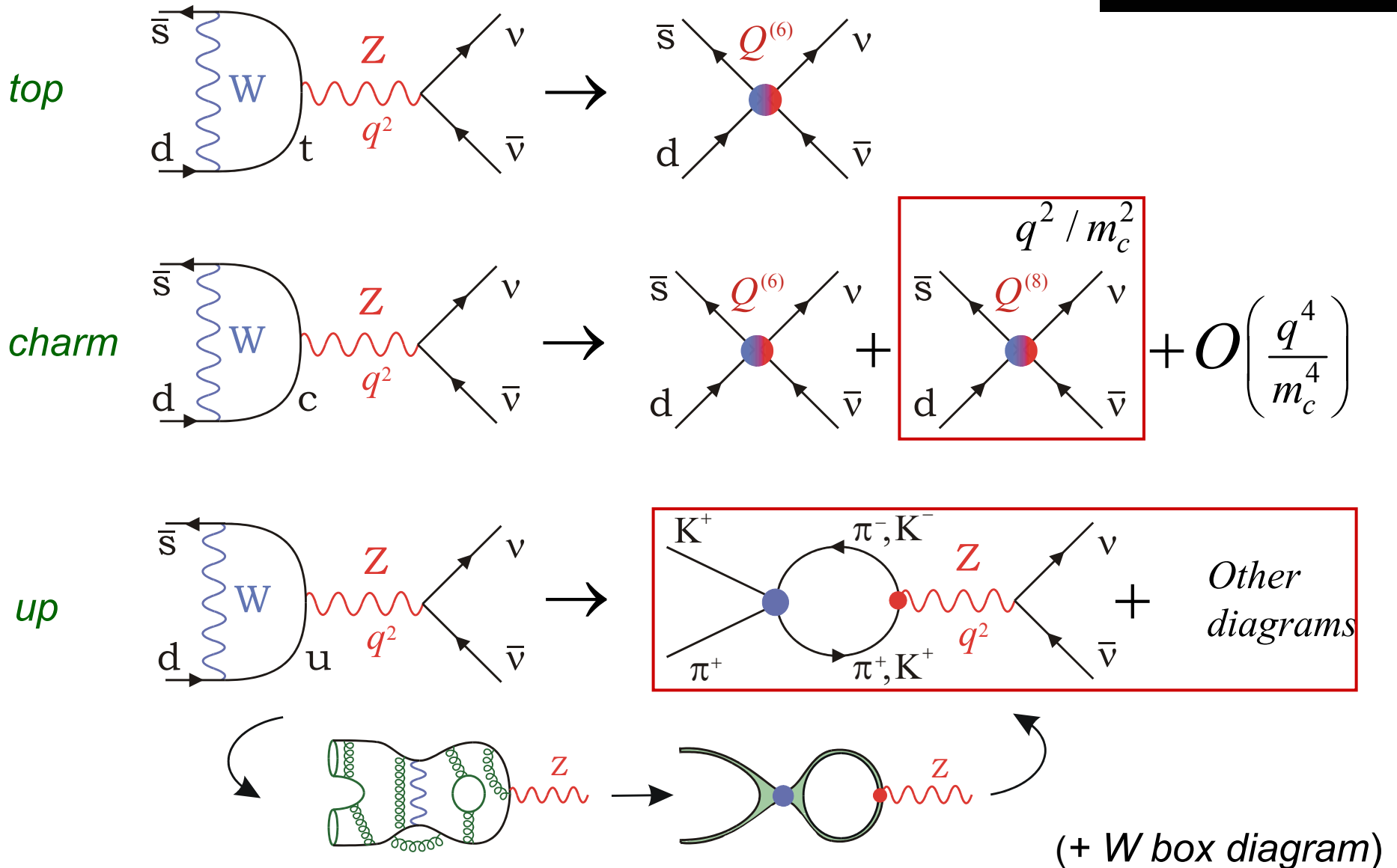
The right estimate is

$\delta P_c / P(x_c) \approx 10\% \rightarrow BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{SM}$ increases by **6%**

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

Isidori, F.M., Smith '05

- Identification of Subleading Corrections



by C. Smith Moriond'05

$$\begin{aligned}
 A_{SD}^Z(q^2 \rightarrow 0) \propto (\bar{s}d)_{V-A} \frac{\lambda_t}{M_W^2} & \left[\begin{array}{l} \text{Dim. 6} \\ X(x_t) \end{array} + x_t \frac{q^2}{m_t^2} (\log x_t + \dots) \right] (\bar{\nu}\nu)_{V-A} \\
 + (\bar{s}d)_{V-A} \frac{\lambda_c}{M_W^2} & \left[\begin{array}{l} \text{Dim. 8} \\ P_c(x_c) \end{array} + \frac{q^2}{M_W^2} \left(\log \frac{m_c^2}{M_W^2} + \dots \right) \right] (\bar{\nu}\nu)_{V-A} \\
 - (\bar{s}d)_{V-A} \frac{\lambda_c}{M_W^2} & \left[\begin{array}{l} \text{Dim. 8} \\ + \frac{q^2}{M_W^2} \left(\log \frac{\mu_{IR}^2}{M_W^2} + \dots \right) \right] (\bar{\nu}\nu)_{V-A}
 \end{aligned}$$

OPE
at $O(q^2/m_c^2)$

- **Identification** of short-distance dimension-eight operators like $Q_c^{(8)} = (\bar{s}d)_{V-A} \partial^2 (\bar{\nu}\nu)_{V-A}$ by an OPE at the charm scale. (Falk, Lewandowski, Petrov ('00))

- **Compute** the one-loop amplitude in ChPT for the **long-distance** u-quark effects.

