



# ${ m B_s^0}$ mass difference $\Delta M_s$ and mixing phase $\phi_s$ at LHCb

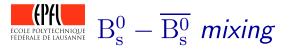
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November  $8^{\rm th}$ , 2005 'Flavour in the era of the LHC workshop', CERN

On behalf of the LHCb collaboration

$$\aleph \ B_s^0 - \overline{B_s^0}$$
 mixing

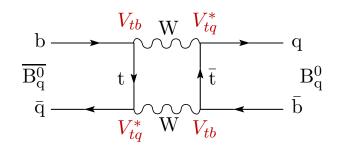
- LHCb full Monte Carlo simulation
- Sensitivity studies
- $\& \Delta M_s \ {
  m from} \ {
  m B}_{
  m s}^0 
  ightarrow {
  m D}_{
  m s} \pi$





Neutral  $B_{\rm q}^0$  are *not* eigenstates of the weak interaction

ightarrow "mixing": particle-anti-particle oscillations ( $|\Delta B=2|$ )



Time evolution of  $B^0_q$  and  $\overline{B^0_q}$ 

$$i\frac{\mathrm{d}}{\mathrm{d}t}\left(\begin{array}{c} \mathrm{B}_{\mathrm{q}}^{0}(t) \\ \overline{\mathrm{B}_{\mathrm{q}}^{0}}(t) \end{array}\right) = \underbrace{\left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)}_{\mathcal{H}_{\mathrm{eff}}}\left(\begin{array}{c} \mathrm{B}_{\mathrm{q}}^{0}(t) \\ \overline{\mathrm{B}_{\mathrm{q}}^{0}}(t) \end{array}\right) \Longrightarrow$$

Physical (mass) eigenstates

$$|\mathbf{B}_{L/H}\rangle = p|\mathbf{B}_{\mathbf{q}}^{0}\rangle \pm q|\overline{\mathbf{B}_{q}^{0}}\rangle$$

$$\mathrm{B_s^0}~\mathrm{CP}~\mathrm{phase}~\phi = \mathrm{arg}\left(-M_{12}^{(s)}/\Gamma_{12}^{(s)}\right) \approx 2\,\mathrm{arg}[V_\mathrm{ts}^*V_\mathrm{tb}] \sim -2\lambda^2\eta = \mathcal{O}(-0.04)~\mathrm{rad}~\mathrm{in}~\mathrm{SM}$$

where 
$$\arg(-\Gamma_{12}^{(s)}) = 2\arg(V_{\rm cb}V_{\rm cs}^*) + \mathcal{O}(\lambda^2)$$
 suppressed contributions  $\rightarrow \arg(-\Gamma_{12}^{(s)}) \simeq 0$ 

- $M_{12}^{(s)}$ : virtual intermediate states  $\Rightarrow$  sensitive to New Physics

En route for NP with  $B_s^0 - \overline{B_s^0}$  mixing?





 $B^0_a - \overline{B^0_a}$  (well measured) versus  $B^0_s - \overline{B^0_s}$  (*Terra incognita*) in SM

- $\Delta M_d \sim 0.5 \text{ ps}^{-1}$
- $\Delta\Gamma_d/\Gamma_d\sim 0$
- $\Delta M_s \sim 20~{\rm ps}^{-1} \sim 40$  times faster than  ${\rm B_d^0}!$
- $\bullet$   $\Delta\Gamma_s/\Gamma_s\sim 10\%$
- $\phi_d \stackrel{\text{SM}}{\equiv} 2 \arg \left[ V_{\text{td}}^* V_{\text{tb}} \right] \approx 2\beta = \mathcal{O}(0.8) \text{ rad}$   $\phi_s \stackrel{\text{SM}}{\equiv} 2 \arg \left[ V_{\text{ts}}^* V_{\text{tb}} \right] \approx -2\beta_s = \mathcal{O}(-0.04) \text{ rad}$ 
  - $\@ifnextchar[{@}]{\@ifnextch$
  - Arr constrain  $V_{
    m td}$ :  $rac{\Delta M_s}{\Delta M_d} \propto rac{{\sf m_{B_s^0}}}{{\sf m_{B_s^0}}} \xi^2 rac{|V_{
    m ts}|^2}{|V_{
    m td}|^2} 
    ightarrow$  theoretical errors from  $\Delta M_q$  partly cancel in ratio
  - $\Delta M_s$  beyond SM prediction ( $\Delta M_s^{\rm SM} > 14.5~{\rm ps}^{-1}$  at 95%):  $\Delta M_s = \Delta M_s^{\rm SM} + \Delta M_s^{\rm NP}$ ?
  - ☆ prerequisite for time-dependent CP-asymmetries!

