

# **Seesaw**

**and**

# **Leptogenesis**

Pasquale Di Bari  
(Max Planck, Munich)

## **Different interests for Leptogenesis:**

## Different interests for Leptogenesis:

- as a **model of baryogenesis** i.e. a way to explain, with a dynamical mechanism, why we live in a **baryon asymmetric universe** (traditional one)

## Different interests for Leptogenesis:

- as a **model of baryogenesis** i.e. a way to explain, with a dynamical mechanism, why we live in a **baryon asymmetric universe** (traditional one)  
from **CMB acoustic peaks (WMAP) + large scale structure (SLOAN)**

$$\eta_B^{CMB} = \frac{n_B - n_{\bar{B}}}{n_\gamma} \Big|_{t_{\text{rec}}} = (6.3 \pm 0.3) \times 10^{-10}$$

## Different interests for Leptogenesis:

- as a **model of baryogenesis** i.e. a way to explain, with a dynamical mechanism, why we live in a **baryon asymmetric universe** (traditional one) from **CMB acoustic peaks (WMAP) + large scale structure (SLOAN)**

$$\eta_B^{CMB} = \frac{n_B - n_{\bar{B}}}{n_\gamma} \Big|_{t_{\text{rec}}} = (6.3 \pm 0.3) \times 10^{-10}$$

- as a **phenomenology of the seesaw mechanism** (more modern one):
  - neutrino mixing;
  - $\beta\beta 0\nu$  decays;
  - $CP$  violation in neutrino mixing;
  - ...
  - **Leptogenesis**

## Different interests for Leptogenesis:

- as a **model of baryogenesis** i.e. a way to explain, with a dynamical mechanism, why we live in a **baryon asymmetric universe** (traditional one) from **CMB acoustic peaks (WMAP) + large scale structure (SLOAN)**

$$\eta_B^{CMB} = \frac{n_B - n_{\bar{B}}}{n_\gamma} \Big|_{t_{\text{rec}}} = (6.3 \pm 0.3) \times 10^{-10}$$

- as a **phenomenology of the seesaw mechanism** (more modern one):
  - neutrino mixing;
  - $\beta\beta 0\nu$  decays;
  - $CP$  violation in neutrino mixing;
  - ...
  - **Leptogenesis**
- as a **probe of very early Universe history** and ‘new’ physics, like BBN (future ?)

# **Baryon asymmetry of the Universe**

## Baryon asymmetry of the Universe

- CMB + cosmic rays exclude a baryon symmetric universe with matter- anti matter domains, on scales as large as the whole horizon

(Cohen, De Rujula, Glashow, '98)



## Baryon asymmetry of the Universe

- CMB + cosmic rays exclude a baryon symmetric universe with matter- anti matter domains, on scales as large as the whole horizon

(Cohen, De Rujula, Glashow, '98)

- from CMB acoustic peaks (WMAP) + large scale structure (SLOAN):

$$\eta_B^{CMB} = (6.3 \pm 0.3) \times 10^{-10}$$

(Tegmark et al. 2003)

## Baryon asymmetry of the Universe

- CMB + cosmic rays exclude a baryon symmetric universe with matter- anti matter domains, on scales as large as the whole horizon

(Cohen, De Rujula, Glashow, '98)

- from CMB acoustic peaks (WMAP) + large scale structure (SLOAN):

$$\eta_B^{CMB} = (6.3 \pm 0.3) \times 10^{-10}$$

(Tegmark et al. 2003)

- ... in very good agreement with the determination from SBBN + primordial Deuterium measurements :

$$\eta_B^{SBBN} = (6.1 \pm 0.5) \times 10^{-10}$$

(Cyburt et al. 2001, Kirkman et al. 2003)

# **Models of Baryogenesis**

## **Models of Baryogenesis**

- at the Planck scale

## Models of Baryogenesis

- at the Planck scale
- from phase transitions
  - Electroweak Baryogenesis
    - \* in the Standard Model
    - \* in the MSSM
    - \* . . .
  - . . .

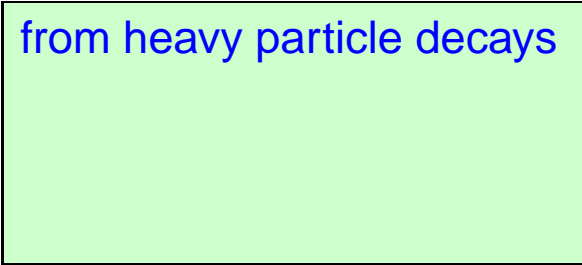
## Models of Baryogenesis

- at the Planck scale
- from phase transitions
  - Electroweak Baryogenesis
    - \* in the Standard Model
    - \* in the MSSM
    - \* . . .
  - . . .
- Affleck-Dine
  - at preheating
  - Q-Balls

## Models of Baryogenesis

- at the Planck scale
- from phase transitions
  - Electroweak Baryogenesis
    - \* in the Standard Model
    - \* in the MSSM
    - \* . . .
  - . . .
- Affleck-Dine
  - at preheating
  - Q-Balls
- from black holes evaporation
- spontaneous baryogenesis
- . . .

## Models of Baryogenesis

- at the Planck scale
- from phase transitions
  - Electroweak Baryogenesis
    - \* in the Standard Model
    - \* in the MSSM
    - \* . . .
  - . . .
- Affleck-Dine
  - at preheating
  - Q-Balls
- from black holes evaporation
- spontaneous baryogenesis
- . . .
- 



## Models of Baryogenesis

- at the Planck scale
- from phase transitions
  - Electroweak Baryogenesis
    - \* in the Standard Model
    - \* in the MSSM
    - \* . . .
  - . . .
- Affleck-Dine
  - at preheating
  - Q-Balls
- from black holes evaporation
- spontaneous baryogenesis
- . . .
- - from heavy particle decays
    - GUT baryogenesis

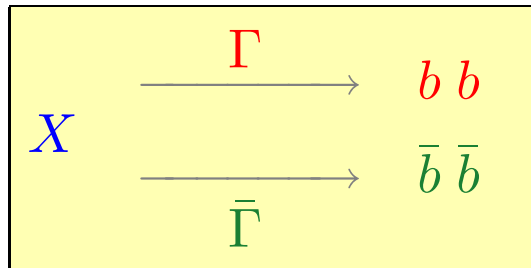
## Models of Baryogenesis

- at the Planck scale
- from phase transitions
  - Electroweak Baryogenesis
    - \* in the Standard Model
    - \* in the MSSM
    - \* . . .
  - . . .
- Affleck-Dine
  - at preheating
  - Q-Balls
- from black holes evaporation
- spontaneous baryogenesis
- . . .
- from heavy particle decays
  - GUT baryogenesis
  - Leptogenesis

## **Heavy particle decays**

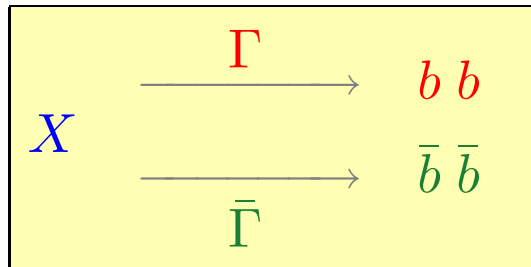
## Heavy particle decays

1. A baryogenesis toy model (Kolb, Turner)

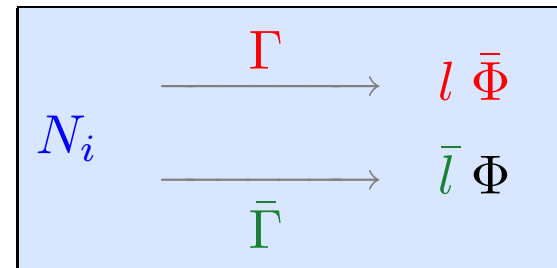


## Heavy particle decays

1. A baryogenesis toy model (Kolb, Turner)

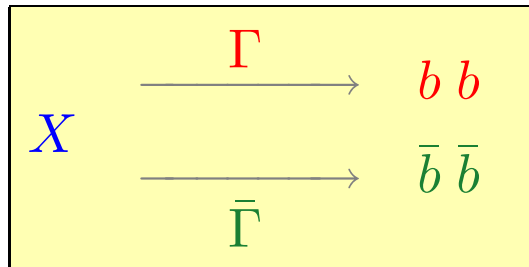


2. Leptogenesis (Fukugita, Yanagida '86)

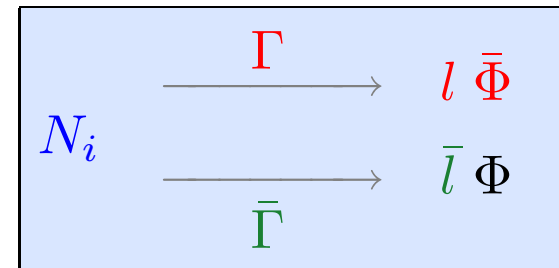


## Heavy particle decays

1. A baryogenesis toy model (Kolb, Turner)



2. Leptogenesis (Fukugita, Yanagida '86)



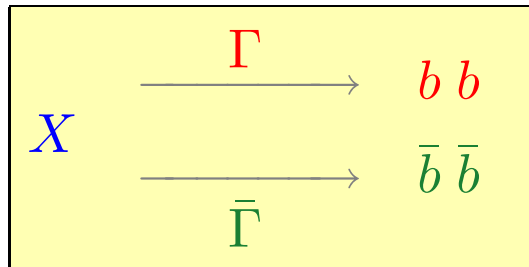
- sphaleron conversion (effective for  $10^{12} \text{ GeV} \gtrsim T \gtrsim 100 \text{ GeV}$ )

$$N_B^f \simeq \frac{1}{3} N_{B-L}^f \simeq -\frac{1}{2} N_L^f$$

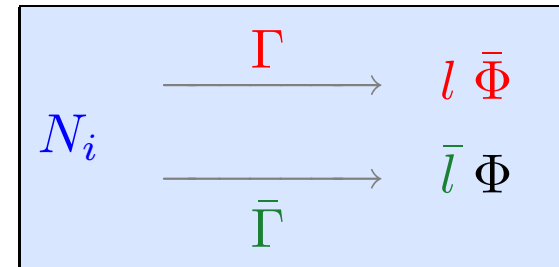
(Kuzmin, Rubakov, Shaposhnikov '85; Khlebnikov, Shaposhnikov '88; Harvey, Turner '90):

## Heavy particle decays

1. A baryogenesis toy model (Kolb, Turner)



2. Leptogenesis (Fukugita, Yanagida '86)



- sphaleron conversion (effective for  $10^{12} \text{ GeV} \gtrsim T \gtrsim 100 \text{ GeV}$ )

$$N_B^f \simeq \frac{1}{3} N_{B-L}^f \simeq -\frac{1}{2} N_L^f$$

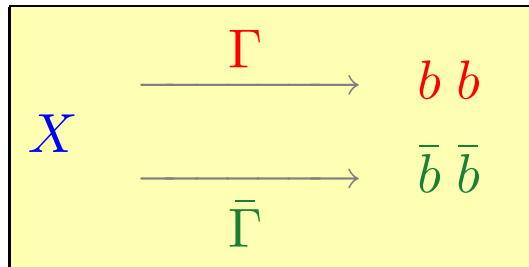
(Kuzmin, Rubakov, Shaposhnikov '85; Khlebnikov, Shaposhnikov '88; Harvey, Turner '90):

- CP asymmetry parameter:

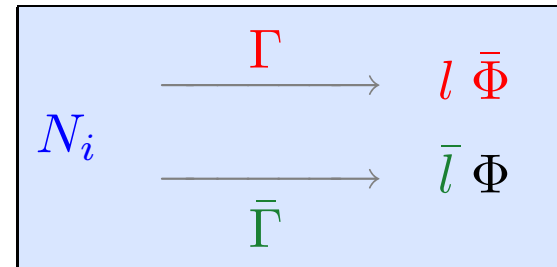
$$\varepsilon = \pm \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} > 0$$

## Heavy particle decays

1. A baryogenesis toy model (Kolb,Turner)



2. Leptogenesis (Fukugita,Yanagida '86)



- sphaleron conversion (effective for  $10^{12} \text{ GeV} \gtrsim T \gtrsim 100 \text{ GeV}$ )

$$N_B^f \simeq \frac{1}{3} N_{B-L}^f \simeq -\frac{1}{2} N_L^f$$

(Kuzmin,Rubakov,Shaposhnikov'85;Khlebnikov,Shaposhnikov'88;Harvey,Turner'90):

- CP asymmetry parameter:

$$\varepsilon = \pm \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} > 0$$

- Total decay parameter:

$$\Gamma_D = \Gamma + \bar{\Gamma} = \Gamma_D^{\text{rest}} \left\langle \frac{1}{\gamma} \right\rangle$$



## Seesaw $\Rightarrow$ Leptogenesis (Fukugita, Yanagida '86)

$$\mathcal{L}_{\text{mass}}^{\nu} = -\frac{1}{2} \left[ (\bar{\nu}_L^c, \bar{\nu}_R) \begin{pmatrix} m_T & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \right] + h.c.$$

## Seesaw $\Rightarrow$ Leptogenesis (Fukugita, Yanagida '86)

$$\mathcal{L}_{\text{mass}}^{\nu} = -\frac{1}{2} \left[ (\bar{\nu}_L^c, \bar{\nu}_R) \begin{pmatrix} \cancel{m_T} & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \right] + h.c.$$

seesaw formula:  $(M_i \gg m_{D i}) \Rightarrow$   $m_{\nu} = -m_D \frac{1}{M} m_D^T$

## Seesaw $\Rightarrow$ Leptogenesis (Fukugita, Yanagida '86)

$$\mathcal{L}_{\text{mass}}^\nu = -\frac{1}{2} \left[ (\bar{\nu}_L^c, \bar{\nu}_R) \begin{pmatrix} \cancel{m_T} & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \right] + h.c.$$

seesaw formula:

$$(M_i \gg m_{D i}) \Rightarrow$$

$$m_\nu = -m_D \frac{1}{M} m_D^T$$

$m_D, m_\nu$  and  $M$  are complex matrices  $\Rightarrow$  natural source of  $CP$  violation

- 3 new heavy RH neutrinos  $N_1, N_2, N_3$  with masses  $M_{\text{ew}} \ll M_1 \leq M_2 \leq M_3$

## Seesaw $\Rightarrow$ Leptogenesis (Fukugita, Yanagida '86)

$$\mathcal{L}_{\text{mass}}^{\nu} = -\frac{1}{2} \left[ (\bar{\nu}_L^c, \bar{\nu}_R) \begin{pmatrix} \cancel{m_T} & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \right] + h.c.$$

seesaw formula:  $(M_i \gg m_{D i}) \Rightarrow$   $m_{\nu} = -m_D \frac{1}{M} m_D^T$

$m_D, m_{\nu}$  and  $M$  are complex matrices  $\Rightarrow$  natural source of  $CP$  violation

- 3 new heavy RH neutrinos  $N_1, N_2, N_3$  with masses  $M_{\text{ew}} \ll M_1 \leq M_2 \leq M_3$
- approximation: only the decays of the  $N_1$ 's contribute to the final asymmetry
  - it typically works well for  $M_2 \gtrsim 2 M_1$  but with some interesting exception !
  - within this approximation the final asymmetry depends only on a few seesaw parameters

## **Out of equilibrium decays**

## Out of equilibrium decays

$$z = \frac{M_{N_1}}{T}, \quad N = n R^3, \quad D = \frac{\Gamma_D}{H z}$$

## Out of equilibrium decays

$$z = \frac{M_{N_1}}{T}, \quad N = n R^3, \quad D = \frac{\Gamma_D}{H z}$$

$$\frac{dN_{N_1}}{dz} = -D(z) N_{N_1}(z)$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon \frac{dN_{N_1}}{dz}$$

- Normalization:  $N_{N_1}^{\text{eq}}(z \ll 1) = 1$

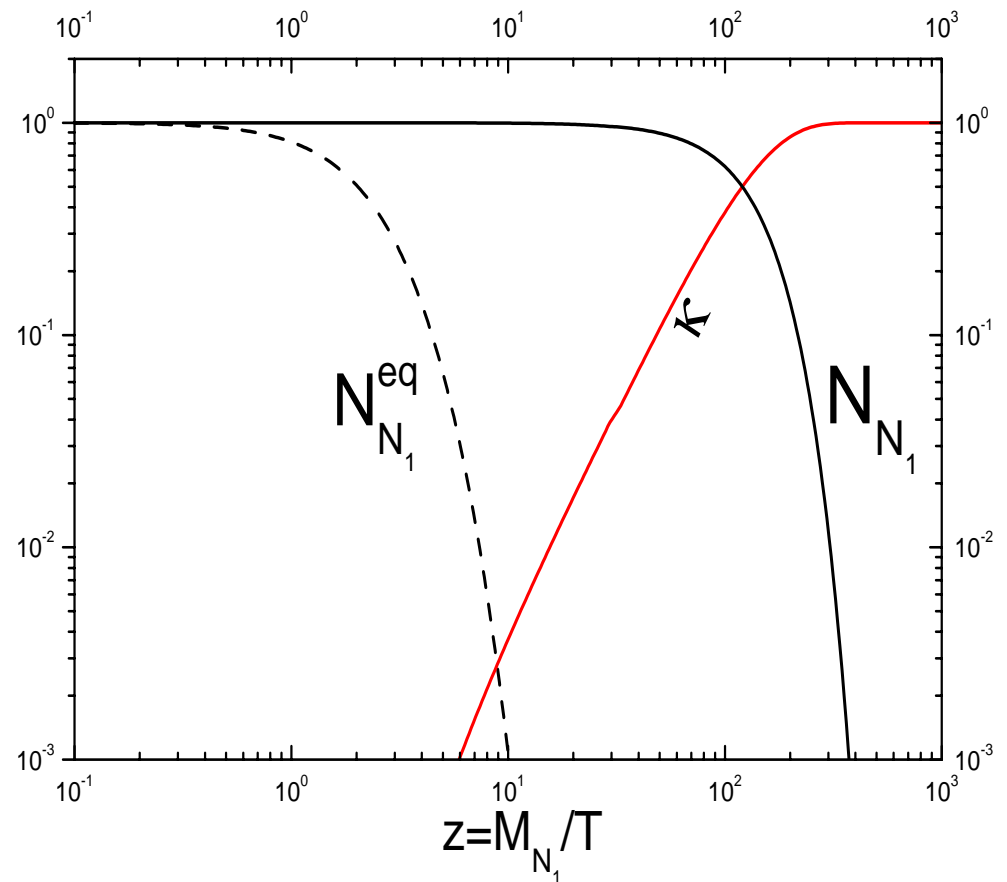
## Out of equilibrium decays

$$z = \frac{M_{N_1}}{T}, \quad N = n R^3, \quad D = \frac{\Gamma_D}{H z}$$

$$\frac{dN_{N_1}}{dz} = -D(z) N_{N_1}(z)$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon \frac{dN_{N_1}}{dz}$$

- Normalization:  $N_{N_1}^{\text{eq}}(z \ll 1) = 1$





## Out of equilibrium decays

$$z = \frac{M_{N_1}}{T}, \quad N = n R^3, \quad D = \frac{\Gamma_D}{H z}$$

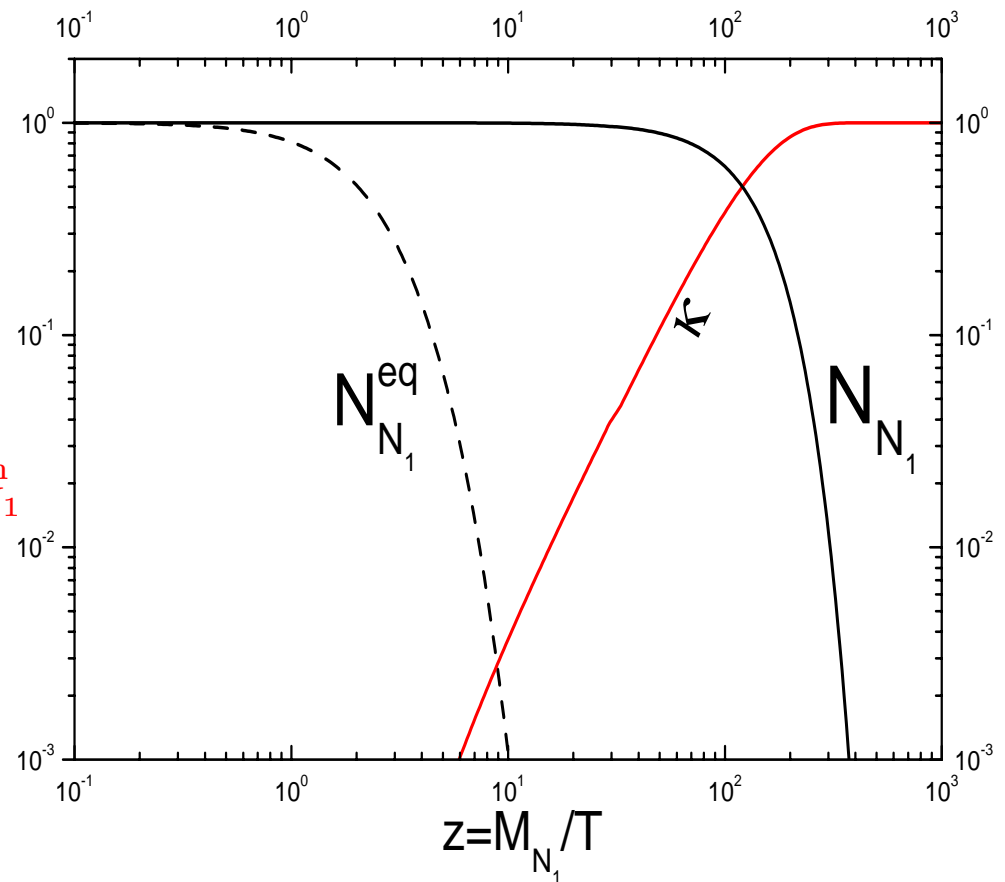
$$\frac{dN_{N_1}}{dz} = -D(z) N_{N_1}(z)$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon \frac{dN_{N_1}}{dz}$$

- Normalization:  $N_{N_1}^{\text{eq}}(z \ll 1) = 1$

- Efficiency factor:

$$\kappa(z) \equiv \frac{N_{B-L}(z) - N_{B-L}^{\text{in}}}{\varepsilon} \Rightarrow \kappa_f = N_{N_1}^{\text{in}}$$