-**Match** ChPT **UV** divergence to SD **IR** one to get $A^Z(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ at $O(G_F^2 \Lambda_{QCD}^2)$


$$\lambda_c \frac{q^2}{M_W^2} \left(\log \frac{m_c^2}{\mu_{IR}^2} - \log \frac{m_c^2}{\mu_{UV}^2} \right) \rightarrow \lambda_c \frac{q^2}{M_W^2} \log \frac{m_c^2}{m_\pi^2}$$

$\mu_{IR} = \mu_{UV} \approx m_c$
 μ -independent

- Z Penguin in ChPT Lu and Wise (1994) is wrong

A new operator must be introduced in ChPT to enforce GIM mechanism:

$$L_{|\Delta S|=1+GIM}^{(2)} = F_\pi^4 G_8 \left\{ \langle \lambda_6 L_\mu L^\mu \rangle - \frac{2ig}{\cos \theta_W} \langle \lambda_6 L_\mu T_3 \rangle Z^\mu \right\}$$

at $O(G_F^2 p^2)$  = 0

With this new piece: 1/ No sensitivity to the singlet part of the Z current.
 2/ At loop level, a structure matching SD arises.

- Conclusion

The result of the analysis is that instead of $m_K^2 / m_c^2 \approx O(15\%)$ for $\langle \pi | Q_c^{(8)} | K \rangle$ and $\Lambda_{QCD}^2 / m_c^2 \approx O(10\%)$ for *u-quark*, both of them scale as

$$\left(\pi F_\pi / m_c \right)^2 \approx O(5\%) \quad P_c(x_c) + \delta P_c : \begin{cases} P_c(x_c) = 0.39 \pm 0.07 \\ \delta P_c = 0.04 \pm 0.02 \end{cases}$$

BSM Scenarios

Compiled by S. Kettel

	$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \times 10^{11}$	$BR(K_L^0 \rightarrow \pi^0 \nu \bar{\nu}) \times 10^{11}$
SM	8.0 ± 1.1	3.0 ± 0.6
MFV hep-ph/0310208	≤ 19.1	≤ 9.9
EEWP NP B697 133	7.5 ± 2.1	31 ± 10
EDSQ hep-ph/0407021	≤ 15	≤ 10
MSSM hep-ph/0408142	≤ 40	≤ 50

- **Complementary programme to the high energy frontier:**
 - **When** new physics will appear at the **Tevatron/LHC**, K rare decays may help to understand the nature of it

The most accurate test of CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

CKM05

$$|V_{ud}| = 0.9739(3) \Rightarrow |V_{us}^{UNI}| = \sqrt{|V_{ud}|^2 + |V_{ub}|^2 - 1} = 0.2269(13)$$

PDG04 $|V_{us}| = 0.2200(26)$ \leftarrow **$\sim 2.4 \sigma$ discrepancy**

$|V_{us}| \rightarrow$ possibly responsible!!

Relies on old experimental and theoretical results of K_{l3}

V_{us} from $K_{\ell 3}$ decays

$$\Gamma(K \rightarrow \pi l \nu(\gamma)) = \frac{G_F^2 M_K^5}{192 \pi^3} C_K^2 |V_{us}|^2 \cdot |f_+^{K^0 \pi^-}(0)|^2 I_K S_{ew} (1 + \delta_{SU(2)}^K + \delta_{em}^{Kl})^2$$

$$S_{ew} = 1.0230(3)$$

(Sirlin 82')

	$\delta_{SU(2)}^K$ (%)	3-body $\leftarrow \delta_{em}^{Kl}$ (%) \rightarrow Full	
K_{e3}^+	2.31 ± 0.22	-0.35 ± 0.16	-0.10 ± 0.16
K_{e3}^0	0	$+0.30 \pm 0.10$	$+0.55 \pm 0.10$
$K_{\mu 3}^+$	2.31 ± 0.22	-0.05 ± 0.20	$+0.20 \pm 0.20$
$K_{\mu 3}^0$	0	$+0.55 \pm 0.20$	$+0.80 \pm 0.20$

