Shell model for weakly bound and unbound nuclear states: Bound and continuum states in one framework

GANIL - ORNL Theory Collaboration

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Paradigm of nuclear structure theory

Binding energy systematics → 'Magic' numbers of nucleons : 2, 8, 20, 28,…

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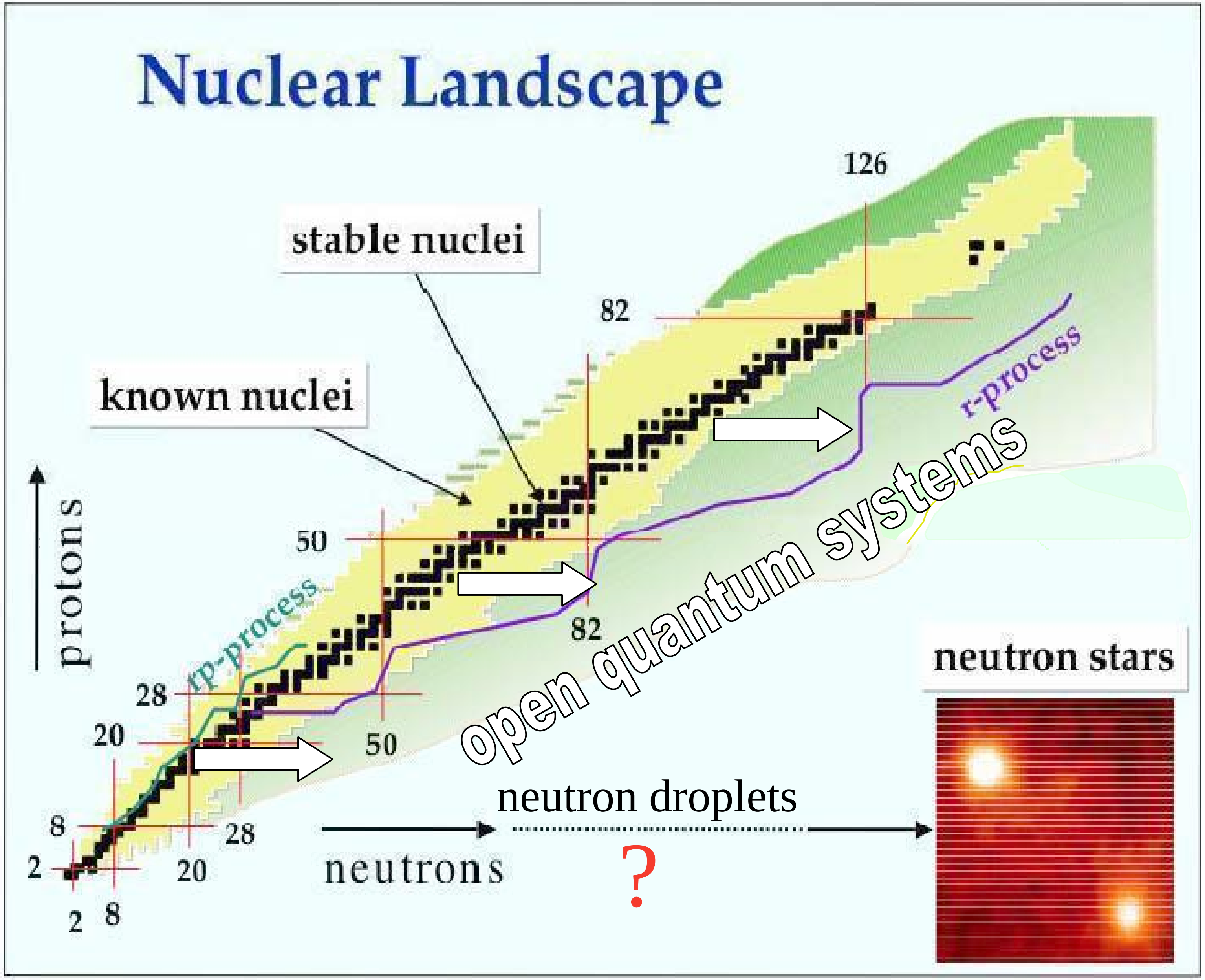
Average one-body potential (1949):
Spherical harmonic oscillator+spin-orbit interaction

How to describe 'non-magic' nuclei? → Effective interactions

Broken symmetry average potential:
Bohr collective Hamiltonian (1952)
Nilsson potential (1955)
… …

Multiconfigurational Shell Model (1953)

Closed quantum many-body systems


Nuclear Landscape
126
stable nuclei
82
known nuclei
r-process
protons
50
open quantum systems
rp-process
82
neutron stars
28
20
50
8
28
2
20
2
8
neutron droplets
neutrons
?

Spectra and matter distribution are modified by the proximity of scattering continuum

New exotic phenomena in weakly bound nuclei:

- continuum anti-odd-even staggering effect
- modification of ‘magic numbers’ and spin-orbit splitting
- halos and correlations, continuum anti-halo effect
- symmetry-breaking effects induced by the coupling to decay channels
- correlated continuum in reactions with multiple weakly bound/unstable subsystems
- influence of the poles of S-matrix on spectra and wave functions (e.g. spectroscopic factors, pair amplitudes, mirror nuclei, etc.)
- new kinds of natural radioactivity (e.g. 2p radioactivity, etc.)
- etc. …..

↓

Open quantum many-body systems

Continuum (real-energy) Shell Model
(1977 - 1999 - 2005)

H.W.Bartz et al, NP A275 (1977) 111
R.J. Philpott, NP A289 (1977) 109
K. Bennaceur et al, NP A651 (1999) 289
J. Rotureau et al, PRL 95 (2005) 042503

Gamow (complex-energy) Shell Model
(2002 -)

N. Michel et al, PRL 89 (2002) 042502
R. Id Betan et al, PRL 89 (2002) 042501
N. Michel et al, PRC 70 (2004) 064311
G. Hagen et al, PRC 71 (2005) 044314

New paradigm is born!

