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10/11/05

Super-Allowed Nuclear β-Decays and CKM Unitarity

Outline

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ii) A Coming β-decay Revolution?

1.) Ancient History: CKM Unitarity (Pre 2004)

V_{CKM} = 3x3 Unitary Quark Mixing Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$\lambda \equiv V_{us} = \sin\theta_{Cabibbo} = ?$ 0.220 or 0.227?

Unitarity Test: $|V_{ud}|^2 + |V_{us}|^2 + \underbrace{|V_{ub}|^2}_{\sim 1\times 10^{-5} \text{ tiny}} = 1$

Deviation → "New Physics" at tree or loop level

Examples: W_R, W^*(extra dim.), Heavy Quark Mixing
Z' loops, SUSY loops ... Leptoquarks, $H^\pm$ ...
* Exotic Muon Decay → $G_\mu \neq 1.16637(1)\times 10^{-5}\,GeV^{-2}$

PDG 2002 (04) $|V_{ud}| = 0.9734(8)$ $0^+\to 0^+$ β-decays, $n \to p e \bar{\nu}$
$|V_{us}| = 0.2196(26)$ K_{e3}: $\Gamma(K \to \pi e \nu)$

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9957(16)(11)}$$ Persistent 2.2σ dev.

Suggests: $V_{ud} = 0.9734 \to 0.9756$ (+0.2%)
or
$V_{us} = 0.2196 \to 0.2291$ (+4.3%)
} Combined Movement?

2.) The "Kaon Revolution" & V_{us}

(2003) BNL (E865): $K^+ \to \pi^0 e^+ \nu_e$
(2004) FNAL (KTeV): $K_L \to \pi^\pm e^\mp \nu, \pi^\pm \mu^\mp \nu$
CERN (NA48): $K_L \to \pi^\pm e^\mp \nu, K^+ \to \pi^0 e^+ \nu$
Frascati (KLOE): $K_L, K_S \to \pi^\pm e^\mp \nu, K^+ \to \pi^0 e^+ \nu$

} ~5-6% Increase! All K_L BR Change!

$$|V_{us}| = 0.2257(9)\left(\frac{0.961}{f_+(0)}\right)$$

Big Increase!

$f_+(0) = 0.961(8)$
Leutwyler & Roos (1984)
Lattice → $f_+(0) \simeq 0.96$
χPT → $f_+(0) \simeq 0.97\text{-}0.98$

Alternative: $\frac{\Gamma(K\to\mu\nu)}{\Gamma(\pi\to\mu\nu)} \to \frac{|V_{us}|}{|V_{ud}|} = \frac{f_\pi}{f_K} \times (0.2761(5))$

Lattice → $\frac{f_K}{f_\pi} = 1.198(3)\left(\begin{smallmatrix}+16\\-5\end{smallmatrix}\right)$ Bernard et al

$$|V_{us}| = 0.2245(5)\left(1.198\frac{f_\pi}{f_K}\right)$$

Consistent with K_{e3} Results

Significant increase in $|V_{us}|$

Uncertainty Dominated By Theory! $f_+(0)$ & $\frac{f_K}{f_\pi}$

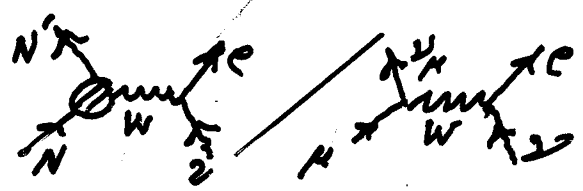
What is $f_+(0)$?

3.) Super-Allowed β-Decays & Vud

Measure $N \to N' e \nu$ for $0^+ \to 0^+$ transitions Vector Current Clear! CVC

Q value → f
$t_{1/2}$ value → t } ft values

Compare with muon decay



$$|V_{ud}|^2 = \frac{2984.48(5)\,\text{sec}}{ft\,(1+RC)}$$

RC = Electroweak Radiative Corrections (~+4%)
+ Nuclear Structure Isospin Corr. QED (~ -0.2% → -0.6%)
Hardy & Towner

For 9 Best Measured Superallowed Decays:
^{10}C, ^{14}O, ^{26}Al, ^{34}Cl, ^{38}K, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co

