Towards a new type of QCD ISR parton shower MC

TeV4LHC, CERN April 29-th, 2005

S. Jadach and M. Skrzypek

stanislaw.jadach@ifj.edu.pl, maciej.skrzypek@ifj.edu.pl

IFJ-PAN, Kraków, Poland

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Work in progress!!!

1) SOLVING CONSTRAINED MARKOVIAN EVOLUTION IN QCD WITH THE HELP OF THE NON-MARKOVIAN MONTE CARLO.

By S. Jadach, M. Skrzypek. IFJPAN-V-04-07, Apr 2005. ** Temporary entry **

e-Print Archive: hep-ph/0504263

2) NON-MARKOVIAN MONTE CARLO ALGORITHM FOR THE CONSTRAINED MARKOVIAN EVOLUTION IN QCD.

By S. Jadach, M. Skrzypek. IFJPAN-V-04-06, Apr 2005. ** Temporary entry **

e-Print Archive: hep-ph/0504205

Perturbative calculations in QCD

Matrix element, QFT textbook approach:

- Finite (fixed) order calculation $\boxed{\sum_N \int d\mathrm{LIPS_N}\, |M_N|^2}$, where
- N = Number of real emissions determined in the beginning LIPS = Lorentz Invariant Phase Space
- The above is convoluted with the strictly collinear PDFs.

Classic Parton shower approach:

- Emission chain built by resurrecting transverse momenta in the LL QCD evolution equations
- MC algorithm of the Markovian type, N determined in the end
- Hard scattering at the Born level
- Very much improved LL approximation but always incomplete NLL in the shower tree and in the hard process ME

Hybrid approaches:

- Andre+Sjöstrand, Frixione+Webber, Nason and more...
- CKKW (Catani et.al.)
- The unsolved problem: NLL parton shower. Hopeless?

R&D on MC solutions of QCD Evolution in Cracow

Monte Carlo modeling of the QCD $\overline{MS}$ DGLAP evolution:

- Markovian MC (forward) precision ($\sim 10^{-3}$) solutions of the full LL DGLAP equations (massless quarks). Acta.Phys.Pol. B35 (2004).
- Markovian MC precision solutions of the full NLL DGLAP equations (massless quarks). IFJPAN-V-04-08, to appear.
- Markovian MC study of the CCFM one-loop evolution. IFJPAN-V-05-03, to appear

Constrained Monte Carlo (CMC) algorithms for DGLAP evolution:

- Constrained MC (non-Markovian) class II. Proc. Loops&Legs 2004, Nucl. Phys. Proc. Suppl. 135 (2004) and IFJPAN-V-04-06, hep-ph/0504205.
- Constrained MC (non-Markovian) class I.
  October 2004 talk at HERA-LHC wshop and IFJPAN-V-04-07, hep-ph/0504263.

People involved:

- K.Golec-Biernat, S.Jadach, W.Płaczek, M.Skrzypek, Z.Wąs

Towards the parton shower (this talk):

- **Constrained MC algorithm (class I) for the HERWIG-style evolution, also referred to as CCFM one-loop. See also talk at HERA-LHC wshop, March 05.**
- NB. Presented CMC techniques are quite general. The problem of constraining the longitudinal momentum in ISR parton shower (QCD and QED) is always present, whichever scenario we may imagine for the future development!

Motivation and background

Known facts:

- Markovian MC implementing the QCD/QED evolution equations is the underlying ingredient in all parton shower type MCs
- Unconstrained forward Markovian MC, with evolution kernels from perturbative QCD/QED, inefficient for ISR.
- Backward evolution MC algorithm of Sjöstrand (Phys.Lett. 157B, 1985) is a widely adopted workaround.
- Backward Markovian MC does not solve the QCD evolution eqs.
  It merely exploits their solutions coming from the external non-MC methods

The old-standing problem:

- Is it possible to invent an efficient MC algorithm, solving internally the evolution eqs. by its own? No use of external PDFs.
- THE ANSWER IS YES! As shown in works listed on the previous page.

Motivation:

- Better modeling the ISR parton shower, possibly more friendly for inclusion of NLL and NNLL into parton shower MCs.
- Possibly easier MC modeling of the unintegrated parton distributions $D_k(p_T, x)$ and CCFM class of the QCD calculations/models.

Vocabulary

- **Markovian MC algorithm**
  The algorithm in which the number of emission (determining the dimension of the dimension of the integral, phase space), is generated as the last variable
- **non-Markovian MC algorithm**
  The algorithm in which the number of emission (the dimension of the integral), is generated as one of the first variables.
- **Constrained MC algorithm = CMC**
  The distributions are the same as in normal Markovian evolution, but the final energy $x = \prod z_i$ and the parton type $k = G, q_j, \bar{q}_j$ are predefined i.e. constrained.
- **HERWIG Evolution (terminology by P. Nason), 1-loop CCFM**:
  Two ingredients:
  $\alpha_S(Q(1-z))$ (Amati+Basetto+Ciafaloni+Marchesini+Veneziano, NPB173, 1980)
  and $\varepsilon_{IR} = Q_0/Q$ where $Q_0 \sim 1$GeV (Webber+Marchesini, NPB310, 1988).
  For simplicity Q_0 coincides with the starting point of the QCD evolution.
- MS-bar DGLAP evolution $\neq$ HERWIG evolution
  At the LL they differ by large NLL and Q_0/Q terms.
  The difference going away at the NLL (Amati at.al., Trentadue+Kodaira, Catani et.al.)

Discussion

- The first efficient CMC algorithm (October 2004) was found for the MS-bar DGLAP evolution.
- Pure bremsstrahlung is the critical part of the CMC algorithm.
- Presently we are upgrading bremsstrahlung part of CMC from DGLAP to HERWIG evolution and this bremsstrahlung version will be presented here.
- We find that the efficiency of CMC for the HERWIG Evolution is satisfactory.
- The rest is modeling of (up to four) Quark$\leftrightarrow$Gluon transitions; this is done for pure DGLAP only, so far.
- In the CMC class I Quark$\leftrightarrow$Gluon transitions are modeled using general purpose MC tool FOAM, hence it should work almost automatically. Still to be checked.
- In the following we show essential part of the CMC algorithm for pure bremsstrahlung and show briefly results for full the DGLAP with Quark$\leftrightarrow$Gluon transitions.