Rigged Hilbert space formulation : Gamow Shell Model (2002)

$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi \quad : \qquad \Psi(0,k) = 0 \;, \quad \Psi(\vec{r},k) \underset{r\to\infty}{\longrightarrow} O_l(kr)$$

outgoing solution

Eigenvalues : $k_n = \sqrt{\frac{2m}{\hbar^2}\left(e_n - i\frac{\Gamma_n}{2}\right)}$ are the poles of the S-matrix :

- Bound states $(k_n = i\kappa_n)$
- Antibound states $(k_n = -i\kappa_n)$
- Resonances $(k_n = \pm\gamma_n - i\kappa_n)$

Completness relation for one-body states:

(T.Berggren (1968))

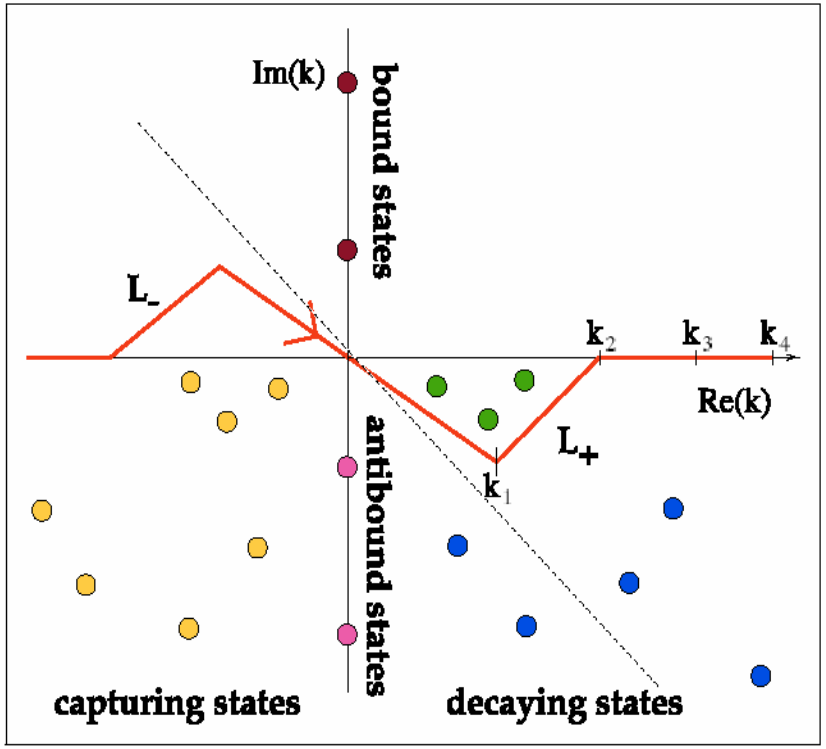
$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1$$

bound, anti-bound, and resonance states

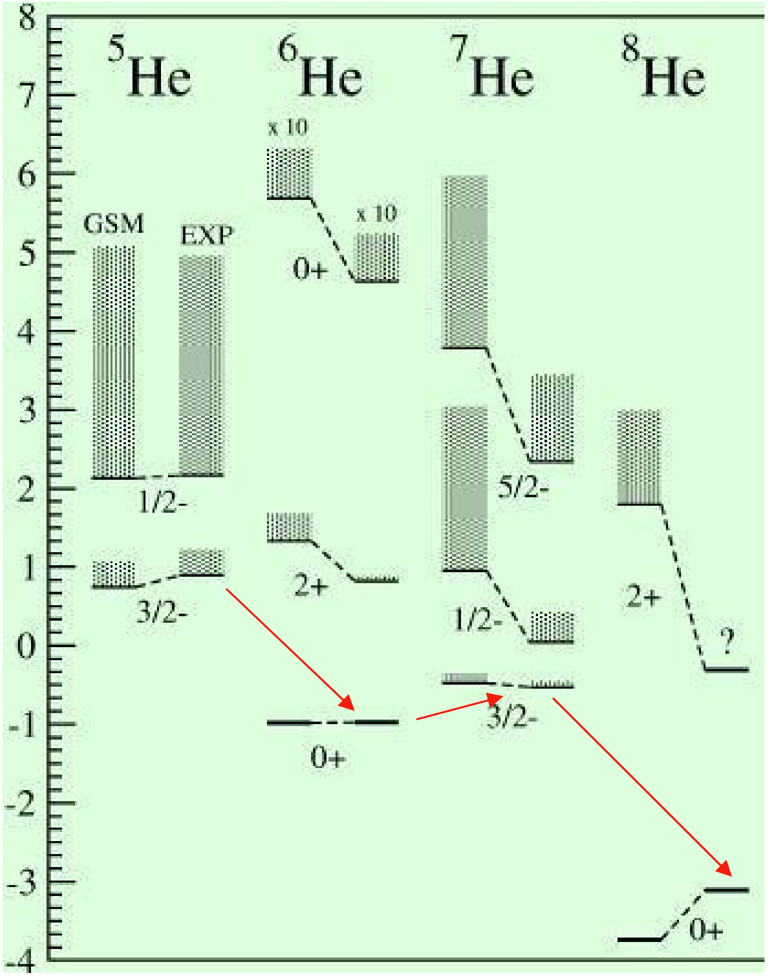
non-resonant continuum

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \sum_{i=1}^{N_d} |u_i\rangle\langle\tilde{u}_i| \cong 1 \;; \quad \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