<u>RC ≃ +3.1 ↔ 3.6%</u>

i) ft Value Updates

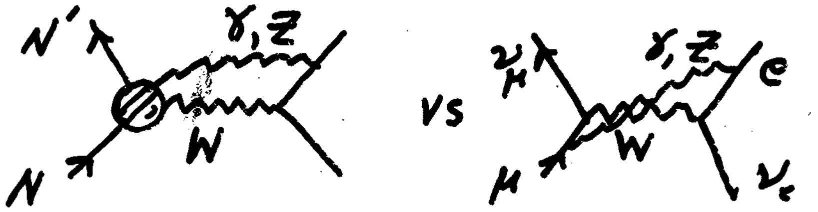
Hardy & Towner (2004) → $V_{ud} = 0.9738(2)_{\text{Nucl.}}(4)_{\text{RC loops}}$

G. Savard et al. (2005) → $V_{ud} = 0.9736(2)_{\text{Nucl}}(4)_{\text{RC loops}}$
^{46}V Qvalue increase

Dominated by Quantum Loop Theory (WM & A. Sirlin 1986)

^{46}V → Potential Q Value Penning Trap Revolution!

(ii) EW Radiative Corrections Revisited A. Sirlin + W.M. preprint



different $U(1)_Y$: $\frac{1}{3}$ vs -1

(Loop) Weak Vector (CVC) + Axial-Vector (Not CVC Protected) Contribute

$$RC^{old}_{EW} = \frac{\alpha}{2\pi}\Big\{ \underbrace{\bar{g}(E_{max}) + 3\ln\frac{m_Z}{m_p}}_{\text{Vector}} + \underbrace{\ln\frac{m_Z}{m_A} + A_g + 2C}_{\text{Axial}} \Big\}$$

$+$ 2loop $= +0.0013(1)$

$\ln\frac{m_Z}{m_A} = 4.3 \pm \underline{0.7}$ $m_A = 1.2\,\text{GeV}$ factor 2 unc. crude error (matching)

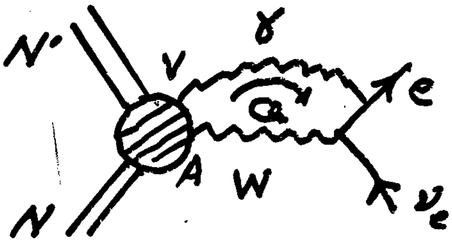
$A_g = -0.34$ $(1-\frac{\alpha_s}{\pi})$ Correction

$C_{Born} = 0.8\, g_A(\mu_p+\mu_n) \simeq 0.89$

Total $\Delta V_{ud}(\text{axial}) = -0.0033 \pm 0.0004$ — unc. for last 20 yr!

Can Axial-Vector loop unc. be improved?

New approach based on Bjorken Sum Rule Analogy
+ Large N_c Approach
(Interpolator Short-Long Distances)



Dangerous γW Box Diagram

$$\mathrm{Box}(\gamma W)_{VA} = \frac{\alpha}{8\pi}\int_0^\infty dQ^2 \frac{m_W^2}{Q^2+m_W^2} F(Q^2)$$

Integration Regions: i) Short-Dis $(1.5\,\mathrm{GeV})^2 \le Q^2 < \infty$
ii) Medium-Dis $(0.82\,\mathrm{GeV})^2 \le Q^2 < (1.5\,\mathrm{GeV})^2$
iii) Long-Dis $0 \le Q^2 < (0.82\,\mathrm{GeV})^2$

i) Short-Distance $(1.5\,\mathrm{GeV})^2 \le Q^2 < \infty$

Same as QCD corrections to Bjorken Sum Rule ($m_g = 0$)!