Pure bremsstrahlung from the “emitter” $k = G, q, \bar{q}$ line

Iterative solution of the QCD evolution equations,
for evolution $t_0 \to t$, where $t = \ln Q$ is the evolution time:

$$x\mathcal{D}_{kk}(t, t_0; x) = e^{-\Phi_k(t,t_0)} \Big\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int_{t_0}^{t} dt_i \; \int_0^1 dz_i \; \mathcal{P}^{\Theta}_{kk}(t_i, z_i) \; \delta_{x=\prod_{i=1}^{n} z_i} \Big\},$$

Notation:

- $\theta_{x>0} = 1$ for $x > y$ and $= 0$ otherwise.
- $\delta_{x=y} \equiv \delta(x - y)$.
- $\mathcal{P}_{kk}(t, z) \equiv \frac{\alpha(t,z)}{\pi} z P_{kk}(t) = -\mathcal{P}^{\delta}_{kk}(t)\delta_{z=1} + \mathcal{P}^{\Theta}_{kk}(t, z)$.
- $\mathcal{P}^{\Theta}_{kk}(t, z) = \mathcal{P}_{kk}(t, z)\theta_{1-z>\varepsilon(t)}$, the same as in LL DGLAP.
- $\mathcal{P}^{\delta}_{kk}(t) = \int\limits_0^{1-\varepsilon(t)} dz \mathcal{P}^{\Theta}_{kk}(z, t)$, from energy sum rule, valid up to NLL.
- Sudakov formfactor: $\Phi_k(t, t_0) = \int\limits_{t_0}^{t} dt' \; \mathcal{P}^{\delta}_{kk}(t')$.
- IR cut $\varepsilon(t) = Q_0/Q$; it is not anymore $<< 1$, as in the standard DGLAP.

Variable mapping more complicated than for normal DGLAP

$$\int_x^{1-\varepsilon(t)} dz_i \int_{t_0}^{t} dt_i \; \mathcal{P}^{\Theta}_{kk}(t_i, z_i) = h_k \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \int_0^1 ds_i, i = 1, 2, ..., n,$$

$$z_i(y_i) = 1 - \exp(\rho^{-1}(y_i)),$$

$$\hat{t}_i(s_i) = \hat{t}_0 \left(\frac{\hat{t} + \ln(1 - z_i)}{\hat{t}_0} \right)^{s_i} - \ln(1 - z_i).$$

where

$$\rho(v) \equiv (\hat{t} + v)\ln(\hat{t} + v) - v - v\ln\hat{t}_0 - \hat{t}\ln\hat{t}, \quad \hat{t} \equiv t - t_\Lambda = \ln Q - \ln\Lambda_0.$$

IMPORTANT: ρ^{-1} is not analytical!
Inversion has to be done numerically. ρ^{-1} will enter the constraint function $\prod z_i$!

The above mapping leads to:

$$x\mathcal{D}_{kk}(t, t_0, x) = e^{-\Phi_k(t,t_0)} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \frac{1}{n!} h_k^n \prod_{i=1}^{n} \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \; \delta_{x=\prod_{i=1}^{n} z_i(y_i)} \int_0^1 ds_i \right\}.$$

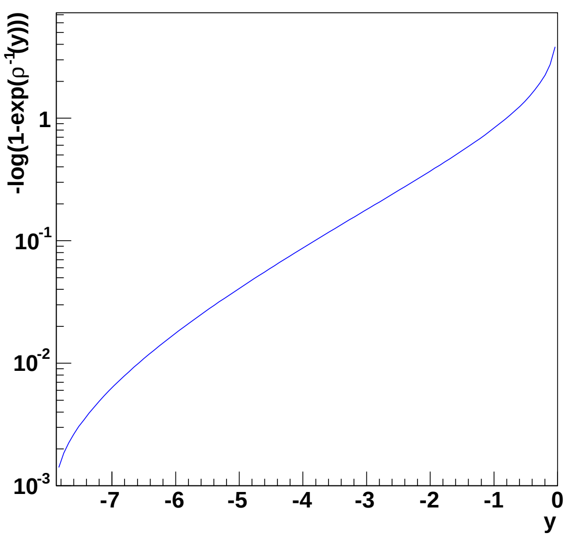
The energy constraint

Using symmetry of the integrand we finally trade the ordering in evolution time variables t_i into ordering in the energy variables y_i ($y_0 \equiv 0$):

$$x\mathcal{D}_{kk}(t,t_0,x) = e^{-\Phi_k(t,t_0)}\Big\{\delta_{x=1}+$$

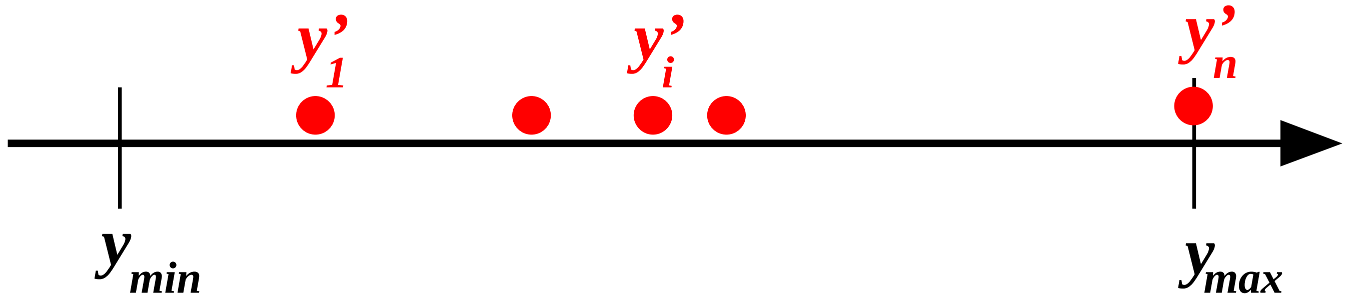
$$+\,x^{-1}\sum_{n=1}^{\infty} h_k^n \prod_{i=1}^{n} \int\limits_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i\;\theta_{y_i>y_{i-1}}\,\delta\left(\ln\frac{1}{x}-\sum_j f(y_j)\right)\int\limits_0^1 ds_i\Big\}.$$