Accurately known

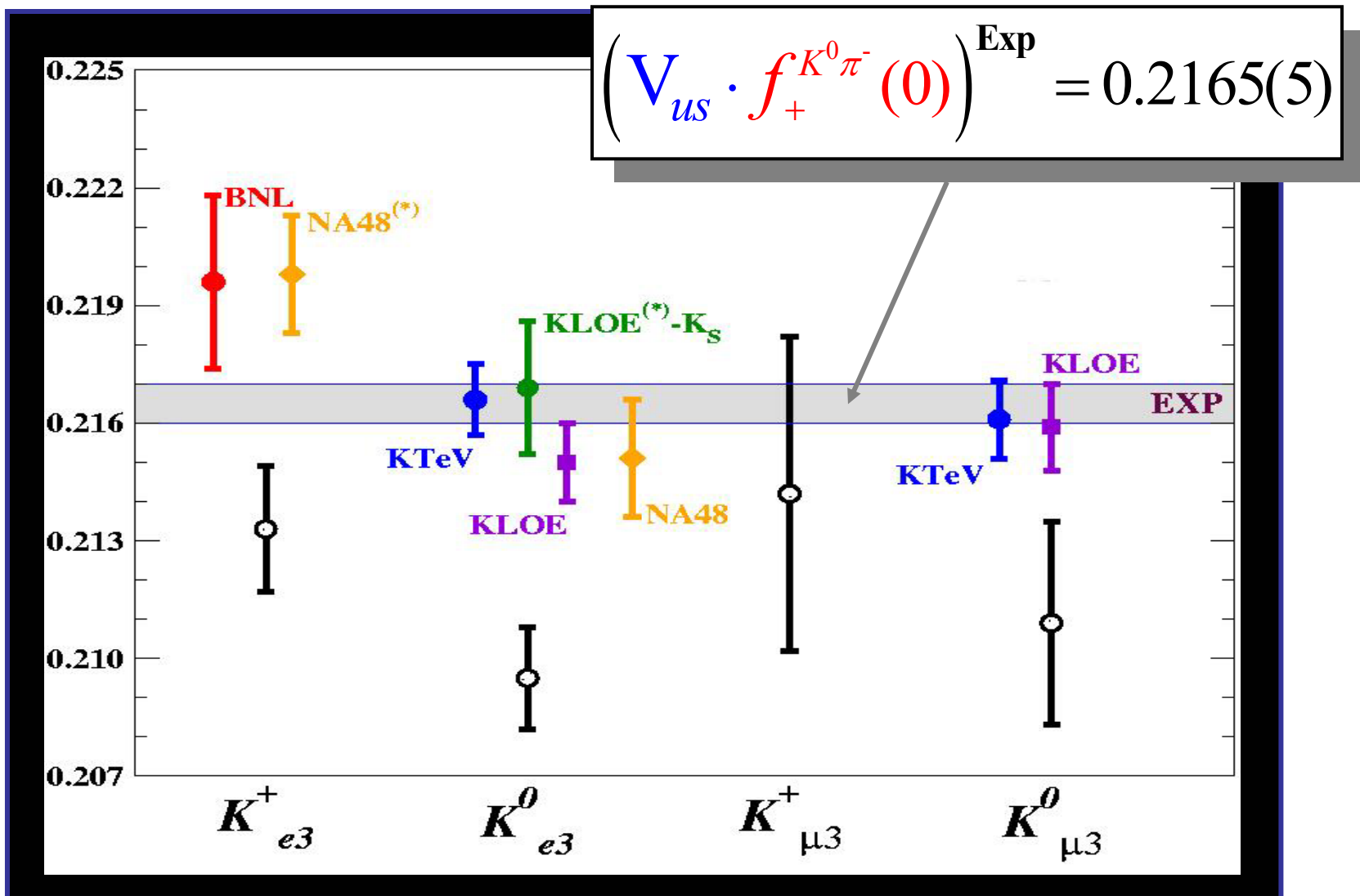
$$\frac{\Delta |V_{us}|}{|V_{us}|} \cong 0.2\%$$

$$f_+^{K^+ \pi^0}(0) = 1.023 f_+^{K^0 \pi^-}(0)$$

(Andre 04', Cirigliano 04'-02')

$$I_K(\lambda_+, \lambda_0) \text{ Accurately measured (ISTRA+, KTeV, NA48) } \quad \Delta |V_{us}| / |V_{us}| \cong 0.3\%$$

Then ... just using inputs from Br , I_K and δ the quantity $(|V_{us}| \cdot f_+^{K^0 \pi^-}(0))$ can be measured with small th. and exp. uncertainties.



Now to estimate V_{us} we need $f_+(0) \equiv f_+^{K^0\pi^-}(0)$

$$\Delta |V_{us}| / |V_{us}| \cong 1\%$$

$f_+(0)$ and The Ademollo-Gatto Theorem

$\chi p T$

$$f_+(0) = 1 + f_2 + f_4 + O(p^8)$$

Vector Current
Conservation
 $m_s = m_u$

-0.023 (NO UNCERTAINTY!)

$O(m_s - m_u)^2$ – AG Theorem
Independent of L_i and μ

DOMINANT UNCERTAINTY !

$O(m_s - m_u)^2$ – AG Theorem
 μ -Independent

MODEL ESTIMATES OF f_4

The PDG-quoted estimate

Leutwyler-Roos (1984), (QUARK MODEL) $f_4 = -0.016 \pm 0.008$

The most recent estimates

Bijnens et al (2003), ($\Delta_{\text{loops}}^{O(p^6)-\chi\text{PT}} + \text{LR}$) $f_4 = -0.001 \pm 0.010$

Jamin et al (2004), ($\Delta_{\text{loops}}^{O(p^6)-\chi\text{PT}} + \text{D. Analysis}$) $f_4 = -0.003 \pm 0.010$

Ambiguity: $\mu = ???$ $\Delta_{\text{loops}}(1\text{GeV}) = 0.004$, $\Delta_{\text{loops}}(M_\rho) = 0.015$, $\Delta_{\text{loops}}(0.5\text{GeV}) = 0.035$

From $O(p^6)$ χPT : Post-Schilcher ('01), Bijnens-Talavera ('03):

$$f_4 = \Delta_{\text{loops}}(\mu) - \frac{8}{F_\pi^4} [C_{12}(\mu) + C_{34}(\mu)] (M_K^2 - M_\pi^2)^2$$

A lattice estimate of f_4 is clearly needed!

!!No Scale Ambiguity!!

f_4 -Lattice QCD Challenge: Our Strategy

D.Becirevic, G.Isidori, V.Lubicz, G.Martinelli,
F.Mescia, S.Simula, C.Tarantino, G.Villadoro.

