Exploring beyond-the-SM phenomena with K physics

Gino Isidori [INFN-Frascati]

Plan:

- General considerations about flavour physics in the LHC era
- The four golden modes of Kaon physics
- Rare K decays beyond the SM
- Anatomy of $K \rightarrow \pi \nu \nu$ within SUSY models
- Not only rare K decays...
- Conclusions
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- Conclusions

Special thanks to:
- Federico Mescia
- Paride Paradisi
- Christopher Smith
- Stephanie Trine
**General considerations about flavour physics in the LHC era**

Given the great phenomenological success of the SM up to LEP energies and the limitations/unsatisfactory-aspects of the model above the e.w. scale

\[ \nu = \langle \phi \rangle \approx 250 \text{ GeV} \]

⇒ natural to consider the SM as an *effective theory*

or the low-energy limit of a more fundamental theory with new degrees of freedom appearing above some energy threshold \( \Lambda \):

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}} (A_i, \psi_i) + \mathcal{L}_{\text{Higgs}} (\phi_i, A_i, \psi_i; Y, \nu) + \ldots
\]

The stability of the Higgs sector

naturally points toward \( \Lambda \sim \text{TeV} \)
⇒ **High-energy experiments** are the key tool to determine the energy scale of the new d.o.f. (or the value of $\Lambda$) via their direct production.

⇒ **Low-energy experiments** are a fundamental ingredient to determine the symmetry properties of the new d.o.f. via indirect effects in precision observables.

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}}(A_i, \psi_i) + \mathcal{L}_{\text{Higgs}}(\phi_i, A_i, \psi_i; Y, \nu) + \sum_{d \geq 5} \frac{c_n}{\Lambda^{d-4}} O_n^d (\phi_i, A_i, \psi_i)
\]

\[\mathcal{L}_{\text{SM}} = \text{renormalizable part of } \mathcal{L}_{\text{eff}}\]

[= all possible operators with $d \leq 4$ compatible with the gauge symmetry]

most general parameterization of the new (heavy) d.o.f. as long as we perform low-energy experiments

Precision measurements in the flavour sector allows us to study the *flavour symmetries* of physics beyond the SM.
\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}} (A_i, \psi_i) + \mathcal{L}_{\text{Higgs}} (\phi_i, A_i, \psi_i; Y, v) + \sum_{d \geq 5} \frac{c_n}{\Lambda^{d-4}} \mathcal{O}_n^d (\phi_i, A_i, \psi_i) \]

3 identical replica of the basic fermion family
⇒ huge flavour-degeneracy [ \( \text{U}(3)^5 \) group ]
\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}}(A_i, \psi_i) + \mathcal{L}_{\text{Higgs}}(\phi_i, A_i, \psi_i; Y, v) + \sum_{d \geq 5} \frac{c_n}{\Lambda^{d-4}} O_n^d(\phi_i, A_i, \psi_i) \]

3 identical replica of the basic fermion family
\[ \Rightarrow \text{huge flavour-degeneracy [ } U(3)^5 \text{ group } \]

Within the SM the flavour-degeneracy is broken only by the \textbf{Yukawa} interaction:

\[ \overline{Q}_L^i Y_D^{ik} d_R^k \Phi \rightarrow \overline{Q}_L^i \begin{pmatrix} M_D^{ik} \\ \Phi \end{pmatrix} d_R^k \]

\[ \overline{Q}_L^i Y_U^{ik} u_R^k \Phi_c \rightarrow \overline{Q}_L^i \begin{pmatrix} M_U^{ik} \\ \Phi_c \end{pmatrix} u_R^k \]

Nowadays we have an \textit{excellent knowledge} of all the physical couplings appearing in the quark- Yukawa sector...

\[ M_D = \text{diag}(m_d, m_s, m_b) \]

\[ M_U = V_{\text{CKM}} \times \text{diag}(m_u, m_c, m_t) \]
\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}} (A_i, \psi_i) + \mathcal{L}_{\text{Higgs}} (\phi_i, A_i, \psi_i; Y, v) + \sum_{d \geq 5} \frac{c_n}{\Lambda^{d-4}} O_n^d (\phi_i, A_i, \psi_i) \]

...but we have only started to investigate the \textit{flavour structure} of the \textit{new degrees of freedom} (which hopefully will show up around the TeV scale):