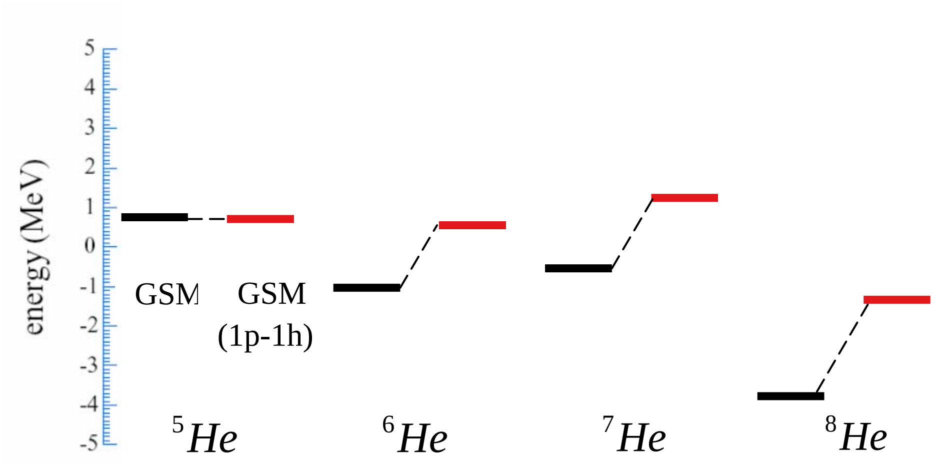
$$\sum_k |SD_k\rangle\langle SD_k| \cong 1$$



complex-symmetric eigenvalue problem for hermitian Hamiltonian



'Helium anomaly'



$H = T + U_{WS} + V_{JT}$ → h_{HF} → s.p. basis (continuum)

Interaction of nucleons in the continuum states is an essential element of binding mechanism in helium isotopes

Hilbert space formulation : Shell Model Embedded in the Continuum (1999)

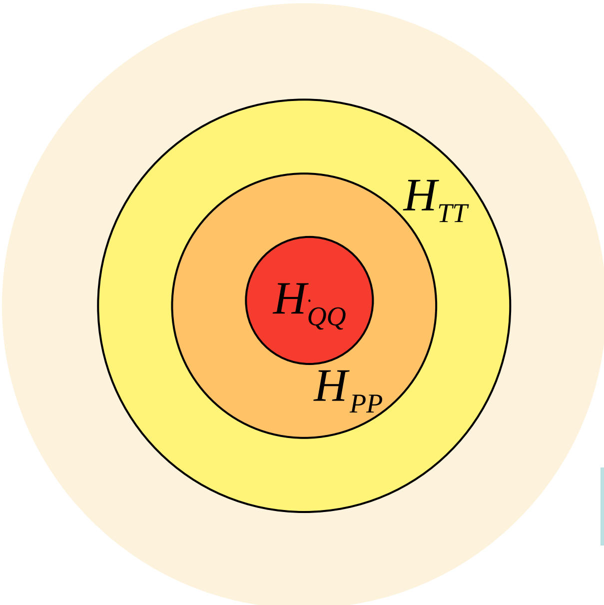
Completeness relation for one-body states:

$$\sum_n |u_n\rangle\langle u_n| + \int_0^{+\infty} |u_k\rangle\langle u_k| dk = 1 \quad \not\Rightarrow \quad \sum_k |SD_k\rangle\langle SD_k| \cong 1$$