Determination of: 
$$\phi_s = \underbrace{\phi^{\rm SM}}_{\mathcal{O}(-0.04)} + \phi^{\rm NP}$$
?
$$\begin{array}{c} b \\ \overline{B}_s^0 \\ \overline{s} \end{array} \begin{array}{c} ??? \\ \vdots ??? \vdots \\ \hline & \vdots ??? \vdots \\ \hline & \overline{b} \end{array}$$





#### Estimate LHCb performances in reconstructing b-decays

- - $\maltese$  proper-time resolution: must be good enough to resolve fast  $B_s^0 \overline{B_s^0}$  oscillations
  - ☆ tagging: knowledge of initial b-hadron flavour, dilution of CP-asymmetries (wrong tag)
- decay channels selection (yields, trigger efficiencies), background sources / levels

#### Full MC simulation main steps:

- **Pythia**: generation of p p collisions at  $\sqrt{s} = 14 \text{ TeV}$  (including pile-up)
- @ Geant: detector response, spill-over and tracking through material





mass resolutions

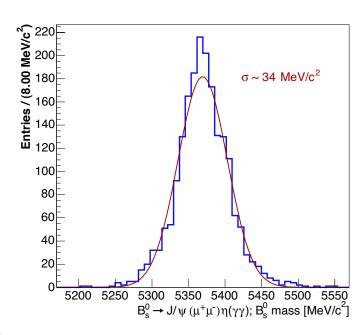
$$Arr$$
 ~ 15  $m MeV/c^2$  for charged final states  $Arr$  ~ 30 − 70  $m MeV/c^2$  when involving  $m \gamma(s)$ 

**№** vertex

- $\Rightarrow$  primaries  $\sigma_z \sim 50 \ \mu \mathrm{m}$
- b-decay vertices  $σ_z < 200 μm$
- proper-time

$$\Rightarrow \sigma_{\tau} \sim 30 - 40 \text{ fs}$$





Flavour tagging power:  $\varepsilon_{\rm eff} = \varepsilon_{\rm tag} (1 - 2\omega_{\rm tag})^2$ 

& for 
$$B_s^0\to$$
 2003 MC:  $\varepsilon_{\rm eff}\sim 6\%$ , 2004 MC (neural network):  $\varepsilon_{\rm eff}\sim 7-9\%$ 

#### Presentation results based on:

- for  $\Delta M_s$ : studies with 2003 MC data (re-opt. TDR CERN/LHCC 2003-030)
- for  $\phi_s$ : new study with recent MC data
  - ightarrow improved tagging, L1 trigger, high-level trigger design ( $\sim 2~\mathrm{kHz}$ ),  $\dots$



# Sensitivity to mixing observables



#### Statistical sensitivities to mixing observables assessed using fast MC

- generate event samples with LHCb expected statistics
- characteristics of samples taken from full simulation (resolutions, acceptance, tagging)

Unbinned maximum likelihood fits to proper-time to extract expected statistical uncertainties

$$\mathcal{L} = \prod_{i} \left[ f_i^{\text{sig}} \mathcal{R}_i^{\text{sig}} + (1 - f_i^{\text{sig}}) \mathcal{R}_i^{\text{bkg}} \right]$$