- Several new sources of \textit{flavour symmetry breaking} are possible
\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}} (A_i, \psi_i) + \mathcal{L}_{\text{Higgs}} (\phi_i, A_i, \psi_i ; Y, v) + \sum_{d \geq 5} \frac{c_n}{\Lambda^{d-4}} O_n^d (\phi_i, A_i, \psi_i) \]

...but we have only started to investigate the flavour structure of the new degrees of freedom (which hopefully will show up around the TeV scale):

- Several new sources of flavour symmetry breaking are possible
- Rare FCNC decays \([q_i \rightarrow q_j + \gamma, l^+ l^−, \nu \nu]\), K-Kbar & B-Bbar mixing \([\Delta F=2]\) are the observables more sensitive to these new flavour-breaking couplings:

\[
A( s \rightarrow d )_{\text{FCNC}} \sim c_{\text{SM}} \frac{y_t^2 V_{ts} V_{td}^*}{16 \pi^2 M_W^2} + c_{\text{new}} \frac{\Delta_{sd}}{\Lambda^2}
\]

- No SM tree-level contributions
- Strong SM suppression due to CKM hierarchy
- Predicted with high precision when dominated by short-distance (e.w.) dynamics

key point!
**FLAVOUR COUPLING:**

<table>
<thead>
<tr>
<th>$\Delta F=2$ box</th>
<th>$b \rightarrow s \ (\sim \lambda^2)$</th>
<th>$b \rightarrow d \ (\sim \lambda^3)$</th>
<th>$s \rightarrow d \ (\sim \lambda^5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(b_L \Gamma s_L)^2$</td>
<td>$(b_L \Gamma d_L)^2$</td>
<td>$(s_L \Gamma d_L)^2$</td>
<td></td>
</tr>
</tbody>
</table>

**ELECTROWEAK STRUCTURE**

- $\Delta F=1$
  - 4-quark box
  - gluon penguin
  - $\gamma$ penguin
  - $Z^0$ penguin
  - $H^0$ penguin

**The FCNC matrix:**

Each box correspond to an indep. combination of dim.-6 $SU(3) \times SU(2) \times U(1)$-invariant operators
Some of the most significant bounds on the scale of new physics, assuming $O(1)$ couplings for the new operators:

<table>
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<tr>
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<th>$b \to s \ (\sim \lambda^2)$</th>
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</table>
| $\Delta F=2$ box | $\Lambda \geq 2 \times 10^3$ TeV \ 
from $A_{CP}(B_d \to \psi K)$ | $\Lambda \geq 4 \times 10^4$ TeV \ 
from $\varepsilon_K$ |
| $\Delta F=1$ 4-quark box | $\Lambda \geq 20$ TeV \ 
from $B(B \to X_s \gamma)$ | $\Lambda \geq 40$ TeV \ 
from $B(B \to X_s \gamma)$ |
| gluon penguin | $\Lambda \geq 40$ TeV \ 
from $B(B \to X_s \gamma)$ | $\Lambda \geq 20$ TeV \ 
from $B(B \to X_s l^+ l^-)$ |
| $\gamma$ penguin | $\Lambda \geq 40$ TeV \ 
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| $Z^0$ penguin | $\Lambda \geq 40$ TeV \ 
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from $B(B \to X_s l^+ l^-)$ |
| $H^0$ penguin | $\Lambda \geq 40$ TeV \ 
from $B(B \to X_s \gamma)$ | $\Lambda \geq 20$ TeV \ 
from $B(B \to X_s l^+ l^-)$ |

(some) of the new eff. couplings must be quite small if $\Lambda \sim \text{TeV}$
Our present knowledge is too limited to draw definite conclusions: only with the help of both high- and low-energy experiments we can hope to solve the puzzle...

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(some) of the new eff. couplings must be quite small if $\Lambda \sim \text{ TeV}$

- MFV
- effective heavy $\Lambda$ for flavour physics [e.g. split SUSY]
- approximate MFV
- non-MFV only in specific flavour structures [e.g. large $b \to s$ coupl.]
- non-MFV only in specific e.w. structures [e.g. large scalar currents]
...and there is also still hope to observe sizable deviations from the SM:

<table>
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<th>$\Delta F=2$ box</th>
<th>$b \to s \ (\sim \lambda^2)$</th>
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| $\Delta F=1$ 4-quark box | $\Lambda \geq 20 \text{ TeV}$ from $B(B \to X_s \gamma)$ | $\Lambda \geq 40 \text{ TeV}$ from $B(B \to X_s \gamma)$ |

