Probing the Flavour Structure of SUSY Breaking With Rare B–Processes
A Beyond Leading Order Analysis

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Supersymmetry

Currently one of the best candidates for new physics at the LHC.

How SUSY is broken, particularly the flavour structure, is currently not known.

Non–trivial flavour structure in the squark can give rise to large deviations from the minimal flavour violation (MFV) scenario.

FCNC processes (e.g. $\bar{B} \rightarrow X_s \gamma$, $\bar{B}_s \rightarrow \mu^+ \mu^-$, $\bar{B}_s - B_s$ mixing) particularly sensitive to the flavour structure of the squark sector.

Provide a useful means of constraining SUSY flavour violation.
General Flavour Mixing

- Flavour violation in the soft terms measured by the dimensionless parameters $\delta^d_{XY}$.

\[
\left(\delta^d_{LL}\right)_{ij} = \frac{\left(m^2_{d,LL}\right)_{ij}}{\sqrt{\left(m^2_{d,LL}\right)_{ii} \left(m^2_{d,LL}\right)_{jj}}}, \quad \left(\delta^d_{LR}\right)_{ij} = \frac{\left(m^2_{d,LR}\right)_{ij}}{\sqrt{\left(m^2_{d,LL}\right)_{ii} \left(m^2_{d,RR}\right)_{jj}}}.\]

- $m^2_{d,XY}$ related to SUSY soft terms ($m^2_Q$, $v_dA_d$, $m^2_D$) by unitary transformations.

- Similar definitions for $\delta^d_{RR}$ and $\delta^d_{RL}$. 
Beyond Leading Order Calculations

- The usefulness of the limits one can place on SUSY flavour violation is tied to the accuracy of the underlying calculation.
- NLO SUSY calculations for a variety of FCNC processes exist but often focus on a particular limit or are incomplete.
- Beyond Leading Order (BLO) calculations provide a means of including large corrections that might arise in a complete NLO calculation.
- Include the resummation of the large logarithms and $\tan \beta$ enhanced terms.
- Large logs ($\sim \log \frac{m_{SUSY}^2}{m_W^2}$) corrections induced by running from the SUSY to the electroweak scale.
Beyond Leading Order Calculations

- $\tan \beta$ enhanced corrections manifest themselves as threshold corrections to the quark masses and Higgs vertices.
- The most well-known of these corrections are those to the bottom quark mass.

$$m_b^{(0)} = \frac{m_b}{1 + \epsilon_b \tan \beta}$$
$$\epsilon_b \tan \beta \sim \mathcal{O}(0.1 - 1)$$

- Similar corrections arise for the charged and neutral Higgs vertices.

$\tilde{d}_I$ $\tilde{d}_I$ $\tilde{d}_R$ $\tilde{d}_R$ $d_L$ $d_L$ $g$ $g$ $d_R$ $d_R$
\textbf{tan \beta Enhanced Effects}

\textit{General Flavour Mixing}

- GFM effects further modify the structure of the corrected vertices and masses present in the theory
- \textit{e.g.} For flavour violation between left handed squarks...

- Similar corrections exist for the remaining three insertions
The good agreement between the experimental result:

\[ \text{BR} (\bar{B} \rightarrow X_s \gamma)_{\text{exp.}} = (3.39 \pm 0.30) \times 10^{-4} \]

and the theoretical prediction

\[ \text{BR} (\bar{B} \rightarrow X_s \gamma)_{\text{SM}} = (3.70 \pm 0.30) \times 10^{-4} \]

places a strong constraint on any model of new physics.

- BLO calculations exist for MFV (Degrassi et al. ’00, Carena et al. ’00) and GFM (OR ’03, FOR ’05).
- The difference between a BLO and LO calculation can be especially large in the GFM case.
Focusing Effects Beyond the Leading Order

$m_{\tilde{q}} = m_{\tilde{g}}/\sqrt{2} = 1 \text{ TeV}$,
$m_A = \mu = -A_u = 500 \text{ GeV}$

- The large difference stems from cancellations between the gluino and chargino contributions to the decay.
- e.g. For the insertion $\delta_{LR}^d$...

- Similar cancellations for $\delta_{RL}^d$ and $\delta_{RR}^d$ (not for $\delta_{LL}^d$ as a LO chargino contribution exists).
Focusing Effect Beyond the Leading Order

$\tan \beta = 40$, $m_{\tilde{q}} = 1 \text{ TeV}$, $m_A = \mu = -A_u = 500 \text{ GeV}$

- Reduction of the gluino contribution and the partial cancellation with the chargino contribution leads to a focusing effect.
Focusing Effect Beyond the Leading Order

\[ \tan \beta = 40, \ m_q = 1 \text{ TeV}, \ m_A = \mu = -A_u = 500 \text{ GeV} \]

Such effects can significantly loosen the bounds placed on the mixing amongst squarks (left–right mixings in particular). e.g.
Focusing Effect Beyond the Leading Order

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\( \bar{B}_s \rightarrow \mu^+ \mu^- \) and \( \bar{B}_s - B_s \) mixing

- SUSY corrections to the neutral Higgs vertex can lead to \( \text{BR}(\bar{B}_s \rightarrow \mu^+ \mu^-) \propto \tan^6 \beta \).

- Models with large \( \tan \beta \) lead to values for \( \text{BR}(\bar{B}_s \rightarrow \mu^+ \mu^-) \) in excess of the SM prediction of \( \sim 3 \times 10^{-9} \).

- Double Higgs penguin diagrams can lead to \( \Delta M_{B_s}^{DP} \propto \tan^4 \beta \).