- $lpha f_i^{
  m sig}$ : signal probability based on reconstructed mass;  $\mathcal{R}_i^{
  m sig}$ ,  $\mathcal{R}_i^{
  m bkg}$ : signal, bkg decay rates
- rates convoluted with proper-time resolution and weighted with acceptance
- proper-time resolution based on per-candidate computed errors from full MC



# Ecole polytechnique $B^0_s \to D_s \pi$ reconstruction



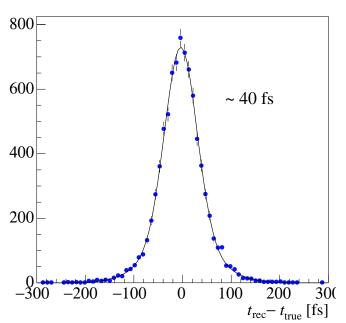
 $\Delta M_s$  measurement with  ${\rm B_s^0} \to {\rm D_s^-}\pi^+$ : flavour specific decay

\* flavour asymmetry  $\mathcal{A}_f^{obs}(t) = -D \cdot \frac{\cos(\Delta M_s t)}{\cosh(\frac{\Delta \Gamma_s t}{2})}$ , D = (1 - 2w) if perfect resolution

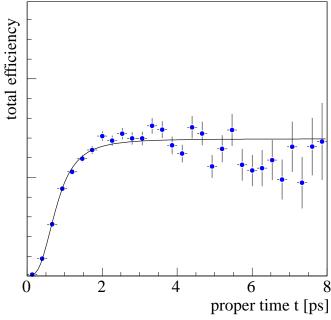
Full MC study (LHCb 2003-127)

- lpha reconstructed with  $D_s \Rightarrow KK\pi$  mode, expect  $\sim 80$  k events per year (2 fb<sup>-1</sup>)
- $\gg B/S \sim 0.3$  from fully-simulated inclusive  ${
  m b \bar b}$  events

#### Proper time resolution $\sim 40$ fs



#### Acceptance (proper-time efficiency)



⇒ characteristics from full MC used as inputs for toy studies (LHCb 2003-103)

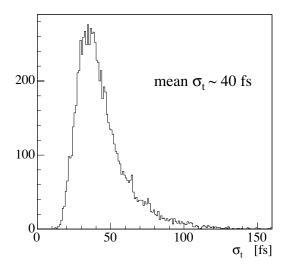


# $B_s^0$ oscillation frequency with $B_s^0 \to D_s \pi$

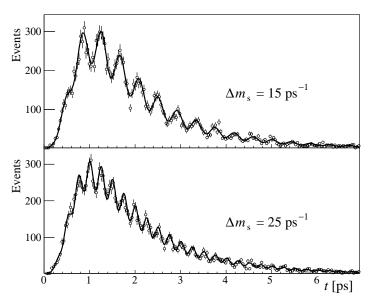
# LHCb

#### Unbinned likelihood fit:

- rates weighted with acceptance, tagging dilution
- proper-time error  $\sigma_t$  obtained from full MC  $\rightarrow$  uncertainty to generated events
- $\Delta \Gamma_s / \Gamma_s = 0.1$



## Once oscillations observed, precise value of $\Delta M_s$ obtained: uncertainty $\sim 0.06\%$ (2 fb<sup>-1</sup>)



Statistical precision on  $\Delta M_s$  after 1 year (2 fb<sup>-1</sup>)

$\Delta M_s \; [\mathrm{ps}^{-1}]$	15	20	25	30
$\sigma(\Delta M_s) \; [\mathrm{ps}^{-1}]$	0.009	0.011	0.013	0.016

- $\sigma(\Delta M_s)$  will probably be dominated by systematics, e.g. t scale
  - ightarrow even if  $\sigma_{
    m sys} \sim 10 \cdot \sigma_{
    m stat}$ , uncertainty < 1%