[NPB 705 (2005) 339, hep-ph/0403217; hep-lat/0411016]

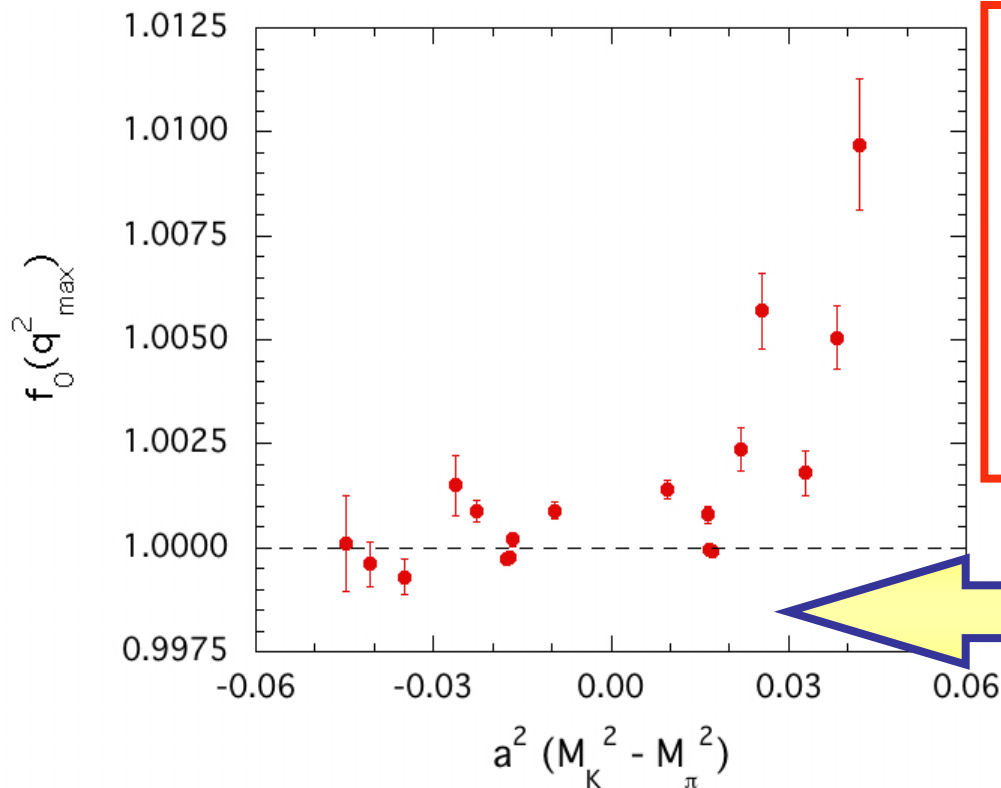
VERY CHALLENGING:

A PRECISION OF $O(1\%)$ MUST BE REACHED ON THE LATTICE !!

1. Evaluation of $f_0[q^2 = (M_K - M_\pi)^2]$ with very high precision ($<1\%$).
2. Extrapolation of $f_0(q^2_{max})$ to $f_0(0)=f_+(0)$ estimating the slope λ_0
3. We consider $\Delta f \equiv f_+(0) - f_2^Q$ (subtraction of the **unphysical** chiral logs) and extrapolate ($m_s/2 \leq m_q \leq m_s$) to the physical meson masses:
Finally, **Δf will be our estimate of f_4 .**

1) $f_0(q^2_{\max})$ - High precision measure (FNAL)

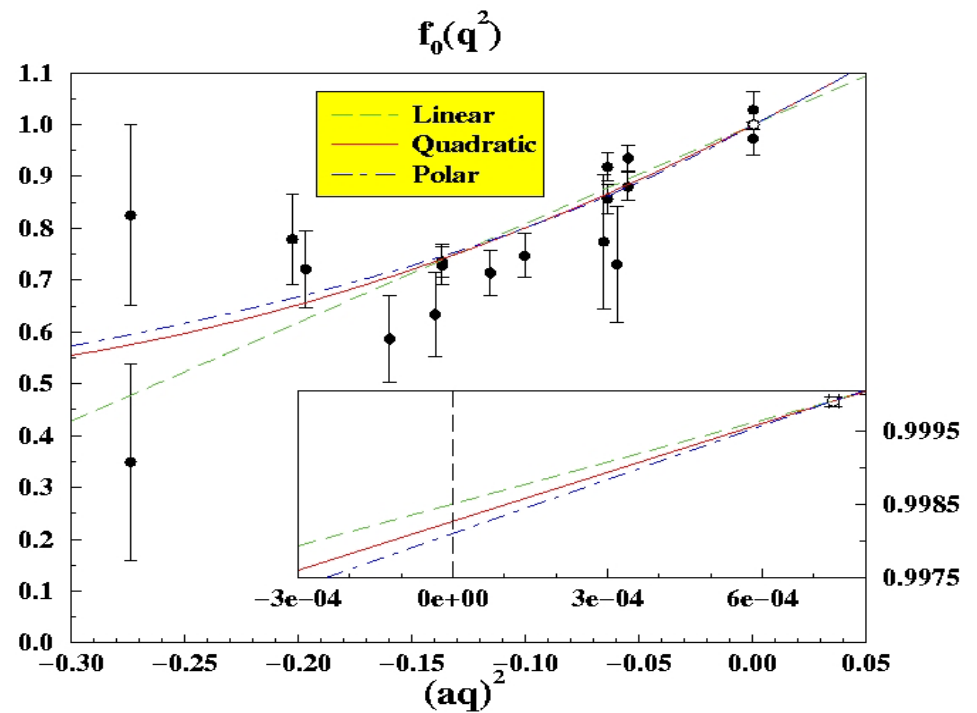
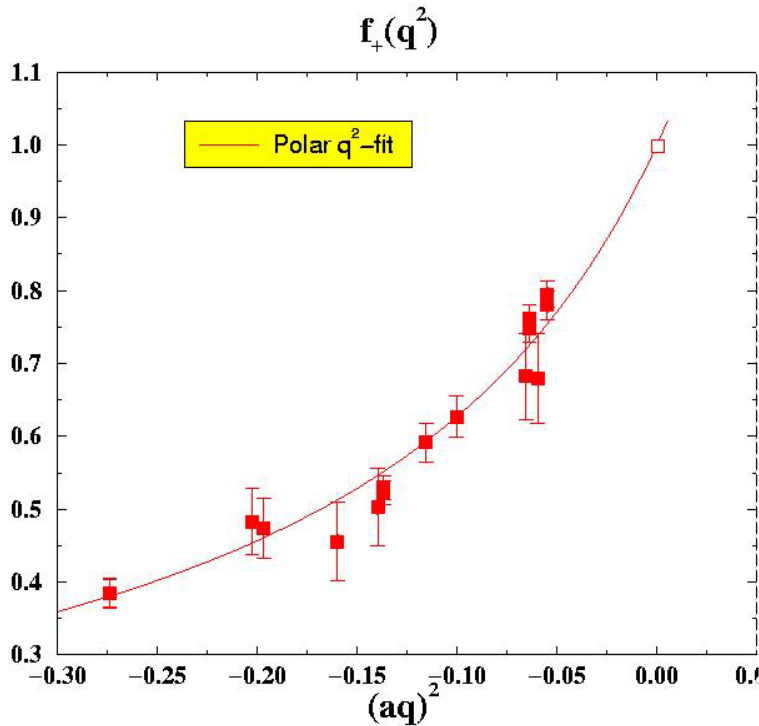
$$R \rightarrow \frac{\langle \pi | \bar{s} \gamma^0 u | K \rangle \cdot \langle K | \bar{u} \gamma^0 s | \pi \rangle}{\langle K | \bar{s} \gamma^0 s | K \rangle \cdot \langle \pi | \bar{u} \gamma^0 u | \pi \rangle} = f_0^2(q^2_{\max}) \frac{(M_K + M_\pi)^2}{4M_K M_\pi}$$



