# Exploring New Physics with B Physics 

Workshop on Flavour in the Era of the LHC CERN - November 7, 2005<br>Yossi Nir (Weizmann Institute of Science)

Thanks to:

- Guy Raz
- Stephane Monteil et al. (CKMfitter)
- Luca Silvestrini (UTfit)


## Plan of Talk

## Plan of Talk

1. Recent Era: Excluding alternatives to the KM mechanism
(a) Is the KM mechanism at work?
(b) Is $\delta_{\mathrm{KM}}$ the only source of CPV in meson decays?
(c) Is CPV in $K \rightarrow \pi \pi$ small because of flavor suppression?
(d) Is there direct CPV?
(e) Is there New Physics in $B^{0}-\bar{B}^{0}$ mixing?
(f) Is there New Physics in $b \rightarrow s$ transitions?
2. Future: Looking for corrections to the KM mechanism $S_{\pi^{0} K_{S}}: S M$ and NP
(a) Factorization related methods
(b) $\mathrm{SU}(3)$ based methods
(c) Supersymmetry as an example

## Is the KM mechanism at work?

- Assume: New Physics in tree decays - negligible
- Define $r_{d}^{2} \exp \left(2 i \theta_{d}\right)=\left\langle B^{0}\right| \mathcal{H}^{\text {full }}\left|\bar{B}^{0}\right\rangle /\left\langle B^{0}\right| \mathcal{H}^{\mathrm{SM}}\left|\bar{B}^{0}\right\rangle$
- Use $\left|V_{u b} / V_{c b}\right|, \mathcal{A}_{D K}, S_{\psi K}, S_{\rho \rho}, \Delta m_{B_{d}}, \mathcal{A}_{\mathrm{SL}}$
- Fit to $\eta, \rho, r_{d}, \theta_{d}$
- Find whether $\eta=0$ is allowed
$S_{\psi K_{S}}, S_{\rho \rho}, \mathcal{A}_{D K} \cdots$


## The KM mechanism is at work!



$$
\delta_{\mathrm{KM}} \neq 0
$$

## Is $\delta_{\mathrm{KM}}$ the only source of CPV in meson decays?



Tree level + CPC observables

$$
\Delta m_{B}, \quad \Delta m_{B_{s}}
$$

Tree level + CPV observables

$$
\varepsilon, \quad S_{\psi K}, \quad \alpha, \gamma
$$

## Very likely, $\delta_{\mathrm{KM}}$ is dominant!

$$
S_{\psi K}=+0.69 \pm 0.03 \Leftrightarrow \sin 2 \beta(\mathrm{CKM} \mathrm{fit})=+0.74_{-0.03}^{+0.07}
$$

The KM mechanism successfully passed its first precision test

## Very likely, $\delta_{\mathrm{KM}}$ is dominant!

$$
S_{\psi K}=+0.69 \pm 0.03 \Leftrightarrow \sin 2 \beta(\mathrm{CKM} \mathrm{fit})=+0.74_{-0.03}^{+0.07}
$$

The KM mechanism successfully passed its first precision test

$$
\alpha(\pi \pi, \pi \rho, \rho \rho)=\left[101_{-9}^{+16}\right]^{o} \Leftrightarrow \alpha(\mathrm{CKM} \text { fit })=96 \pm 16^{o}
$$

The KM mechanism successfully passed its second precision test

## Very likely, $\delta_{\mathrm{KM}}$ is dominant!

$$
S_{\psi K}=+0.69 \pm 0.03 \Leftrightarrow \sin 2 \beta(\mathrm{CKM} \mathrm{fit})=+0.74_{-0.03}^{+0.07}
$$

The KM mechanism successfully passed its first precision test

$$
\alpha(\pi \pi, \pi \rho, \rho \rho)=\left[101_{-9}^{+16}\right]^{o} \Leftrightarrow \alpha(\mathrm{CKM} \text { fit })=96 \pm 16^{o}
$$

The KM mechanism successfully passed its second precision test

$$
\gamma(D K)=\left[63_{-13}^{+15}\right]^{o} \Leftrightarrow \gamma(\mathrm{CKM} \mathrm{fit})=\left[57_{-14}^{+7}\right]^{o}
$$

The KM mechanism successfully passed its third precision test

## Is CPV in $K \rightarrow \pi \pi$ small because of flavor?

SM:

- $\epsilon \sim 10^{-3}, \epsilon^{\prime} \sim 10^{-5}$ because of flavor suppression
- Some CP violating phases are order one

Approximate CP:

- All CPV phases are small
- All CP asymmetries are small


## Is CPV in $K \rightarrow \pi \pi$ small because of flavor?

SM:

- $\epsilon \sim 10^{-3}, \epsilon^{\prime} \sim 10^{-5}$ because of flavor suppression
- Some CP violating phases are order one

Approximate CP:

- All CPV phases are small
- All CP asymmetries are small

B Physics:

- $S_{\psi K} \sim 0.7$
$\Longrightarrow$ Some CP violating phases are indeed $\mathcal{O}(1)$

$$
\mathcal{A}_{K \mp \pi \pm}, \mathcal{A}_{\rho \pi}^{-+}
$$

## Is CP violated in $\Delta B=1$ processes?

SM:

- Indirect $\left(M_{12}\right)$ and direct $\left(A_{f}\right)$ CP violations are both large

Superweak:

- There is no direct $\left(A_{f}\right) \mathrm{CP}$ violation

K Physics:

- $\epsilon^{\prime} / \epsilon=(1.72 \pm 0.18) \times 10^{-3}$
$\Longrightarrow \mathrm{CP}$ is violated in $\Delta S=1$ processes $(s \rightarrow u \bar{u} d)$

$$
\mathcal{A}_{K \mp \pi \pm}, \mathcal{A}_{\rho \pi}^{-+}
$$

## Is CP violated in $\Delta B=1$ processes?

SM:

- Indirect $\left(M_{12}\right)$ and direct $\left(A_{f}\right)$ CP violations are both large

Superweak:

- There is no direct $\left(A_{f}\right) \mathrm{CP}$ violation

K Physics:

- $\epsilon^{\prime} / \epsilon=(1.72 \pm 0.18) \times 10^{-3}$
$\Longrightarrow \mathrm{CP}$ is violated in $\Delta S=1$ processes $(s \rightarrow u \bar{u} d)$

B Physics:

- $\mathcal{A}_{K \mp \pi^{ \pm}}=-0.115 \pm 0.018, \mathcal{A}_{\rho \pi}^{-+}=-0.48 \pm 0.14$
$\Longrightarrow \mathrm{CP}$ is violated in $\Delta B=1$ processes $(b \rightarrow u \bar{u} s, b \rightarrow u \bar{u} d)$
$S_{\psi K_{S}}, \Delta m_{B}, \mathcal{A}_{\mathrm{SL}}$


## Is there NP in $B^{0}-\bar{B}^{0}$ mixing?

- Assume: New Physics in tree decays - negligible
- Define $r_{d}^{2} \exp \left(2 i \theta_{d}\right) \equiv 1+h e^{i \sigma} \equiv\left\langle B^{0}\right| \mathcal{H}^{\text {full }}\left|\bar{B}^{0}\right\rangle /\left\langle B^{0}\right| \mathcal{H}^{\mathrm{SM}}\left|\bar{B}^{0}\right\rangle$
- Use $\left|V_{u b} / V_{c b}\right|, \mathcal{A}_{D K}, S_{\psi K}, S_{\rho \rho}, \Delta m_{B_{d}}, \mathcal{A}_{\mathrm{SL}}$
- Fit to $\eta, \rho, r_{d}, \theta_{d}($ or $h, \sigma)$
- Find whether $h \neq 0\left(r_{d} \neq 1\right)$ is allowed


## Very likely, the KM mechanism dominates




CKMFitter
Agashe et al., hep-ph/0509117
For arbitrary phase, $h=\left|A_{\mathrm{NP}} / A_{\mathrm{SM}}\right| \sim 0.2 \pm 0.2$

## Is there NP in $b \rightarrow s$ transitions?

- Rare $b \rightarrow s$ processes consistent with the SM predictions
$\Longrightarrow$ New Physics contributions to certain operators are strongly constrained ( $Z$-penguin, magnetic)
$\Longrightarrow$ New physics contributions to other operators are still very weakly constrained (chromomagnetic, dim-6)


## Is there NP in $b \rightarrow s$ transitions?

$$
\mathrm{SM}: \Delta S \equiv-\eta_{\mathrm{CP}} S-S_{\psi K} \approx 0, \quad C \approx 0
$$

| $f_{\mathrm{CP}}$ | $\Delta S$ | $C$ |
| :---: | :---: | :---: |
| $\phi K_{S}$ | $-0.22 \pm 0.19$ | $-0.09 \pm 0.15$ |
| $\eta^{\prime} K_{S}$ | $-0.19 \pm 0.09^{\dagger}$ | $-0.07 \pm 0.07$ |
| $f_{0} K_{S}$ | $+0.06 \pm 0.24$ | $+0.06 \pm 0.21$ |
| $\pi^{0} K_{S}$ | $-0.38 \pm 0.26$ | $-0.02 \pm 0.13$ |
| $\omega K_{S}$ | $-0.06 \pm 0.30$ | $-0.44 \pm 0.24$ |
| $K_{S} K_{S} K_{S}$ | $-0.09 \pm 0.23$ | $-0.31 \pm 0.17$ |
| $K^{+} K^{-} K_{S}^{\ddagger}$ | $-0.16 \pm 0.17$ | $+0.09 \pm 0.10$ |

$\dagger$ Belle and Babar not quite consistent $(\Longrightarrow-0.19 \pm 0.13) ~ \ddagger$ Not a CP eigenstate
$\Longrightarrow$ How good is $\approx$ ?