Vector Part $\langle p | T[VA] | n \rangle$ = Axial Part $\{\langle p | T[VV] | p \rangle - \langle n | T[VV] | n \rangle\}$

Bj Sum Rule: $\int_0^1 dx\,[g_1^p(x) - g_1^n(x)] = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} \cdots\right]$ — Calculated to $\left(\frac{\alpha_s}{\pi}\right)^3$!

$$F(Q^2) = \frac{1}{Q^2}\left[1 - \frac{\alpha_s(Q^2)}{\pi} - C_2\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - C_3\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3 \cdots\right]$$

($\frac{1}{Q^2}$ ↑ $\ln \frac{m_Z}{m_A}$; $\frac{\alpha_s}{\pi}$ ↑ A_g)

$C_2 = 4.583 - 0.333 N_F$
$C_3 = 41.440 - 7.607 N_F + 0.177 N_F^2$
} Reduces RC slightly

ii) <u>Intermediate Distances</u>: $(0.826\,GeV)^2 < Q^2 < (1.5\,GeV)^2$

$$F(Q^2) = \frac{-1.490}{Q^2+m_\rho^2} + \frac{6.855}{Q^2+m_A^2} - \frac{4.414}{Q^2+m_{\rho'}^2} \quad \text{Interpolator}$$

$m_\rho = 776$ MeV, $m_A = 1230$ MeV, $m_{\rho'} = 1465$ MeV

iii) <u>Long Distance</u>: $0 \leq Q^2 \leq (0.826\,GeV)^2$

Dipole Form Factors: $\frac{m^4}{(Q^2+m^2)^2}$ EM + Weak

$$C_{Born} = 0.89 \longrightarrow 0.83 \pm 0.08$$

<u>Small Changes in RC</u>

$$\Delta V_{ud} = 0.00007 \pm 0.00008 \pm 0.00016 \pm 0.00005$$

C_{Born} $\pm 10\%$ Interpolator $\pm 100\%$ 2loop

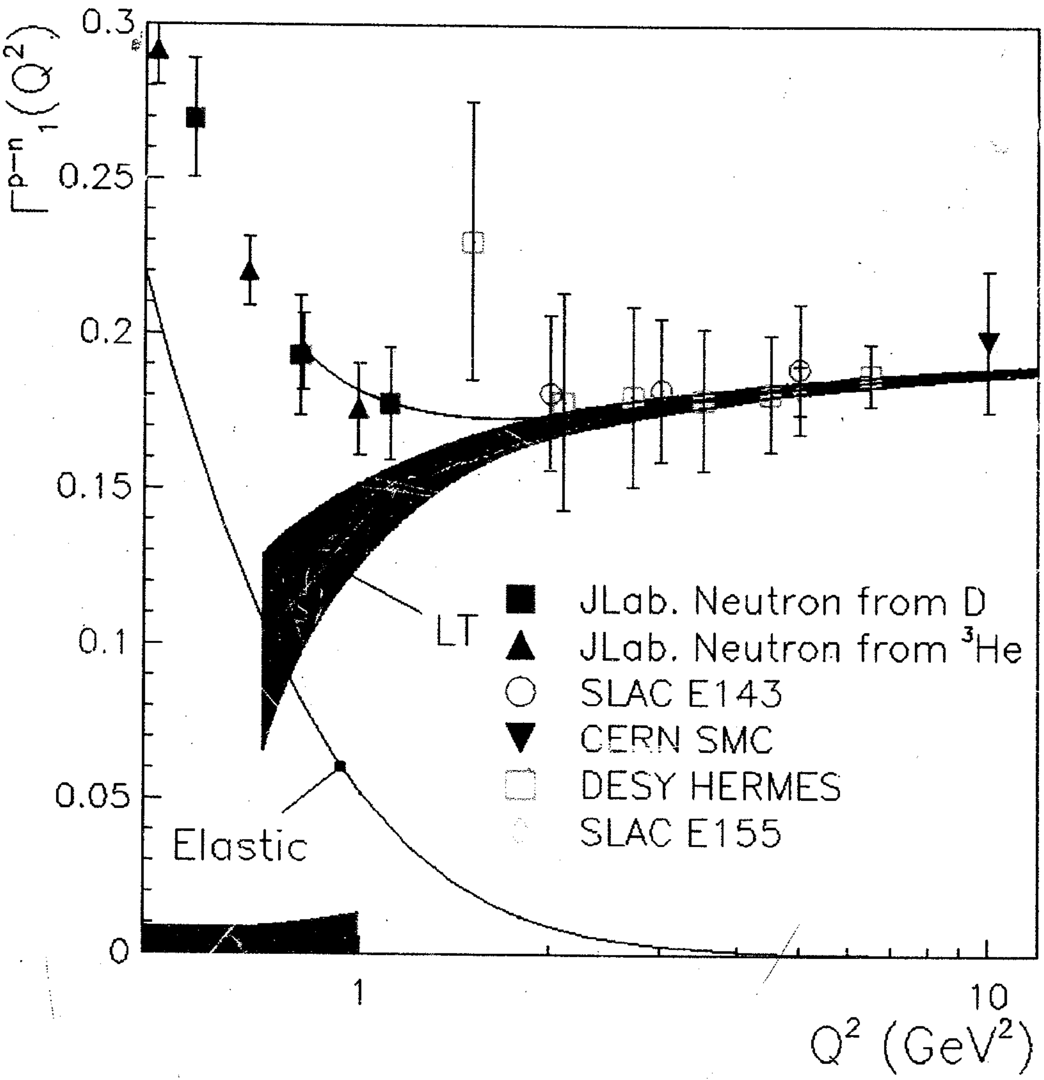
Total Unc. $= \pm 0.00019$ Old ± 0.00040