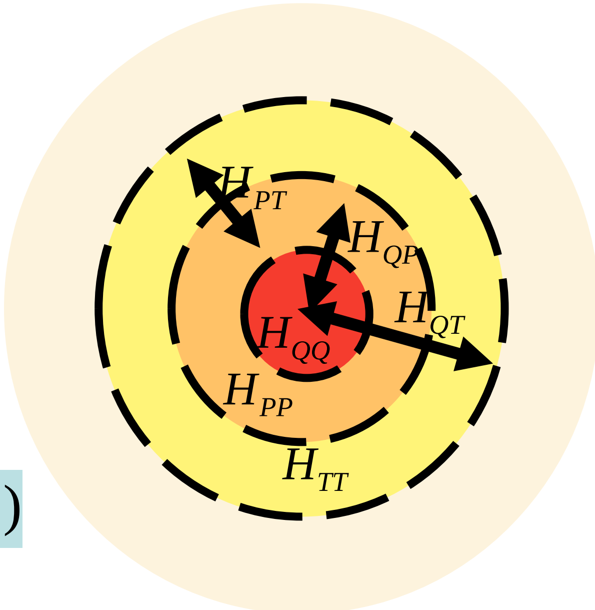
bound states

non-resonant (real) continuum

but… different emission thresholds open at distinctly different energies



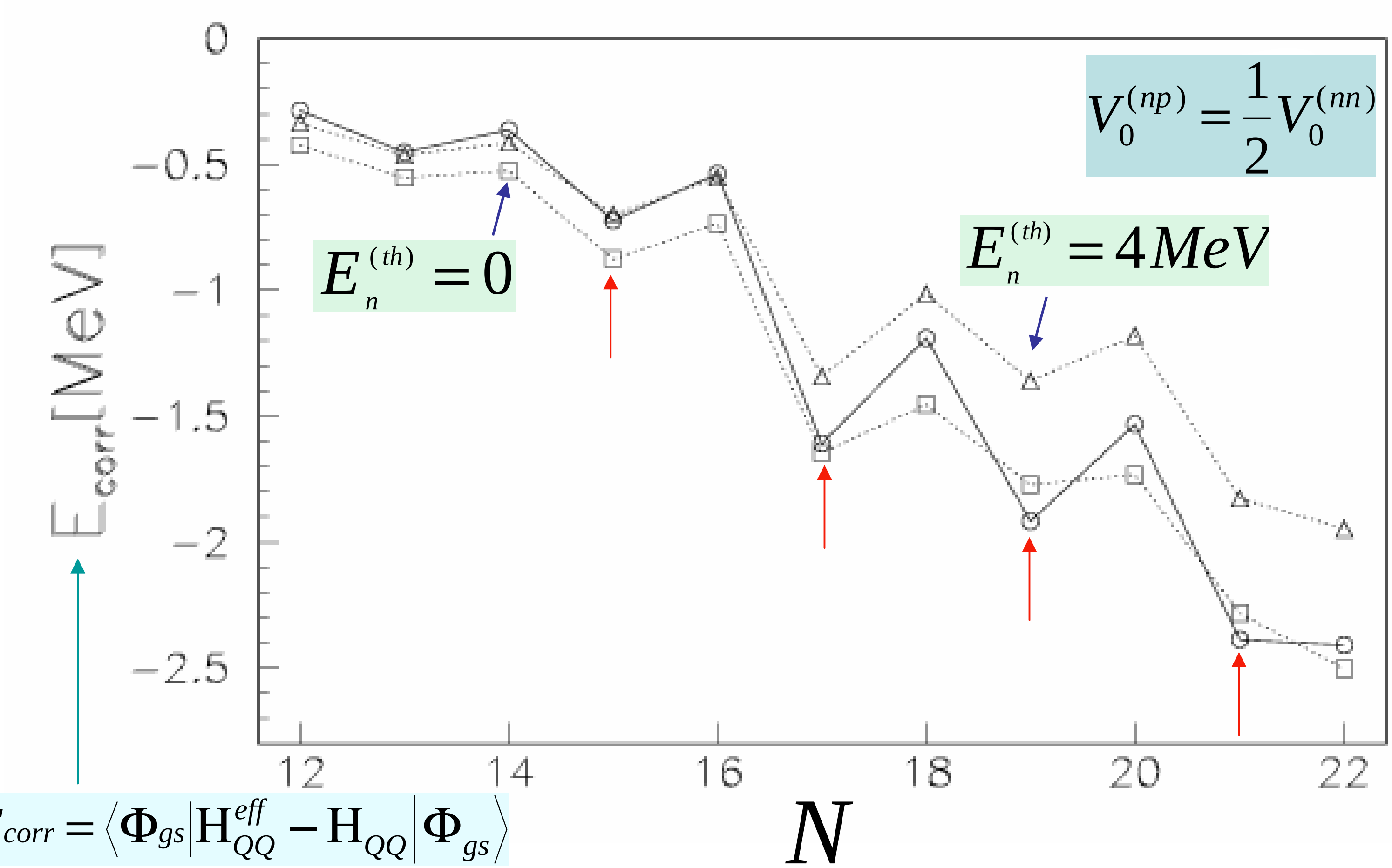
$$Q = [A]$$
$$P = [A-1] \otimes [1]$$
$$T = [A-2] \otimes [2]$$
$$\ldots = \ldots$$
$$Q + P + T + \ldots = 1$$



$$H_{QQ} \rightarrow H_{QQ}^{eff}(E)$$

Incompatible symmetries of H_{QQ} and $H_{QQ}^{eff}(E)$

Reduction of the odd-even staggering close to the drip-line



$E_{corr} = \langle \Phi_{gs} | H_{QQ}^{eff} - H_{QQ} | \Phi_{gs} \rangle$

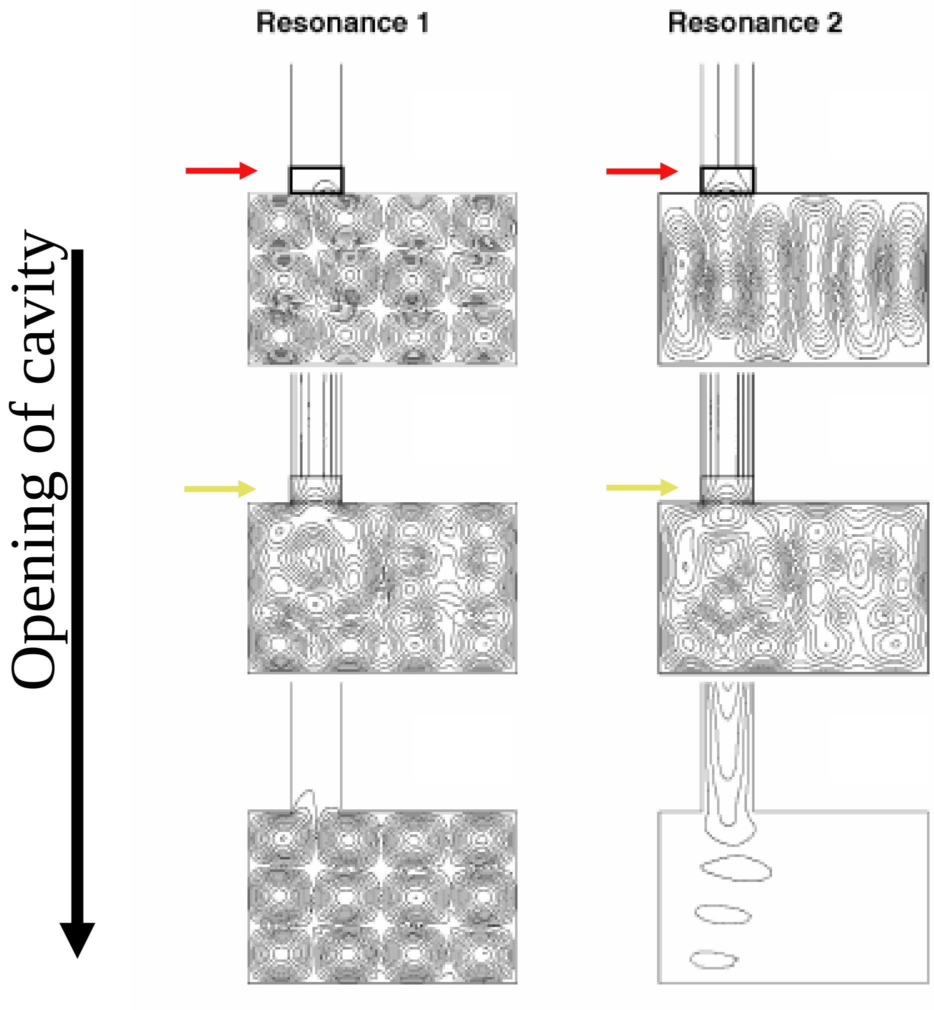
Continuum correction to the binding energy of closed QS favors:

$(\mathrm{odd} - N, \mathrm{odd} - Z)$ - nuclei (np - coupling)

$(\mathrm{even} - N, Z)$ - nuclei (nn - coupling)

$(N, \mathrm{even} - Z)$ - nuclei (pp - coupling)

Continuum couplings are essential in the overlapping regime: failure of RMT!
How important are continuum couplings for isolated states (low-density regime)?