Here, $f(y_i)$ is very steeply (exponentially) rising,
see plot below for $q_0 = 1$GeV and $q = 1000$GeV



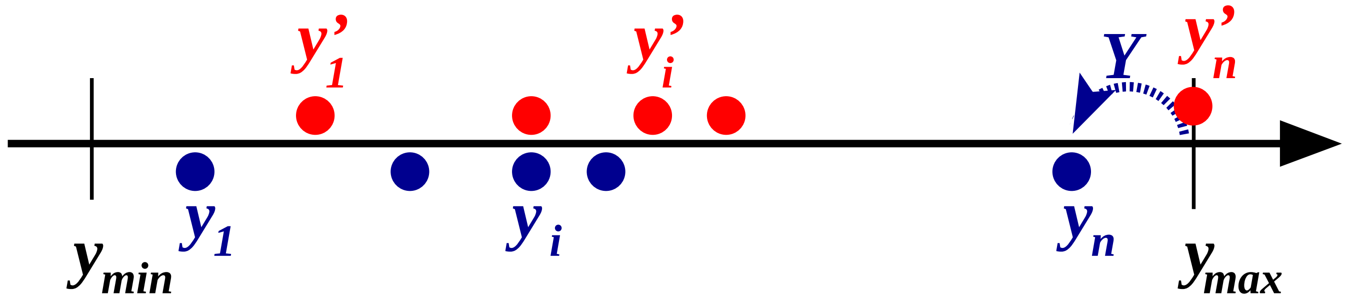
hence the constraint $x = \prod_{i=1}^{n} z_i(y_i)$ is “saturated” by a single z.

Linear shift: $y'_i \rightarrow y_i = y'_i - Y$



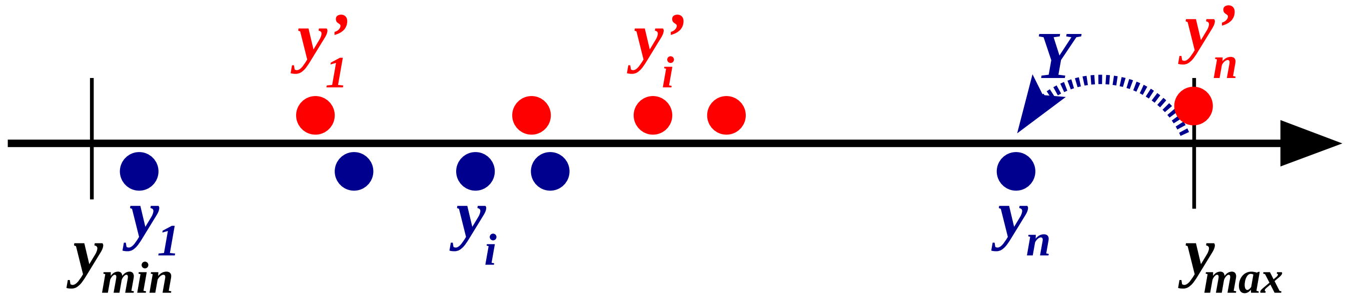
- Begin with y'_i such that one of them $y_n \equiv y_{\max}$

Linear shift: $y'_i \to y_i = y'_i - Y$



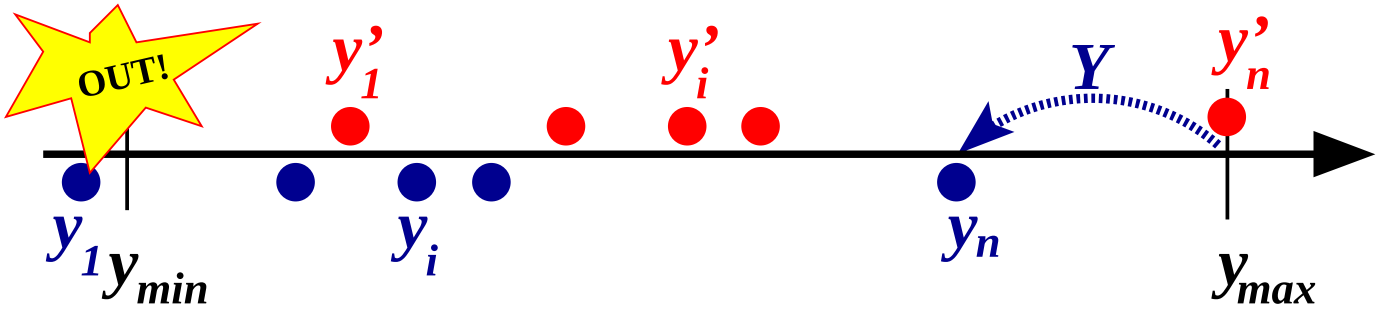
- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$

Linear shift: $y'_i \to y_i = y'_i - Y(y'_1, y'_2, ..., y'_n)$



- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i

Linear shift: $y'_i \to y_i \; = y'_i - Y(y'_1, y'_2, ..., y'_n)$



- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i
- Sometimes the smallest y'_i is shifted OUT of the phase space, below IR the limit $y_{\min}$. Such an event gets MC weight $w = 0$