Decay rate for unmixed  $B_{\mathrm{s}}^{0}$ 

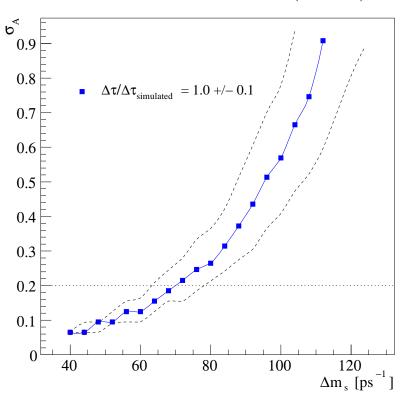
November 8<sup>th</sup>, 2005





'Amplitude method' used to evaluate maximum value of  $\Delta M_s$  measurable (2  ${
m fb}^{-1}$ )

 $\rightarrow$  fit factor A in front of  $\cos{(\Delta M_s t)}$  term in asymmetry for different  $\Delta M_s$  values



Statistical uncertainty on amplitude factor A ( $\sigma_A$ ) versus  $\Delta M_s$ 

Sensitivity limit:  $\Delta M_s$  for which  $5 \cdot \sigma_A = 1 = A$ 

In 1 year,  $\geq 5\sigma$  observation of  $B_s^0$  oscillations up to  $\Delta M_s = 68~{\rm ps}^{-1}$ 

 $\rightarrow$  could exclude full SM range

'Immediate' measure of  $\Delta M_s$  if small: 1/8 year LHCb running! (0.25 fb<sup>-1</sup>,  $\Delta M_s = 40 \text{ ps}^{-1}$ )



## *CP* violation and $\bar{b} \rightarrow \bar{c}c\bar{s}$ transitions



- $aisebox{8.5}{
  m B}_{
  m s}^0 
  ightarrow {
  m J}/\psi \; \phi$  : admixture of CP eigenstates ( $\eta_{{
  m J}/\psi\phi}=+1,-1,+1$ )
  - $\rightarrow$  one-angle  $\theta_{tr}$  angular analysis (Phys.Rev. D63 (2001) 114015, hep-ph/0012219)
- fraction of CP-odd decays defined as  $R_T \equiv \left|A_\perp(0)\right|^2/\sum_{i=0,\parallel,\perp}\left|A_f(0)\right|^2 \sim \mathcal{O}(0.2)$
- $\hbox{\it \&} \ B_s^0 \to \eta_c \ \phi \ \hbox{, } B_s^0 \to J/\psi \eta^{(')} \hbox{: pure CP-even eigenstates} \to \hbox{no angular analysis needed}$
- Mixing-induced CP violation: phase mismatch  $\phi_s-2\phi_D\approx\phi_s\neq0,\pi$  "first mix, then decay"

 $\overline{B}_{s}^{0} = \overline{V}_{tb} \quad t \quad V_{ts}^{*} \\
\overline{B}_{s}^{0} = \overline{V}_{tb}^{0} \quad \overline{b} = \overline{V}_{cs}^{0} = \overline{S}_{s} \quad \phi, \eta^{(')} \\
\overline{B}_{s}^{0} = \overline{V}_{tb}^{0} \quad \overline{b} = \overline{V}_{tb}^{0} = \overline{C}_{s}^{0} \quad J/\psi, \eta_{c}$ 

ightarrow CP-asymmetry directly measures  $\phi_s = \mathcal{O}(-0.04) \ \mathrm{rad}$  (for given  $\eta_{f_{\mathrm{CP}}}$ )

$$\mathcal{A}_{\mathrm{CP}}(t) = \frac{-\eta_{f_{\mathrm{CP}}} \sin{(\phi_s)} \sin{(\Delta M_s t)}}{\cosh{(\frac{\Delta \Gamma_s t}{2})} - \eta_{f_{\mathrm{CP}}} \cos{(\phi_s)} \sinh{(\frac{\Delta \Gamma_s t}{2})}}$$