- For $M_K \rightarrow M_\pi$, $R \rightarrow 1 + \mathcal{O}(M_K^2 - M_\pi^2)^2$
- Stat. and Syst. errors scale as $(M_K^2 - M_\pi^2)^2$, like the physical SU(3) breaking effects.
- Independent of Z_V and b_V

Stat. errors well below 1%

2) Extrapolation of $f_0(q_{MAX})$ to $f_+(0)$



$$f_+(q^2) = \frac{f_+(0)}{1 - \lambda_+ q^2}$$

(Vector Meson Dominance)

$$f_0^{(lin)}(q^2) = f_+(0) \cdot (1 + \lambda_0^{(lin)} \cdot q^2)$$

$$f_0^{(quad)}(q^2) = f_+(0) \cdot (1 + \lambda_0^{(quad)} \cdot q^2 + c_0 \cdot q^2)$$

$$f_0^{(pol)}(q^2) = \frac{f_+(0)}{1 - \lambda_0^{(pol)} q^2}$$

Systematic uncertainty for the determination of $f_+(0)$

Check of the theory: form factor vs experiment

The theoretical estimate of $f_+(0)$ can be checked comparing $f_+(q^2)$ vs experiment (Bijnens-Talavera ('03)):

<u>Polar fit:</u>	Our (March'04)	KTeV (June'04)
	$\lambda_+ = 0.025(2)$	$\lambda_+ = 0.0250(4)$
	$\lambda_0 = 0.012(2)$	$\lambda_0 = 0.0141(10)$

REAL
PREDICTION

good agreement between data and theory

Chiral-QCD also predicts similar values.

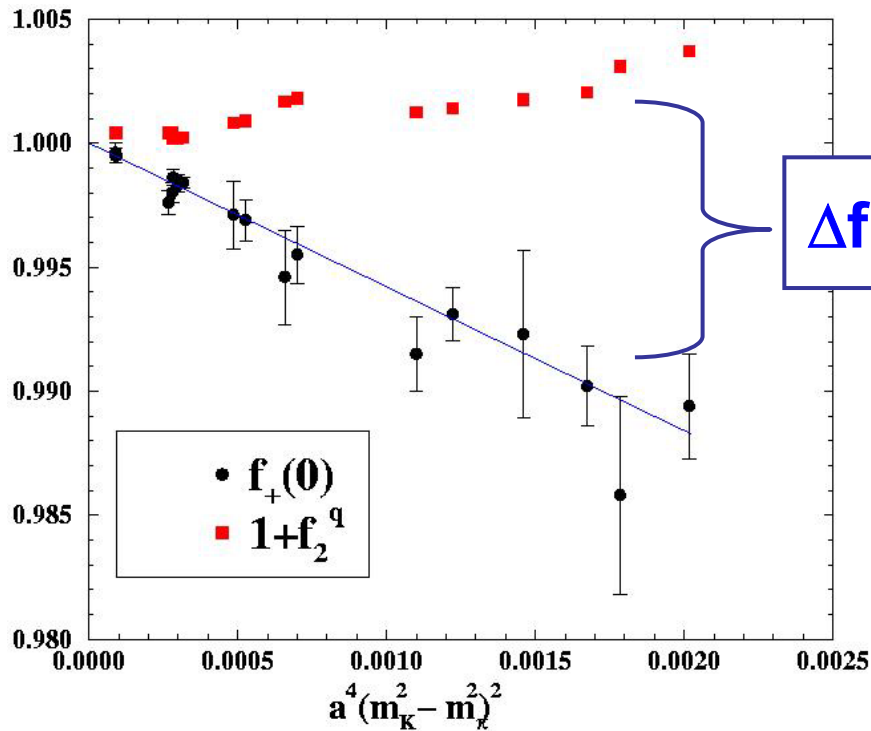
High precision measurements

ISTRA+ (hep-ex/0404030- K^+_{e3}): Polar fit not available but curvature visible for $f_+(q^2)$.

λ_+ and λ_0 from a linear fit consistent with KTeV.