1) Validates Old Result ($\Delta V_{ud} = +0.00007$)

2) Unc. Reduced By Factor 2, $\pm 0.00034 \rightarrow \pm 0.00019$

*3) Tested by Bj Sum Rule Exp Data (see Fig)

From A. Deur et al.
hep ex/0407007 Nov 2004



iii) V_{ud} & CKM Unitarity Update

Savard et al ft values (2005)

Hardy & Towner: Nuclear Corrections (2002-2004)

A. Sirlin & W.M.: RC Improvement

$$V_{ud} = 0.97377\underbrace{(11)(15)}_{ft\ \&\ Nucl.}\underbrace{(19)}_{EW\ Rad.\ Corr.}$$

total error ± 0.00027

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9992(5)_{V_{ud}}(4)_{V_{us}}(8)_{f_+(0)}$$

Good Agreement With Unitarity to $\pm 0.09\%$!

4.) Future Outlook

i) Lattice Hopes & V_{us}

lattice $f_+(0)$ & $f_K/f_\pi \rightarrow \pm 1/2\%$ (better?)

* Chiral Pert $\rightarrow f_+(0) \simeq 0.98 \rightarrow V_{us} = 0.2213$? Why?

Nucleus	ft (sec) (Savard et al)	V_{ud}
^{10}C	3039.5(47)	0.97381(77)(15)(19)
^{14}O	3043.3(19)	0.97368(39)(15)(19)
^{26}Al	3036.8(11)	0.97406(23)(15)(19)
^{34}Cl	3050.0(12)	0.97412(26)(15)(19)
^{38}K	3051.1(10)	0.97404(26)(15)(19)
^{42}Sc	3046.8(12)	0.97330(32)(15)(19)
^{46}V	3050.7(12)	0.97280(34)(15)(19)
^{50}Mn	3045.8(16)	0.97367(41)(15)(19)
^{54}Co	3048.4(11)	0.97373(40)(15)(19)

Ave 0.97377(11)(15)(19)

Recent Change: ^{46}V Q Value Change Penning Trap

$ft = 3045.5(22) \rightarrow 3050.7(12)$

$V_{ud} = 0.97363(44)(15)(19) \rightarrow 0.97280(34)(15)(19)$

Major Shift!

Other Q Values Need Remeasurement!

Big Problem?

(Major Revolution?)

ii) A Coming β-Decay Revolution?

^{46}V Q value and other recent Penning Trap Measurement Suggest Problems with old Q values

→ **Potential ft & V_{ud} Shifts**

* All Q & $t_{1/2}$ Should be remeasured using modern techniques
* Nuclear Theory should be reexamined!
* Neutron Decay: τ_n & g_A → V_{ud} (= 0.9729(4)(11)(2) in 2004)

What if: $V_{us} \rightarrow 0.222(1)$ i.e. $f_+(0) = 0.98$

$V_{ud} \rightarrow 0.9732(3)$

Back to the Past!

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \rightarrow 0.9964(7)$$

5σ deviation possible!

It ain't over till it's over!