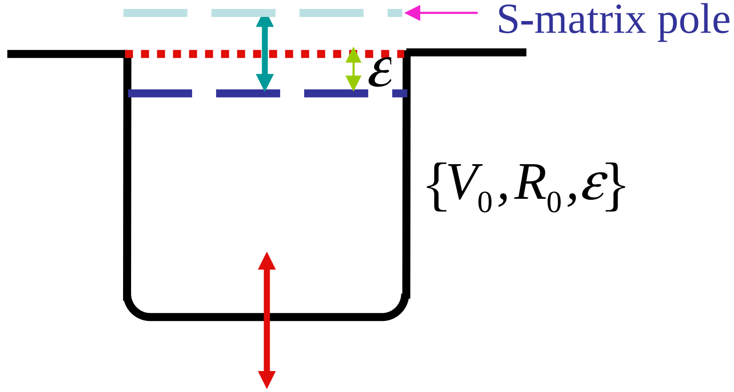
'Perturbative' continuum coupling

'Non-perturbative' continuum coupling

$$E_i^{(corr)}(E,\varepsilon) = \langle \Phi_i | H_{QQ}^{eff}(E,\varepsilon) - H_{QQ} | \Phi_i \rangle$$

total energy

position of S-matrix pole

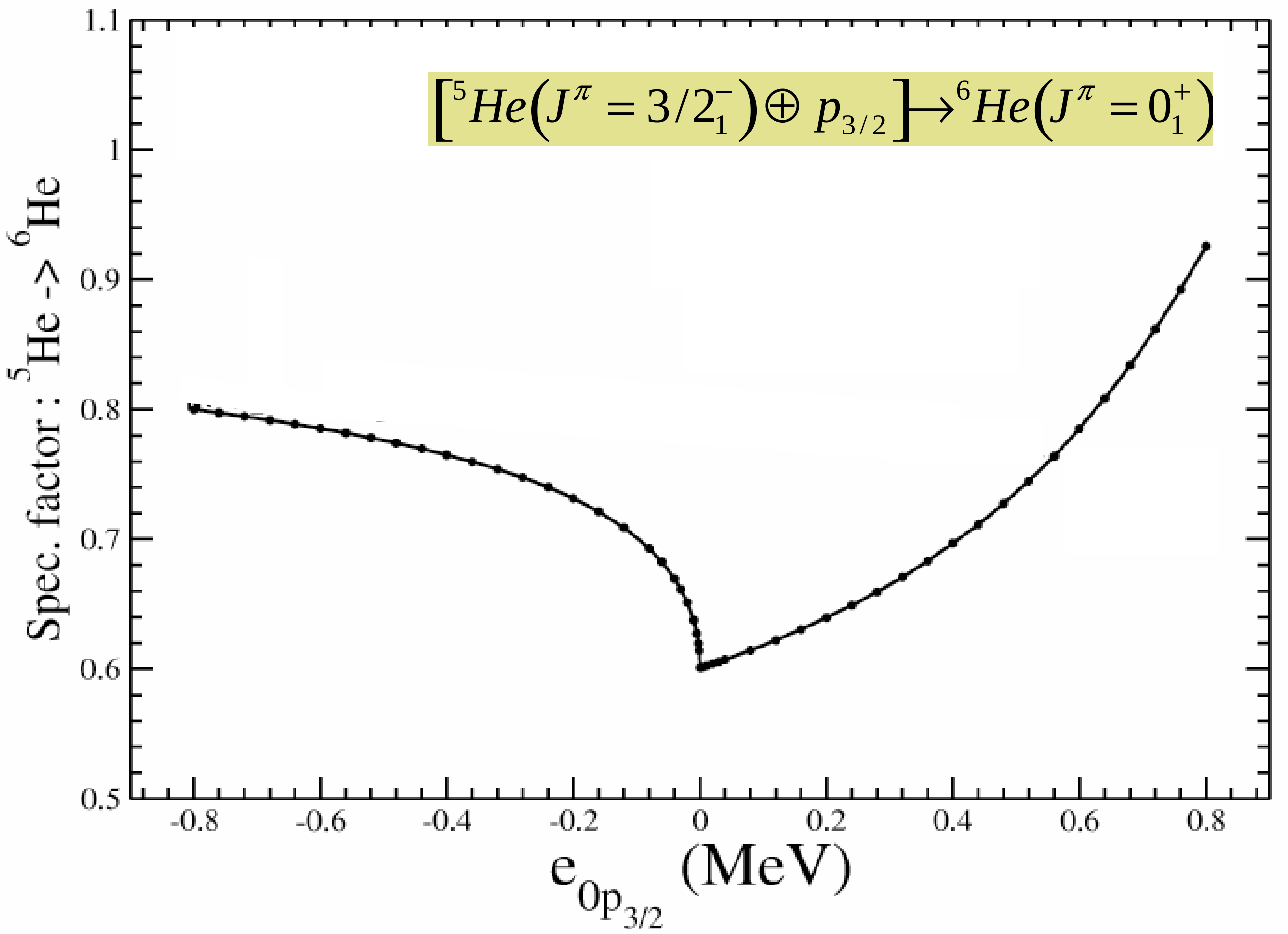


$$E_\ell^{(corr)}(E=0,\varepsilon) = -const|\varepsilon|^{-1+\ell/2} + O(|\varepsilon|^0)$$

Singularity for $\ell = 0,1$ poles of the S-matrix!