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<th>gluon penguin</th>
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<td>$B_s \to \mu\mu$</td>
<td>$B_d \to \mu\mu$</td>
<td>$K \to \pi \nu \nu$</td>
<td>$K_L \to \pi^0 \ell^+\ell^-$</td>
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Three very interesting corners which, for different & well-motivated reasons, could hide sizable effects.
The four golden modes of Kaon physics

Channels where short-distance dynamics (W - top quark loops) constitutes the dominant contribution (or a large fraction) of the decay amplitude:

\[
K^+ \rightarrow \pi^+ \nu \nu \quad K_L \rightarrow \pi^0 \nu \nu \quad K_L \rightarrow \pi^0 e^+e^- \quad K_L \rightarrow \pi^0 \mu^+\mu^-
\]

common leading short-distance amplitude within the SM:
The four golden modes of Kaon physics

Channels where short-distance dynamics (W - top quark loops) constitutes the dominant contribution (or a large fraction) of the decay amplitude:

\[
\begin{align*}
K^+ &\to \pi^+ \nu \nu \\
K_L &\to \pi^0 \nu \nu \\
K_L &\to \pi^0 e^+e^- \\
K_L &\to \pi^0 \mu^+\mu^-
\end{align*}
\]

I. Clean electroweak short-distance amplitude

- similar -within the SM- for all the channels
  potentially different beyond the SM

II. Long-distance contributions due to charm & light quarks

- potentially large effects of e.m. origin in \( K \to \pi \ll \)
  [but under good th. control in the \( K_L \to \pi^0 \) case]

- small effects in \( K \to \pi \nu \nu \) modes
  [totally negligible in the \( K_L \to \pi^0 \) case]
In the last few years there has been a substantial progress in the control of the theory error associated to charm & light-quark loops:

- $K_L \to \pi^0 l^+ l^-$ Complete analysis of the long-distance effects of e.m. origin:

\[
B(K_L \to \pi^0 l^+ l^-)^{[\text{SM}]} = \left[ C_{\text{mix}} + C_{\text{int}} y_t + C_{\text{dir}} y_t^2 + C_{\text{CPC}} \right] \times 10^{-12}
\]

\[
y_t = \frac{\text{Im}(V_{ts}^* V_{td})}{10^{-4}}
\]

\[
\begin{align*}
[e^+ e^-] & \approx 23 + (10 + 4) + 0 \quad \Rightarrow \quad (3.7 \pm 1.0) \times 10^{-11} \\
[\mu^+ \mu^-] & \approx 5.4 + (2.5 + 1.8) + 5.2 \quad \Rightarrow \quad (1.5 \pm 0.3) \times 10^{-11}
\end{align*}
\]
In the last few years there has been a substantial progress in the control of the theory error associated to charm & light-quark loops:

- $K_L \to \pi^0 l^+l^-$ Complete analysis of the long-distance effects of e.m. origin
- $K^+ \to \pi^+ \nu\nu$ NNLO analysis of the leading (dim.-6) charm contribution:

\[ \Delta(BR)_{NLO} = \pm 10\% \]
\[ \Delta(BR)_{NNLO} = \pm 6\% \]

\[ \text{totally dominated by the param. uncertainty of mc} \]
In the last few years there has been a substantial progress in the control of the theory error associated to charm & light-quark loops:

- **$K_L \rightarrow \pi^0 l^+l^-$** Complete analysis of the long-distance effects of e.m. origin
- **$K^+ \rightarrow \pi^+ \nu \nu$** NNLO analysis of the leading (dim.-6) charm contribution & first consistent analysis of the $O(m_K^2/m_c^2)$ corrections:

  **amplitude decomposition:**

  - $\approx 66\%$
  - $\approx 31\%$
  - $\approx 3\%$

  $\Delta(\text{BR})_{\text{long}} = \pm 3\%$

G.I., Mescia, Smith '05

$Q_v^{(8)} \sim (s \bar{d})(\nu \bar{\nu})$
Summary of the *irreducible theoretical uncertainties*:

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<th>Process</th>
<th>Short-distance (e.w.) contrib. to the total rate $(\Gamma - \Gamma_{\text{no s.d.}}) / \Gamma$</th>
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<td>$K_L \to \pi^0 e^+ e^-$</td>
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Very few observables, also in the B system, have a similar degree of short-distance sensitivity & theoretical cleanliness.
Rare Kaon decays beyond the SM [general properties]