# Physics model: $\bar{b} \to \bar{c}c\bar{s}$ to pure CP eigenstates



- $\ref{eq:Final states} f = \eta_c \phi, J/\psi \eta^{(')}$  CP-even eigenstates:  $(\mathcal{CP})|f\rangle = \eta_f |f\rangle$ ,  $\eta_f = +1$
- $\ref{Transition rates}$  of initially pure  $B^0_s$  and  $\overline{B^0_s}$  states (perfect resolution)

$$R\left(\mathbf{B}_{s}^{0}(t) \to f\right) = |A_{f}(0)|^{2} \times e^{-\Gamma_{s}t}$$

$$\times \left[\cosh\left(\frac{\Delta\Gamma_{s}t}{2}\right) - \eta_{f}\cos(\phi_{s})\sinh\left(\frac{\Delta\Gamma_{s}t}{2}\right) + qD\,\eta_{f}\sin(\phi_{s})\sin\left(\Delta M_{s}t\right)\right]$$

- $\Delta D = (1 2\omega)$ : tagging dilution;  $\omega$ : wrong tag fraction
- & Both D and  $\phi_s$  modulate the oscillating term: need a control channel to extract  $\omega \to B^0_s \to D_s \pi$  is used
- $\red{w}$  Untagged events: access to  $\Delta\Gamma_s$  and  $\phi_s$  (small sensitivity to  $\phi_s$ , since  $\mathcal{O}(\phi_s^2)$  in SM)



## Inputs from full MC simulation for $\phi_s$ study



Decay channels considered to assess LHCb sensitivity to  $\phi_s$ :

$$B_{\rm s}^0 \to {\rm J}/\psi(\mu^+\mu^-)\phi({\rm K}^+{\rm K}^-)$$

$$aisebox{0.85}{\ }B^0_s o J/\psi(\mu^+\mu^-)\eta(\gamma\gamma,\pi^+\pi^-\pi^0)$$
: pure CP eigenstate

$$\&~B_s^0 \to \eta_c(\pi^+\pi^-\pi^+\pi^-,\pi^+\pi^-K^+K^-,K^+K^-K^+K^-)\phi(K^+K^-)$$
: pure CP eigenstate

These channels were studied in the full MC (2004 MC data), and inputs used for toys Most relevant parameters (yields after high-level trigger):

Parameters	$J/\psi  \eta(\gamma\gamma)$	$J/\psi  \eta(\pi^+\pi^-\pi^0)$	$\eta_{ m c}\phi$	$J/\psi \phi$
Untagged yield [k events] $(2  ext{ fb}^{-1})$	8.9	3.1	3	125
B/S	2.0	3.0	0.7	0.3
Mean $\sigma_{t_i^{rec}}$ [fs]	30.4	25.5	26.2	35.8
$\omega_{tag} \ / \ \varepsilon_{tag} [\%]$	35/63	30/62	31/66	33/60

These  $\bar b\to \bar c c\bar s$  transitions will be fitted simultaneously with  $B^0_s\to D_s\pi$  sample





- $aisebox{0.8}{\circ}$  Generate and fit  $\sim 250$  toy experiments corresponding to 1 year data taking at  $m 2 \ fb^{-1}$
- **W** Unbinned (extended) likelihood fit to  $\mathcal{L}_{tot}^{ar{\mathrm{b}} o ar{\mathrm{c}} \mathrm{c} ar{\mathrm{s}}}$

$$\mathcal{L}_{tot}^{ar{ ext{b}} 
ightarrow ar{ ext{ccs}}} = \prod_{i \in ext{B}_{ ext{s}}^0 
ightarrow f} \mathcal{L}_{i}^{ar{ ext{b}} 
ightarrow ar{ ext{ccs}}}(m_i, heta_{ ext{tr}i}, t_i^{rec}, \sigma_{t_i}, q_i)$$