## Out of equilibrium decays

$$z = \frac{M_{N_1}}{T}, \quad N = n R^3, \quad D = \frac{\Gamma_D}{H z}$$

$$\frac{dN_{N_1}}{dz} = -D(z) N_{N_1}(z)$$

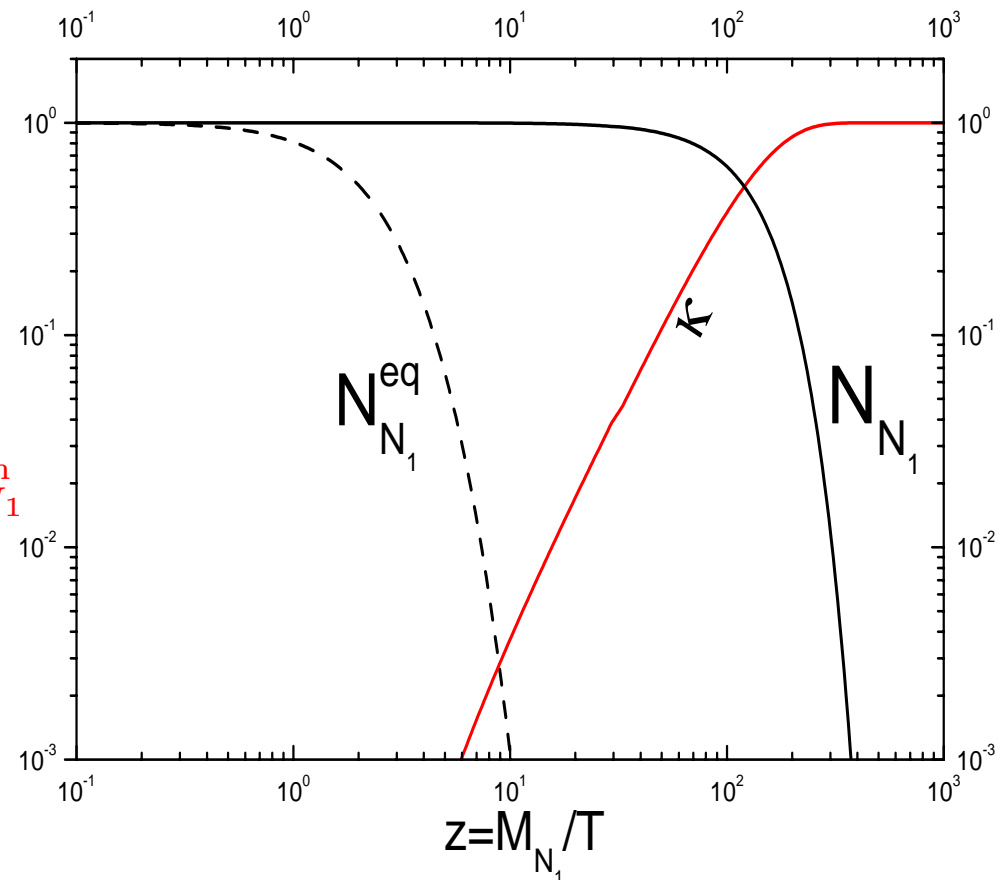
$$\frac{dN_{B-L}}{dz} = -\varepsilon \frac{dN_{N_1}}{dz}$$

- Normalization:  $N_{N_1}^{\text{eq}}(z \ll 1) = 1$

- Efficiency factor:

$$\kappa(z) \equiv \frac{N_{B-L}(z) - N_{B-L}^{\text{in}}}{\varepsilon} \Rightarrow \kappa_f = N_{N_1}^{\text{in}}$$

- $N_{B-L}^{\text{fin}} = N_{B-L}^{\text{in}} + \varepsilon N_{N_1}^{\text{in}}$



## Out of equilibrium decays

$$z = \frac{M_{N_1}}{T}, \quad N = n R^3, \quad D = \frac{\Gamma_D}{H z}$$

$$\frac{dN_{N_1}}{dz} = -D(z) N_{N_1}(z)$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon \frac{dN_{N_1}}{dz}$$

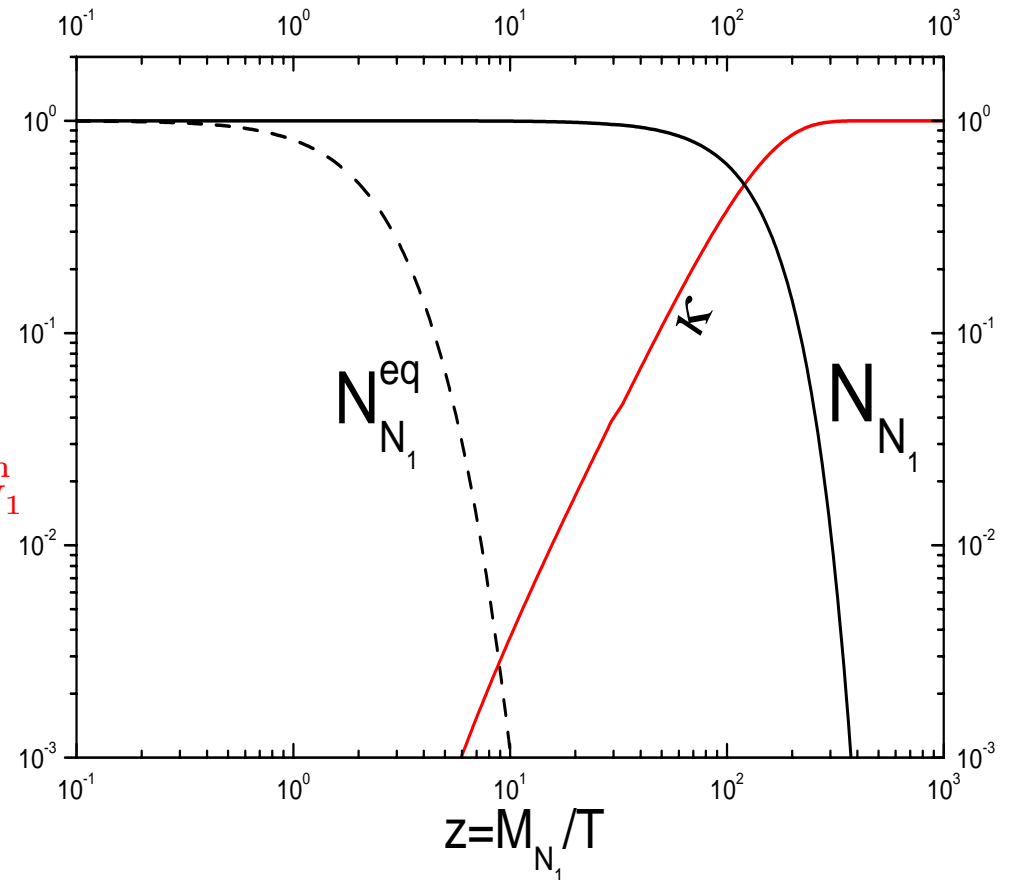
- Normalization:  $N_{N_1}^{\text{eq}}(z \ll 1) = 1$

- Efficiency factor:

$$\kappa(z) \equiv \frac{N_{B-L}(z) - N_{B-L}^{\text{in}}}{\varepsilon} \Rightarrow \kappa_f = N_{N_1}^{\text{in}}$$

- $N_{B-L}^{\text{fin}} = N_{B-L}^{\text{in}} + \varepsilon N_{N_1}^{\text{in}}$

$$\eta_B^f \simeq \frac{1}{3} \frac{N_{B-L}^{\text{fin}}}{N_\gamma^{\text{rec}}} \simeq 10^{-2} \varepsilon \kappa_f$$



## Out of equilibrium decays

$$z = \frac{M_{N_1}}{T}, \quad N = n R^3, \quad D = \frac{\Gamma_D}{H z}$$

$$\frac{dN_{N_1}}{dz} = -D(z) N_{N_1}(z)$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon \frac{dN_{N_1}}{dz}$$

- Normalization:  $N_{N_1}^{\text{eq}}(z \ll 1) = 1$

- Efficiency factor:

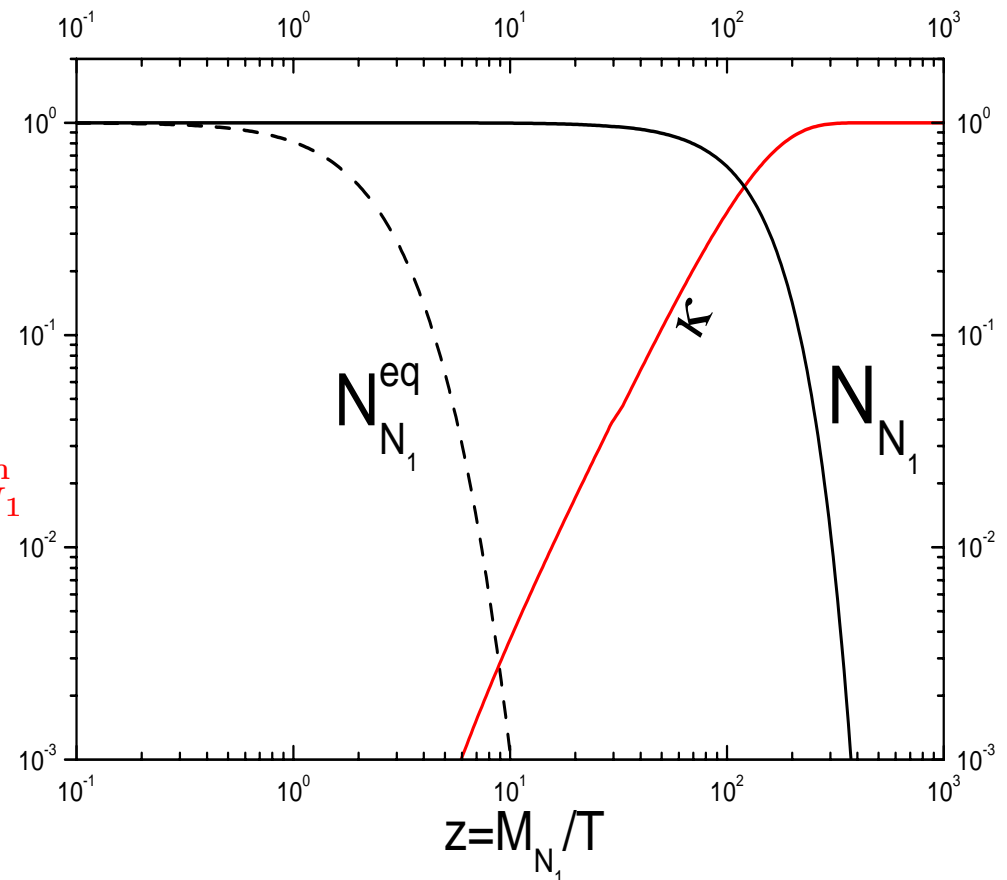
$$\kappa(z) \equiv \frac{N_{B-L}(z) - N_{B-L}^{\text{in}}}{\varepsilon} \Rightarrow \kappa_f = N_{N_1}^{\text{in}}$$

- $N_{B-L}^{\text{fin}} = N_{B-L}^{\text{in}} + \varepsilon N_{N_1}^{\text{in}}$

$$\eta_B^f \simeq \frac{1}{3} \frac{N_{B-L}^{\text{fin}}}{N_\gamma^{\text{rec}}} \simeq 10^{-2} \varepsilon \kappa_f$$

- Decay parameter

$$K = \frac{\Gamma_D^{\text{rest}}}{H|_{z=1}} = \frac{2 t_U(z=1)}{\tau_{N_1}}$$



## Out of equilibrium decays

$$z = \frac{M_{N_1}}{T}, \quad N = n R^3, \quad D = \frac{\Gamma_D}{H z}$$

$$\frac{dN_{N_1}}{dz} = -D(z) N_{N_1}(z)$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon \frac{dN_{N_1}}{dz}$$

- Normalization:  $N_{N_1}^{\text{eq}}(z \ll 1) = 1$

- Efficiency factor:

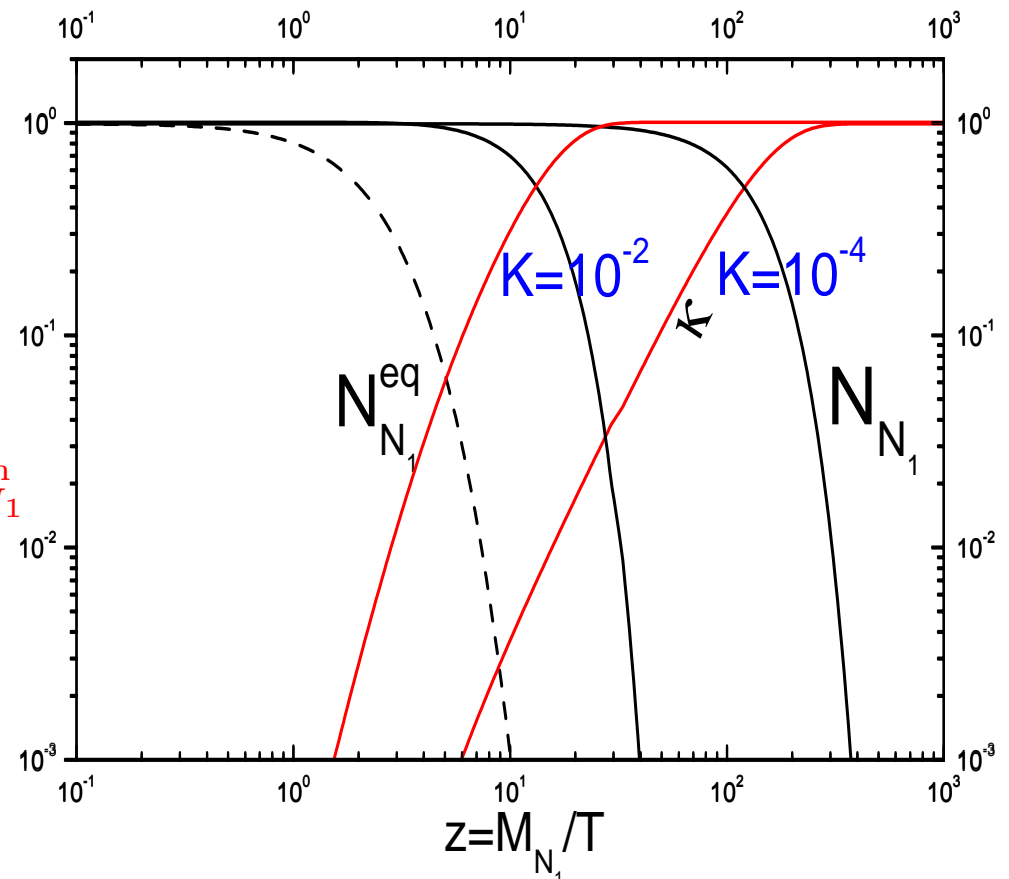
$$\kappa(z) \equiv \frac{N_{B-L}(z) - N_{B-L}^{\text{in}}}{\varepsilon} \Rightarrow \kappa_f = N_{N_1}^{\text{in}}$$

- $N_{B-L}^{\text{fin}} = N_{B-L}^{\text{in}} + \varepsilon N_{N_1}^{\text{in}}$

$$\eta_B^f \simeq \frac{1}{3} \frac{N_{B-L}^{\text{fin}}}{N_\gamma^{\text{rec}}} \simeq 10^{-2} \varepsilon \kappa_f$$

- Decay parameter

$$K = \frac{\Gamma_D^{\text{rest}}}{H|_{z=1}} = \frac{2 t_U(z=1)}{\tau_{N_1}}$$



## Decays and Inverse Decays

$$\begin{aligned} \frac{dN_{N_1}}{dz} &= -D (N_{N_1} - N_{N_1}^{\text{eq}}) \\ \frac{dN_{B-L}}{dz} &= -\varepsilon \frac{dN_{N_1}}{dz} - W_{ID} N_{B-L} \end{aligned}$$

$$D = \frac{\Gamma_D}{H z} = K z \left\langle \frac{1}{\gamma} \right\rangle, \quad W_{ID} = \frac{1}{2} \frac{N_{N_1}^{\text{eq}}}{N_{b,l}^{\text{eq}}} D \propto K$$

## Decays and Inverse Decays

$$\begin{aligned} \frac{dN_{N_1}}{dz} &= -D (N_{N_1} - N_{N_1}^{\text{eq}}) \\ \frac{dN_{B-L}}{dz} &= -\varepsilon \frac{dN_{N_1}}{dz} - W_{ID} N_{B-L} \end{aligned}$$

$$D = \frac{\Gamma_D}{H z} = K z \left\langle \frac{1}{\gamma} \right\rangle, \quad W_{ID} = \frac{1}{2} \frac{N_{N_1}^{\text{eq}}}{N_{b,l}^{\text{eq}}} D \propto K$$

$$N_{B-L}(z; K, z_i) = N_{B-L}^{\text{in}} e^{-\int_{z_i}^z dz' W_{ID}(z')} + \varepsilon \kappa(z)$$

$$\kappa(z; K, z_i) = - \int_{z_i}^z dz' \left[ \frac{dN_{N_1}}{dz'} \right] e^{-\int_{z'}^z dz'' W_{ID}(z'')}$$

## Decays and Inverse Decays

$$\begin{aligned} \frac{dN_{N_1}}{dz} &= -D (N_{N_1} - N_{N_1}^{\text{eq}}) \\ \frac{dN_{B-L}}{dz} &= -\varepsilon \frac{dN_{N_1}}{dz} - W_{ID} N_{B-L} \end{aligned}$$

$$D = \frac{\Gamma_D}{H z} = K z \left\langle \frac{1}{\gamma} \right\rangle, \quad W_{ID} = \frac{1}{2} \frac{N_{N_1}^{\text{eq}}}{N_{b,l}^{\text{eq}}} D \propto K$$

$$N_{B-L}(z; K, z_i) = N_{B-L}^{\text{in}} e^{-\int_{z_i}^z dz' W_{ID}(z')} + \varepsilon \kappa(z)$$

$$\kappa(z; K, z_i) = - \int_{z_i}^z dz' \left[ \frac{dN_{N_1}}{dz'} \right] e^{-\int_{z'}^z dz'' W_{ID}(z'')}$$

- Weak wash-out regime for  $K \lesssim 1$  (out-of-equilibrium picture recovered for  $K \rightarrow 0$ )
- Strong wash-out regime for  $K \gtrsim 1$



Close-to-equilibrium approximation

$$\frac{dN_{N_1}}{dz} \simeq \frac{dN_{N_1}^{\text{eq}}}{dz}$$

Close-to-equilibrium approximation

$$\frac{dN_{N_1}}{dz} \simeq \frac{dN_{N_1}^{\text{eq}}}{dz}$$

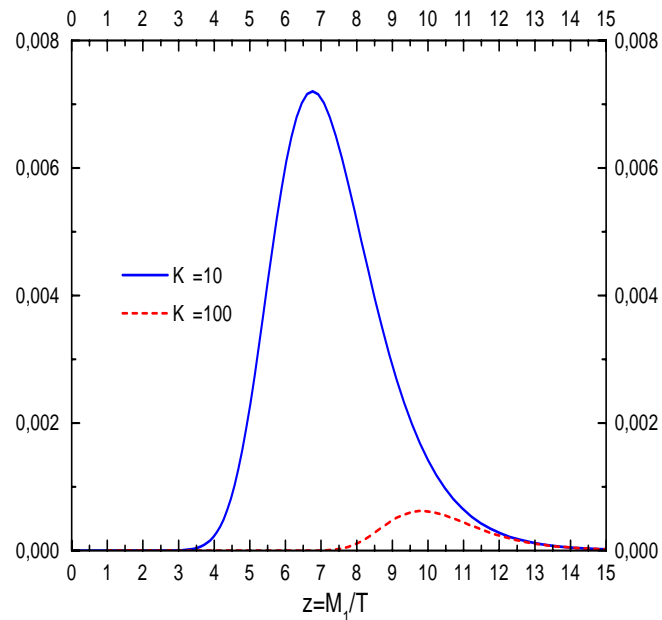
$$\begin{aligned} \kappa_f &= \int_0^\infty dz' e^{-\psi(z', \infty)} \\ &\simeq \int_{z_B - \Delta z_B}^{z_B + \Delta z_B} dz' e^{-\psi(z', \infty)} \end{aligned}$$

Close-to-equilibrium approximation

$$\frac{dN_{N_1}}{dz} \simeq \frac{dN_{N_1}^{\text{eq}}}{dz}$$

$$\begin{aligned} \kappa_f &= \int_0^\infty dz' e^{-\psi(z', \infty)} \\ &\simeq \int_{z_B - \Delta z_B}^{z_B + \Delta z_B} dz' e^{-\psi(z', \infty)} \end{aligned}$$

Baryogenesis temperature  $T_B = M_1/z_B$



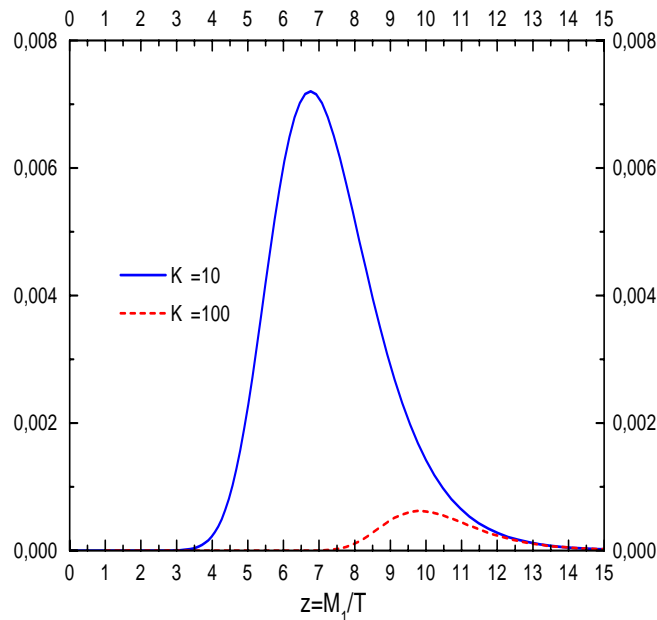
Close-to-equilibrium approximation

$$\frac{dN_{N_1}}{dz} \simeq \frac{dN_{N_1}^{\text{eq}}}{dz}$$

$$\kappa_f = \int_0^\infty dz' e^{-\psi(z', \infty)}$$

$$\simeq \int_{z_B - \Delta z_B}^{z_B + \Delta z_B} dz' e^{-\psi(z', \infty)}$$

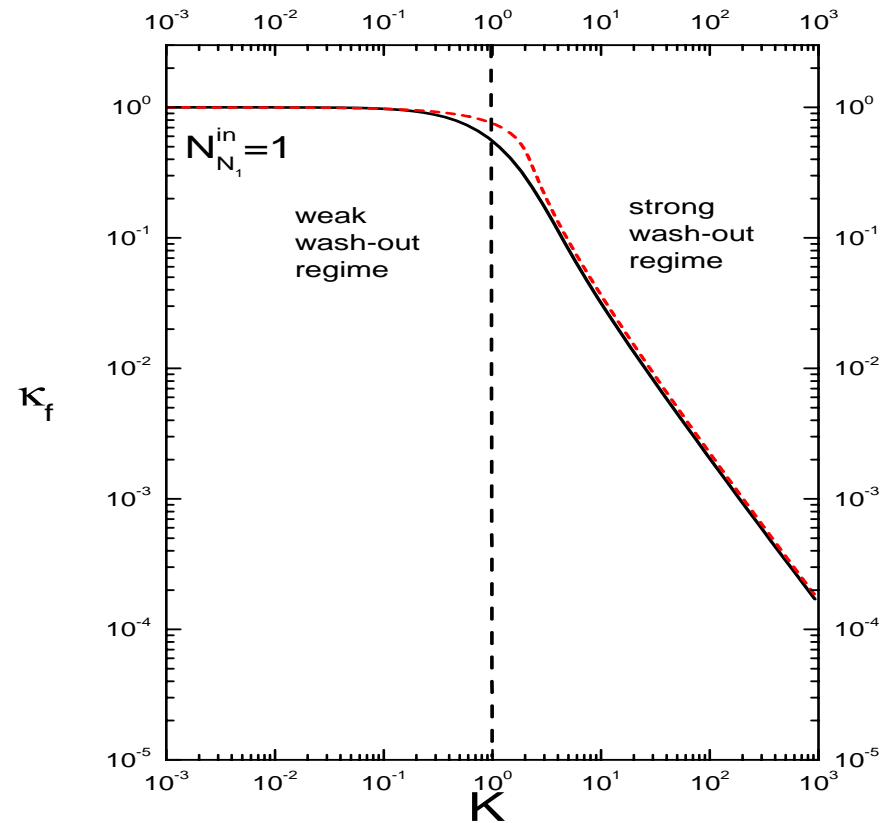
Baryogenesis temperature  $T_B = M_1/z_B$