- MFV corrections: Deviations from the SM prediction of \( \sim 18 \text{ ps}^{-1} \) towards the experimental limit (95% C.L.) of \( 14.5 \text{ ps}^{-1} \) (HFAG).

- Suppressed by factor of \( m_s \). GFM effects can bypass this factor.
BLO effects $\bar{B}_S \rightarrow \mu^+ \mu^-$ and $\Delta M_{B_S}$

$m_q = 1$ TeV, $m_A = \mu = -A_u = 500$ GeV, $\tan \beta = 50$, $m_{\tilde{q}} = [m_{\tilde{q}}/\sqrt{2}, \sqrt{2} m_{\tilde{q}}]$

- BLO effects decrease the contributions arising from LL and RR insertions for both processes (for $\mu > 0$).
- Mixed double Higgs penguin diagrams – one mediated by chargino exchange, the other mediated by gluino exchange.
BLO effects – $\bar{B}_s \rightarrow \mu^+ \mu^-$ and $\Delta M_{B_s}$

$m_q = 1\text{ TeV}, m_A = \mu = -A_u = 500\text{ GeV}, \tan \beta = 50, m_g = [m_q/\sqrt{2}, \sqrt{2} m_q]$

- At LO the contributions due to LR and RL insertions to the neutral Higgs penguin accidentally cancel.
- BLO effects reintroduce a dependence on these parameters.
The constraints imposed by $\bar{B}_s \rightarrow \mu^+ \mu^-$ are already proving useful if $m_A$ is small and $\tan \beta$ is large.

Constraints Imposed by $\bar{B}_s \rightarrow \mu^+ \mu^-$

$m_{q} = m_{g} = 1 \text{ TeV}$, $\mu = -A_u = 500 \text{ GeV}$, $\tan \beta = 40$
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Constraints Imposed by $\bar{B}_S - B_S$ Mixing

$m_{\tilde{q}} = m_{\tilde{g}} = 1 \text{ TeV}, \mu = -A_u = 500 \text{ GeV}, \tan \beta = 40$

▶ The lower bound on $\Delta M_{B_s}$ also proves to be rather useful.
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Limits on Flavour Violation
\[ \tan \beta = 40, \ m_\tilde{q} = \sqrt{2}m_{\tilde{q}}, \ A_u = -m_{\tilde{q}}, \ \mu = m_{\tilde{q}}/\sqrt{2}, \ m_A = 500 \text{ GeV} \]

- \( \bar{B}_s \to \mu^+ \mu^- \) and \( \Delta M_{B_s} \) already provide useful constraints on a variety of insertions: \( \delta_{RR}^d \) in particular.

\begin{align*}
\bar{B} \to X_{s\gamma} & \quad \bar{B}_s \to \mu^+ \mu^- \\
\delta_{LR} & \quad \Delta M_{B_s} \\
\delta_{RR} & \quad \text{Allowed}
\end{align*}
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- $\bar{B}_{s} \rightarrow \mu^{+}\mu^{-}$
- $\bar{B} \rightarrow X_{s}\gamma$
- $\Delta M_{B_{s}}$
- $m_{\tilde{q}}$ (GeV)
Limits on Flavour Violation

\( m_{\tilde{q}} = m_{\tilde{g}} = 1 \text{ TeV}, \mu = -A_u = 500 \text{ GeV}, m_A = 500 \text{ GeV}, \tan \beta = 40 \)

- The combination of the three decays can also play a role in constraining combinations of insertions.
Limits on Flavour Violation

\( m_q = m_g = 1 \text{ TeV}, \mu = -A_u = 500 \text{ GeV}, m_A = 500 \text{ GeV}, \tan \beta = 40 \)
Supersymmetry
Beyond Leading Order Calculations
Limits
Summary

**Single Insertions**

**Multiple Insertions**

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**Limits on Flavour Violation**

\[
\tan \beta = 40, \ m_{\tilde{g}} = m_{\tilde{q}} = 500 \text{ GeV}, \ A_t = -0.5m_{\tilde{q}}, \ \mu = 0.5m_{\tilde{q}}, \ m_A = 250 \text{ GeV}
\]

- Successful measurement of either \( BR(\bar{B}_s \rightarrow \mu^+\mu^-) \) or \( \Delta M_{B_s} \) will place an extremely useful constraint on general flavour mixing.

- \( \delta^d_{LL}, \delta^d_{RR} = [-0.8, 0.8], \delta^d_{LR}, \delta^d_{RL} = [-0.08, 0.08]. \)
Limits on Flavour Violation
\[ \tan \beta = 40, \ m_g = m_{\tilde{q}} = 500 \text{ GeV}, \ A_t = -0.5m_{\tilde{q}}, \ \mu = 0.5m_{\tilde{q}}, \ m_A = 250 \text{ GeV} \]
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- \( \delta^d_{LL}, \delta^d_{RR} = [-0.8, 0.8], \delta^d_{LR}, \delta^d_{RL} = [-0.08, 0.08] \).
Limits on Flavour Violation

$\tan \beta = 40, \ m_g = m_{\bar{q}} = 1 \text{ TeV}, \ A_t = -0.5 m_{\bar{q}}, \ \mu = 0.5 m_{\bar{q}}, \ m_A = 500 \text{ GeV}$
Summary

- Rare B–decays provide an ideal probe of the flavour structure of soft SUSY breaking.
- Beyond Leading Order corrections play an important role in MFV and GFM frameworks.
- The B physics programs at the Tevatron and the LHC will be able to probe a large range of allowed parameter space for GFM models with large $\tan \beta$. 