Master formula for the bremsstrahlung CMC

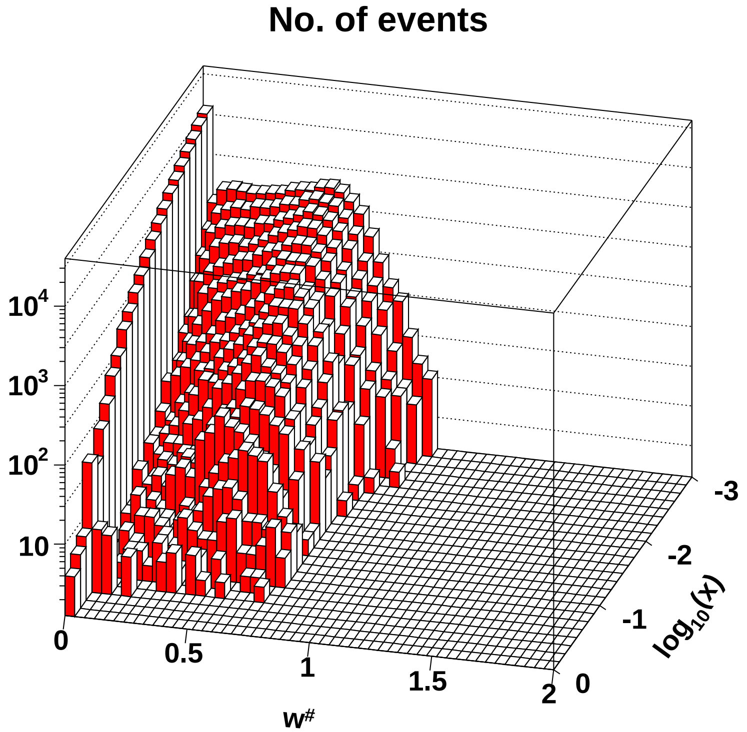
$$x\mathcal{D}_{kk}(\tau,\tau_0;x) = e^{(\tau-\tau_0)a_k} \sum_{n=0}^{\infty} \Big\{ e^{b_k \mathcal{R}(\varepsilon)} \delta_{n=0}\delta_{x=1} + \delta_{n>0}\theta_{1-x>\varepsilon} e^{b_k\mathcal{R}(1-x)} \frac{b_k x^{\omega_k - 1}}{x g(x)}$$
$$\times P_n\big(b_k[\mathcal{R}(1-x)-\mathcal{R}(\varepsilon)]\big) \prod_{i=1}^{n} \int_0^1 dr_i \, \frac{\delta(1-\max r_j)}{n} \int_0^1 ds_i \; w^{\#} \Big\}$$

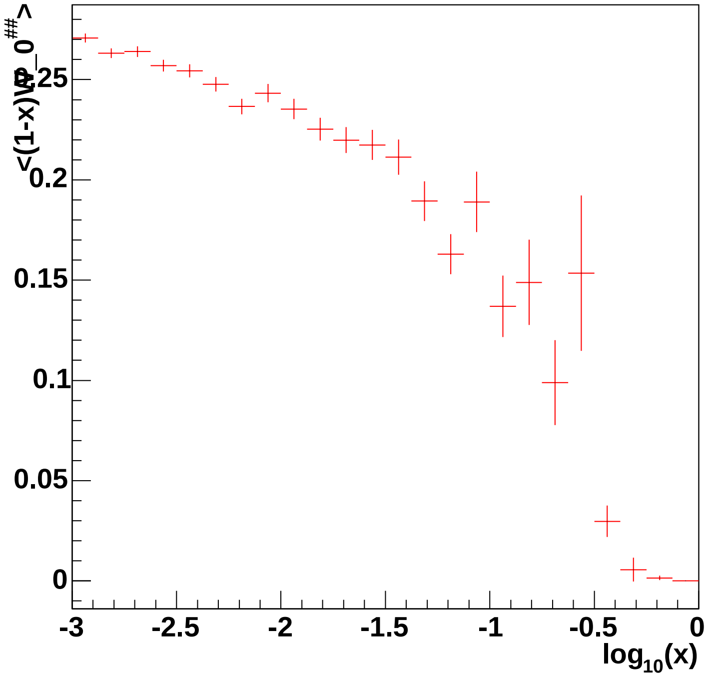
NOTATION:

- Mapping $z_i(y_i) = 1 - \exp(\rho^{-1}(y_i))$.
- Mapping $\hat{t}_i(s_i) = \hat{t}_0 \left(\frac{\hat{t}+\ln(1-z_i)}{\hat{t}_0}\right)^{s_i} - \ln(1-z_i)$.
- Poisson distribution: $P_n(\lambda) = e^{-\lambda}\lambda^n/n!, \quad \lambda =< n >$.
- $\mathcal{R}(1-z) \equiv \rho(\ln(1-z))$, (implicitly depends on t and t_0).
- MC weight: $\boxed{w^{\#} = w_P \, \frac{x g(x)}{|\partial_Y \ln F(Y_0)|} \, \theta_{y'_1 - Y_0 > y_{\min}}}$,
- where $g(x) = |\partial_y \ln z(y)|_{z=x} = \frac{1-x}{x}$ is to stabilize the MC weight.
- Ordering of y'_i is here relaxed (to get explicit $1/(n-1)!$ of Poisson).

Prototype Monte Carlo I.d

The starting parton distribution is that of gluon in the proton.