- only non relativistic stage matters

- $\kappa_f(z_i = 0) \simeq \kappa_f(z_i = z_B - \Delta z_B)$

- $\kappa_f \simeq \frac{2}{K z_B} \left( 1 - e^{-\frac{K z_B}{2}} \right)$



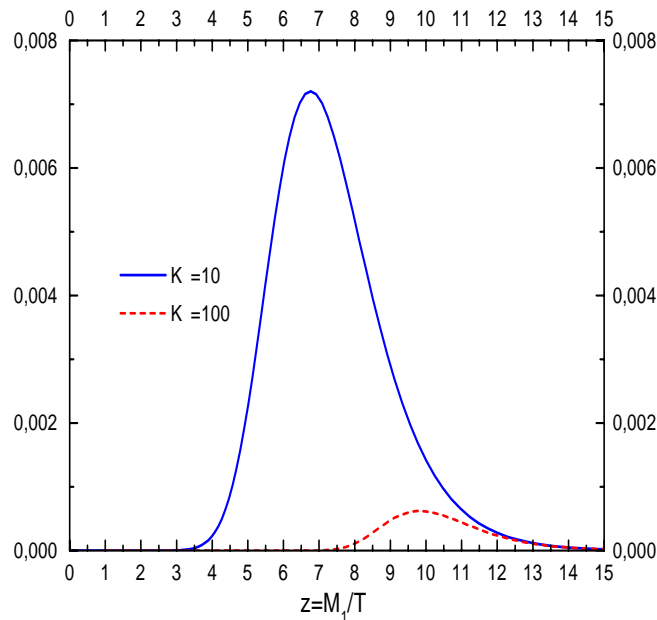
Close-to-equilibrium approximation

$$\frac{dN_{N_1}}{dz} \simeq \frac{dN_{N_1}^{\text{eq}}}{dz}$$

$$\kappa_f = \int_0^\infty dz' e^{-\psi(z', \infty)}$$

$$\simeq \int_{z_B - \Delta z_B}^{z_B + \Delta z_B} dz' e^{-\psi(z', \infty)}$$

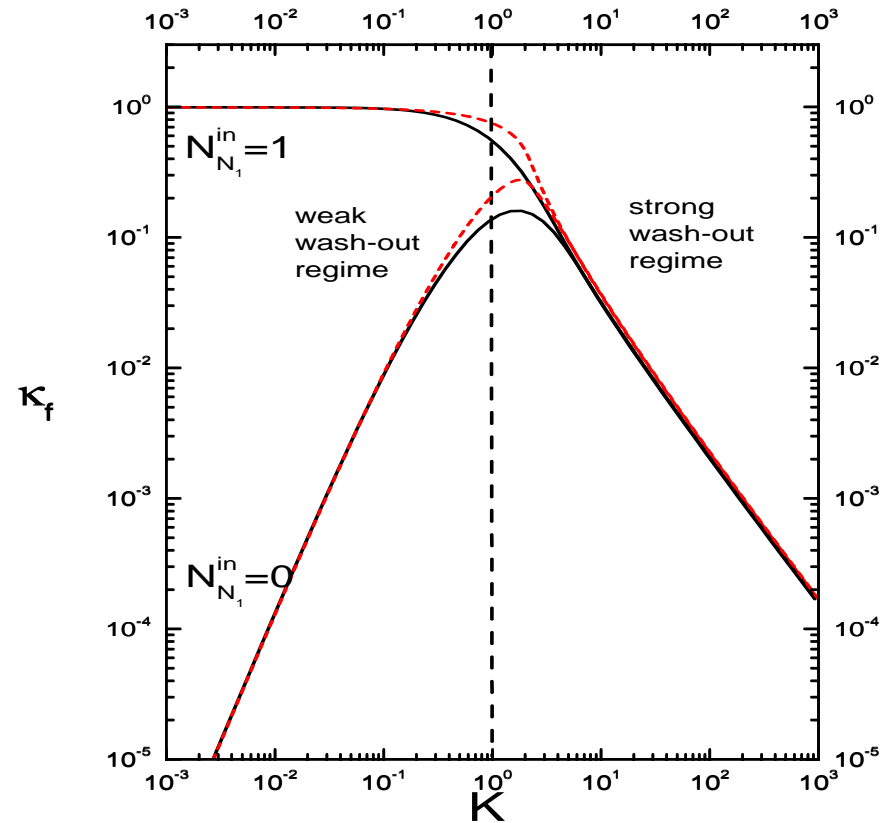
Baryogenesis temperature  $T_B = M_1/z_B$



- only non relativistic stage matters

- $\kappa_f(z_i = 0) \simeq \kappa_f(z_i = z_B - \Delta z_B)$

- $\kappa_f \simeq \frac{2}{K z_B} \left( 1 - e^{-\frac{K z_B}{2}} \right)$



## Effective neutrino mass

$$\tilde{m}_1 \equiv \frac{(m_D^\dagger m_D)_{11}}{M_1}$$

## Effective neutrino mass

$$\tilde{m}_1 \equiv \frac{(m_D^\dagger m_D)_{11}}{M_1}$$

$$\Gamma_D^{\text{rest}} = \frac{\tilde{m}_1 M_1^2}{8 \pi v^2} \Rightarrow K = \frac{\Gamma_D^{\text{rest}}}{H|_{z=1}} = \frac{\tilde{m}_1}{m_\star}$$

## Effective neutrino mass

$$\tilde{m}_1 \equiv \frac{(m_D^\dagger m_D)_{11}}{M_1}$$

$$\Gamma_D^{\text{rest}} = \frac{\tilde{m}_1 M_1^2}{8 \pi v^2} \Rightarrow K = \frac{\Gamma_D^{\text{rest}}}{H|_{z=1}} = \frac{\tilde{m}_1}{m_\star}$$

- equilibrium neutrino mass

$$m_\star = \text{const} \frac{v^2 \sqrt{g_\star}}{M_{Pl}} \simeq 10^{-3} \text{ eV}$$



## **Seesaw orthogonal matrix**

(Casas, Ibarra'01)

## Seesaw orthogonal matrix

(Casas,Ibarra'01)

$$M \rightarrow D_M = \text{diag}(M_1, M_2, M_3) , \quad U^\dagger m_\nu U^* = -D_m$$

## Seesaw orthogonal matrix

(Casas,Ibarra'01)

$$M \rightarrow D_M = \text{diag}(M_1, M_2, M_3) , \quad U^\dagger m_\nu U^* = -D_m$$

$$m_\nu = -m_D \frac{1}{M} m_D^T \Leftrightarrow \Omega^T \Omega = I$$

## Seesaw orthogonal matrix

(Casas,Ibarra'01)

$$M \rightarrow D_M = \text{diag}(M_1, M_2, M_3) , \quad U^\dagger m_\nu U^* = -D_m$$

$$m_\nu = -m_D \frac{1}{M} m_D^T \Leftrightarrow \Omega^T \Omega = I$$

$$\Omega = D_m^{-1/2} U^\dagger m_D D_M^{-1/2} \Rightarrow \Omega_{ij} = \frac{(U^\dagger m_D)_{ij}}{\sqrt{m_i M_j}}$$

**Weak or strong wash-out?**

## Weak or strong wash-out?

- $\tilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} = \sum_{j=1}^3 m_j |\Omega_{j1}^2| \geq m_1$

## Weak or strong wash-out?

- $\tilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} = \sum_{j=1}^3 m_j |\Omega_{j1}^2| \geq m_1$

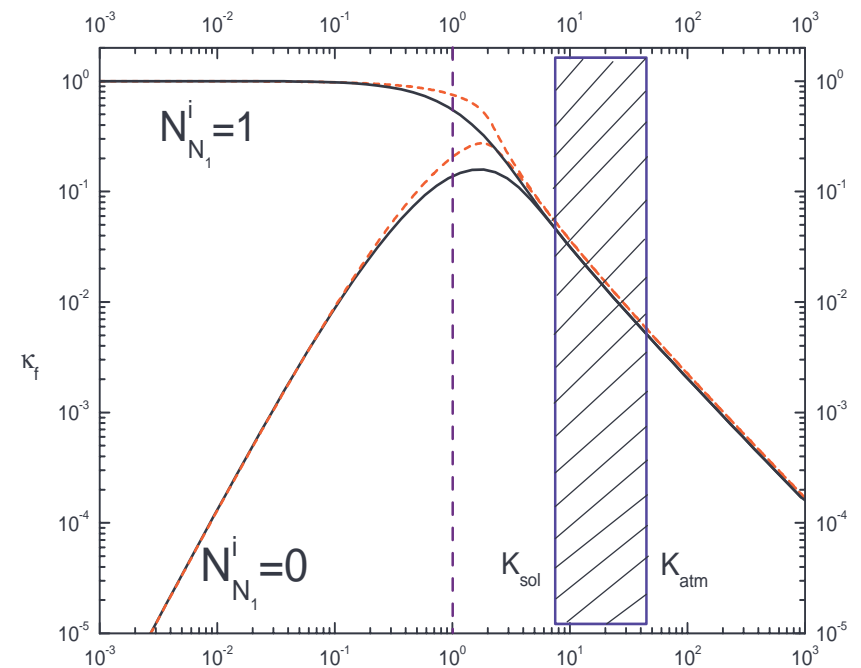
- typically  $\tilde{m}_1 \gg m_\star \simeq 10^{-3} \text{ eV}$

$\mathcal{O}(m_{\text{sol}} \simeq 0.008 \text{ eV}) < \tilde{m}_1 < \mathcal{O}(m_{\text{atm}} \simeq 0.05 \text{ eV})$

## Leptogenesis K range

Translating  $\tilde{m}_1$  in terms of  $K = \tilde{m}_1 / m_\star$ :

$$8 \simeq K_{\text{sol}} \lesssim K \lesssim K_{\text{atm}} \simeq 50$$



Neutrino mixing data favor leptogenesis

to lie in the **strong wash-out regime**

## Weak or strong wash-out?

- $\tilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} = \sum_{j=1}^3 m_j |\Omega_{j1}^2| \geq m_1$

- typically  $\tilde{m}_1 \gg m_\star \simeq 10^{-3} \text{ eV}$

$\mathcal{O}(m_{\text{sol}} \simeq 0.008 \text{ eV}) < \tilde{m}_1 < \mathcal{O}(m_{\text{atm}} \simeq 0.05 \text{ eV})$

- conditions for weak wash-out:

1. fully hierarchical neutrinos

2.  $m_1 \leq \tilde{m}_1 \ll m_\star \ll m_{\text{sol}} \Rightarrow$

$$\Omega \simeq R_{23} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \Omega_{22} & \sqrt{1 - \Omega_{22}^2} \\ 0 & -\sqrt{1 - \Omega_{22}^2} & \Omega_{22} \end{pmatrix}$$

3.  $\tilde{m}_1 \simeq m_1 \Rightarrow \varepsilon_1 \simeq 1 - m_1/\tilde{m}_1 \Rightarrow$

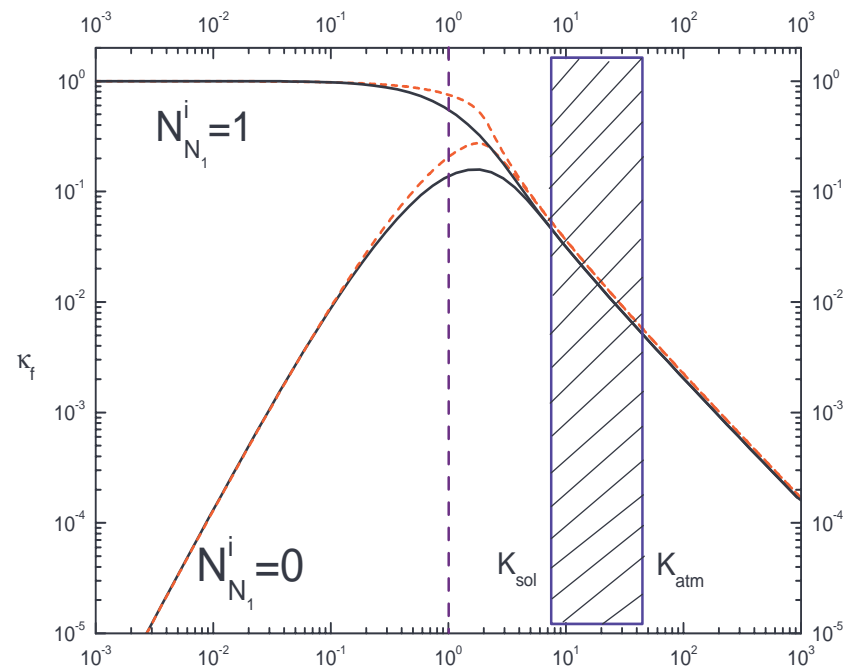
$\tilde{m}_1 \gg m_1 \Rightarrow \Omega \neq R_{23}$

$\Rightarrow$  **fine-tuned conditions !**

## Leptogenesis K range

Translating  $\tilde{m}_1$  in terms of  $K = \tilde{m}_1/m_\star$ :

$8 \simeq K_{\text{sol}} \lesssim K \lesssim K_{\text{atm}} \simeq 50$

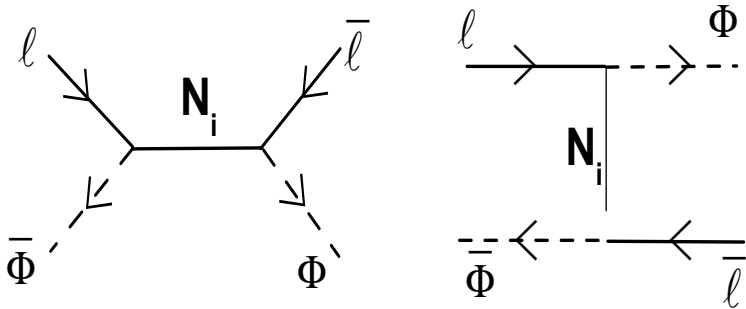


Neutrino mixing data favor leptogenesis

to lie in the strong wash-out regime



## $\Delta L = 2$ processes

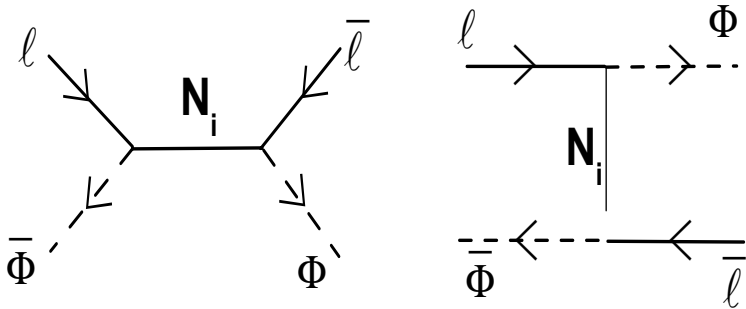


$$W_{\Delta L=2}(z) = \cancel{W_{\Delta L=2}^{\text{res}}(z)} + \Delta W(z)$$

- $\Delta W$  is important only in the non relativistic regime

$$\Delta W(z \ll 1) \propto \frac{M_1 \bar{m}^2}{z^2}, \quad \boxed{\bar{m}^2 \equiv \sum_i m_{\nu_i}^2}$$

## $\Delta L = 2$ processes



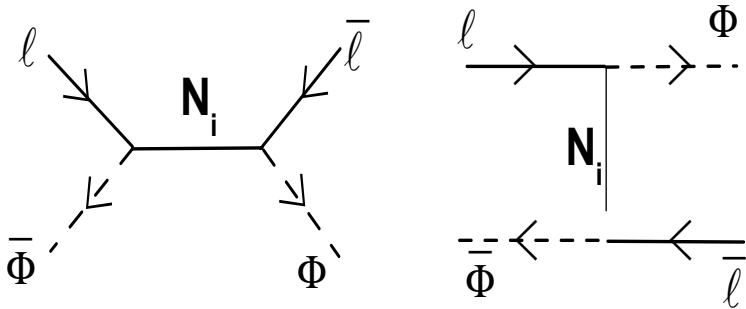
$$\kappa_f(\tilde{m}_1) \rightarrow \kappa_f(\tilde{m}_1) e^{-\frac{\Delta\omega}{z_B} \frac{M_1}{10^{14} \text{ GeV}} \frac{\bar{m}^2}{m_{\text{atm}}^2}}$$

$$W_{\Delta L=2}(z) = \cancel{W_{\Delta L=2}^{\text{res}}(z)} + \Delta W(z)$$

- $\Delta W$  is important only in the non relativistic regime

$$\Delta W(z \ll 1) \propto \frac{M_1 \bar{m}^2}{z^2}, \quad \boxed{\bar{m}^2 \equiv \sum_i m_{\nu_i}^2}$$

## $\Delta L = 2$ processes



$$\kappa_f(\tilde{m}_1) \rightarrow \kappa_f(\tilde{m}_1) e^{-\frac{\Delta\omega}{z_B} \frac{M_1}{10^{14} \text{ GeV}} \frac{\bar{m}^2}{m_{\text{atm}}^2}}$$

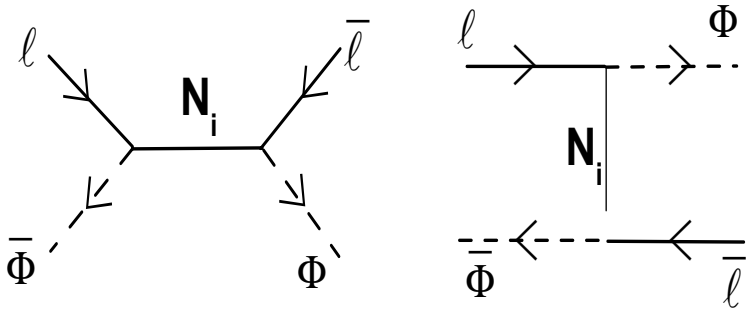
- $\bar{m}^2 \gg m_{\text{atm}}^2 \Rightarrow$  contributes to an **upper bound** on the **absolute neutrino mass scale**

$$W_{\Delta L=2}(z) = \cancel{W_{\Delta L=2}^{\text{res}}(z)} + \Delta W(z)$$

- $\Delta W$  is important only in the **non relativistic regime**

$$\Delta W(z \ll 1) \propto \frac{M_1 \bar{m}^2}{z^2}, \quad \boxed{\bar{m}^2 \equiv \sum_i m_{\nu_i}^2}$$

## $\Delta L = 2$ processes



$$\kappa_f(\tilde{m}_1) \rightarrow \kappa_f(\tilde{m}_1) e^{-\frac{\Delta\omega}{z_B} \frac{M_1}{10^{14} \text{ GeV}} \frac{\bar{m}^2}{m_{\text{atm}}^2}}$$

- $\bar{m}^2 \gg m_{\text{atm}}^2 \Rightarrow$  contributes to an **upper bound** on the **absolute neutrino mass scale**

$$W_{\Delta L=2}(z) = \cancel{W_{\Delta L=2}^{\text{res}}(z)} + \Delta W(z)$$

- in the case of hierarchical neutrinos is negligible for  $M_1 \ll 10^{14} \text{ GeV}$

- $\Delta W$  is important only in the non relativistic regime

$$\Delta W(z \ll 1) \propto \frac{M_1 \bar{m}^2}{z^2}, \quad \bar{m}^2 \equiv \sum_i m_{\nu_i}^2$$

## CP asymmetry

- Interference between tree level and (vertex + self energy) one-loop diagrams

$$\Rightarrow \varepsilon_1 \simeq \frac{1}{8\pi v^2 (m_D^\dagger m_D)_{11}} \sum_{i=2,3} \text{Im} \left[ (m_D^\dagger m_D)_{1i}^2 \right] \times \left[ f_V \left( \frac{M_i^2}{M_1^2} \right) + f_S \left( \frac{M_i^2}{M_1^2} \right) \right]$$

(Flanz,Paschos,Sarkar'95; Covi,Roulet,Vissani'96; Buchmüller,Plümacher'98)

## CP asymmetry

- Interference between tree level and (vertex + self energy) one-loop diagrams

$$\Rightarrow \varepsilon_1 \simeq \frac{1}{8\pi v^2 (m_D^\dagger m_D)_{11}} \sum_{i=2,3} \text{Im} \left[ (m_D^\dagger m_D)_{1i}^2 \right] \times \left[ f_V \left( \frac{M_i^2}{M_1^2} \right) + f_S \left( \frac{M_i^2}{M_1^2} \right) \right]$$

(Flanz, Paschos, Sarkar'95; Covi, Roulet, Vissani'96; Buchmüller, Plümacher'98)

- for hierarchical RH neutrino masses:

$$\varepsilon_1 \simeq \varepsilon_1^{\max}(M_1, m_1, \tilde{m}_1) \sin \delta_L(m_1, \tilde{m}_1, \Omega_{j1}^2)$$

## CP asymmetry

- Interference between tree level and (vertex + self energy) one-loop diagrams

$$\Rightarrow \varepsilon_1 \simeq \frac{1}{8\pi v^2 (m_D^\dagger m_D)_{11}} \sum_{i=2,3} \text{Im} \left[ (m_D^\dagger m_D)_{1i}^2 \right] \times \left[ f_V \left( \frac{M_i^2}{M_1^2} \right) + f_S \left( \frac{M_i^2}{M_1^2} \right) \right]$$

(Flanz,Paschos,Sarkar'95; Covi,Roulet,Vissani'96; Buchmüller,Plümacher'98)

- for hierarchical RH neutrino masses:

$$\varepsilon_1 \simeq \varepsilon_1^{\max}(M_1, m_1, \tilde{m}_1) \sin \delta_L(m_1, \tilde{m}_1, \Omega_{j1}^2)$$

$$\varepsilon_1^{\max}(M_1, m_1, \tilde{m}_1) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1), \quad \beta(m_1, \tilde{m}_1) \leq 1$$

## CP asymmetry

- Interference between tree level and (vertex + self energy) one-loop diagrams

$$\Rightarrow \varepsilon_1 \simeq \frac{1}{8\pi v^2 (m_D^\dagger m_D)_{11}} \sum_{i=2,3} \text{Im} \left[ (m_D^\dagger m_D)_{1i}^2 \right] \times \left[ f_V \left( \frac{M_i^2}{M_1^2} \right) + f_S \left( \frac{M_i^2}{M_1^2} \right) \right]$$

(Flanz,Paschos,Sarkar'95; Covi,Roulet,Vissani'96; Buchmüller,Plümacher'98)

- for hierarchical RH neutrino masses:

$$\varepsilon_1 \simeq \varepsilon_1^{\max}(M_1, m_1, \tilde{m}_1) \sin \delta_L(m_1, \tilde{m}_1, \Omega_{j1}^2)$$

$$\varepsilon_1^{\max}(M_1, m_1, \tilde{m}_1) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1), \quad \beta(m_1, \tilde{m}_1) \leq 1$$

$$\beta(m_1 = 0, \tilde{m}_1) = 1 \Rightarrow \varepsilon_1 \text{ maximum for fully hierarchical neutrinos}$$