Two basic scenarios:

**Minimal Flavour Violation**
flavour symmetry broken only by the (SM) Yukawa couplings

- Small deviations (10-20%) from SM
- Stringent correlations with other rare decays in B physics \([B_d \to X_{s,d} \nu\nu, B_d \to X_{s,d} l^+l^−, B_{s,d} \to l^+l^−]\)

**New sources of Flavour Symmetry breaking around the TeV scale**

- Potentially large effects, especially in the three CPV \(K_L\) decays (no \(\lambda^5\) suppression)
- Correlations with observables in B physics not obvious
**Rare Kaon decays beyond the SM** [general properties]

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Recent (almost) model-indep. analysis :

Consistent with results of specific models:
- Constrained MSSM [Buras et al. '01]
- One universal extra dim. [Buras et al. '03]
- Littlest-Higgs [Buras et al. '05]
Rare Kaon decays beyond the SM [general properties]

Two basic scenarios:

E.g.: II. Generic MSSM

New sources of Flavour Symmetry breaking around the TeV scale

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- Correlations with observables in B physics not obvious

Grossman-Nir bound:
$$\Gamma(K_L \to \pi^0\nu\bar{\nu}) < \Gamma(K^+ \to \pi^+\nu\bar{\nu})$$
**Rare Kaon decays beyond the SM** [general properties]

Two basic scenarios:

- **E.g.:**
  - II. Enhanced e.w. penguins
  - New sources of Flavour Symmetry breaking around the TeV scale

- Potentially large effects, especially in the three CPV $K_L$ decays (no $\lambda^5$ suppression)
- Correlations with observables in B physics not obvious

G.I., Smith, Unterdorfer '04
Rare Kaon decays beyond the SM [general properties]

Two basic scenarios:

**Minimal Flavour Violation**
flavour symmetry broken only by the (SM) Yukawa couplings

- Small deviations (10-20%) from SM
- Stringent correlations with other rare decays in B physics: $B_d \rightarrow X_{s,d} \nu \nu$, $B_d \rightarrow X_{s,d} l^+ l^-$, $B_{s,d} \rightarrow l^+ l^-$

A precise exp. info from one of the two $K \rightarrow \pi \nu \nu$ modes is a key ingredient to verify or disproof the MFV hypothesis

**New sources of Flavour Symmetry breaking around the TeV scale**

- Potentially large effects, especially in the three CPV $K_L$ decays (no $\lambda^5$ suppression)
- Correlations with observables in B physics not obvious

In presence of sizable non-MFV couplings mandatory to explore also the $K_L \rightarrow \pi ll$ modes
Anatomy of $K \rightarrow \pi \nu\nu$ within low-energy SUSY models

...or better within supersymmetric extensions of the SM with

- minimal particle content
- R-parity conservation
- soft-breaking terms in the TeV range:

$$\mathcal{L}_{\text{soft}} = (M_f)_{ij} \chi_i \chi_j + (M^2_s)_{ij} \phi_i \phi_j + A_{ijk} \phi_i \phi_j \phi_k$$

- gaugino/higgsino masses
- squark/slepton masses
- trilinear scalar couplings

$$M^2_{\text{squarks}} = \begin{bmatrix}
(M_Q^2)_{LL} & (M^2_{U,D})_{LR} \\
(M^2_{U,D})^+_{LR} & (M^2_{U,D})_{RR}
\end{bmatrix}$$

Five very different structures

- which naturally contain new sources of flavor-symmetry breaking
- which contribute in different ways to different FCNC processes
**Anatomy of $K \to \pi \nu \nu$ within low-energy SUSY models**

Main distinctive features $K \to \pi \nu \nu$ amplitudes [wide literature]:

- Gluino-type amplitudes essentially negligible, even in presence of new sources of flavour mixing [contrary to $\varepsilon_K$, $b \to s\gamma$, $\Delta M_{Bd}$, CPV in B decays]
  $\Rightarrow$ reduced sensitivity to LL, RR and LR-down type mixings

- Appreciable deviations from SM induced only by *chargino -- up-squark* diagrams $\Rightarrow$ minor effects within pure MFV [except for very light sparticles]