NA48 (hep-ex/0410065- K^{L+}_{e3}): λ_+ from a linear and a polar fit consistent with KTeV.

3) Δf and subtraction of the chiral logs



- At the simulated masses

$$\Delta f^q \equiv f_+(0) - 1 - f_2^q \neq 0$$

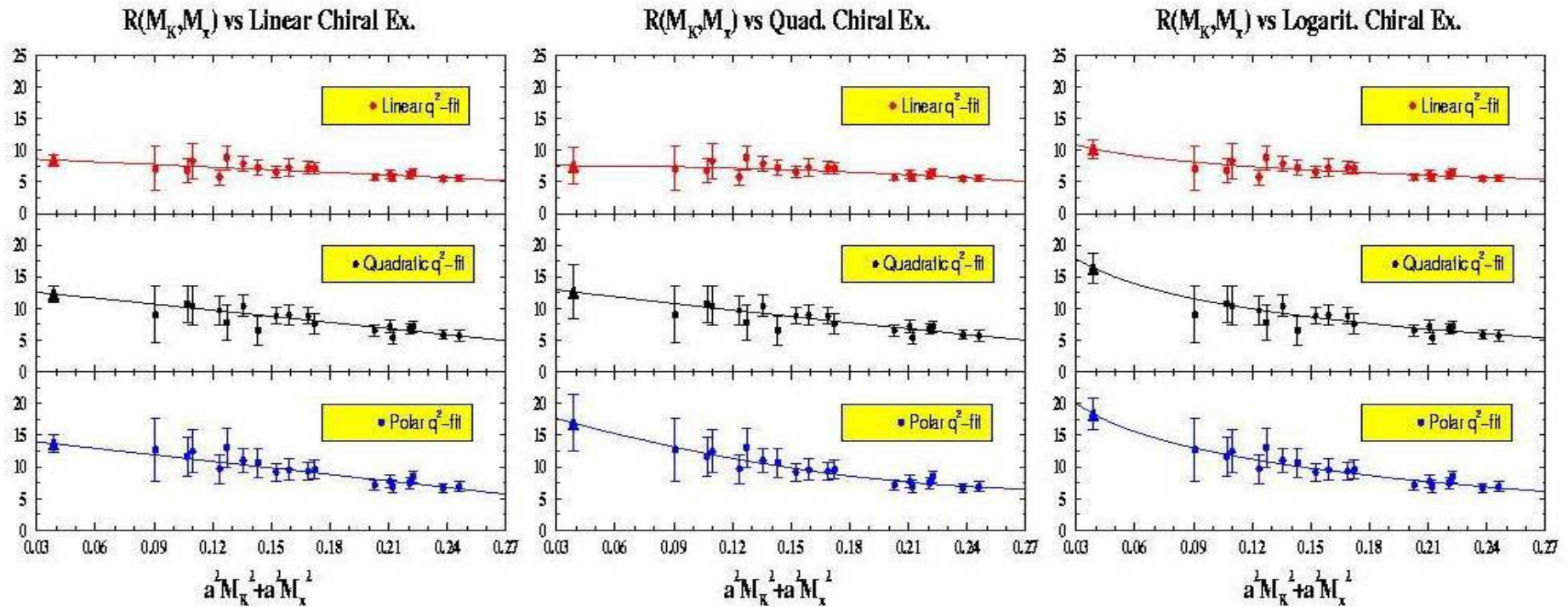
- f_2^q = quenched artefacts:
well defined but non-trivial chiral
behaviour

$$\Delta f^q \propto B(M_K^2 - M_\pi^2)^2 \quad (\text{Ademollo-Gatto})$$

- Scale independent and no leading quenched artefacts
- Hopefully suited for a smooth chiral behaviour

3) Chiral extrapolation

$$R(M_K, M_\pi) \equiv \frac{\Delta f^q}{(M_K^2 - M_K^2)^2} \equiv \frac{f_+(0) - 1 - f_2^{quenched}}{(M_K^2 - M_K^2)^2}$$



Having subtracted the leading log. correction,
several extrapolations are tried

The **final result** is: $\Delta f = -0.017 \pm 0.005_{\text{stat}} \pm 0.007_{\text{syst}}$
[Leutwyler and Roos (PDG2002): $\Delta f^{\text{LR}} = -0.016 \pm 0.008$]


$$f_+^{K^0\pi^-}(0) = 0.960 \pm 0.005_{\text{stat}} \pm 0.007_{\text{syst}}$$

+ quenching
error at $O(p^6)$!

Systematic error: The dominant contributions come from the uncertainties on the q^2 and **mass dependencies** of the form factor

D. Becirevic, G. Isidori, V. Lubicz, G. Martinelli, F. Mescia, S. Simula,
C. Tarantino, G. Villadoro. [NPB 705 (2005) 339]

Two preliminary unquenched estimates have been recently presented

$$\Delta f = -0.015 \pm 0.006_{\text{stat}} \pm 0.009_{\text{syst}} \quad \text{MILC- [Dec-04] } N_F=2+1$$

our strategy (good!) but staggered fermions (less good!!)