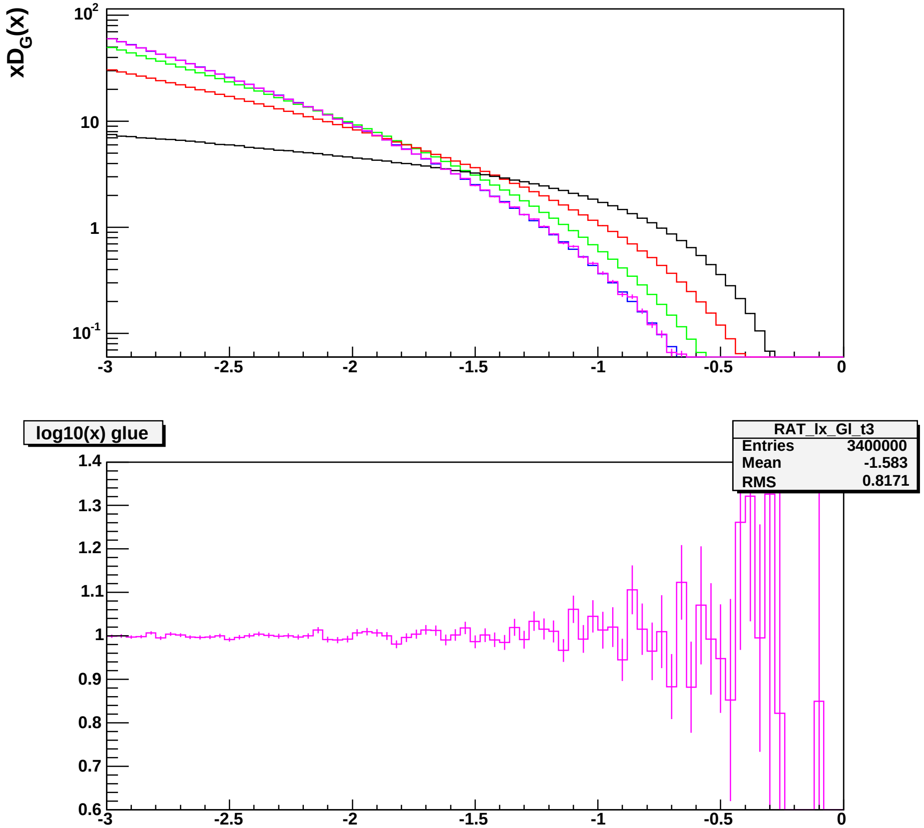


Plotted are weight distribution and the average weight as a function of x.
The maximum weight is below one and the acceptance rate 0.25 is surprisingly good!
About 1/3 of events has zero weight.

THIS IS THE MAIN NEW RESULT IN THIS TALK

The comparison with the simpler Markovian MC

Below is the comparison of CMC I.d (HERWIG evolution, gluonstrahlung) with the unconstrained Markovian MC:



There is a reasonable agreement, within the statistical error, for small statistics, so far.

THIS IS THE MAIN NEW RESULT IN THIS TALK

Full DGLAP: Hierarchic reorganization of the emission chain

The full DGLAP Constrained MC requires two-level organization of the emission chain:
(S) Flavor transmutation super-level $G \to Q \to G \to Q \to G \to \dots$
(B) Bremsstrahlung sub-level, any No. of gluon emissions ($Q \to Q$, $G \to G$).

The starting point is the usual iterative solution ($k \equiv k_n$) of the QCD evolution equations:

$$xD_k(\tau,x) = e^{-(\tau-\tau_0)R_k} xD_k(\tau_0,x)+$$

$$+\sum_{n=1}^{\infty} \sum_{k_{n-1}\dots k_1 k_0} \left[\prod_{j=1}^{n} \int_{\tau_0}^{\tau} d\tau_j \; \theta_{\tau_j>\tau_{j-1}}\right] \int_0^1 dx_0 \left[\prod_{i=1}^{n} \int_0^1 dz_i\right]$$

$$\times\, e^{-(\tau-\tau_n)R_k} \left[\prod_{i=1}^{n} \mathcal{P}^{\Theta}_{k_i k_{i-1}}(z_i)\; e^{-(\tau_i-\tau_{i-1})R_{k_{i-1}}}\right] x_0 D_{k_0}(\tau_0,x_0)\delta_{x=x_0\prod_{i=1}^{n} z_i},$$

Notation:

- Kernels: $\mathcal{P}^{\Theta}_{k_i k_{i-1}}(z_i) = \frac{\alpha(t_0)}{\pi}\; z_i P_{k_i k_{i-1}}(z_i)\theta_{1-z_i>\varepsilon}$
- Transition rates: $R_k = \sum_j \int_0^{1-\epsilon} dz\; \mathcal{P}^{\Theta}_{jk}(z) = \sum_j R_{jk}$

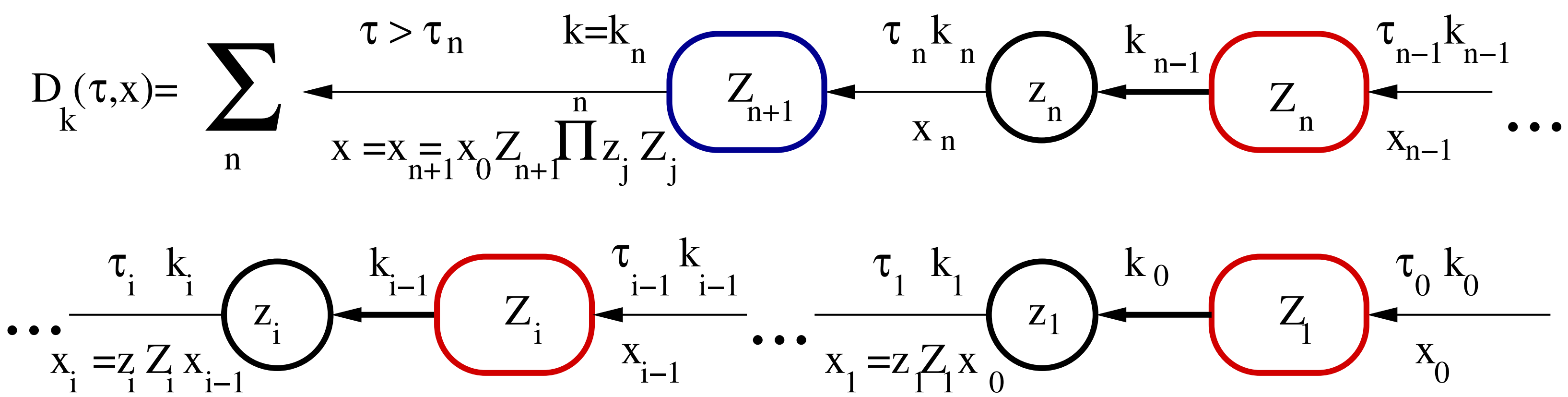
Two-level hierarchic: $\mathcal{D}_{kk}$ are also multi-integrals!