![Diagram](image-url)
**Anatomy of K → πνν within low-energy SUSY models**

Main distinctive features $K \rightarrow \pi\nu\nu$ amplitudes [wide literature]:

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- Appreciable deviations from SM induced only by **chargino -- up-squark** diagrams $\Rightarrow$ minor effects within pure MFV [except for very light sparticles]

\[K \rightarrow \pi\nu\nu\text{ decays are the best probe of the flavour structure of the up-type trilinear terms which are still largely unknown:}\]

\[\mathcal{L}_{soft} \subset (A^U Y^U)_{ij} Q_L^i U_R^j \phi\]
Anatomy of $K \to \pi \nu \nu$ within low-energy SUSY models

Main distinctive features $K \to \pi \nu \nu$ amplitudes [wide literature]:

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  $\Rightarrow$ reduced sensitivity to LL, RR and LR-down type mixings

- Appreciable deviations from SM induced only by chargino -- up-squark diagrams $\Rightarrow$ minor effects within pure MFV [except for very light sparticles]

\[ K \to \pi \nu \nu \text{ decays are the best probe of the flavour structure of the up-type trilinear terms which are still largely unknown:} \]

\[ \mathcal{L}_{\text{soft}} \subset (A^U Y^U)_{ij} Q^i_L U^j_R \phi \]

N.B.: The LR mixing in the up sector contains at least one large source of flavour symmetry breaking ($y_t$) + box
Systematic investigation of the correlations of SUSY effects in $K \to \pi \nu \nu$, other FCNCs, e.w. & future high-energy observables

[G.I., F.Mescia, P.Paradisi, C.Smith, S.Trine]:

- $B(K^+ \to \pi^+ \nu \nu)$
- $B(B_d \to \mu \mu)$
- $\Delta M_{B_d}$

$\Delta M_{B_d}$

$(\tilde{m}_\chi)_{\text{min}} = 250 \pm 10 \text{ GeV}$
$(\tilde{m}_t)_{\text{min}} = 500 \pm 10 \text{ GeV}$
$(\tilde{m}_q)_{\text{heavy}} = 1000 \text{ GeV}$
$tan(\beta) = 2$

$A_0 = 1 \text{ TeV}$
$|A_{13}| \leq \lambda A_0$
$|A_{23}| \leq \lambda A_0$
Non-standard effects induced by chargino-squarks amplitudes largely dominant in $K \to \pi \nu \nu$ with respect to similar effects in B physics

The $A$ terms are still largely unconstrained

\begin{align*}
B(K^+ \to \pi^+ \nu \nu) \\
B(B_d \to \mu \mu) \\
\Delta M_{B_d} \\
B(B_d \to X_s l^+ l^-)
\end{align*}

$\Delta M_{B_d}$

$\begin{align*}
(m_{\chi})_{\text{min}} &= 250 \pm 10 \text{ GeV} \\
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(m_q)_{\text{heavy}} &= 1000 \text{ GeV} \\
\tan(\beta) &= 2
\end{align*}$

$A_0 = 1 \text{ TeV}$

$|A_{13}| \leq \lambda A_0$

$|A_{23}| \leq \lambda A_0$
☆ Even if the mass spectrum is fully determined by LHC, large uncertainties in FCNCs due to several FV couplings ⇒ important to combine different observables

☆ The single MIA (mass insert. approx.) is bad approximation for \( K \rightarrow \pi \nu \nu \), even in presence of a single flavour-violating coupling ⇒ dominance of the double-MIA [Colangelo & G.I. '98]
At fixed magnitude of the $A$ terms, there is a larger room for deviations from the SM in the CPV observables $\Rightarrow$ great interest of $K_L \to \pi^0 \nu \nu$
• At fixed magnitude of the A terms, there is a larger room for deviations from the SM in the CPV observables ⇒ great interest of $K_L \rightarrow \pi^0\nu\nu$

• Slower decoupling of penguins ($K \rightarrow \pi\nu\nu$) with respect to boxes ($\Delta F = 2$)
Complete analysis still in progress [more observables & more correlations under investigation] but general conclusions already clear:

✶ The flavour structure of the up-type trilinear terms is still largely unknown.

✶ $K \to \pi \nu \nu$ decays are a unique probe of these soft-breaking terms, while they are marginally sensitive to flavour-blind structures compared to $\varepsilon_K$ and $b \to s \gamma$.