## CP asymmetry

- Interference between tree level and (vertex + self energy) one-loop diagrams

$$\Rightarrow \varepsilon_1 \simeq \frac{1}{8\pi v^2 (m_D^\dagger m_D)_{11}} \sum_{i=2,3} \text{Im} \left[ (m_D^\dagger m_D)_{1i}^2 \right] \times \left[ f_V \left( \frac{M_i^2}{M_1^2} \right) + f_S \left( \frac{M_i^2}{M_1^2} \right) \right]$$

(Flanz,Paschos,Sarkar'95; Covi,Roulet,Vissani'96; Buchmüller,Plümacher'98)

- for hierarchical RH neutrino masses:

$$\varepsilon_1 \simeq \varepsilon_1^{\max}(M_1, m_1, \tilde{m}_1) \sin \delta_L(m_1, \tilde{m}_1, \Omega_{j1}^2)$$

$$\varepsilon_1^{\max}(M_1, m_1, \tilde{m}_1) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1), \quad \beta(m_1, \tilde{m}_1) \leq 1$$

$$\beta(m_1 = 0, \tilde{m}_1) = 1 \Rightarrow \varepsilon_1 \text{ maximum for fully hierarchical neutrinos}$$

$$\varepsilon_1^{\max}(M_1) \equiv \frac{3}{16\pi} \frac{M_1 m_{\text{atm}}}{v^2} \simeq 10^{-6} \left( \frac{M_1}{10^{10} \text{ GeV}} \right)$$

# CMB constraints in the full hierarchical case

(Buchmuller,PDB,Plumacher '02)

$$\eta_B^{\max}(M_1, \tilde{m}_1)|_{m_1=0} \simeq d \varepsilon_1^{\max}(M_1) \kappa_f(M_1, \tilde{m}_1)$$

$$\eta_B^{\max}(M_1, \tilde{m}_1)|_{m_1=0} \propto M_1 e^{-\frac{M_1}{10^{14} \text{ GeV}}}$$

$$d \simeq \frac{1}{3 N_\gamma^{\text{rec}}} \simeq 10^{-2}$$

CMB bound:

$$\eta_B^{\max}(M_1, \tilde{m}_1)|_{m_1=0} \geq \eta_B^{\text{CMB}}$$

# CMB constraints in the full hierarchical case

(Buchmuller,PDB,Plumacher '02)

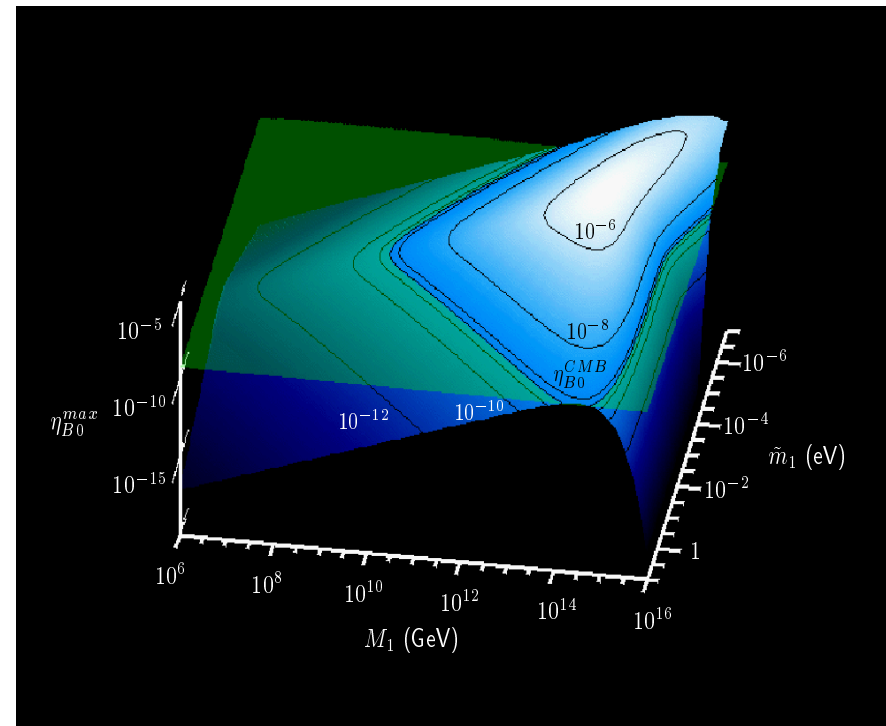
$$\eta_B^{\max}(M_1, \tilde{m}_1)|_{m_1=0} \simeq d \varepsilon_1^{\max}(M_1) \kappa_f(M_1, \tilde{m}_1)$$

$$\eta_B^{\max}(M_1, \tilde{m}_1)|_{m_1=0} \propto M_1 e^{-\frac{M_1}{10^{14} \text{ GeV}}}$$

$$d \simeq \frac{1}{3 N_\gamma^{\text{rec}}} \simeq 10^{-2}$$

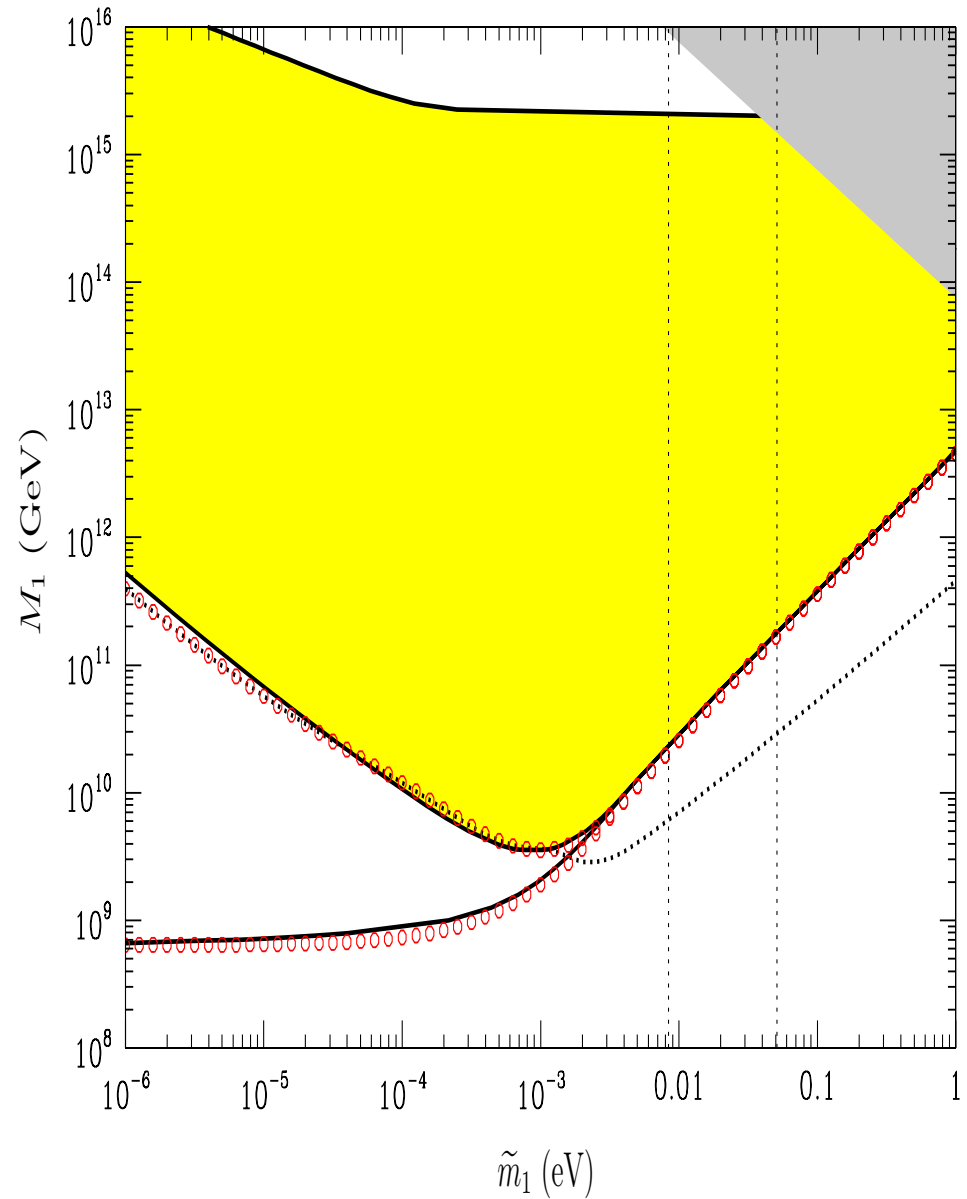
CMB bound:

$$\eta_B^{\max}(M_1, \tilde{m}_1)|_{m_1=0} \geq \eta_B^{\text{CMB}}$$



# Lower bound on $M_1$ and on $T_{\text{in}}$

(Davidson, Ibarra '02; Buchmuller, PDB, Plumacher '02, '04;  
Giudice, Notari, Raidal, Riotto, Strumia, '03)



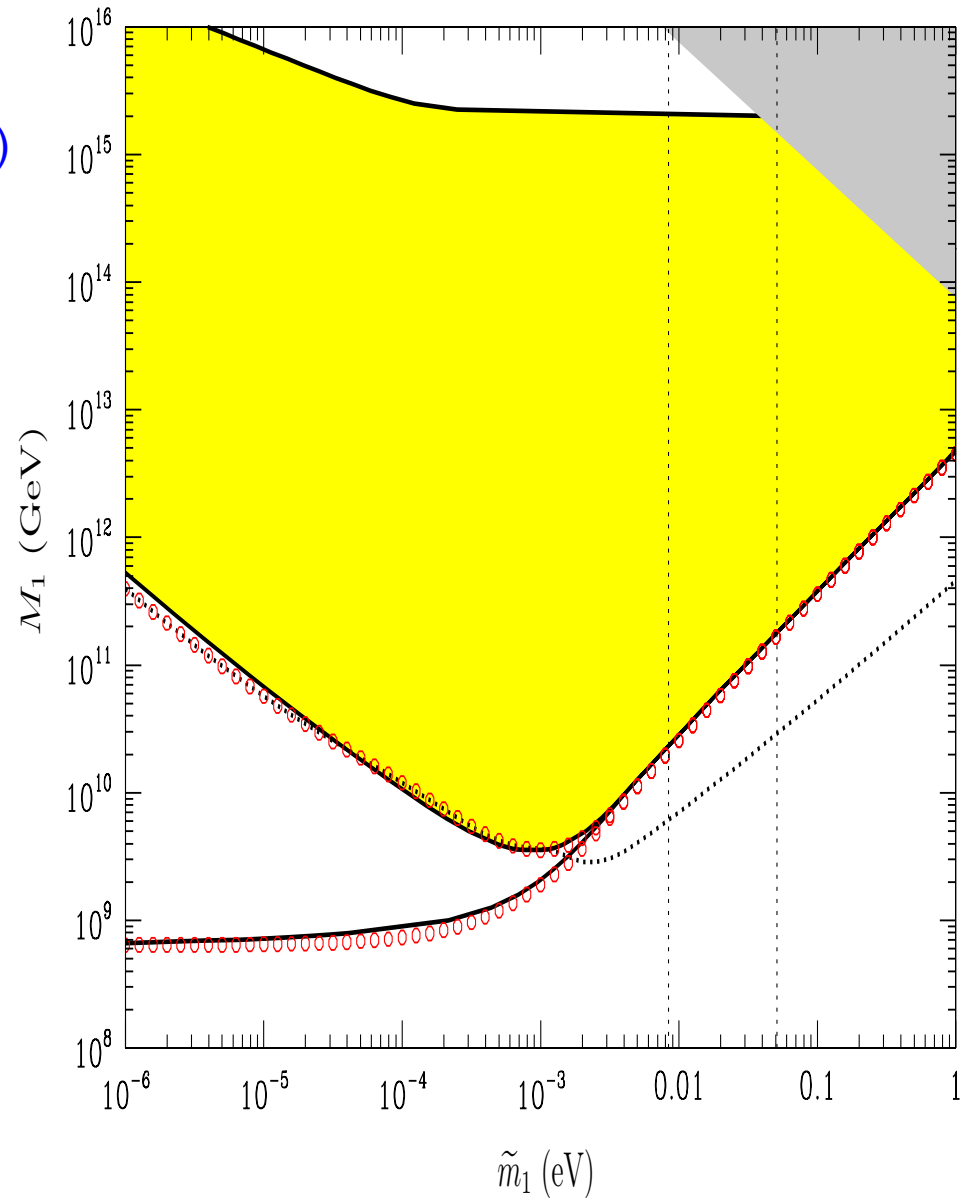
# Lower bound on $M_1$ and on $T_{\text{in}}$

(Davidson, Ibarra '02; Buchmuller, PDB, Plumacher '02, '04;  
Giudice, Notari, Raidal, Riotto, Strumia, '03)

Strong wash-out regime ( $\tilde{m}_1 \gtrsim 10^{-3}$  eV)

$$M_1, T_i \gtrsim 2 \times 10^9 \text{ GeV}$$

(within inflation  $T_i = T_{\text{reh}}$ )



# Lower bound on $M_1$ and on $T_{\text{in}}$

(Davidson, Ibarra '02; Buchmuller, PDB, Plumacher '02, '04;  
Giudice, Notari, Raidal, Riotto, Strumia, '03)

Strong wash-out regime ( $\tilde{m}_1 \gtrsim 10^{-3}$  eV)

$$M_1, T_i \gtrsim 2 \times 10^9 \text{ GeV}$$

(within inflation  $T_i = T_{\text{reh}}$ )

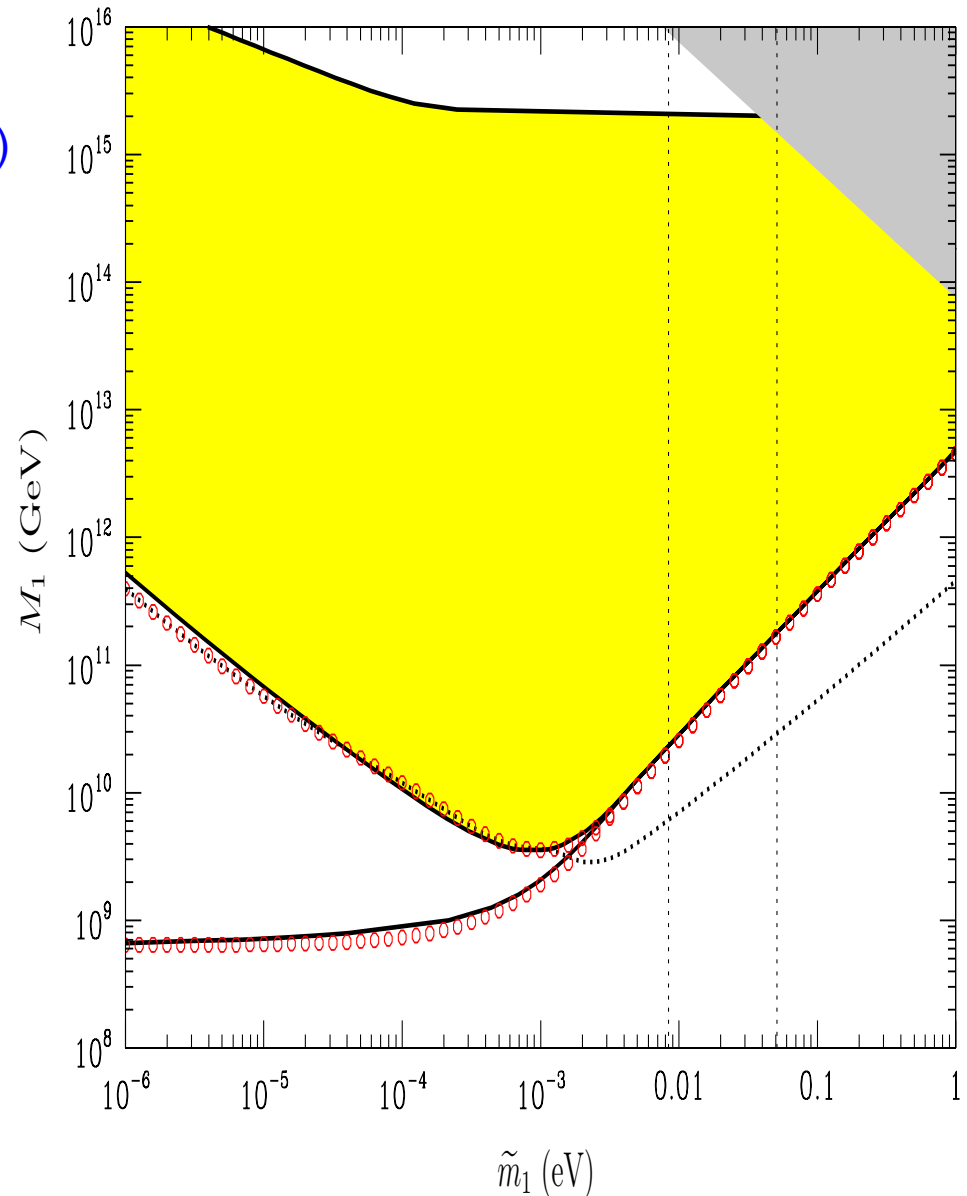
For  $\tilde{m}_1$  between  $m_{\text{sol}}$  and  $m_{\text{atm}}$ :

$$M_1 \gtrsim (10^{10} - 10^{11}) \text{ GeV}$$

$\Rightarrow$  **problem for many neutrino models !**

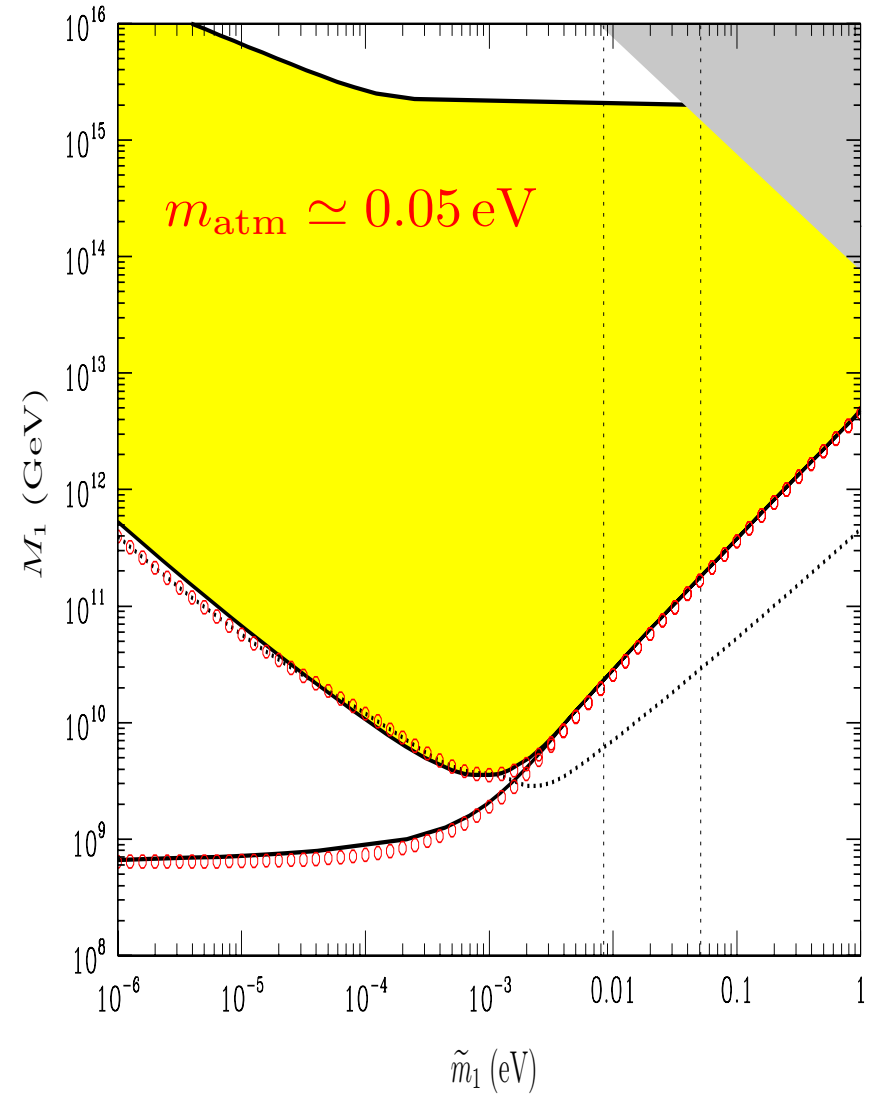
(Branco et al. '02, Davidson '02, Akhmedov et al. '03, ...)

$$T_{\text{reh}} \gtrsim \frac{M_1^{\text{min}}(\tilde{m}_1)}{z_B(\tilde{m}_1) - 2} \simeq \frac{M_1^{\text{min}}}{5}$$



# Leptogenesis 'conspiracy'

(Buchmuller,PDB,Plumacher,'04)

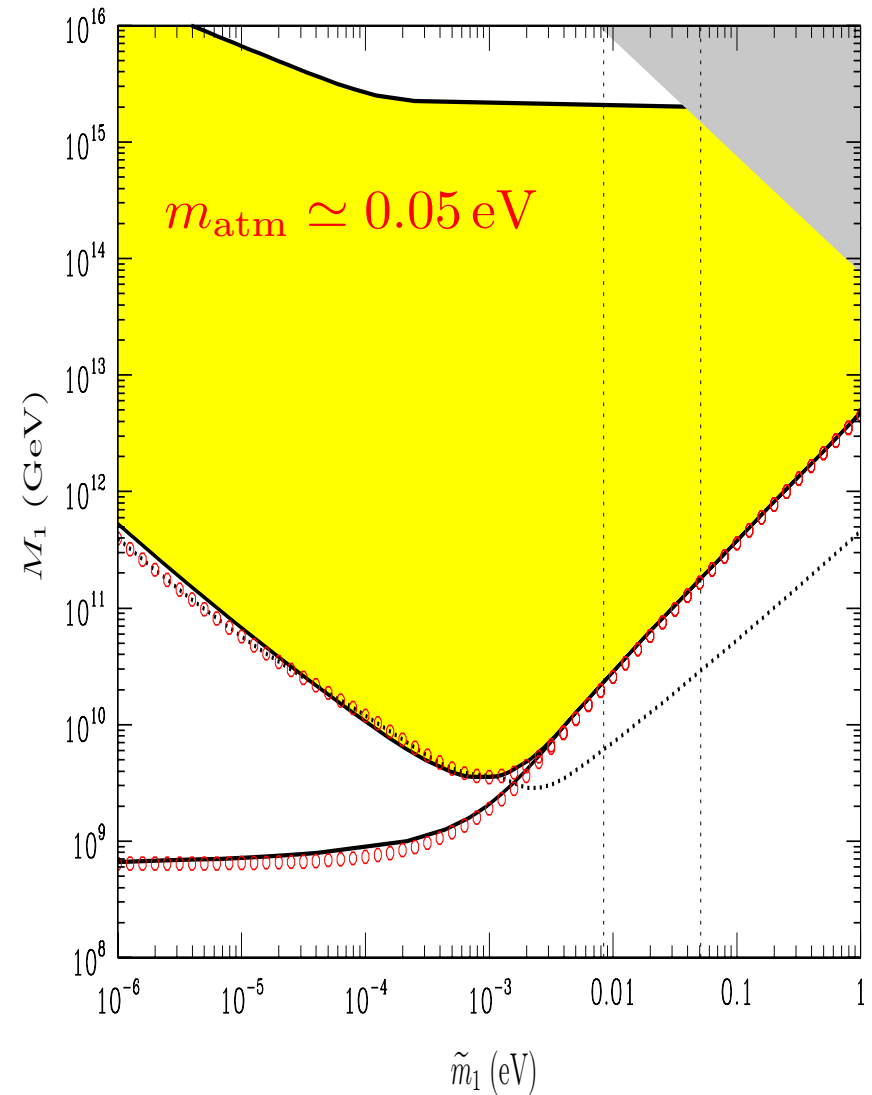


## Leptogenesis 'conspiracy'

(Buchmuller,PDB,Plumacher,'04)

$$m_3^2 = m_{\text{atm}}^2 + m_1^2$$

What if we **do not** use the information from neutrino mixing data:  $m_3^{\text{min}} = m_{\text{atm}}$  ?