$$\Delta f = -0.023 \pm 0.009_{\text{syst}} \quad \text{JLQCD- [Mach-05] } N_F=2$$

$$f_+^{K^0\pi^-}(0) = 0.960 \pm 0.005_{\text{stat}} \pm 0.007_{\text{syst}}$$

$$V_{us} \cdot f_+^{K^0\pi^-} - \text{CKM 2005}$$

$$\left(V_{us} \cdot f_+^{K^0\pi^-} \right)^{\text{Exp}} = 0.2165(5)$$

•CKM-Unitarity recovered:

$$|V_{us}|^2 + |V_{ud}|^2 + |V_{ub}|^2 = 0.9993(11)$$

- $|V_{ud}| = 0.9739(5)$

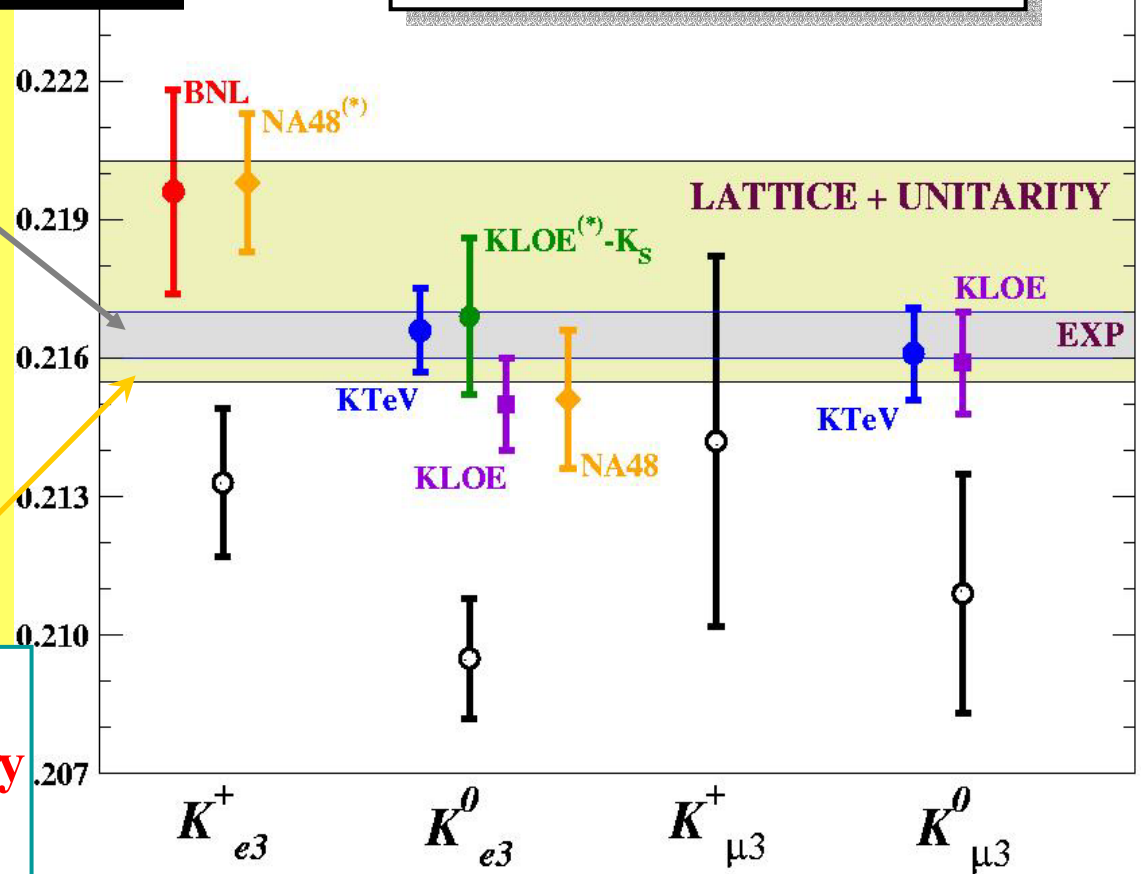
(Updated average: CKM 2005)

$$V_{us}^{\text{uni}} \cdot f_+^{\text{four-LR}} = 0.2179(24)$$

$$V_{us}^{\text{uni}} \cdot f_+^{\text{Bijens-Jamin}} = 0.2215(26)$$

Theory

↖ 2σ - higher ↗



$$\tau_{K^+}^{\text{PDG}} = 1.2384(24) \cdot 10^{-8} \text{ s}, \tau_L^{\text{PDG}} = 5.15(4) \cdot 10^{-8} \text{ s}, \tau_S^{\text{PDG}} = 8.935(8) \cdot 10^{-11} \text{ s}$$

$$|V_{us}|^{K_{l3}} = (0.2253 \pm 0.0020) \quad \longrightarrow \quad \delta|V_{us}| \sim 1\%$$

(dominated by the $f_+(0)$ theoretical uncertainty)


Comments on other routes to V_{us}

$$|V_{us}|^{\text{Uni}} = (0.2269 \pm 0.0013) \quad |V_{us}|^{\text{K13}} = (0.2253 \pm 0.0020)$$

$$\frac{\Gamma(K \rightarrow \mu \bar{\nu}_\mu (\gamma))}{\Gamma(\pi \rightarrow \mu \bar{\nu}_\mu (\gamma))} = \frac{|V_{us}|^2 f_K^2 m_K \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2}{|V_{ud}|^2 f_\pi^2 m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} 0.9930(35) \quad [\text{Rad. Corr.}]$$

◆ $f_K/f_\pi - 1 = 0.210(4)(13)$

(MILC hep-lat/0407028)

Unquenched

 Staggered-Fermions

$|V_{us}| = (0.2219 \pm 0.0026)$

(Marciano hep-ph/0406324)