$$D_k(\tau,x)=\int dZ\; dx_0\; \mathcal{D}_{kk}(\tau,Z|\tau_0)\; D_k(\tau_0,x_0)\delta_{x=Zx_0}+$$

$$+\sum_{n=1}^{\infty}\sum_{\substack{k_{n-1}\dots,k_1 k_0\\ k_n\neq k_{n-1}\neq\dots\neq k_1\neq k_0}}\int_0^1 dZ_{n+1}\left[\prod_{j=1}^{n}\int_{\tau_0}^{\tau} d\tau_j\;\theta_{\tau_j>\tau_{j-1}}\int_0^1 dz_j\int_0^1 dZ_j\right]\int_0^1 dx_0$$

$$\times\;\mathcal{D}_{kk}(\tau,Z_{n+1}|\tau_n)\left[\prod_{i=1}^{n}\mathbf{P}^{\Theta}_{k_i k_{i-1}}(z_i)\;\mathcal{D}_{k_{i-1}k_{i-1}}(\tau_i,Z_i|\tau_{i-1})\right]$$

$$\times\; D_{k_0}(\tau_0,x_0)\delta\Big(x-x_0 Z_{n+1}\prod_{i=1}^{n} z_i Z_i\Big),\qquad k\equiv k_n.$$



Red oval is pure bremsstrahlung segment; Blue oval is Q↔G transition.

Two-level hierarchic: $\mathcal{D}_{kk}$ are also multi-integrals!

$$D_k(\tau,x) = \int dZ\; dx_0\; \mathcal{D}_{kk}(\tau,Z|\tau_0)\; D_k(\tau_0,x_0)\delta_{x=Zx_0} +$$

$$+\sum_{n=1}^{\infty} \sum_{\substack{k_{n-1}\dots,k_1 k_0 \\ k_n\neq k_{n-1}\neq\dots\neq k_1\neq k_0}} \int_0^1 dZ_{n+1} \left[\prod_{j=1}^{n} \int_{\tau_0}^{\tau} d\tau_j\; \theta_{\tau_j>\tau_{j-1}} \int_0^1 dz_j \int_0^1 dZ_j\right] \int_0^1 dx_0$$

$$\times\, \mathcal{D}_{kk}(\tau, Z_{n+1}|\tau_n) \left[\prod_{i=1}^{n} \mathbf{P}^{\Theta}_{k_i k_{i-1}}(z_i)\; \mathcal{D}_{k_{i-1}k_{i-1}}(\tau_i, Z_i|\tau_{i-1})\right]$$

$$\times\, D_{k_0}(\tau_0,x_0)\delta\Big(x - x_0 Z_{n+1}\prod_{i=1}^{n} z_i Z_i\Big), \qquad k\equiv k_n,$$

$$\mathcal{D}_{kk}(\tau,Z|\tau_0) = \frac{e^{\Phi_k(\tau,\tau_0)}}{Z}\Big\{\delta_{Z=1} + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\int_{\tau_0}^{\tau} d\tau_i\; \theta_{\tau_i>\tau_{i-1}} \int_0^1 dz_i\; z_i \mathbf{P}^{\Theta}_{kk}(z_i)\delta_{Z=\prod_{i=1}^{n} z_i}\Big\}$$

NOTATION:

- Pure bremss. Sudakov formfactor $\Phi_k(\tau,\tau_0) = (\tau-\tau_0)(a_k + b_k \ln\varepsilon)$
- Kernel $\times$ coupling const: $\mathbf{P}^{\Theta}_{k_1k_2}(z) = \frac{2}{\beta_0} P_{k_1k_2}(z)\theta_{1-z>\varepsilon}$
- Other: $b_k \equiv \frac{2}{\beta_0} B_{kk}, \quad a_k \equiv \frac{2}{\beta_0} A_{kk},$

CMC for full DGLAP; top level integrand for FOAM

- Neglecting temporarily $w^{\#}$ inside the segments $\mathcal{D}_{kk}$, gluon bremsstrahlung sub-level, we can integrate/sum analytically over all variables of the sub-level
- The overall (energy) x-constraint δ-function is eliminated using $\int dx_0$
- We are left with the $3n+1$-dim. integrals (n= No. of flavor changes) of the flavor-changing super-level, the INTEGRAND FOR FOAM is the following:

$$D_k(\tau, x) = x^{-1} \int_x^1 dZ \int\limits_0^{R(x)} dR_1 \; Z(R_1)^{\omega_k - 2} e^{a_k(\tau-\tau_0)} \; x_0 D_k(\tau_0, x_0) +$$

$$+ x^{-1} \sum_{n=1}^{\infty} \sum_{\substack{k_{n-1}\dots,k_1 k_0 \\ k_n \neq k_{n-1} \neq \dots \neq k_1 \neq k_0}} \left[\prod_{j=1}^{n} \int\limits_{\tau_0}^{\tau} d\tau_j \; \theta_{\tau_j > \tau_{j-1}} \right] \int\limits_0^{R(x)} dR_{n+1} \; Z(R_{n+1})^{\omega_k - 2} e^{a_k(\tau - \tau_n)}$$

$$\times \left[\prod_{i=1}^{n} \int\limits_{x_{i+1}}^{1} dz_i \; \mathbf{P}^{\Theta}_{k_i k_{i-1}}(z_i) \int\limits_0^{R(x_{i+1}/z_i)} dR_i \; Z(R_i)^{\omega_{k_{i-1}} - 2} e^{a_{k_{i-1}}(\tau_i - \tau_{i-1})} \right]$$

$$\times \; x_0 D_{k_0}(\tau_0, x_0),$$

$$R(Z_i) = (1 - Z_i)^{b_{k_{i-1}}(\tau_i - \tau_{i-1})}, \quad Z(R_i) = 1 - \exp\left((b_{k_{i-1}}(\tau_i - \tau_{i-1}))^{-1} \ln R_i \right),$$