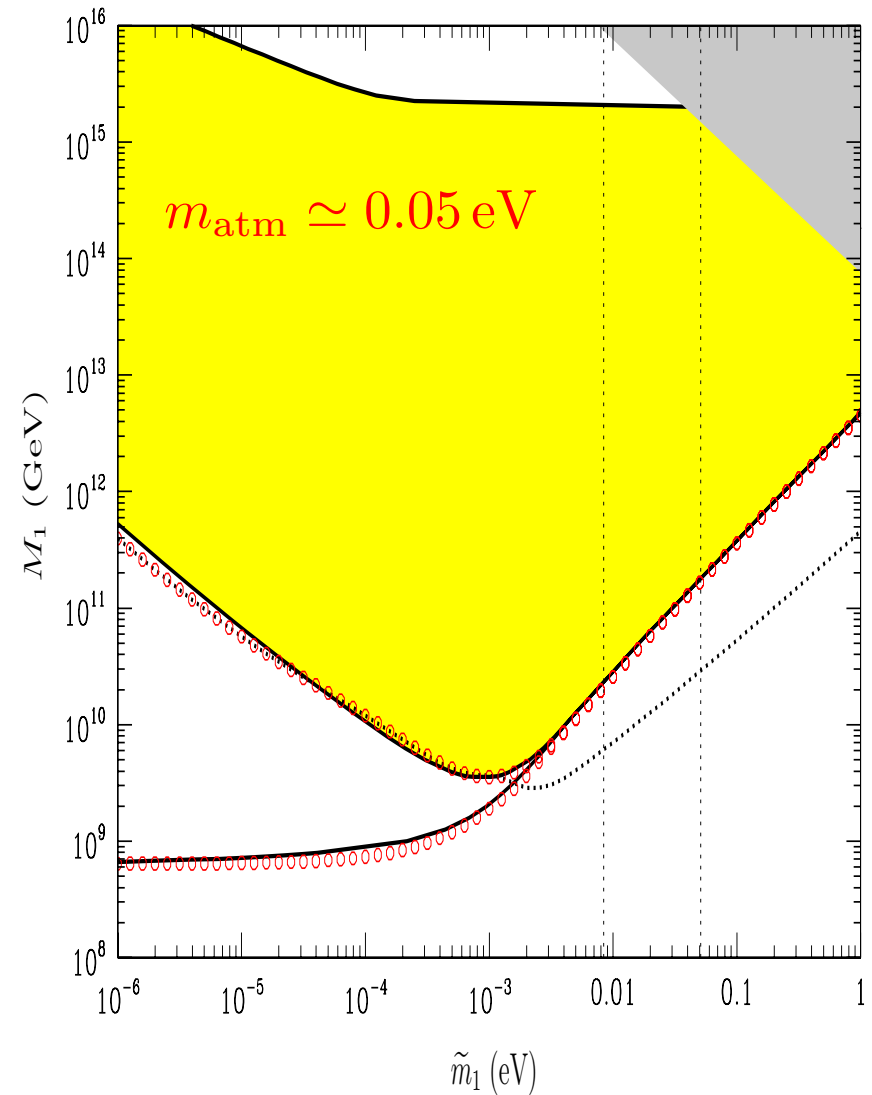


## Leptogenesis 'conspiracy'

(Buchmuller,PDB,Plumacher,'04)

$$m_3^2 = m_{\text{atm}}^2 + m_1^2$$

What if we **do not** use the information from neutrino mixing data:  $m_3^{\text{min}} = m_{\text{atm}}$  ?



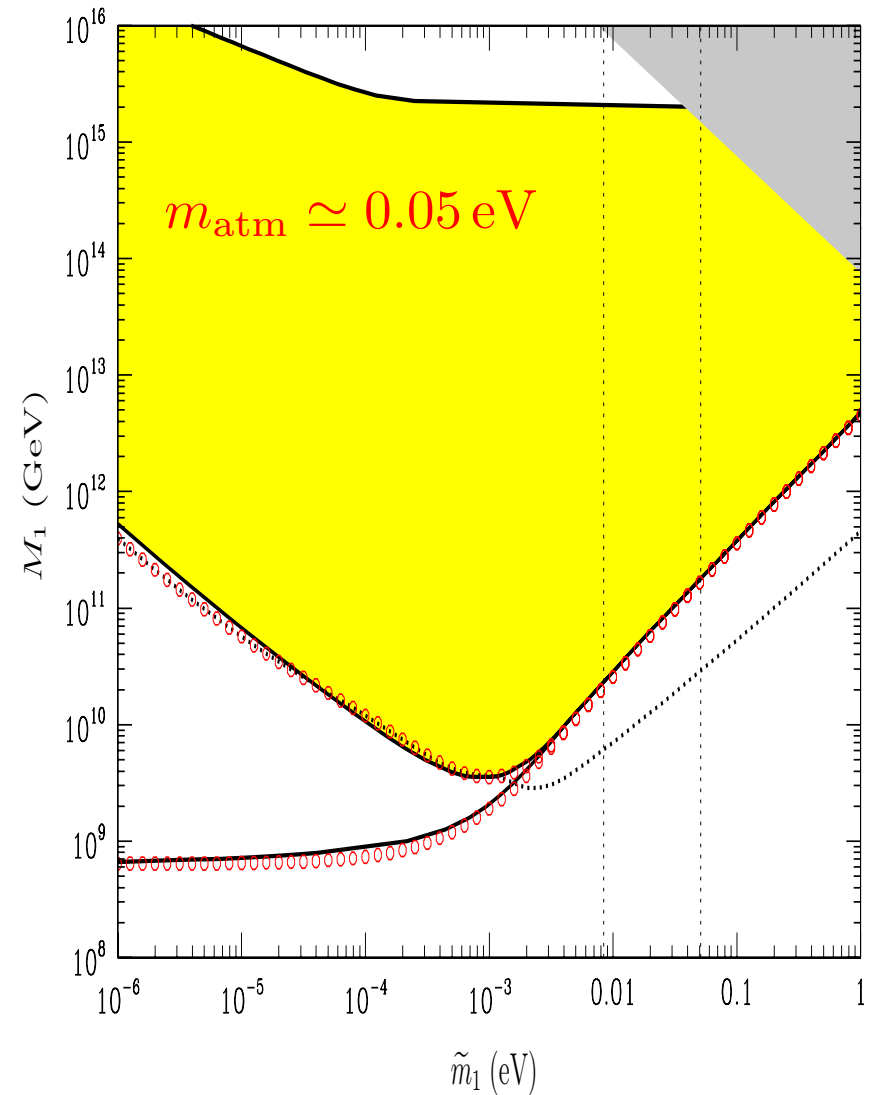
## Leptogenesis 'conspiracy'

(Buchmuller,PDB,Plumacher,'04)

$$m_3^2 = m_{\text{atm}}^2 + m_1^2$$

What if we **do not** use the information from neutrino mixing data:  $m_3^{\text{min}} = m_{\text{atm}}$  ?

- if  $m_3^{\text{min}} \ll 10^{-3}$  eV:
  - weak wash-out regime favored  
( $\Rightarrow$  problem with initial conditions)
  - $M_1^{\text{min}} \gg 10^{11}$  GeV



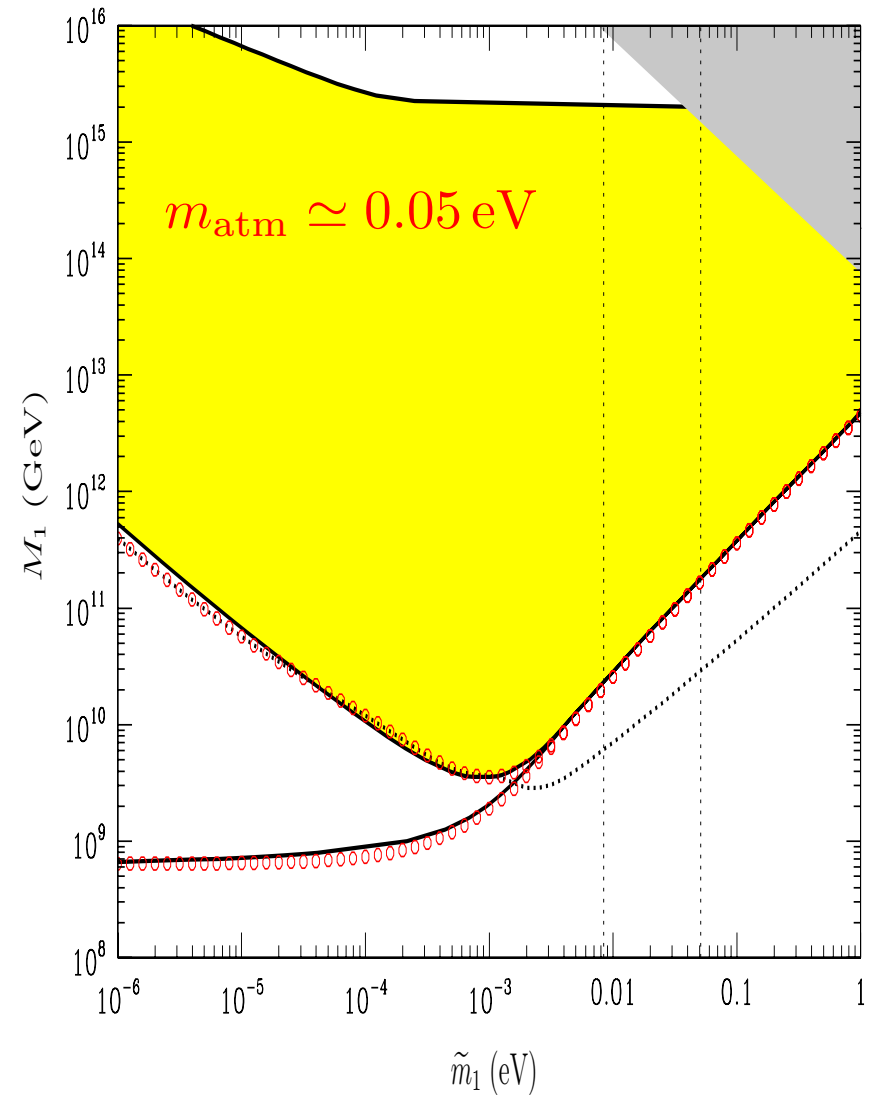
## Leptogenesis 'conspiracy'

(Buchmuller,PDB,Plumacher,'04)

$$m_3^2 = m_{\text{atm}}^2 + m_1^2$$

What if we **do not** use the information from neutrino mixing data:  $m_3^{\text{min}} = m_{\text{atm}}$  ?

- if  $m_3^{\text{min}} \ll 10^{-3}$  eV:
  - weak wash-out regime favored  
( $\Rightarrow$  problem with initial conditions)
  - $M_1^{\text{min}} \gg 10^{11}$  GeV
- if  $m_3^{\text{min}} \gg 1$  eV  $\Rightarrow \tilde{m}_1^{\text{max}} \ll m_3^{\text{min}}$ :



## Leptogenesis 'conspiracy'

(Buchmuller,PDB,Plumacher,'04)

$$m_3^2 = m_{\text{atm}}^2 + m_1^2$$

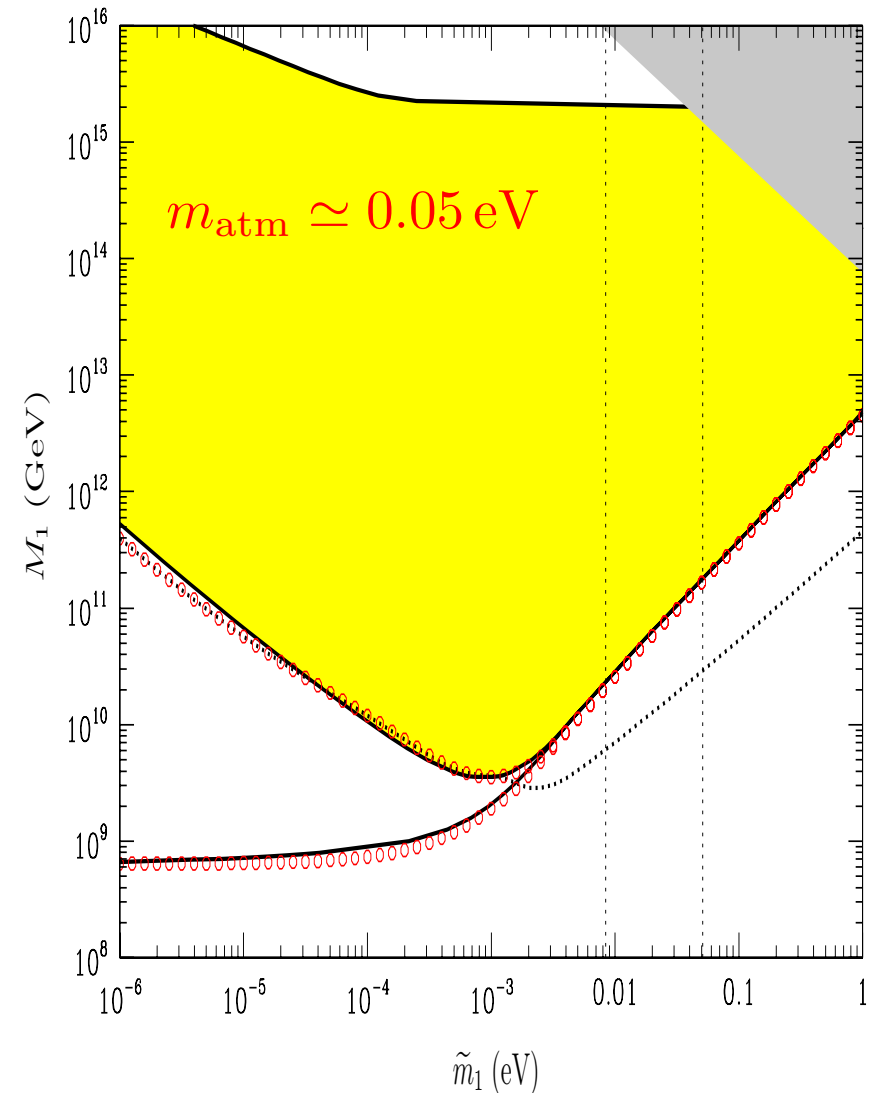
What if we **do not** use the information from neutrino mixing data:  $m_3^{\text{min}} = m_{\text{atm}}$  ?

- if  $m_3^{\text{min}} \ll 10^{-3}$  eV:
  - weak wash-out regime favored  
( $\Rightarrow$  problem with initial conditions)
  - $M_1^{\text{min}} \gg 10^{11}$  GeV
- if  $m_3^{\text{min}} \gg 1$  eV  $\Rightarrow \tilde{m}_1^{\text{max}} \ll m_3^{\text{min}}$ :

$\Rightarrow$  the experimental result:

$$\mathcal{O}(10^{-3} \text{ eV}) < m_{\text{atm}} < \mathcal{O}(1 \text{ eV})$$

is a **successful test for thermal leptogenesis** !



# **Testing the seesaw mechanism with leptogenesis**

## Testing the seesaw mechanism with leptogenesis

$$m_D = U \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \Omega \begin{pmatrix} \sqrt{M_1} & 0 & 0 \\ 0 & \sqrt{M_2} & 0 \\ 0 & 0 & \sqrt{M_3} \end{pmatrix} \quad \left( \begin{array}{l} U^\dagger U = I \\ \Omega \Omega^T = I \end{array} \right)$$

## Testing the seesaw mechanism with leptogenesis

$$\boxed{m_D} = U \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \Omega \begin{pmatrix} \sqrt{M_1} & 0 & 0 \\ 0 & \sqrt{M_2} & 0 \\ 0 & 0 & \sqrt{M_3} \end{pmatrix} \quad \left( \begin{array}{l} U^\dagger U = I \\ \Omega \Omega^T = I \end{array} \right)$$

$\uparrow$   $\uparrow$

*theory* “observables”

## Testing the seesaw mechanism with leptogenesis

$$\boxed{m_D} = U \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \Omega \begin{pmatrix} \sqrt{M_1} & 0 & 0 \\ 0 & \sqrt{M_2} & 0 \\ 0 & 0 & \sqrt{M_3} \end{pmatrix} \quad \left( \begin{array}{l} U^\dagger U = I \\ \Omega \Omega^T = I \end{array} \right)$$

$\uparrow$   
*theory*

$\uparrow$   
*“observables”*

- parameter counting:  $6 + 3 + 6 + 3 = 18$

- some of them we measure:  $\theta_{12}, \theta_{23}, m_{\text{sol}}, m_{\text{atm}}$

future experiments could measure or constraint:  $m_1, \theta_{13}, \delta, \varphi_1, \varphi_2$



## Testing the seesaw mechanism with leptogenesis

$$\boxed{m_D} = U \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \Omega \begin{pmatrix} \sqrt{M_1} & 0 & 0 \\ 0 & \sqrt{M_2} & 0 \\ 0 & 0 & \sqrt{M_3} \end{pmatrix} \quad \left( \begin{array}{l} U^\dagger U = I \\ \Omega \Omega^T = I \end{array} \right)$$

$\uparrow$   
*theory*

$\uparrow$   
*“observables”*

- parameter counting:  $6 + 3 + 6 + 3 = 18$

- some of them we measure:  $\theta_{12}, \theta_{23}, m_{\text{sol}}, m_{\text{atm}}$

future experiments could measure or constraint:  $m_1, \theta_{13}, \delta, \varphi_1, \varphi_2$

- **leptogenesis**  $\Rightarrow$  information on  $\Omega, M_i$  (but also on  $m_1$  !)

## Testing the seesaw mechanism with leptogenesis

$$\boxed{m_D} = U \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \Omega \begin{pmatrix} \sqrt{M_1} & 0 & 0 \\ 0 & \sqrt{M_2} & 0 \\ 0 & 0 & \sqrt{M_3} \end{pmatrix} \quad \left( \begin{array}{l} U^\dagger U = I \\ \Omega \Omega^T = I \end{array} \right)$$

↑  
*theory*

↑  
“observables”

- parameter counting:  $6 + 3 + 6 + 3 = 18$

- some of them we measure:  $\theta_{12}, \theta_{23}, m_{\text{sol}}, m_{\text{atm}}$

future experiments could measure or constraint:  $m_1, \theta_{13}, \delta, \varphi_1, \varphi_2$

- leptogenesis  $\Rightarrow$  information on  $\Omega, M_i$  (but also on  $m_1$  !)

- $\varepsilon_i = \varepsilon_i(m_D^\dagger m_D) \Rightarrow U$  cancels out

$\Rightarrow$  in general only  $\Omega$  is the source of the  $CP$  violation necessary for leptogenesis

$\Rightarrow$  (no) CP violation in neutrino mixing does not imply  $\varepsilon_i \neq 0$  ( $\varepsilon_i = 0$ )

however with extra theoretical input  $U$  and  $\Omega$  can be related

## Seesaw geometry

(PDB '04)

For hierarchical RH neutrinos the dependence on many parameters is marginal and the predicted asymmetry depends approximately only on **6 parameters**

$$\eta_B \simeq -10^{-2} \kappa_f(m_1, \tilde{m}_1, M_1) \varepsilon_1(M_1, m_1, \tilde{m}_1, \Omega_{j1}^2)$$

$$\left\{ \begin{array}{l} \Omega \Omega^T = I \Rightarrow \Omega_{31}^2 + \Omega_{21}^2 + \Omega_{11}^2 = 1 \\ \tilde{m}_1 = \sum_{j=1,3} m_j |\Omega_{j1}^2| \end{array} \right.$$

## Seesaw geometry

(PDB '04)

For hierarchical RH neutrinos the dependence on many parameters is marginal and the predicted asymmetry depends approximately only on **6 parameters**

$$\eta_B \simeq -10^{-2} \kappa_f(m_1, \tilde{m}_1, M_1) \varepsilon_1(M_1, m_1, \tilde{m}_1, \Omega_{j1}^2) = \eta_B^{CMB}$$

$$\left\{ \begin{array}{l} \Omega \Omega^T = I \Rightarrow \Omega_{31}^2 + \Omega_{21}^2 + \Omega_{11}^2 = 1 \\ \tilde{m}_1 = \sum_{j=1,3} m_j |\Omega_{j1}^2| \end{array} \right.$$

## Seesaw geometry

(PDB '04)

For hierarchical RH neutrinos the dependence on many parameters is marginal and the predicted asymmetry depends approximately only on **6 parameters**

$$\eta_B \simeq -10^{-2} \kappa_f(m_1, \tilde{m}_1, M_1) \varepsilon_1(M_1, m_1, \tilde{m}_1, \Omega_{j1}^2) = \eta_B^{CMB}$$

$$\left\{ \begin{array}{l} \Omega \Omega^T = I \Rightarrow \Omega_{31}^2 + \Omega_{21}^2 + \Omega_{11}^2 = 1 \\ \tilde{m}_1 = \sum_{j=1,3} m_j |\Omega_{j1}^2| \end{array} \right.$$

We have seen that:

$$\varepsilon_1^{\max}(M_1, \tilde{m}_1, \Omega_{j1}^2) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1)$$

and that  $\beta = 1$  for  $m_1 = 0$ , but:

## Seesaw geometry

(PDB '04)

For hierarchical RH neutrinos the dependence on many parameters is marginal and the predicted asymmetry depends approximately only on **6 parameters**

$$\eta_B \simeq -10^{-2} \kappa_f(m_1, \tilde{m}_1, M_1) \varepsilon_1(M_1, m_1, \tilde{m}_1, \Omega_{j1}^2) = \eta_B^{CMB}$$

$$\begin{cases} \Omega \Omega^T = I \Rightarrow \Omega_{31}^2 + \Omega_{21}^2 + \Omega_{11}^2 = 1 \\ \tilde{m}_1 = \sum_{j=1,3} m_j |\Omega_{j1}^2| \end{cases}$$

We have seen that:

$$\varepsilon_1^{\max}(M_1, \tilde{m}_1, \Omega_{j1}^2) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1)$$

and that  $\beta = 1$  for  $m_1 = 0$ , but:

which configuration of  $\Omega_{j1}^2$  maximize  $\varepsilon_1$  ?

which is the  $\varepsilon_1$  bound for any  $m_1$  ?