- 1. Mass distributions fitted to determine signal and background probabilities
- 2. Sidebands: background parameters determined, acceptance fitted
- 3. Signal window: physics parameters  $\vec{\alpha} = (\Delta \Gamma_s/\Gamma_s, \Delta M_s, \phi_s, \tau_{\rm B_s^0}(,R_T))$  and wrong tag fraction  $\omega$  fitted
- $\&~\mathcal{L}_{tot}^{ar{b} oar{c}car{s}}$  simultaneously maximized with likelihood of the  $B_s^0 o D_s\pi$  control sample



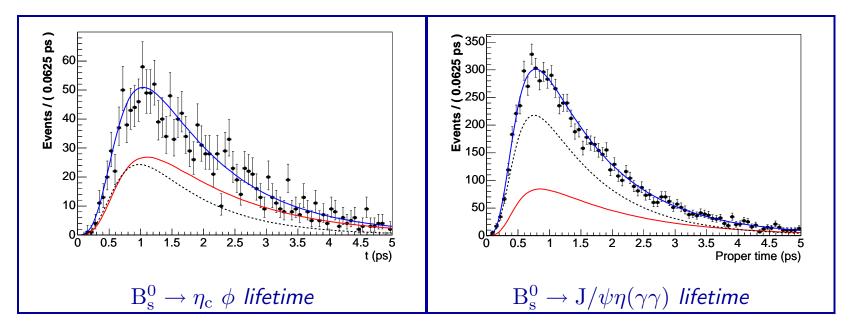


$$\mathcal{L}_{t,even}^{sig}(t_{i}^{rec},\sigma_{t_{i}},q_{i}|\overrightarrow{\alpha},\omega,acc_{s}) \propto A(t_{i}^{rec}) \times \left[ (1-\omega)\Gamma_{\mathbf{B}_{s}^{0}\to f}^{even}(t_{i}^{true}) + \omega\Gamma_{\overline{\mathbf{B}}_{s}^{0}\to f}^{even}(t_{i}^{true}) \right]$$

$$\otimes G(t_{i}^{rec} - t_{i}^{true},S\sigma_{t_{i}},\mu\sigma_{t_{i}})$$

$$\mathcal{L}_{t}^{bkg}(t_{i}^{rec};\tau_{bkg},acc_{s}) \propto A(t_{i}^{rec}) \times E(t_{i}^{true};\tau_{bkg}) \otimes \delta(t_{i}^{rec} - t_{i}^{true})$$

 $\vec{\alpha} = (\Delta\Gamma_s/\Gamma_s, \Delta M_s, \phi_s, \tau_{\rm B_s^0}(R_T))$ : vector of physics parameters



Signal: red, Background: black, Total: blue





## Physics input values

$$\phi_s \ [{
m rad}] \ \Delta M_s \ [{
m ps}^{-1}] \ \Delta \Gamma_s / \Gamma_s \ au_{
m B_s^0} \ [{
m ps}] \ R_T$$
 -0.04 20.0 0.1 1.472 0.2

## Fit results $(2 \text{ fb}^{-1})$

Sensitivity
 
$$J/\psi \ \eta(\gamma\gamma)$$
 $J/\psi \ \eta(3\pi)$ 
 $\eta_c \phi$ 
 $J/\psi \ \phi$ 
 $\sigma(\Delta\Gamma_s/\Gamma_s)$ 
 0.019
 0.024
 0.025
 0.011