✶ The maximal sensitivity to the up-type trilinear terms is obtained for:
  - Light stop & charginos [but non-negligible effects up to masses ~ 1 TeV]
  - Small tan $\beta$
Other interesting and peculiar aspects of rare $K$ decays arise in more specific corners of the MSSM parameter-space.

E.g.: RR Higgs-mediated $Z$ penguins at large $\tan\beta$

\begin{equation}
\sim y_b (\delta_{RR})_3 \varepsilon_{\text{loop}} (\tan\beta)^2
\end{equation}

$(\tan\beta)^4$ – contribution to the decay amplitude

If $\tan\beta \sim 50$ sizable corrections to the BR's even for $(\delta_{RR})_{31}(\delta_{RR})_{32} = O(\lambda^5)$
Not only $K \rightarrow \pi \nu \nu$ ...

The four golden FCNC modes provide a key information for any model with new d.o.f. at the TeV scale carrying quark-flavour quantum numbers,

but there are other observables in the kaon sector which provides useful infos about more specific BSM scenarios, which is still worth to search / measure with better precision in the future:

- Precision tests of Lepton-Flavour Universality in charged-current interactions [e.g.: $\Gamma(K^+ \rightarrow \mu^+\nu)/\Gamma(K^+ \rightarrow e^+\nu)$ ]

- Search for LFV decays [e.g.: $\Gamma(K_L \rightarrow \mu e)$, ... ]

- T-violating observables [e.g.: $P_T^{\mu}(K^+ \rightarrow \pi^0 \mu^+\nu)$, ... ]

- Precision tests of CPT and QM by means of interference measurements
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The neutral kaon system is by far the best probe of these type of effects
Violations of Lepton-Flavour Universality in charged-current interactions

Interesting effect generated within the MSSM

Key ingredients:
- large tan$\beta$
- sizable LFV terms [slepton sector]

\[
\Gamma(K \rightarrow \mu\nu)^{\text{exp}} = \Gamma(K \rightarrow \mu\nu_\mu) + \Gamma(K \rightarrow \mu\nu_e) + \Gamma(K \rightarrow \mu\nu_\tau) \\
\approx \text{SM} \quad \approx 0
\]

scalar LFV amplitude

The best probe of this effect is obtained by means of

\[
R^K_{\mu e} = \frac{\Gamma(K^+ \rightarrow \mu^+\nu)}{\Gamma(K^+ \rightarrow e^+\nu)}
\]

sizable one-loop eff. coupl.
because of 3$^{\text{rd}}$ generation & large mixing in the lepton sector
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\[
R^{K}_{\mu e} = \frac{\Gamma(K^+ \to \mu^+\nu)}{\Gamma(K^+ \to e^+\nu)}
\]

\[
R^{K}_{\mu e} \approx (R^{K}_{\mu e})^{\text{SM}} \left[ 1 + 0.013 \left( \frac{\tan\beta}{40} \right)^6 \left( \frac{500 \text{ GeV}}{M_H} \right)^4 \left( \frac{\Delta_{13}}{5 \times 10^{-4}} \right)^2 \right] \\
< 0.017 \quad [\text{NA48/2 @ ICHEP'05}]
\]

\[\text{N.B.: } \Delta_{13} \sim 5 \times 10^{-4} \Leftrightarrow \text{LFV } \tau \text{ decays of } O(10^{-10}) \Rightarrow R^{k}_{\mu e} \text{ deep probe of LFV}\]
CPT tests & neutral kaon interferometry

CPT symmetry is linked to the basic mathematical tools that we use in particle physics:

$$\text{QFT} + \text{Lorentz invariance} + \text{Locality} \Rightarrow \text{CPT}$$

These tools have intrinsic limitations [we are not able to include gravity in consistent way] ⇒ we should expect CPT at some level

But we do not have a consistent & predictive theory if we abandon these tools ⇒ hard to define a reference scale/size for CPT

$$|M_{K^-} - M_K| < 10^{-18} M_K$$

Very suggestive... (but should not be over-emphasized)

Main message:
- The neutral kaon system offer a unique framework to test CPT (& QM)
- The reference scale in this type of search is set by the most significant experimental bounds ⇒ worth to improve
Conclusions

There is no doubt that Kaon physics will still be very interesting in the LHC era.

The still-open question is if we will have dedicated experiments [hopefully more the one...] able to fully exploit its discovery potential.