## Seesaw geometry

(PDB '04)

For hierarchical RH neutrinos the dependence on many parameters is marginal and the predicted asymmetry depends approximately only on **6 parameters**

$$\eta_B \simeq -10^{-2} \kappa_f(m_1, \tilde{m}_1, M_1) \varepsilon_1(M_1, m_1, \tilde{m}_1, \Omega_{j1}^2) = \eta_B^{CMB}$$

$$\Omega_{j1}^2 \equiv X_j + iY_j \equiv \rho_j e^{i\varphi_j}$$

$$\left\{ \begin{array}{l} \Omega \Omega^T = I \Rightarrow \Omega_{31}^2 + \Omega_{21}^2 + \Omega_{11}^2 = 1 \\ \tilde{m}_1 = \sum_{j=1,3} m_j |\Omega_{j1}^2| \end{array} \right.$$

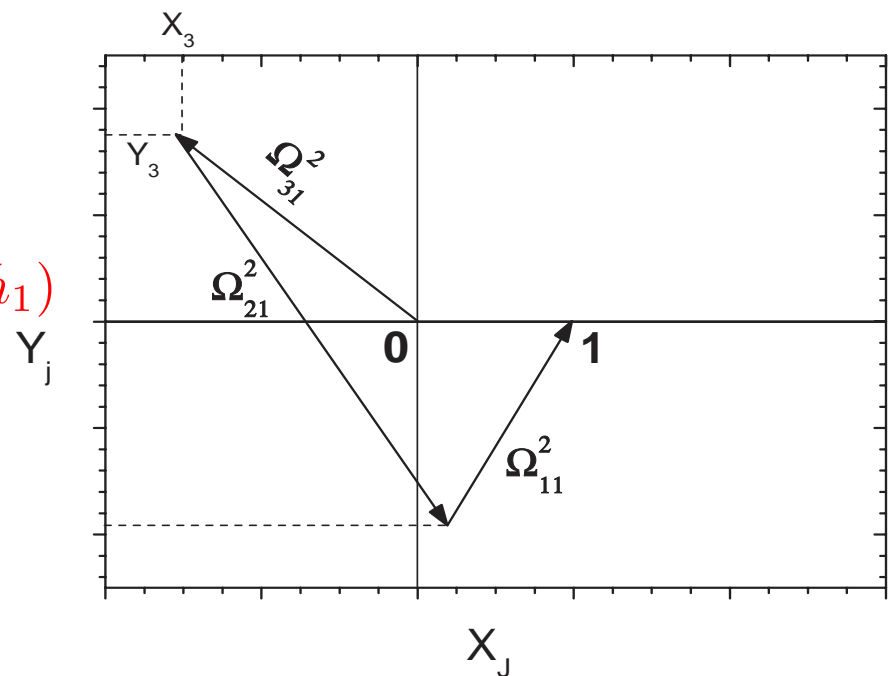
We have seen that:

$$\varepsilon_1^{\max}(M_1, \tilde{m}_1, \Omega_{j1}^2) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1)$$

and that  $\beta = 1$  for  $m_1 = 0$ , but:

which configuration of  $\Omega_{j1}^2$  maximize  $\varepsilon_1$  ?

which is the  $\varepsilon_1$  bound for any  $m_1$  ?



## *CP* asymmetry bound for any $m_1$

$$\varepsilon_1(M_1, m_1, \tilde{m}_1, \Omega_{j1}^2) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1) \sin \delta_L(m_1, \tilde{m}_1, \Omega_{j1}^2), \quad Y_j \equiv \text{Im}(\Omega_{j1}^2)$$

$$[\beta \times \sin \delta_L](m_1, \tilde{m}_1, \Omega_{j1}^2) = \frac{\Delta m_{31}^2 Y_3 + \Delta m_{21}^2 Y_2}{m_{\text{atm}} \tilde{m}_1},$$



## $CP$ asymmetry bound for any $m_1$

$$\varepsilon_1(M_1, m_1, \tilde{m}_1, \Omega_{j1}^2) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1) \sin \delta_L(m_1, \tilde{m}_1, \Omega_{j1}^2), \quad Y_j \equiv \text{Im}(\Omega_{j1}^2)$$

$$[\beta \times \sin \delta_L](m_1, \tilde{m}_1, \Omega_{j1}^2) = \frac{\Delta m_{31}^2 Y_3 + \Delta m_{21}^2 Y_2}{m_{\text{atm}} \tilde{m}_1},$$

- in general:  $\beta(m_1, \tilde{m}_1) = \frac{m_{\text{atm}}}{m_3 + m_1} f(m_1, \tilde{m}_1)$

$$(\tilde{m}_1 = m_1) \quad 0 \leq f(m_1, \tilde{m}_1) \leq 1 \quad (\tilde{m}_1/m_1 \rightarrow \infty)$$

(Davidson, Ibarra '02; Buchmüller, PDB, Plümacher '03)

## $CP$ asymmetry bound for any $m_1$

$$\varepsilon_1(M_1, m_1, \tilde{m}_1, \Omega_{j1}^2) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1) \sin \delta_L(m_1, \tilde{m}_1, \Omega_{j1}^2), \quad Y_j \equiv \text{Im}(\Omega_{j1}^2)$$

$$[\beta \times \sin \delta_L](m_1, \tilde{m}_1, \Omega_{j1}^2) = \frac{\Delta m_{31}^2 Y_3 + \Delta m_{21}^2 Y_2}{m_{\text{atm}} \tilde{m}_1},$$

- in general:  $\beta(m_1, \tilde{m}_1) = \frac{m_{\text{atm}}}{m_3 + m_1} f(m_1, \tilde{m}_1)$

$$(\tilde{m}_1 = m_1) \quad 0 \leq f(m_1, \tilde{m}_1) \leq 1 \quad (\tilde{m}_1/m_1 \rightarrow \infty)$$

(Davidson, Ibarra '02; Buchmüller, PDB, Plümacher '03)

- fully hierarchical neutrinos ( $m_1 = 0$ ):

$$f = \beta = 1 \Rightarrow \varepsilon_1 = \varepsilon_1^{\max}(M_1)$$

(Hamaguchi, Murayama, Yanagida '01)

## CP asymmetry bound for any $m_1$

$$\varepsilon_1(M_1, m_1, \tilde{m}_1, \Omega_{j1}^2) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1) \sin \delta_L(m_1, \tilde{m}_1, \Omega_{j1}^2), \quad Y_j \equiv \text{Im}(\Omega_{j1}^2)$$

$$[\beta \times \sin \delta_L](m_1, \tilde{m}_1, \Omega_{j1}^2) = \frac{\Delta m_{31}^2 Y_3 + \Delta m_{21}^2 Y_2}{m_{\text{atm}} \tilde{m}_1},$$

- in general:  $\beta(m_1, \tilde{m}_1) = \frac{m_{\text{atm}}}{m_3 + m_1} f(m_1, \tilde{m}_1)$

$$(\tilde{m}_1 = m_1) \quad 0 \leq f(m_1, \tilde{m}_1) \leq 1 \quad (\tilde{m}_1/m_1 \rightarrow \infty)$$

(Davidson, Ibarra '02; Buchmüller, PDB, Plümacher '03)

- fully hierarchical neutrinos ( $m_1 = 0$ ):

$$f = \beta = 1 \Rightarrow \varepsilon_1 = \varepsilon_1^{\max}(M_1)$$

(Hamaguchi, Murayama, Yanagida '01)

- quasi-degenerate neutrinos ( $m_1/m_{\text{atm}} \gg 1$ ):

$$f(m_1, \tilde{m}_1) = \sqrt{1 - \frac{\tilde{m}_1^2}{m_1^2}}$$

(Hambye, Lin, Notari, Papucci, Strumia '04; PDB '04)

## CP asymmetry bound for any $m_1$

$$\varepsilon_1(M_1, m_1, \tilde{m}_1, \Omega_{j1}^2) = \varepsilon_1^{\max}(M_1) \beta(m_1, \tilde{m}_1) \sin \delta_L(m_1, \tilde{m}_1, \Omega_{j1}^2), \quad Y_j \equiv \text{Im}(\Omega_{j1}^2)$$

$$[\beta \times \sin \delta_L](m_1, \tilde{m}_1, \Omega_{j1}^2) = \frac{\Delta m_{31}^2 Y_3 + \Delta m_{21}^2 Y_2}{m_{\text{atm}} \tilde{m}_1},$$

- in general:  $\beta(m_1, \tilde{m}_1) = \frac{m_{\text{atm}}}{m_3 + m_1} f(m_1, \tilde{m}_1)$   
 $(\tilde{m}_1 = m_1) \quad 0 \leq f(m_1, \tilde{m}_1) \leq 1 \quad (\tilde{m}_1/m_1 \rightarrow \infty)$

(Davidson, Ibarra '02; Buchmüller, PDB, Plümacher '03)

- fully hierarchical neutrinos ( $m_1 = 0$ ):

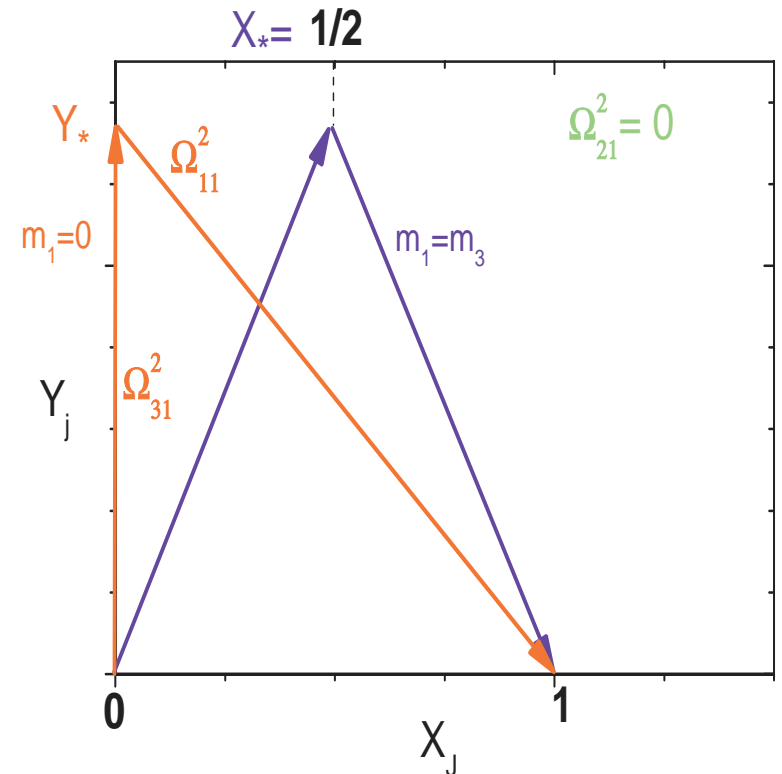
$$f = \beta = 1 \Rightarrow \varepsilon_1 = \varepsilon_1^{\max}(M_1)$$

(Hamaguchi, Murayama, Yanagida '01)

- quasi-degenerate neutrinos ( $m_1/m_{\text{atm}} \gg 1$ ):

$$f(m_1, \tilde{m}_1) = \sqrt{1 - \frac{\tilde{m}_1^2}{m_1^2}}$$

(Hambye, Lin, Notari, Papucci, Strumia '04; PDB '04)

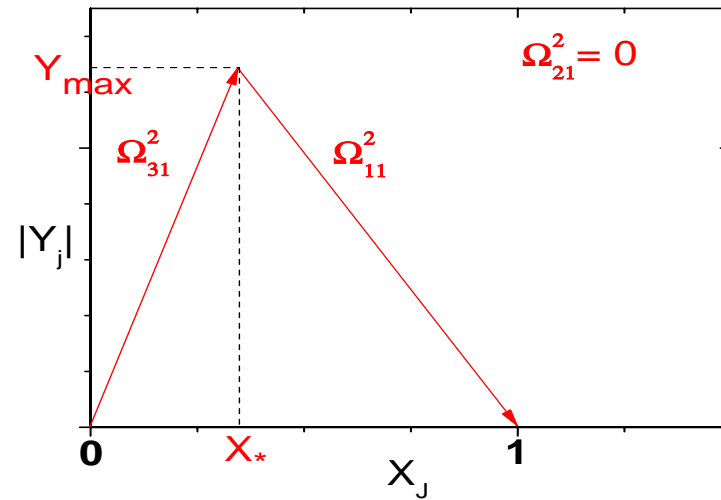


- also for a general choice of  $m_1$  and  $\tilde{m}_1$  the  $CP$  asymmetry is maximized for  $\Omega_{21}^2 = 0$

(PDB '04)

- also for a general choice of  $m_1$  and  $\tilde{m}_1$  the  $CP$  asymmetry is maximized for  $\Omega_{21}^2 = 0$

(PDB '04)

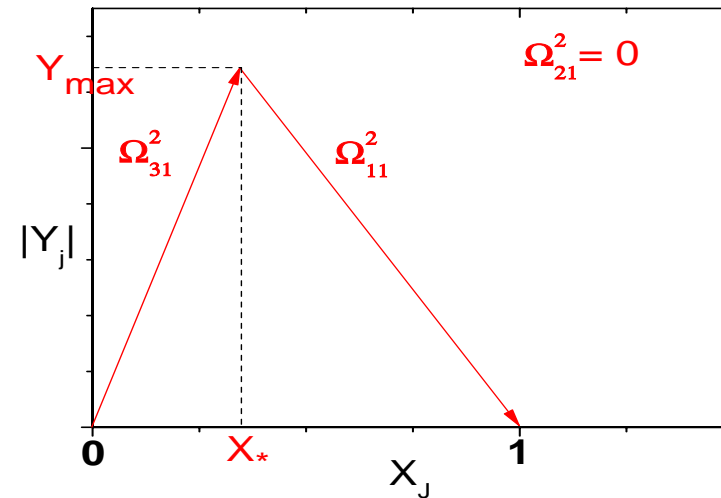


- also for a general choice of  $m_1$  and  $\tilde{m}_1$  the  $CP$  asymmetry is maximized for  $\Omega_{21}^2 = 0$

$\Rightarrow f(m_1, \tilde{m}_1)$  is given by (PDB '04):

$$f(m_1, \tilde{m}_1) = \frac{m_3 + m_1}{\tilde{m}_1} Y_{\max}(m_1, \tilde{m}_1)$$

$$\begin{cases} m_1 \rho_1 + m_3 \rho_3 \\ m_3 \cos \varphi_3 = m_1 \cos \varphi_1 \end{cases}$$



- also for a general choice of  $m_1$  and  $\tilde{m}_1$  the  $CP$  asymmetry is maximized for  $\Omega_{21}^2 = 0$

$\Rightarrow f(m_1, \tilde{m}_1)$  is given by (PDB '04):

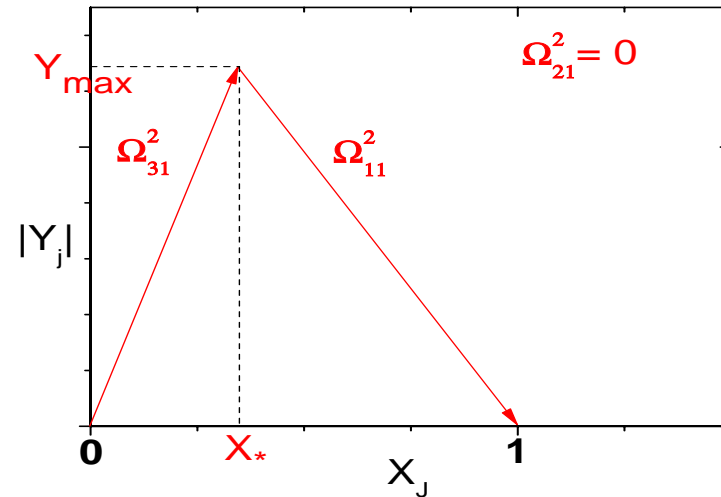
$$f(m_1, \tilde{m}_1) = \frac{m_3 + m_1}{\tilde{m}_1} Y_{\max}(m_1, \tilde{m}_1)$$

$$\begin{cases} m_1 \rho_1 + m_3 \rho_3 \\ m_3 \cos \varphi_3 = m_1 \cos \varphi_1 \end{cases}$$

- all limits can be reproduced, including an approximate expression valid for  $m_1/m_{\text{atm}} \ll 1$

(Buchmüller, PDB, Plümacher '03):

$$f \simeq f_{\text{h.n.}} \equiv \frac{m_3 - m_1 \sqrt{1 + \frac{m_3^2 - m_1^2}{\tilde{m}_1^2}}}{m_3 - m_1}$$





- also for a general choice of  $m_1$  and  $\tilde{m}_1$  the  $CP$  asymmetry is maximized for  $\Omega_{21}^2 = 0$

$\Rightarrow f(m_1, \tilde{m}_1)$  is given by (PDB '04):

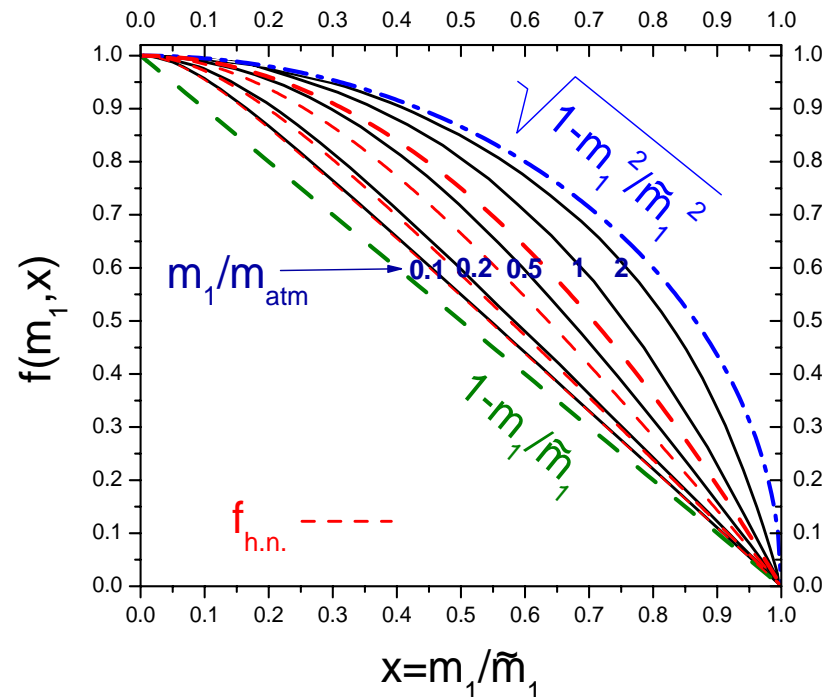
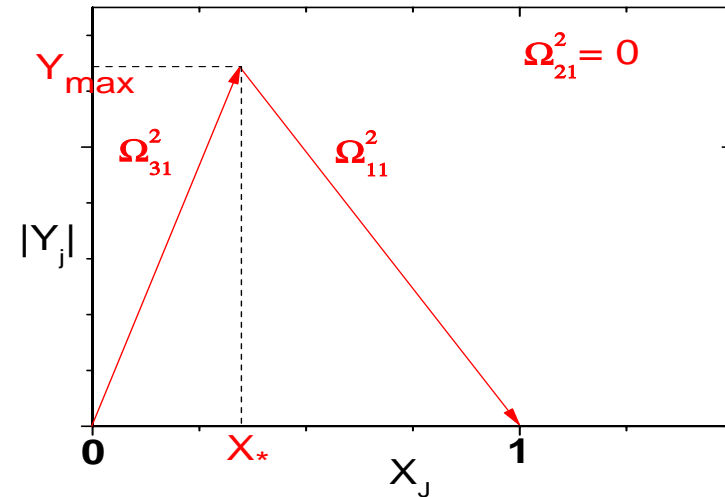
$$f(m_1, \tilde{m}_1) = \frac{m_3 + m_1}{\tilde{m}_1} Y_{\max}(m_1, \tilde{m}_1)$$

$$\begin{cases} m_1 \rho_1 + m_3 \rho_3 \\ m_3 \cos \varphi_3 = m_1 \cos \varphi_1 \end{cases}$$

- all limits can be reproduced, including an approximate expression valid for  $m_1/m_{\text{atm}} \ll 1$

(Buchmüller, PDB, Plümacher '03):

$$f \simeq f_{\text{h.n.}} \equiv \frac{m_3 - m_1 \sqrt{1 + \frac{m_3^2 - m_1^2}{\tilde{m}_1^2}}}{m_3 - m_1}$$



- also for a general choice of  $m_1$  and  $\tilde{m}_1$  the  $CP$  asymmetry is maximized for  $\Omega_{21}^2 = 0$

$\Rightarrow f(m_1, \tilde{m}_1)$  is given by (PDB '04):

$$f(m_1, \tilde{m}_1) = \frac{m_3 + m_1}{\tilde{m}_1} Y_{\max}(m_1, \tilde{m}_1)$$

$$\begin{cases} m_1 \rho_1 + m_3 \rho_3 \\ m_3 \cos \varphi_3 = m_1 \cos \varphi_1 \end{cases}$$

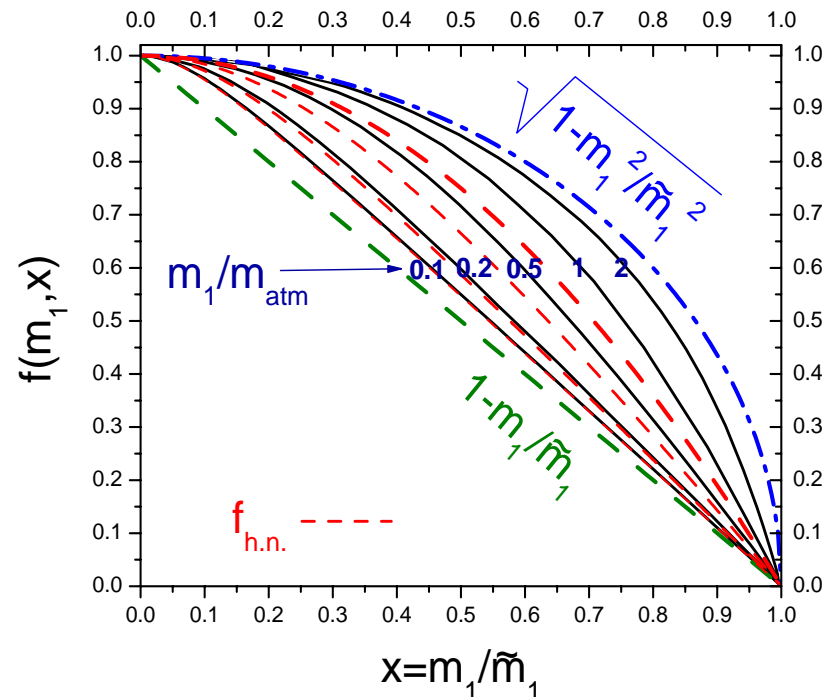
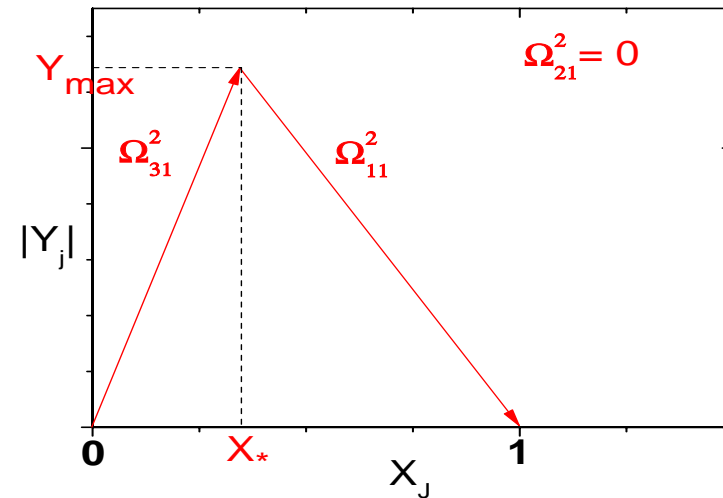
- all limits can be reproduced, including an approximate expression valid for  $m_1/m_{\text{atm}} \ll 1$