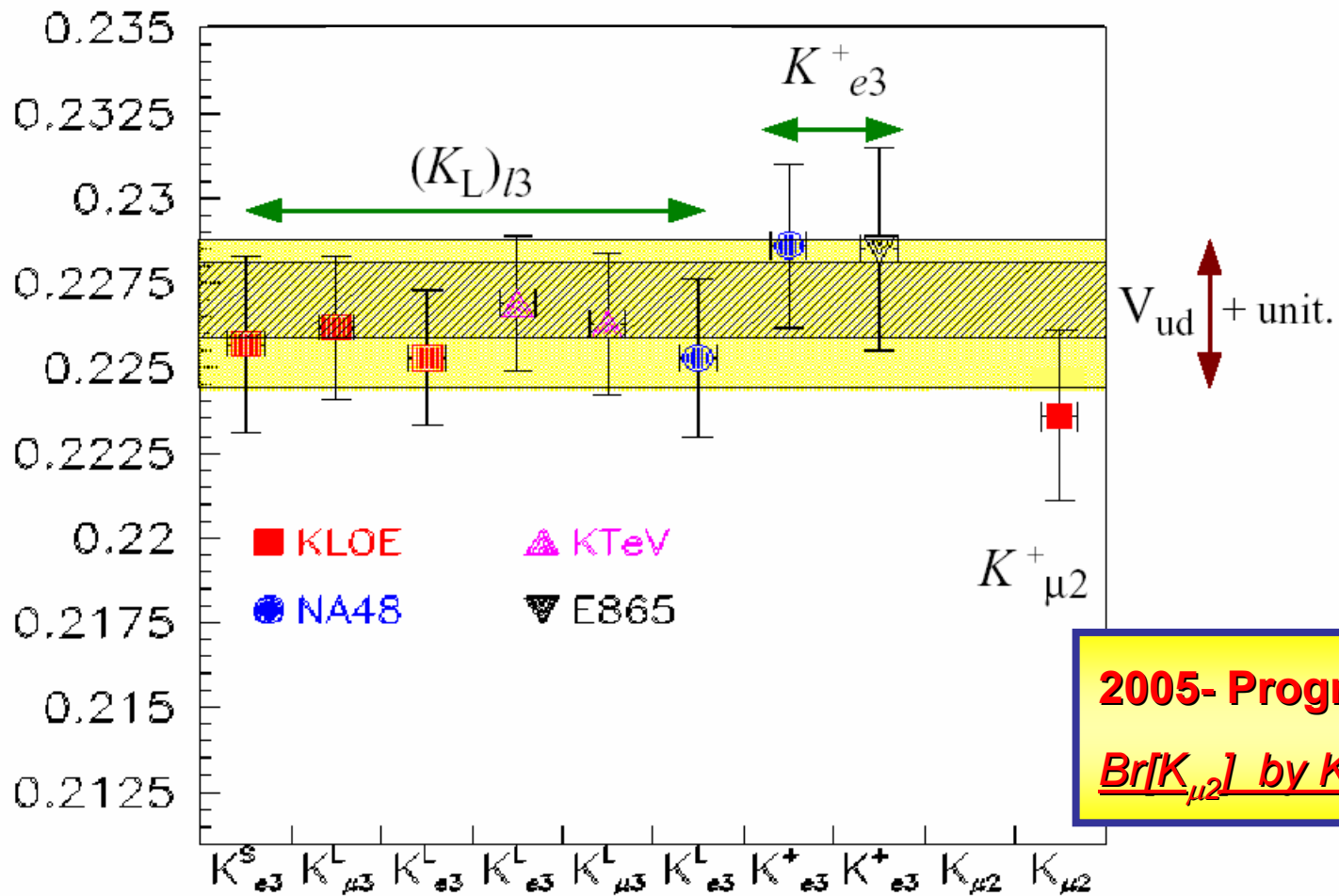
Charming: Unquenched & 6.5% uncertainty

Disappointing: not better, $\delta|V_{us}| \sim 1.2\%$

The dominant source of systematic error comes from the lattice calculation. VERY DIFFICULT TO REDUCE !!

Summary of V_{us} from from K decays :

3rd CKM Workshop
 [March 2005, San Diego]



V_{us} -CONCLUSIONS

- ◆ K_{β} decays offer a good opportunity to estimate V_{us} and test the CKM relation, thanks to Ademollo-Gatto Theorem.
- ◆ Over the years, a great deal of activity has been devoted to reach higher precision and to reduce model-assumptions.
- ◆ We have presented a **methodology** to reach **1% accuracy** for $f_+(0)$
- ◆ Our calculation of $f_+(0)$ is the **first one** obtained by using a non-perturbative method **based only on QCD**, albeit in the quenched approximation (which can be in principle removed)
- ◆ Our **final result**, $f_+(0) = 0.960 \pm 0.005_{\text{stat}} \pm 0.007_{\text{syst}}$ is in **good agreement with** the estimate made by **Leutwyler and Roos** (PDG)
- ◆ The most important step is to remove the **quenched approximation**
- ◆ Further steps: using lower masses (considering finite volume effects)

KAON PHYSICS HAS REPRESENTED
SO FAR A HUGE SOURCE OF
INFORMATION FOR PARTICLE PHYSICS.
NEW IMPORTANT RESULTS ARE
EXPECTED IN THE FUTURE.

Grazie mille e Buon Lavoro

BACKUP

TH very clean

: $\left(\begin{array}{l} \text{With improved} \\ \text{CKM parameters} \\ \sim 2008 \end{array} \right) \rightarrow$

$$\begin{array}{l} \sigma(\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})) < 5\% \\ \sigma(\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})) < 5\% \end{array}$$

Very clean
determination
of Unitarity
Triangle

$$\sigma(\text{Br}) \cong 10\%$$

$$\sigma(\text{Br}) \cong 5\%$$

$$\begin{array}{l} \sigma(\sin 2\beta \cong 0.04) \mid \sigma(\gamma) = 9^\circ \mid \sigma(|V_{td}|) = 7\% \\ \sigma(\sin 2\beta \cong 0.025) \mid \sigma(\gamma) = 5^\circ \mid \sigma(|V_{td}|) = 4\% \end{array}$$

by Buras CKM05