'Non-perturbative' continuum coupling: instability of the Q subspace

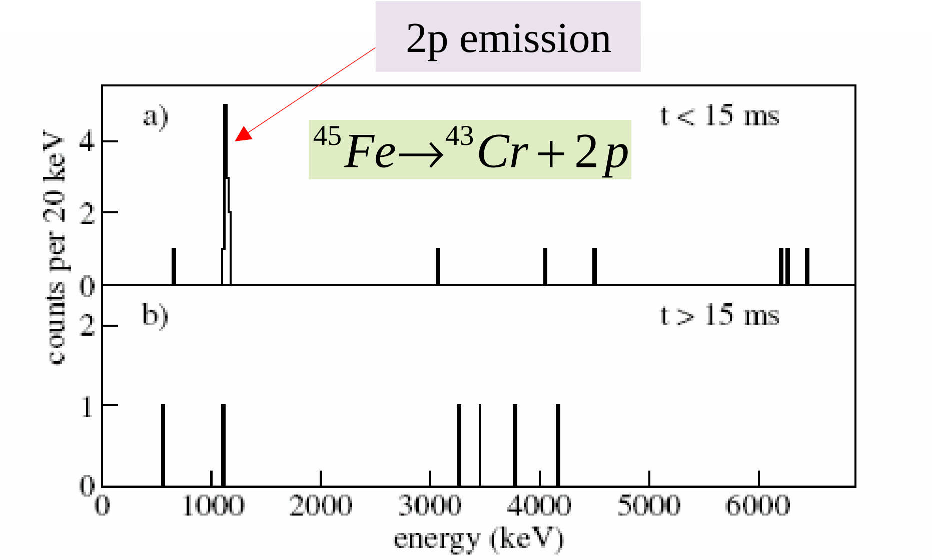
Spectroscopic factors



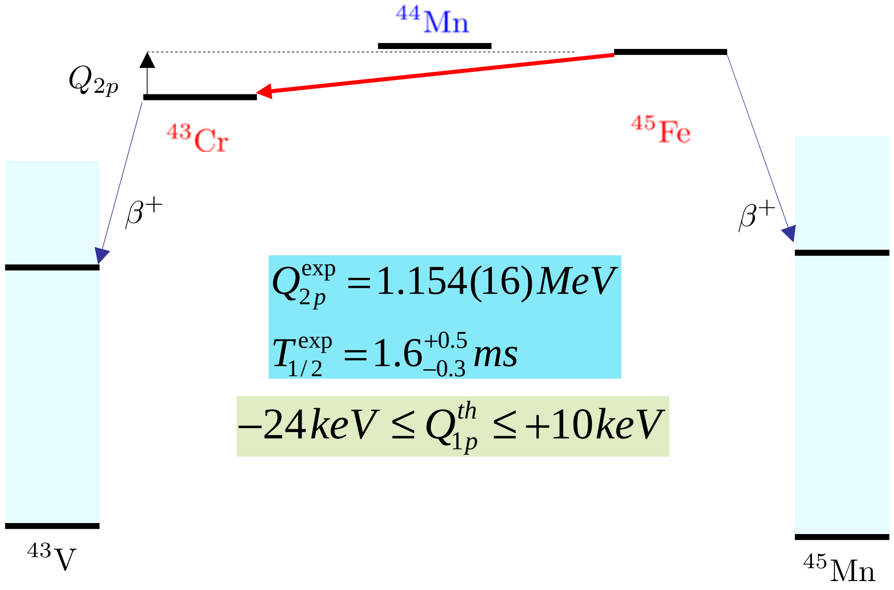
$$S_{(lj)}=\frac{1}{2J_A+1}\left[\sum_{n\in(b,r)}\left\langle\Psi_A\left\|a^{+}(n;lj)\right\|\Psi_{A-1}\right\rangle^2+\underbrace{\int_{L^+}\left\langle\Psi_A\left\|a^{+}(k;lj)\right\|\Psi_{A-1}\right\rangle^2 dk}_{\text{continuum contribution}}\right]$$

2p decay from the ground state of ^{45}Fe, ^{48}Ni and ^{54}Zn

M. Pfutzner et al, Eur. Phys. J. A14 (2002) 279; J. Giovinazzo et al, Phys. Rev. Lett. 89 (2002) 102501



$$^{45}Fe \rightarrow {}^{43}Cr + 2p$$

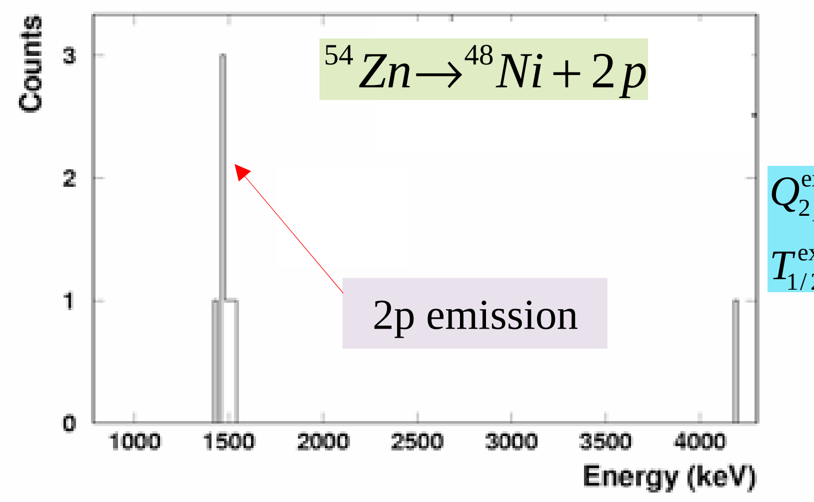


$$Q_{2p}^{exp} = 1.154(16) MeV$$

$$T_{1/2}^{exp} = 1.6_{-0.3}^{+0.5} ms$$

$$-24 keV \le Q_{1p}^{th} \le +10 keV$$

B. Blank et al, Phys. Rev. Lett. 94 (2005) 232501



$$^{54}Zn \rightarrow {}^{48}Ni + 2p$$

$$Q_{2p}^{exp} = 1.48(2) MeV$$

$$T_{1/2}^{exp} = 3.7_{-1.0}^{+2.2} ms$$

Two-proton emission

J. Rotureau et al., PRL 95 (2005) 042503

$$H_{QQ}^{eff}(E) = H_{QQ} + H_{QP}G_P^{(+)}(E)H_{PQ} + [H_{QT} + H_{QP}G_P^{(+)}(E)H_{PT}]\tilde{G}_T^{(+)}(E)[H_{TQ} + H_{TP}G_P^{(+)}(E)H_{PQ}]$$