(Buchmüller, PDB, Plümacher '03):

$$f \simeq f_{\text{h.n.}} \equiv \frac{m_3 - m_1 \sqrt{1 + \frac{m_3^2 - m_1^2}{\tilde{m}_1^2}}}{m_3 - m_1}$$

- good agreement with recent results obtained in a different way using the approx.  $m_{\text{sol}} = 0$

(Hambye, Lin, Notari, Papucci, Strumia '04)



## Upper bound on the absolute neutrino mass scale

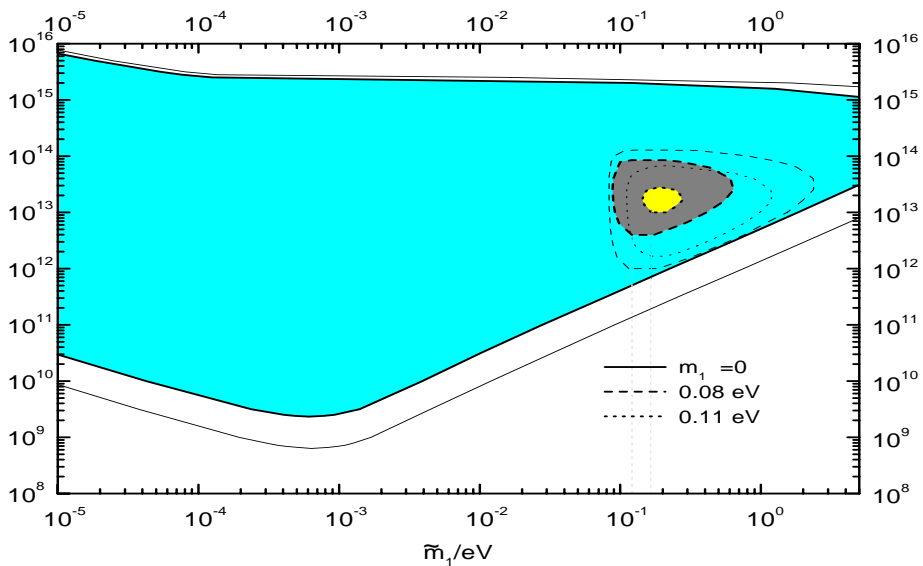
(Buchmüller,PDB,Plümacher '02,'03,'04)

$$\eta_B^{\max}(m_1, \tilde{m}_1, M_1) \simeq 10^{-2} \varepsilon_1^{\max}(M_1) \frac{m_{\text{atm}}}{m_3 + m_1} f(m_1, \tilde{m}_1) \kappa_f(\tilde{m}_1) e^{-w M_1 \sum_i m_i^2} \geq \eta_B^{\text{CMB}}$$

# Upper bound on the absolute neutrino mass scale

(Buchmüller,PDB,Plümacher '02,'03,'04)

$$\eta_B^{\max}(m_1, \tilde{m}_1, M_1) \simeq 10^{-2} \varepsilon_1^{\max}(M_1) \frac{m_{\text{atm}}}{m_3 + m_1} f(m_1, \tilde{m}_1) \kappa_f(\tilde{m}_1) e^{-w M_1} \sum_i m_i^2 \geq \eta_B^{\text{CMB}}$$



Analytically one can maximize  $\eta_B^{\max}$  and using

$f \simeq f_{hn}$  and central values of  $\eta_B^{\text{CMB}}$ ,  $m_{\text{atm}}$

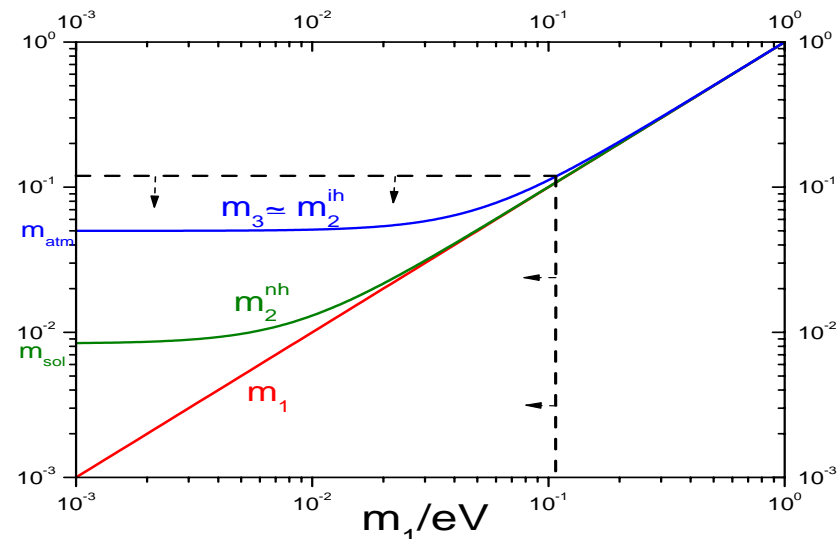
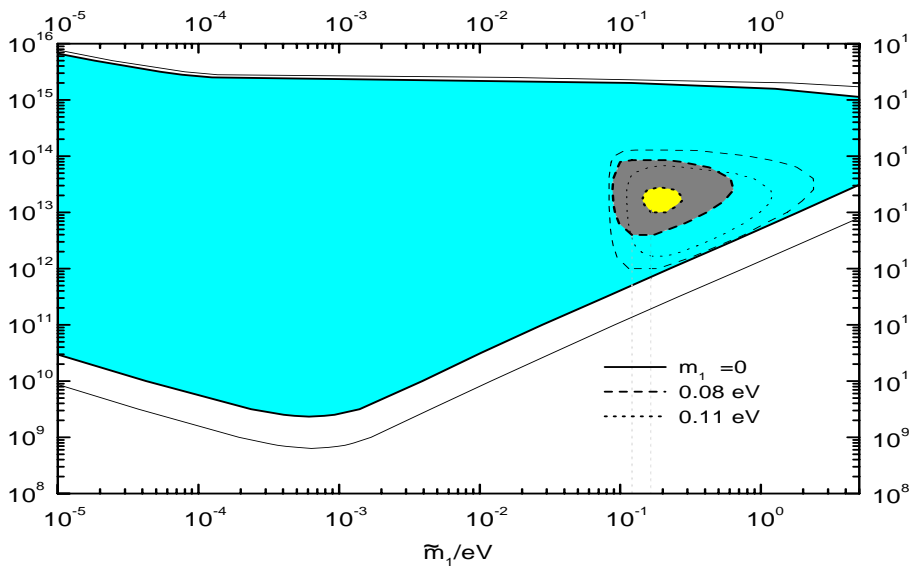
$\Rightarrow$

$$m_i < \left( \frac{10^{25} g_{\text{rec}} a_{\text{sph}} \pi^6 v^4}{3^{9/2} e M_{\text{Pl}}^2 \text{GeV}^2} \right)^{1/4} \text{eV} \simeq 0.12 \text{eV}$$

# Upper bound on the absolute neutrino mass scale

(Buchmüller,PDB,Plümacher '02,'03,'04)

$$\eta_B^{\max}(m_1, \tilde{m}_1, M_1) \simeq 10^{-2} \varepsilon_1^{\max}(M_1) \frac{m_{\text{atm}}}{m_3 + m_1} f(m_1, \tilde{m}_1) \kappa_f(\tilde{m}_1) e^{-w M_1} \sum_i m_i^2 \geq \eta_B^{\text{CMB}}$$



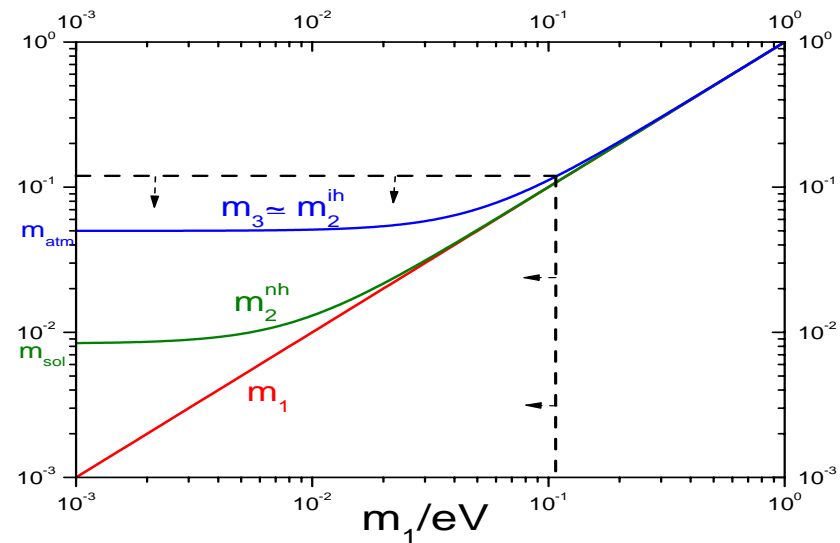
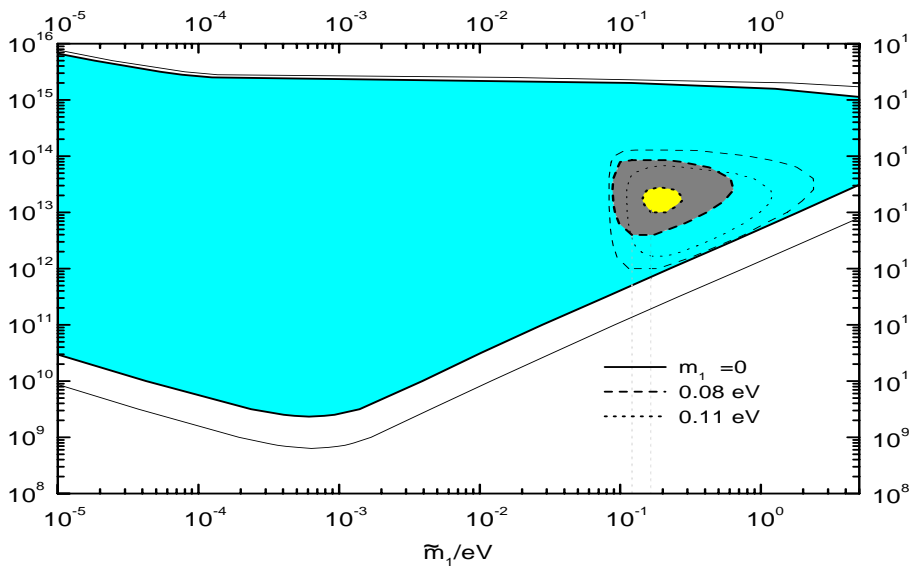
Analytically one can maximize  $\eta_B^{\max}$  and using  $f \simeq f_{hn}$  and central values of  $\eta_B^{\text{CMB}}$ ,  $m_{\text{atm}}$

$$\Rightarrow m_i < \left( \frac{10^{25} g_{\text{rec}} a_{\text{sph}} \pi^6 v^4}{3^{9/2} e M_{\text{Pl}}^2 \text{GeV}^2} \right)^{1/4} \text{eV} \simeq 0.12 \text{eV}$$

# Upper bound on the absolute neutrino mass scale

(Buchmüller, PDB, Plümacher '02,'03,'04)

$$\eta_B^{\max}(m_1, \tilde{m}_1, M_1) \simeq 10^{-2} \varepsilon_1^{\max}(M_1) \frac{m_{\text{atm}}}{m_3 + m_1} f(m_1, \tilde{m}_1) \kappa_f(\tilde{m}_1) e^{-w M_1} \sum_i m_i^2 \geq \eta_B^{\text{CMB}}$$



Analytically one can maximize  $\eta_B^{\max}$  and using

$f \simeq f_{hn}$  and central values of  $\eta_B^{\text{CMB}}$ ,  $m_{\text{atm}}$

$\Rightarrow$

$$m_i < \left( \frac{10^{25} g_{\text{rec}} a_{\text{sph}} \pi^6 v^4}{3^{9/2} e M_{\text{Pl}}^2 \text{GeV}^2} \right)^{1/4} \text{eV} \simeq 0.12 \text{eV}$$

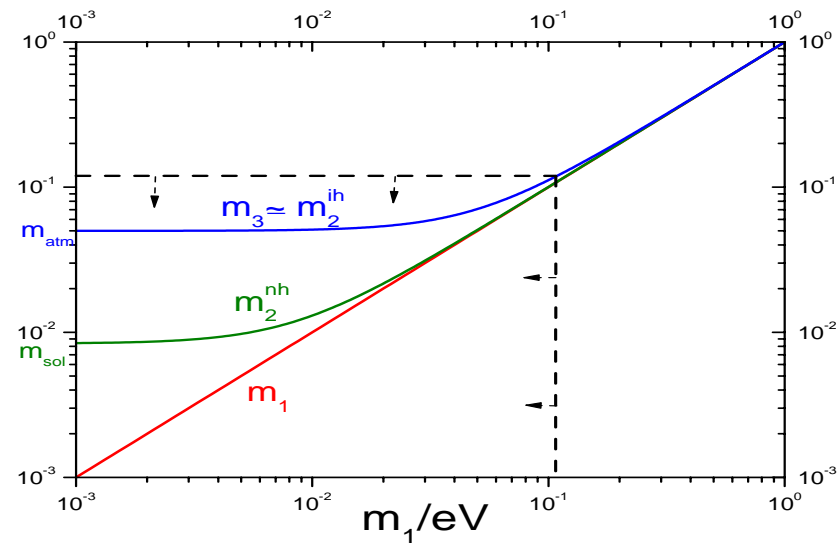
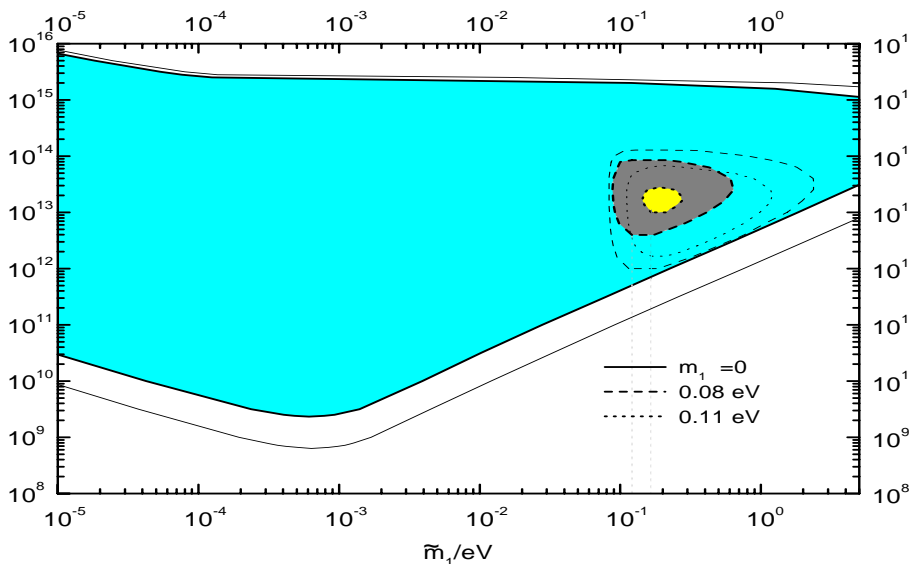
accounting for : (i) more precise bound on  $f$  (Hambye et al. 04), (ii) statistical errors ( $3\sigma$  bound),

(iii) running of neutrino masses (Antusch et al'03)  $\Rightarrow m_i \leq 0.12 \text{ eV}$

# Upper bound on the absolute neutrino mass scale

(Buchmüller,PDB,Plümacher '02,'03,'04)

$$\eta_B^{\max}(m_1, \tilde{m}_1, M_1) \simeq 10^{-2} \varepsilon_1^{\max}(M_1) \frac{m_{\text{atm}}}{m_3 + m_1} f(m_1, \tilde{m}_1) \kappa_f(\tilde{m}_1) e^{-w M_1} \sum_i m_i^2 \geq \eta_B^{\text{CMB}}$$



Analytically one can maximize  $\eta_B^{\max}$  and using

$f \simeq f_{hn}$  and central values of  $\eta_B^{\text{CMB}}$ ,  $m_{\text{atm}}$

$\Rightarrow$

$$m_i < \left( \frac{10^{25} g_{\text{rec}} a_{\text{sph}} \pi^6 v^4}{3^{9/2} e M_{\text{Pl}}^2 \text{GeV}^2} \right)^{1/4} \text{eV} \simeq 0.12 \text{eV}$$

accounting for : (i) more precise bound on  $f$  (Hambye et al. 04), (ii) statistical errors ( $3\sigma$  bound),

(iii) running of neutrino masses (Antusch et al'03)  $\Rightarrow m_i \leq (0.12 \pm 0.03) \text{eV} \leq 0.1 \text{eV}$

# **Stability of neutrino mass bounds**

(Buchmuller,PDB,Plumacher '03)



## Stability of neutrino mass bounds

(Buchmuller,PDB,Plumacher '03)

Suppose that:

$$\eta_B^{\max} \longrightarrow \xi \eta_B^{\max}$$

How the bounds change ?

## Stability of neutrino mass bounds

(Buchmuller,PDB,Plumacher '03)

Suppose that:

$$\eta_B^{\max} \longrightarrow \xi \eta_B^{\max}$$

How the bounds change ?

$$M_1^{\min}, T_i^{\min} \longrightarrow \frac{M_1^{\min}, T_i^{\min}}{\xi}$$

## Stability of neutrino mass bounds

(Buchmuller,PDB,Plumacher '03)

Suppose that:

$$\eta_B^{\max} \longrightarrow \xi \eta_B^{\max}$$

How the bounds change ?

$$M_1^{\min}, T_i^{\min} \longrightarrow \frac{M_1^{\min}, T_i^{\min}}{\xi}$$

$$m_1^{\text{bound}} \longrightarrow m_1^{\text{bound}} \xi^{1/4}$$

## Stability of neutrino mass bounds

(Buchmuller,PDB,Plumacher '03)

Suppose that:

$$\eta_B^{\max} \longrightarrow \xi \eta_B^{\max}$$

How the bounds change ?

$$M_1^{\min}, T_i^{\min} \longrightarrow \frac{M_1^{\min}, T_i^{\min}}{\xi}$$

$$m_1^{\text{bound}} \longrightarrow m_1^{\text{bound}} \xi^{1/4}$$

The **lower bound on the RH neutrino mass** is much more **sensitive** to some **variation** than the upper bound on the light neutrino masses

## The supersymmetric (MSSM) case

(Davidson et al. '92; Covi,Roulet,Vissani '96; Plumacher '97; Giudice et al. '03; PDB '04)

1.  $N_1 \longrightarrow N_1, \tilde{N}_1^c$
2.  $N_\gamma^{\text{rec}} \longrightarrow \sim 2 N_\gamma^{\text{rec}}$
3.  $\varepsilon_1^{\text{max}} \longrightarrow 2 \varepsilon_1^{\text{max}}$
4.  $g_\star \longrightarrow 2 g_\star \Rightarrow H(1) \longrightarrow \sqrt{2} H(1)$
5.  $\Gamma_D^{\text{rest}} \longrightarrow 2 \Gamma_D^{\text{rest}}$
6.  $\Gamma_{\Delta L=2} \longrightarrow 5 (\Gamma_{\Delta L=2})/3$
7. running of neutrino masses halved

## The supersymmetric (MSSM) case

(Davidson et al. '92; Covi,Roulet,Vissani '96; Plumacher '97; Giudice et al. '03; PDB '04)

1.  $N_1 \longrightarrow N_1, \tilde{N}_1^c$
2.  $N_\gamma^{\text{rec}} \longrightarrow \sim 2 N_\gamma^{\text{rec}}$
3.  $\varepsilon_1^{\text{max}} \longrightarrow 2 \varepsilon_1^{\text{max}}$
4.  $g_\star \longrightarrow 2 g_\star \Rightarrow H(1) \longrightarrow \sqrt{2} H(1)$
5.  $\Gamma_D^{\text{rest}} \longrightarrow 2 \Gamma_D^{\text{rest}}$
6.  $\Gamma_{\Delta L=2} \longrightarrow 5 (\Gamma_{\Delta L=2})/3$
7. running of neutrino masses halved

$$1 + 2 + 3 + 4 + 5 \Rightarrow (T_{\text{reh}}^{\text{min}})^{\text{MSSM}} \simeq (1/\sqrt{2}) (T_{\text{reh}}^{\text{min}})^{\text{SM}} \simeq 1.5 \times 10^9 \text{ GeV}$$

## The supersymmetric (MSSM) case

(Davidson et al. '92; Covi,Roulet,Vissani '96; Plumacher '97; Giudice et al. '03; PDB '04)

1.  $N_1 \longrightarrow N_1, \tilde{N}_1^c$
2.  $N_\gamma^{\text{rec}} \longrightarrow \sim 2 N_\gamma^{\text{rec}}$
3.  $\varepsilon_1^{\text{max}} \longrightarrow 2 \varepsilon_1^{\text{max}}$
4.  $g_\star \longrightarrow 2 g_\star \Rightarrow H(1) \longrightarrow \sqrt{2} H(1)$
5.  $\Gamma_D^{\text{rest}} \longrightarrow 2 \Gamma_D^{\text{rest}}$
6.  $\Gamma_{\Delta L=2} \longrightarrow 5 (\Gamma_{\Delta L=2})/3$
7. running of neutrino masses halved

$$1 + 2 + 3 + 4 + 5 \Rightarrow (T_{\text{reh}}^{\text{min}})^{\text{MSSM}} \simeq (1/\sqrt{2}) (T_{\text{reh}}^{\text{min}})^{\text{SM}} \simeq 1.5 \times 10^9 \text{ GeV}$$

tension with the upper bound on  $T_{\text{reh}}$  from gravitino problem

$\Rightarrow$  motivation for models of **non thermal leptogenesis**

(Lazarides,Shafi'91;Murayama,Yanagida'94; Giudice,Peloso,Riotto,Tkachev'99)

## The supersymmetric (MSSM) case

(Davidson et al. '92; Covi,Roulet,Vissani '96; Plumacher '97; Giudice et al. '03; PDB '04)

1.  $N_1 \longrightarrow N_1, \tilde{N}_1^c$
2.  $N_\gamma^{\text{rec}} \longrightarrow \sim 2 N_\gamma^{\text{rec}}$
3.  $\varepsilon_1^{\text{max}} \longrightarrow 2 \varepsilon_1^{\text{max}}$
4.  $g_\star \longrightarrow 2 g_\star \Rightarrow H(1) \longrightarrow \sqrt{2} H(1)$
5.  $\Gamma_D^{\text{rest}} \longrightarrow 2 \Gamma_D^{\text{rest}}$
6.  $\Gamma_{\Delta L=2} \longrightarrow 5 (\Gamma_{\Delta L=2})/3$
7. running of neutrino masses halved

$$1 + 2 + 3 + 4 + 5 \Rightarrow (T_{\text{reh}}^{\text{min}})^{\text{MSSM}} \simeq (1/\sqrt{2}) (T_{\text{reh}}^{\text{min}})^{\text{SM}} \simeq 1.5 \times 10^9 \text{ GeV}$$

tension with the upper bound on  $T_{\text{reh}}$  from gravitino problem

$\Rightarrow$  motivation for models of **non thermal leptogenesis**

(Lazarides,Shafi'91;Murayama,Yanagida'94; Giudice,Peloso,Riotto,Tkachev'99)

$$1+2+3+4+5+6+7 \Rightarrow m_i^{\text{MSSM}} < 0.15 \text{ eV}$$



## Phase suppression

$$\sin \delta_L = \frac{\rho_3 \sin \varphi_3 + (m_{\text{sol}}^2/m_{\text{atm}}^2) \rho_2 \sin \varphi_2}{\rho_3 + (m_{\text{sol}}/m_{\text{atm}}) \rho_2} \Rightarrow M_1 \gtrsim \frac{4 \times 10^8 \text{ GeV}}{\kappa_f \sin \delta_L}$$