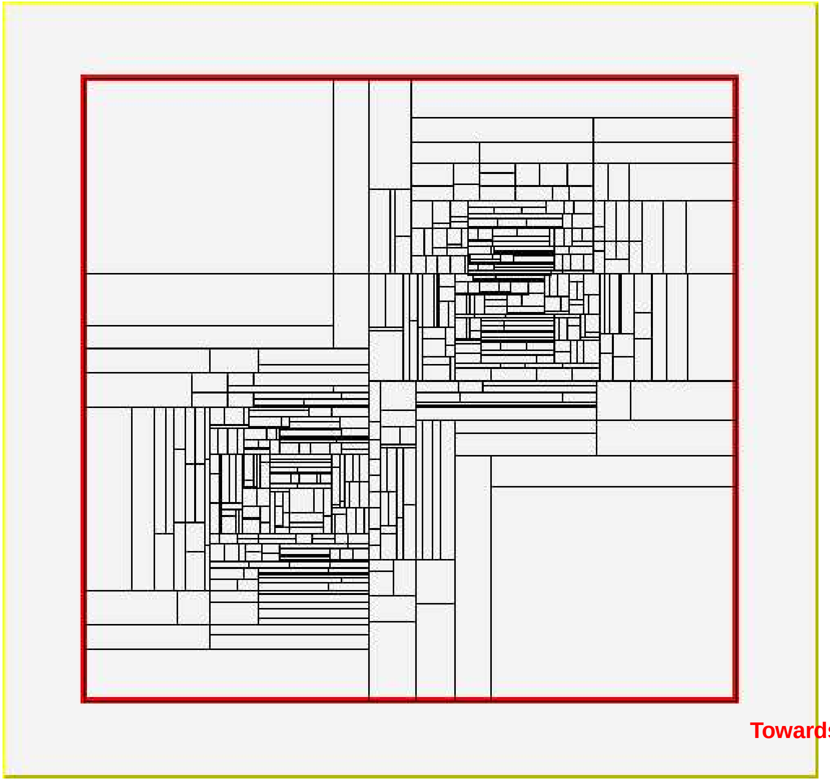
FOAM 1.02M, integrated with ROOT! S.Jadach and P.Sawicki

What is FOAM for?

- Suppose you want to generate randomly points (vectors) according to an arbitrary probability distribution in n dimensions, for which you supply your own subprogram. FOAM can do it for you! Even for distributions with strong peaks and discontinuous!
- FOAM generates random points with weight one or with variable weight.
- FOAM is capable to integrate using efficient "adaptive" MC method.

How does it work?

- It creates hyper-rectangular "foam of cells", which is more dense around its peaks.
- See the following 2-dim. example of the map of 1000 cells for doubly peaked distribution:



CMC algorithm of type I, full DGLAP

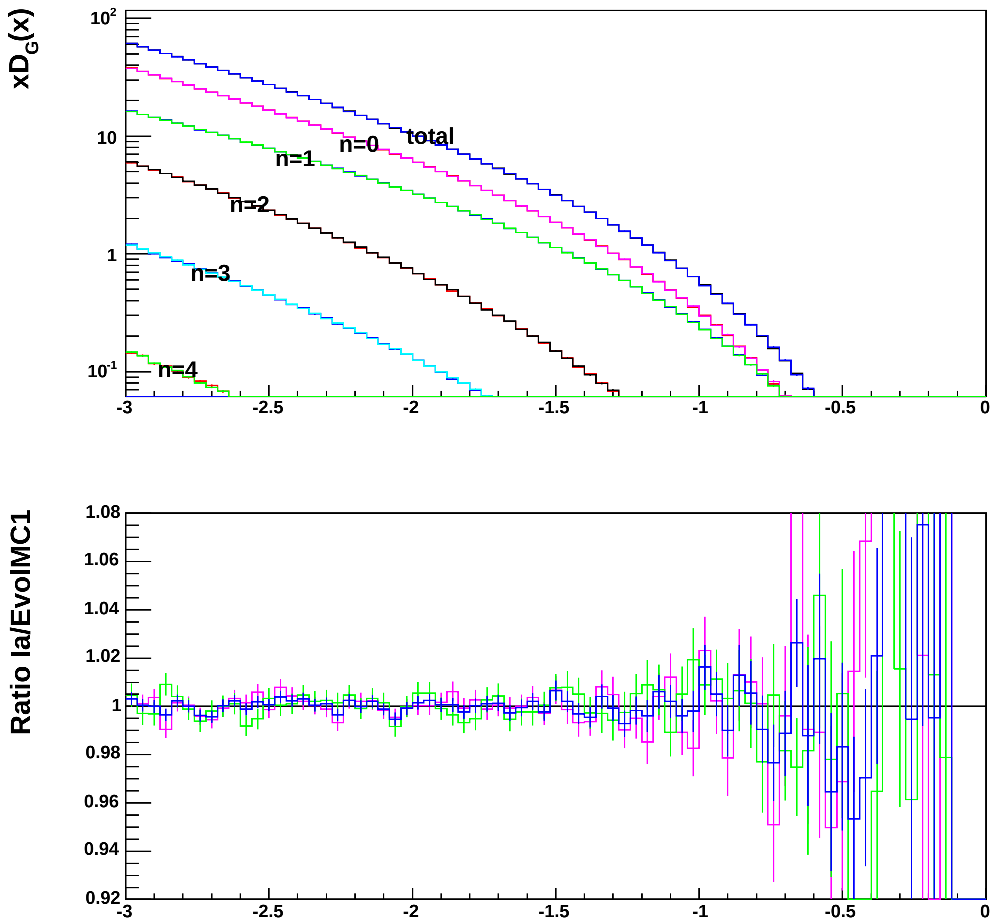
CMC algorithm description

- Generate super-level variables n, k_i, τ_i Z_i and z_i using Foam general purpose MC tool.
- Limiting no. of flavor transition ($G \to Q$ and $Q \to G$) to $n = 0, 1, 2, 3$ is enough, for the $\sim 0.2\%$ precision.
- For each pure gluon bremsstralung segment defined by Z_i and (τ_i, τ_{i-1}), $i = 1, 2, ..., n+1$, gluon emission variable $(z_j^{(i)}, \tau_j^{(i)},\ j = 1, 2, ..., n^{(i)}$, are generated using previously described dedicated CMC.
- Weight$= 1$ events available!