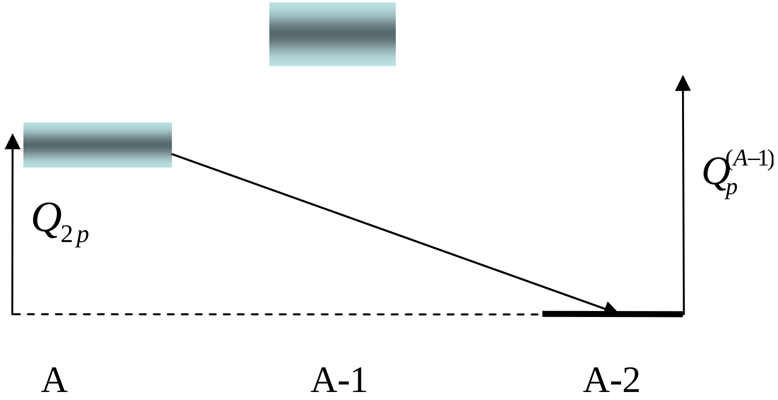
coupling with one particle in the continuum

coupling with two particles in the continuum

I. Direct emission:

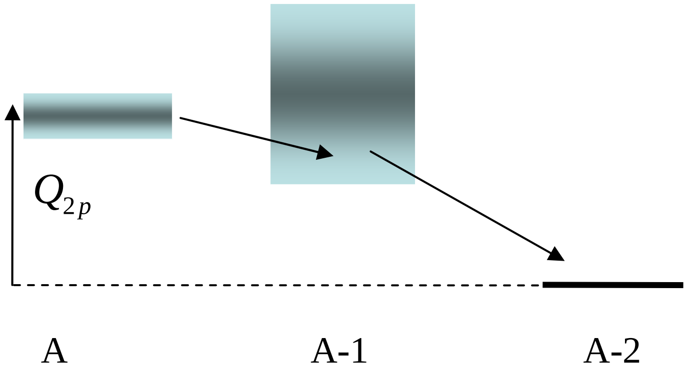
$$\{H_{QP} \ , \ H_{PT}\} \rightarrow 0$$

$$\rightarrow \quad H_{QQ}^{eff}(E) = H_{QQ} + H_{QT}G_T^{(+)}(E)H_{TQ}$$



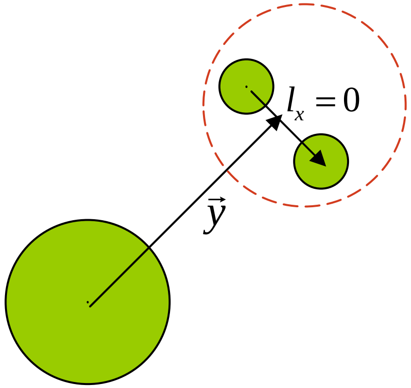
II. Indirect emission :

$$\{H_{QT}\} \rightarrow 0$$



$$\rightarrow \quad H_{QQ}^{eff}(E) = H_{QQ} + H_{QP}G_P^{(+)}(E)H_{PQ} + [H_{QP}G_P^{(+)}(E)H_{PT}]\tilde{G}_T^{(+)}(E)[H_{TP}G_P^{(+)}(E)H_{PQ}]$$

Direct 2p emission



+ final state interaction in terms of $(l_x = 0)$ s-wave phase shift

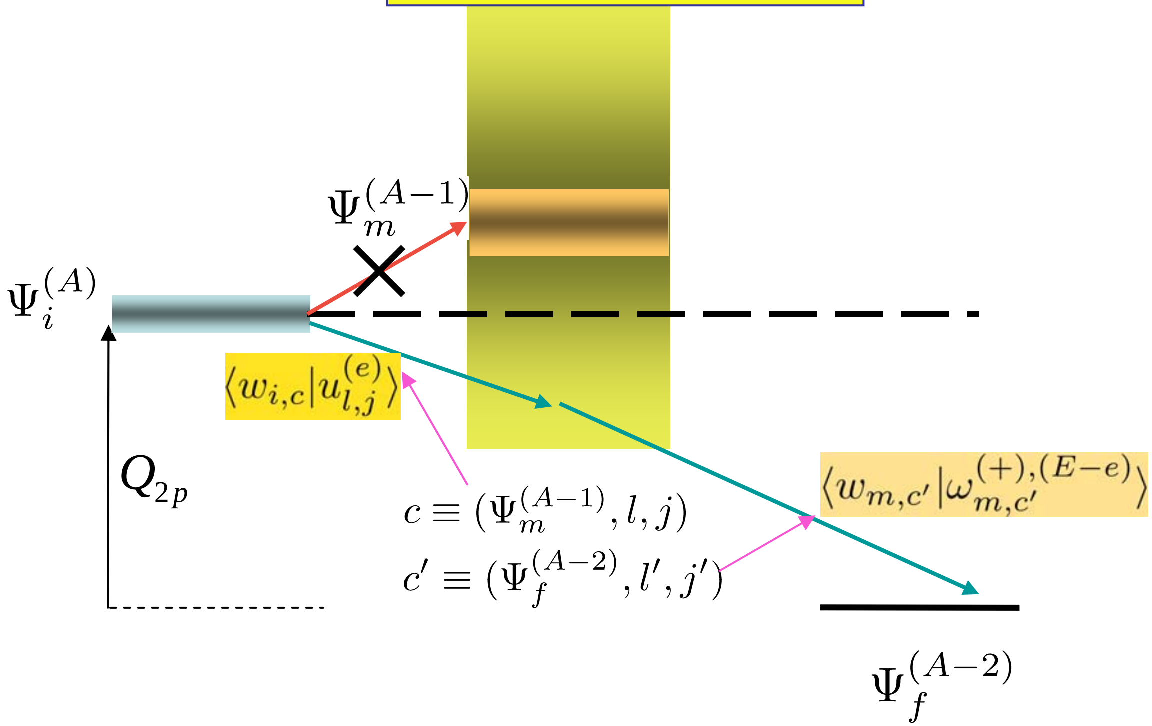
Diproton emission channel:

$$c = \left(\theta^{(\mathrm{int})}_{J_f^{(A-2)}}, l_x = 0, S = 0, L = 0\right)$$

$$\Gamma = \int_0^{Q_{2p}} \Gamma(U)\rho(U)dU$$

$$\Gamma(U) = -2Im\left[\left\langle \omega^{T,(+)}_{i,U} \middle| w^T_i \right\rangle\right]$$

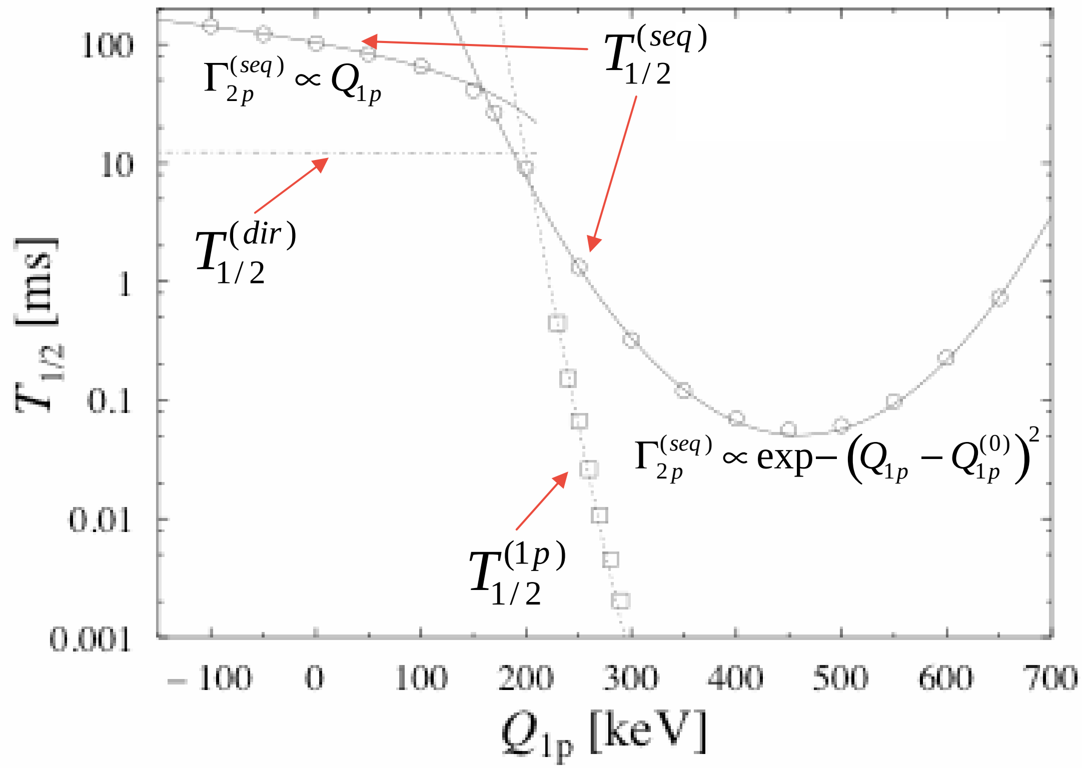
Indirect 2p emission



$$^{45}_{26}Fe \rightarrow ^{43}_{24}Cr + 2p$$

weak Q - P coupling $\Gamma_{2p}^{(dir)} >> \Gamma_{2p}^{(seq)} >> \Gamma_{1p}$

strong Q - P coupling $\Gamma_{1p} >> \Gamma_{2p}^{(seq)} >> \Gamma_{2p}^{(dir)}$



Exp: $T_{1/2} = 1.6^{+0.5}_{-0.3} (ms)$

SMEC: $T_{1/2}^{(dir)} = 12.3^{+7.5}_{-4.6} (ms)$ $T_{1/2}^{(seq)} = 91.8^{+107.4}_{-42.8} (ms)$

$Q_{2p}^{(exp)} = 1.154^{+0.016}_{-0.016} (MeV)$ $Q_{1p} = 0.0^{+0.05}_{-0.1} (MeV)$

Precise knowledge of Q_{2p}, Q_{1p} is indispensable!

Conclusions

- Continuum shell model : Gamow (complex-energy) Shell Model or Shell Model Embedded in the Continuum, provide a consistent description of the of weakly bound nuclei

-Priorities (personal choice):

* study of pairing correlations via final state pp correlations in 2p decays (masses!)
* exotic decays (2n, neutron droplets, $^{3}He, \ldots$) and exotic clustering phenomena
* 2p-capture via correlated continuum
* reactions with multiple $(n > 1)$ weakly bound partners: dynamics in (multiple) correlated continua vs structure of weakly bound systems and resonances
* new 'magic' nuclei & spin-orbit splitting: 3-body & tensor forces vs continuum coupling correlations
* effective symmetries in weakly bound/unbound nuclei (open quantum systems)
* strong NN correlations beyond mean-field : $S_{n,p} \to 0 \leftrightarrow \lambda_{n,p} - \Delta_{n,p} \to 0$

  ⇒ new limit of nuclear mean-field
* …   …   …

Nuclear structure enters in a new, exciting era!