- $\sin \delta_L = 1 \Rightarrow$

$$\rho_2 = 0, \text{Re}[\Omega_{11}^2] \simeq 1, \phi_3 = \frac{\pi}{2}$$

$m_1$  is dominated by  $N_1$

- models with  $m_3$  dominated by  $N_1$ :

$$\tilde{m}_1 \geq m_{\text{atm}}, \sin \delta_L < 1:$$

$$M_1 \gtrsim 1.5 \times 10^{11} \text{ GeV},$$

$$T_{\text{reh}} \gtrsim 2 \times 10^{10} \text{ GeV};$$

- models with  $m_2$  dominated by  $N_1$ :

$$\tilde{m}_1 \geq m_{\text{sol}}, \sin \delta_L \lesssim m_{\text{sol}}/m_{\text{atm}}$$

$$M_1 \gtrsim 4 \times 10^{10} \text{ GeV},$$

$$T_{\text{reh}} \gtrsim 7 \times 10^9 \text{ GeV};$$

## Phase suppression

$$\sin \delta_L = \frac{\rho_3 \sin \varphi_3 + (m_{\text{sol}}^2/m_{\text{atm}}^2) \rho_2 \sin \varphi_2}{\rho_3 + (m_{\text{sol}}/m_{\text{atm}}) \rho_2} \Rightarrow M_1 \gtrsim \frac{4 \times 10^8 \text{ GeV}}{\kappa_f \sin \delta_L}$$

- $\sin \delta_L = 1 \Rightarrow$

$$\rho_2 = 0, \text{Re}[\Omega_{11}^2] \simeq 1, \phi_3 = \frac{\pi}{2}$$

$m_1$  is dominated by  $N_1$

- models with  $m_3$  dominated by  $N_1$ :

$$\tilde{m}_1 \geq m_{\text{atm}}, \sin \delta_L < 1:$$

$$M_1 \gtrsim 1.5 \times 10^{11} \text{ GeV},$$

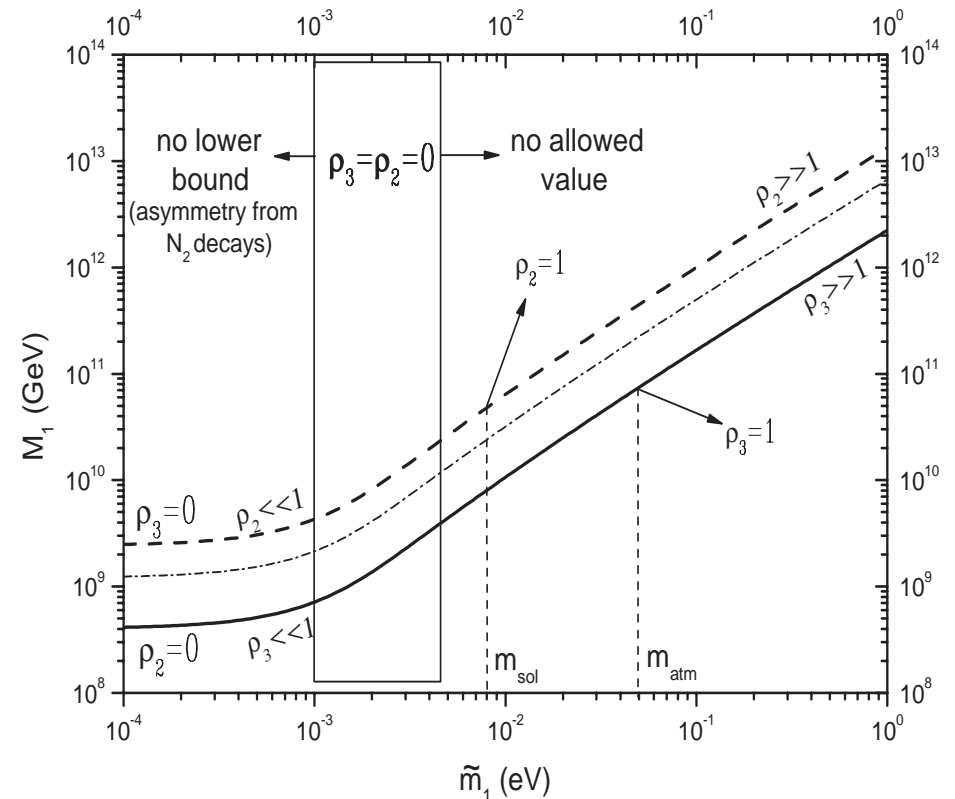
$$T_{\text{reh}} \gtrsim 2 \times 10^{10} \text{ GeV};$$

- models with  $m_2$  dominated by  $N_1$ :

$$\tilde{m}_1 \geq m_{\text{sol}}, \sin \delta_L \lesssim m_{\text{sol}}/m_{\text{atm}}$$

$$M_1 \gtrsim 4 \times 10^{10} \text{ GeV},$$

$$T_{\text{reh}} \gtrsim 7 \times 10^9 \text{ GeV};$$



## A new scenario of thermal leptogenesis

$$\Omega \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \Omega_{22} & \sqrt{1 - \Omega_{22}^2} \\ 0 & -\sqrt{1 - \Omega_{22}^2} & \Omega_{22} \end{pmatrix} \Rightarrow \tilde{m}_1 = m_1, \varepsilon_1 = 0 \text{ but...}$$

## A new scenario of thermal leptogenesis

$$\Omega \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \Omega_{22} & \sqrt{1 - \Omega_{22}^2} \\ 0 & -\sqrt{1 - \Omega_{22}^2} & \Omega_{22} \end{pmatrix} \Rightarrow \tilde{m}_1 = m_1, \varepsilon_1 = 0 \text{ but...}$$

$$\varepsilon_2 = \varepsilon_{\max}(M_2) \frac{m_3 - m_2}{m_{\text{atm}}} f(m_2, \tilde{m}_2) \sin \delta_L^{(2)}$$

## A new scenario of thermal leptogenesis

$$\Omega \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \Omega_{22} & \sqrt{1 - \Omega_{22}^2} \\ 0 & -\sqrt{1 - \Omega_{22}^2} & \Omega_{22} \end{pmatrix} \Rightarrow \tilde{m}_1 = m_1, \varepsilon_1 = 0 \text{ but...}$$

$$\varepsilon_2 = \varepsilon_{\max}(M_2) \frac{m_3 - m_2}{m_{\text{atm}}} f(m_2, \tilde{m}_2) \sin \delta_L^{(2)}$$

The two problems with  $\tilde{m}_1 \ll m_*$  are solved:

- **no fine-tuning:**  $m_* \gg \tilde{m}_1 \geq m_1$
- $\tilde{m}_2 \geq m_{\text{sol}} \gg m_*$ : **strong wash-out !**

## A new scenario of thermal leptogenesis

$$\Omega \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \Omega_{22} & \sqrt{1 - \Omega_{22}^2} \\ 0 & -\sqrt{1 - \Omega_{22}^2} & \Omega_{22} \end{pmatrix} \Rightarrow \tilde{m}_1 = m_1, \varepsilon_1 = 0 \text{ but...}$$

$$\varepsilon_2 = \varepsilon_{\max}(M_2) \frac{m_3 - m_2}{m_{\text{atm}}} f(m_2, \tilde{m}_2) \sin \delta_L^{(2)}$$

The two problems with  $\tilde{m}_1 \ll m_*$  are solved:

- **no fine-tuning**:  $m_* \gg \tilde{m}_1 \geq m_1$
- $\tilde{m}_2 \geq m_{\text{sol}} \gg m_*$ : **strong wash-out !**
- ... **no lower bound on  $M_1$  !** But ...

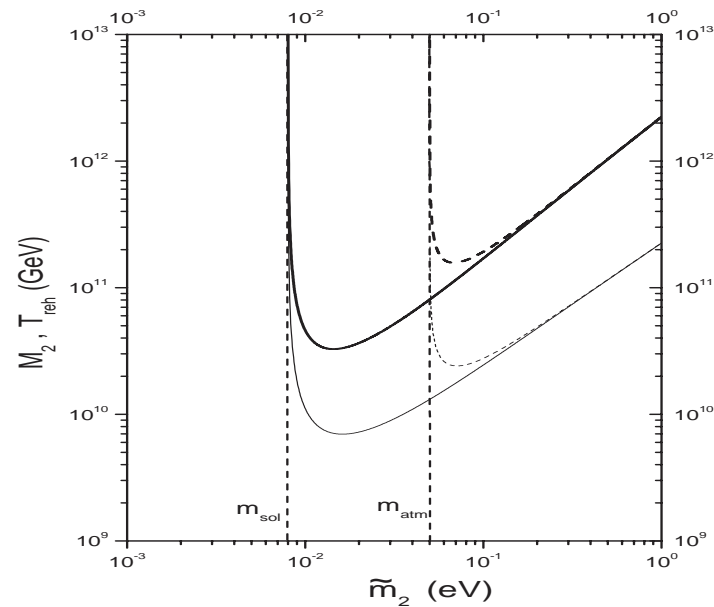
## A new scenario of thermal leptogenesis

$$\Omega \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \Omega_{22} & \sqrt{1 - \Omega_{22}^2} \\ 0 & -\sqrt{1 - \Omega_{22}^2} & \Omega_{22} \end{pmatrix} \Rightarrow \tilde{m}_1 = m_1, \varepsilon_1 = 0 \text{ but...}$$

$$\varepsilon_2 = \varepsilon_{\max}(M_2) \frac{m_3 - m_2}{m_{\text{atm}}} f(m_2, \tilde{m}_2) \sin \delta_L^{(2)}$$

The two problems with  $\tilde{m}_1 \ll m_*$  are solved:

- **no fine-tuning:**  $m_* \gg \tilde{m}_1 \geq m_1$
- $\tilde{m}_2 \geq m_{\text{sol}} \gg m_*$ : **strong wash-out !**
- ... **no lower bound on  $M_1$  !** But ...



## Beyond a hierarchical heavy neutrino spectrum

Two conditions for the validity of the hierarchical heavy neutrino (below 10%):

$$1) \frac{M_2}{M_1} \geq \frac{z_B(K_2)+2}{z_B(K_1)-2} \gtrsim 5 \quad 2) \xi_\varepsilon \equiv \frac{\varepsilon_1}{\varepsilon_{\max}(M_1)} \lesssim 0.1$$



## Beyond a hierarchical heavy neutrino spectrum

Two conditions for the validity of the hierarchical heavy neutrino (below 10%):

$$1) \frac{M_2}{M_1} \geq \frac{z_B(K_2)+2}{z_B(K_1)-2} \gtrsim 5 \quad 2) \xi_\varepsilon \equiv \frac{\varepsilon_1}{\varepsilon_{\max}(M_1)} \lesssim 0.1$$

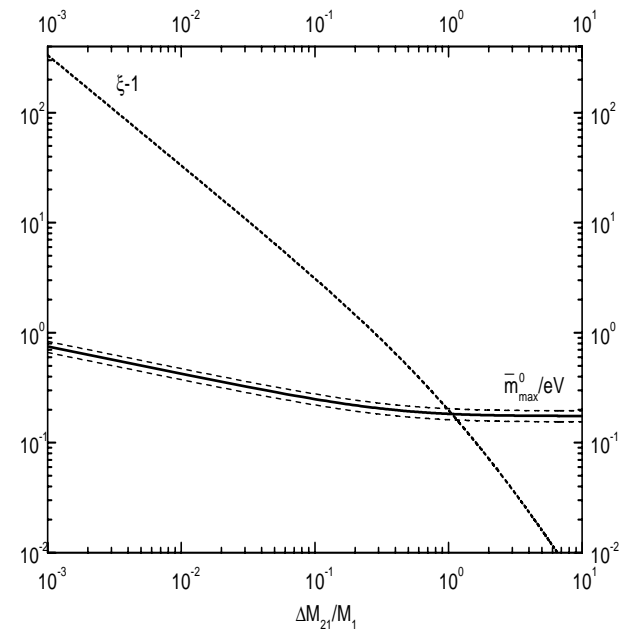
$$\xi_\varepsilon = \xi \left( \frac{\Delta_{21}}{M_1} \right) \beta(m_1, \tilde{m}_1) \sin \delta_L + \left[ \xi \left( \frac{\Delta_{31}}{M_1} \right) - \xi \left( \frac{\Delta_{21}}{M_1} \right) \right] \frac{\text{Im}[\sum_h m_h \Omega_{h1}^* \Omega_{h3}]^2}{m_{\text{atm}} \tilde{m}_1}$$

## Beyond a hierarchical heavy neutrino spectrum

Two conditions for the validity of the hierarchical heavy neutrino (below 10%):

$$1) \frac{M_2}{M_1} \geq \frac{z_B(K_2)+2}{z_B(K_1)-2} \gtrsim 5 \quad 2) \xi_\varepsilon \equiv \frac{\varepsilon_1}{\varepsilon_{\max}(M_1)} \lesssim 0.1$$

$$\xi_\varepsilon = \xi \left( \frac{\Delta_{21}}{M_1} \right) \beta(m_1, \tilde{m}_1) \sin \delta_L + \left[ \xi \left( \frac{\Delta_{31}}{M_1} \right) - \xi \left( \frac{\Delta_{21}}{M_1} \right) \right] \frac{\text{Im}[\sum_h m_h \Omega_{h1}^* \Omega_{h3}]^2}{m_{\text{atm}} \tilde{m}_1}$$



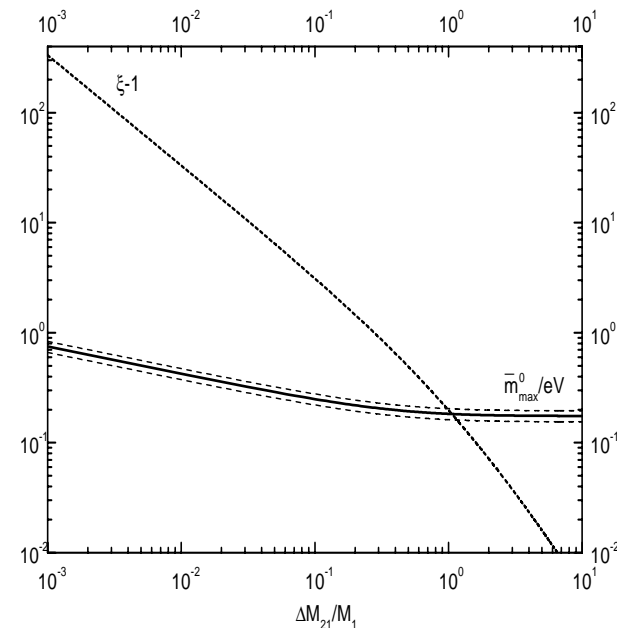
## Beyond a hierarchical heavy neutrino spectrum

Two conditions for the validity of the hierarchical heavy neutrino (below 10%):

$$1) \frac{M_2}{M_1} \geq \frac{z_B(K_2)+2}{z_B(K_1)-2} \gtrsim 5 \quad 2) \xi_\varepsilon \equiv \frac{\varepsilon_1}{\varepsilon_{\max}(M_1)} \lesssim 0.1$$

$$\xi_\varepsilon = \xi \left( \frac{\Delta_{21}}{M_1} \right) \beta(m_1, \tilde{m}_1) \sin \delta_L + \left[ \xi \left( \frac{\Delta_{31}}{M_1} \right) - \xi \left( \frac{\Delta_{21}}{M_1} \right) \right] \frac{\text{Im}[\sum_h m_h \Omega_{h1}^* \Omega_{h3}]^2}{m_{\text{atm}} \tilde{m}_1}$$

- $M_2 \gtrsim 5 M_1 \Rightarrow \xi_\varepsilon \ll 0.1$  unless huge values of the imaginary parts  
i.e. fine-tuned phase cancellations
- second term disappears for  $M_2 = M_3$  ('inverted' heavy neutrino spectrum) maximum for  $M_3 \gg M_1, M_2$  ('normal' h.n.s.)
- the second term can become important for **quasi-degenerate neutrinos**



## 'Degenerate' leptogenesis

- $\varepsilon_1^{\max} \rightarrow \xi_\varepsilon \varepsilon_1 \Rightarrow \eta_B^{\max} \rightarrow \xi_\varepsilon \eta_B^{\max}$
- dependence on the other seesaw parameters  $\Rightarrow$  great model dependence
- extreme situation: **resonant leptogenesis** :almost no bounds at all !

(Pilaftsis, Underwood '04)

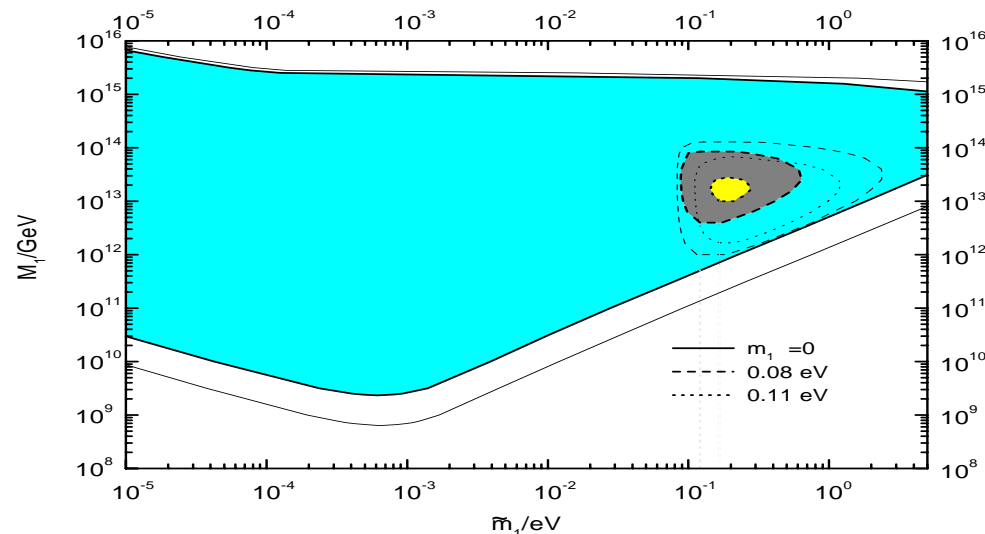
Assuming equal degeneracies of light and heavy neutrinos:

- 'normal' heavy neutrino spectrum  $\Rightarrow m_i^{\text{bound}} \lesssim 0.2 \text{ eV}$
  - 'inverted' heavy neutrino spectrum  $\Rightarrow m_i^{\text{bound}} \lesssim 0.6 \text{ eV}$
- they use the resonant regime !**

(Hambye, Lin, Notari, Papucci, Strumia '04)

## A 'too-short-blanket' problem

(Buchmüller,PDB,Plümacher'03,PDB '04)



For  $T_{\text{reh}}^{\text{max}} \sim (5 \times 10^9 - 10^{12}) \text{ GeV}$  (MSSM case):

$$\frac{m_1}{m_{\text{atm}}} \lesssim \frac{A}{\sqrt{1 + 2A}} \quad \text{with} \quad A \simeq 0.2 \frac{T_{\text{reh}}^{\text{max}}}{10^{10} \text{ GeV}} \quad (M_1^{\text{max}} \simeq 5 T_{\text{reh}}^{\text{max}})$$

Example:  $T_{\text{reh}}(M_1) \lesssim 3 (15) \times 10^{10} \text{ GeV} \Rightarrow m_{1(3)} \lesssim 0.02 (0.055) \text{ eV}$

The assumption of hierarchical heavy neutrino spectrum seems to be reasonable for the most interesting region of the allowed parameter space ! More investigation is needed.

## **Conclusions**

## **Conclusions**

- The minimal model of leptogenesis can explain the matter-anti matter asymmetry: this is a not trivial result given the current information from neutrino mixing data;

## Conclusions

- The minimal model of leptogenesis can explain the matter-anti matter asymmetry: this is a not trivial result given the current information from neutrino mixing data;
- Well known problem with a minimum allowed value of  $M_1$  and of  $T_{\text{reh}}$ ;



## Conclusions

- The minimal model of leptogenesis can explain the matter-anti matter asymmetry: this is a not trivial result given the current information from neutrino mixing data;
- Well known problem with a minimum allowed value of  $M_1$  and of  $T_{\text{reh}}$ ;
- The determination of the absolute neutrino mass scale will be an important test for leptogenesis: if  $m_1 \gtrsim 0.1 \text{ eV}$  it is necessary to go beyond the minimal set of assumptions;

## Conclusions

- The minimal model of leptogenesis can explain the matter-anti matter asymmetry: this is a not trivial result given the current information from neutrino mixing data;
- Well known problem with a minimum allowed value of  $M_1$  and of  $T_{\text{reh}}$ ;
- The determination of the absolute neutrino mass scale will be an important test for leptogenesis: if  $m_1 \gtrsim 0.1 \text{ eV}$  it is necessary to go beyond the minimal set of assumptions;
- **Seesaw geometry**: nice and simple tool to study leptogenesis and make connections with neutrino mass models :

## Conclusions

- The minimal model of leptogenesis can explain the matter-anti matter asymmetry: this is a not trivial result given the current information from neutrino mixing data;
- Well known problem with a minimum allowed value of  $M_1$  and of  $T_{\text{reh}}$ ;
- The determination of the absolute neutrino mass scale will be an important test for leptogenesis: if  $m_1 \gtrsim 0.1 \text{ eV}$  it is necessary to go beyond the minimal set of assumptions;
- **Seesaw geometry**: nice and simple tool to study leptogenesis and make connections with neutrino mass models :
  - $CP$  asymmetry bound for any  $m_1$ ;
  - expression for the effective leptogenesis phase;
  - useful correspondence with neutrino mass models

*IFAE, Catania, March 30 2005*

*Seesaw and Leptogenesis*

- lower bounds on  $M_1$  and on  $T_{\text{reh}}$  more severe for typically considered neutrino SD mass models; models with  $m_1$  dominated by  $M_1$  allow  $T_{\text{reh}} \ll 10^{10}$  GeV but do not have connections with quark masses;

- lower bounds on  $M_1$  and on  $T_{\text{reh}}$  more severe for typically considered neutrino SD mass models; models with  $m_1$  dominated by  $M_1$  allow  $T_{\text{reh}} \ll 10^{10}$  GeV but do not have connections with quark masses;
- new scenario of thermal leptogenesis: asymmetry generated by  $N_2$   
 $\Rightarrow$  no lower bound on  $M_1$ ;

- lower bounds on  $M_1$  and on  $T_{\text{reh}}$  more severe for typically considered neutrino SD mass models; models with  $m_1$  dominated by  $M_1$  allow  $T_{\text{reh}} \ll 10^{10}$  GeV but do not have connections with quark masses;
- new scenario of thermal leptogenesis: asymmetry generated by  $N_2$   
 $\Rightarrow$  no lower bound on  $M_1$ ;
- for  $M_2 \gtrsim 5 M_1$  the assumption of hierarchical heavy neutrinos works very well for hierarchical light neutrinos; the quasi-degenerate case is more subtle.