 $\sigma(R_T) = 0.0047$ 

Channels	$oldsymbol{\sigma}(\phi_{oldsymbol{s}})$ [rad]	Weight $(\sigma/\sigma_i)^2$ [%]
$B_s^0 \to J/\psi \ \eta(\gamma \ \gamma)$	0.112	6.4
$B_s^0 \to J/\psi \; \eta(\pi^+ \; \pi^- \; \pi^0)$	0.148	3.6
$\mathrm{B_s^0}  ightarrow \eta_\mathrm{c} \ \phi$	0.106	7.1
Combined three pure CP eigenstates channels	0.068	17.1
$B_s^0 \to J/\psi \ \phi$	0.031	82.9
Combined all four CP eigenstates channels	0.028	100.0

#### Contribution from pure CP eigenstates: $\sim 17\%$

With 10 fb<sup>-1</sup> (5 years): 
$$\sigma(\phi_s) \sim 0.013$$
 rad  $\longrightarrow \sim 3\sigma$  for  $\phi_s = -0.04$  rad (SM)





 $\Delta M_s$  with  $\mathrm{B_s^0} \to \mathrm{D_s}\pi$ 

- $aisebox{ very good precision after 1 year LHCb. If SM <math>\Delta M_s$ , do not need  $2~{
  m fb}^{-1}$  to measure it
- could exclude full SM range in 1 year

$$\phi_s$$
 with  $B_s^0 \to J/\psi \phi(K^+K^-)$ ,  $B_s^0 \to J/\psi \eta(\gamma\gamma,\pi^+\pi^-\pi^0)$ ,  $B_s^0 \to \eta_c(4h)\phi(K^+K^-)$ 

- $\gg 3\sigma$  measurement within 5 year for SM  $\phi_s$ ,  $\sim 17\%$  contribution from pure CP eigenstates
- $lpha \geq 5\sigma$  after 1 year if  $\phi_s \sim -0.2$  rad
- $\ref{eq:condition}$  other channels could be added:  $B^0_s\to J/\psi(e^+e^-)\phi(K^+K^-)$  ,  $B^0_s\to J/\psi\eta'$
- lifetime unbiased selections and trigger to be explored (flat proper-time efficiency)
  - $\Rightarrow B_s^0 \overline{B_s^0}$  represents a sensitive probe for New Physics  $\Leftarrow$





# **BACK-UP SLIDES**



# Physics model: $B_s^0 \to J/\psi \ \phi$



 $\ensuremath{\cancel{\&}}$  Final state f is an admixture of CP eigenstates

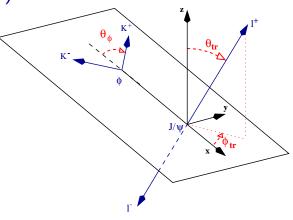
🖈 
$$f=0,\parallel$$
: CP-even,  $\eta_f=+1$  ,  $f=\perp$ : CP-odd,  $\eta_f=-1$ 

- $\red{w}$  Linear polarization amplitudes:  $A_f(t)$ 
  - Arr fraction of CP-odd decays defined as  $R_T \equiv \left|A_{\perp}(0)\right|^2/\sum_{i=0,\parallel,\perp}\left|A_f(0)\right|^2 \sim \mathcal{O}(0.2)$
  - $R_T = (0.2 \pm 0.1)$ , CDF: Phys.Rev.Lett. 94 (2005) 101803 (hep-ex/0412057)
- $\ref{thm:properties}$  The one-angle  $heta_{tr}$  distribution enables to disentangle the different CP eigenstates

$$\frac{d\Gamma(t)}{d(\cos(\theta_{tr}))} \propto \left[ |A_0(t)|^2 + |A_{\parallel}(t)|^2 \right] \frac{3}{8} (1 + \cos^2 \theta_{tr}) + |A_{\perp}(t)|^2 \frac{3}{4} \sin^2 \theta_{tr}$$

(Phys.Rev. D63 (2001) 114015, hep-ph/0012219)

Transversity angle  $\theta_{tr}$ : angle between positive lepton from the  $J/\psi$  and the normal to the  $\phi$  decay plane, in the  $J/\psi$  rest frame