Numeric tests

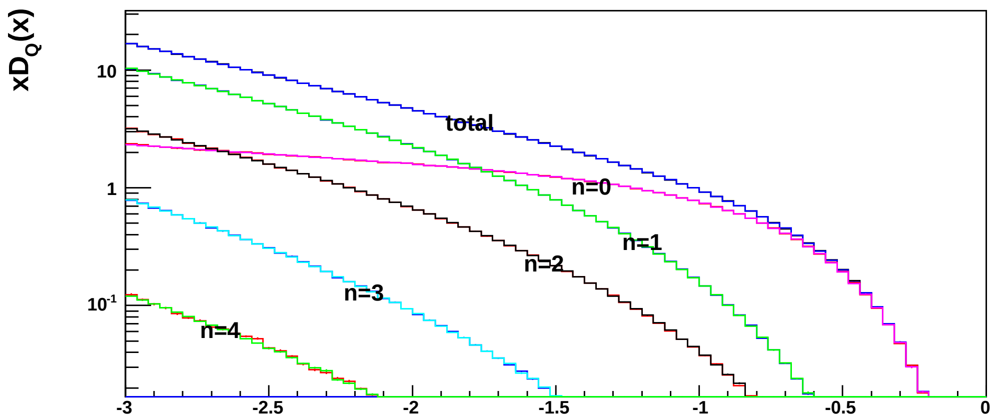
- In the next slides we show numerical results from such a non-Markovian CMC EvolCMC for "evolution" ranging from $Q = 1$GeV to $Q = 1$TeV, $x > 10^{-3}$,
- They are compare them with the results of the Markovian unconstrained evolution of our own EvolFMC
- EvolFMC was previously x-checked with QCDnum16 and ACHEB
- The agreement of Nonmarkovian EvolCMC and Markovian EvolFMC is excelent, $\sim 0.25\%$.

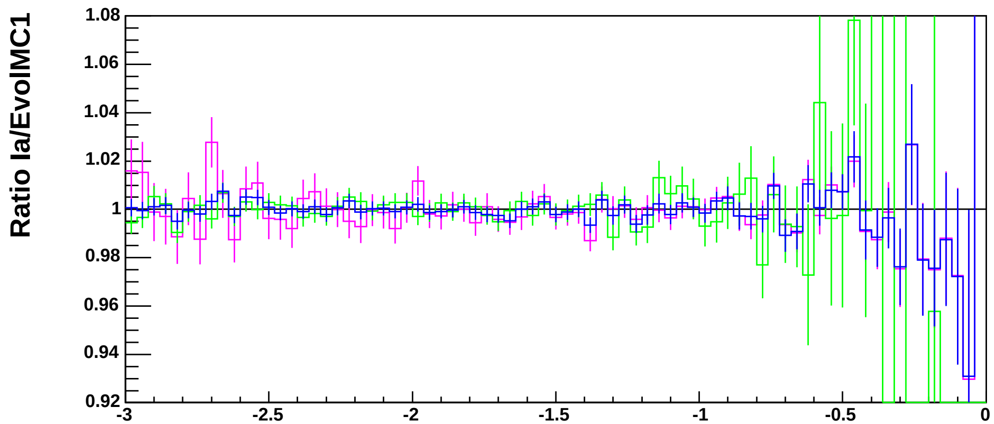
Test of non-Markovian Constrained MC, DGLAP case



$n = 0$: $G \to G$
$n = 1$: $Q \to G$ and any no. of gluon emissions out of Q and G,
$n = 2$: $G \to Q \to G$, etc.
$n = 3$: $Q \to G \to Q \to G$, etc.
$n = 4$: $G \to Q \to G \to Q \to G$, etc. “Total” is the sum of $n = 0, 1, 2, 3, 4$.
Evolution from proton at $Q = 1$GeV up to 1TeV. New non-Markovian CMC (EvolCMC) agrees with unconstrained Markovian MC (EvolFMC) to within $\sim 0.25\%$! (100M)

Test of non-Markovian Constrained MC, DGLAP case





$n = 0$: $Q \to Q$
$n = 1$: $G \to Q$ and any no. of gluon emissions out of Q and G,
$n = 2$: $G \to Q \to G \to Q$, etc.
$n = 3$: $G \to Q \to G \to Q$, etc.
$n = 4$: $Q \to G \to Q \to G \to Q$, etc. “Total” is the sum of $n = 0, 1, 2, 3, 4$.
Evolution from proton at $Q = 1$GeV up to 1TeV. New non-Markovian CMC (EvolCMC) agrees with unconstrained Markovian MC (EvolFMC) to within $\sim 0.25\%$! (210M)

Summary and outlook

- It is demonstrated using prototype program (bremsstrahlung) that the Constrained MC works in practice for the HERWIG evolution and for standard LL DGLAP with Quark—Gluon transitions.
- Still to be done: implementing Quark—Gluon transitions for the HERWIG evolution, as it was already done for the MS-bar LL DGLAP
- Including the rest of NLL corrections into CMC, and more...
- However, most difficult technical problems in constructing Constrained MCs for DGLAP-like evolutions are now solved!
- How to exploit this new technology in the construction of the full scale parton shower MC? To be seen...