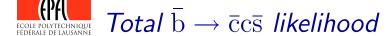


Scans: input values to nominal, except for parameter under study  $(2 \text{ fb}^{-1})$ 

$\sigma(\phi_s)$ [rad]	Nominal	$\Delta M_s = 15 \mathrm{ps}^{-1}$	$\Delta M_s = 25 \mathrm{ps}^{-1}$	$\Delta\Gamma_s/\Gamma_s = 0.2$	$R_T = 0$	$R_T = 0.5$
$J/\psi  \eta(\gamma\gamma)$	0.112	0.102	0.126	0.099	_	_
$J/\psi  \eta(3\pi)$	0.148	0.136	0.161	0.139	_	_
$\eta_c \phi$	0.106	0.100	0.113	0.097	_	_
$\mathrm{J}/\psi \; \phi$	0.031	0.028	0.034	0.030	0.021	0.062

- $\ensuremath{\text{@}}$  sensitivity to  $\phi_s$  increases by  $\sim 10\%$  per  $5~\mathrm{ps^{-1}}$  step in  $\Delta M_s$
- $R_T=0$ : pure CP eigenstate limit for  $B^0_s o J/\psi \ \phi$  ,  $\sigma(\phi_s)$  1.5 times better w.r.t nominal
- $R_T = 0.5$ :  $\sigma(\phi_s)$  gets 2 times worse for equal CP-even and CP-odd fractions

Good precision for larger  $\phi_s \sim -0.2$  rad: more than  $5\sigma$  in one year





$$\mathcal{L}_{tot}^{\bar{\mathbf{b}} \to \bar{\mathbf{c}} c \bar{\mathbf{s}}} = \prod_{i \in \mathbf{B}_{\mathbf{s}}^{0} \to f} \mathcal{L}_{i}^{\bar{\mathbf{b}} \to \bar{\mathbf{c}} c \bar{\mathbf{s}}}(m_{i}, \theta_{\mathrm{tr}i}, t_{i}^{rec}, \sigma_{t_{i}}, q_{i})$$

$$\mathcal{L}_{i}^{\bar{b} \to \bar{c}c\bar{s}}(m_{i}, \theta_{tri}, t_{i}^{rec}, \sigma_{t_{i}}, q_{i}) = \mathcal{L}_{m}^{sig}(m_{i}) \Big[ R_{T} \mathcal{L}_{\theta_{tr}}^{sig,odd}(\theta_{tri}) \mathcal{L}_{t,odd}^{sig}(t_{i}^{rec}, \sigma_{t_{i}}, q_{i}) + (1 - R_{T}) \mathcal{L}_{\theta_{tr}}^{sig,even}(\theta_{tri}) \mathcal{L}_{t,even}^{sig}(t_{i}^{rec}, \sigma_{t_{i}}, q_{i}) \Big] \times \mathcal{L}_{m}^{bkg}(m_{i}) \mathcal{L}_{\theta_{tr}}^{bkg}(\theta_{tri}) \mathcal{L}_{t}^{bkg}(t_{i}^{rec})$$

- $\mathcal{L}_m^{sig}(m_i), \mathcal{L}_m^{bkg}(m_i)$ : signal, background probabilities based on the reconstructed mass  $m_i$ 
  - ☆ Gaussian for signal, exponential for background
- $\mathcal{L}_t^{sig}(t_i^{rec}, \sigma_{t_i}, q_i), \mathcal{L}_t^{bkg}(t_i^{rec})$ : signal, background decay rates
- $\mathcal{L}_{\theta_{\mathrm{tr}}}^{sig}(\theta_{\mathrm{tr}i}), \mathcal{L}_{\theta_{\mathrm{tr}}}^{bkg}(\theta_{\mathrm{tr}i})$ : angular parts for  $B_{\mathrm{s}}^{0} \to \mathrm{J}/\psi \ \phi$  with transversity angle  $\theta_{\mathrm{tr}i}$