## Basics

- Formalism:
- Effective $\mathcal{H}$ for $b \rightarrow s q \bar{q}$ decays:

$$
\begin{aligned}
& \mathcal{H}=\frac{G_{F}}{\sqrt{2}} \sum_{p=u, c} V_{p s}^{*} V_{p b}\left(\sum_{i=1}^{2} C_{i} O_{i}^{p}\right. \\
& \left.+\sum_{i=3}^{10} C_{i} O_{i}+C_{7 \gamma} O_{7 \gamma}+C_{8 g} O_{8 g}\right)
\end{aligned}
$$

- Decay amplitudes:

$$
A_{f}=\langle f| \mathcal{H}\left|B^{0}\right\rangle, \bar{A}_{f}=\langle f| \mathcal{H}\left|\bar{B}^{0}\right\rangle, \quad \lambda_{f}=e^{-i \phi_{B}}\left(\bar{A}_{f} / A_{f}\right)
$$

- CP asymmetries:

$$
S_{f}=2 \mathcal{I} m\left(\lambda_{f}\right) /\left(1+\left|\lambda_{f}\right|^{2}\right), C_{f}=\left(1-\left|\lambda_{f}\right|^{2}\right) /\left(1+\left|\lambda_{f}\right|^{2}\right)
$$

- SM:
$-A_{f}^{\mathrm{SM}}=A_{f}^{c}+A_{f}^{u}$ with $A_{f}^{c} \propto V_{c b}^{*} V_{c s}$ and $A_{f}^{u} \propto V_{u b}^{*} V_{u s}$
$-A_{f}^{\mathrm{SM}}=A_{f}^{c}\left(1+a_{f}^{u} e^{i \gamma}\right)$


## The Problem

- Simple NP:
$-C_{i}\left(m_{W}\right)=C_{i}^{\mathrm{SM}}\left(m_{W}\right)+x_{i} \varepsilon e^{i \theta}, x_{i}$ known
$-A_{f}^{\mathrm{SM}}=A_{f}^{c}\left[1+a_{f}^{u} e^{i \gamma}+b_{f}^{c} \varepsilon e^{i \theta}\right]$
- CP asymmetries:
$-\Delta S_{f}=+2 \cos 2 \beta_{\mathrm{eff}}\left[\mathcal{R} e\left(b_{f}^{c}\right) \varepsilon \sin \theta+\mathcal{R} e\left(a_{f}^{u}\right) \sin \gamma\right]$
$-C_{f}=-2 \mathcal{I} m\left(b_{f}^{c}\right) \varepsilon \sin \theta-2 \operatorname{IIm}\left(a_{f}^{u}\right) \sin \gamma$
- Problem:
- To be concvinced that $\varepsilon \neq 0$, we need to know $a_{f}^{u}$
$-a_{f}^{u}=\left|\left(V_{u b} V_{u s}\right) /\left(V_{c b} V_{c s}\right)\right| \times$ hadronic parameters


## Factorization-related methods

Example: $B \rightarrow K^{0} \pi^{0}$ :

$$
\begin{aligned}
A_{K^{0} \pi^{0}}^{c} \approx & i V_{c b}^{*} V_{c s} \frac{G_{F}}{2} f_{K} F^{B \rightarrow \pi}\left(m_{K}^{2}\right)\left(m_{B}^{2}-m_{\pi}^{2}\right)\left(a_{4}+r_{\chi} a_{6}\right) \\
A_{K^{0} \pi^{0}}^{u} \approx & i V_{u b}^{*} V_{u s} \frac{G_{F}}{2}\left[f_{K} F^{B \rightarrow \pi}\left(m_{K}^{2}\right)\left(m_{B}^{2}-m_{\pi}^{2}\right)\left(a_{4}+r_{\chi} a_{6}\right)\right. \\
& \left.-f_{\pi} F^{B \rightarrow K}\left(m_{\pi}^{2}\right)\left(m_{B}^{2}-m_{K}^{2}\right) a_{2}\right]
\end{aligned}
$$

where $r_{\chi}=2 m_{K}^{2} /\left[m_{b}\left(m_{s}+m_{d}\right)\right], a_{i} \equiv C_{i}+C_{i \pm 1} / N_{c}$


$$
a_{K \pi}^{u} \approx \lambda^{2} R_{u}\left(1-\frac{f_{\pi}}{f_{K}} \frac{F^{B \rightarrow K}}{F^{B \rightarrow \pi}} \frac{a_{2}}{a_{4}+r_{\chi} a_{6}}\right) \approx 2.75 \lambda^{2} R_{u} \approx 0.052
$$

$\Longrightarrow \Delta S_{\pi K_{S}} \approx 0.06, \quad C_{\pi K_{S}} \approx 0$

## $\Delta S_{\pi^{0} K_{S}}$ in Factorization-related methods

| $\Delta S_{\pi^{0} K_{S}}$ | Method | hep-ph/ | Authors |
| :---: | :---: | :---: | :---: |
| $+0.06 \pm 0.04$ | NF | 0503151 | Buchalla, Hiller, Nir, Raz |
| $+0.04 \pm 0.03$ | $\mathrm{NF}+$ model | 0502235 | Cheng, Chua, Soni |
| $+0.07 \pm 0.04$ | QCDF | 0505075 | Beneke |
| $+0.06 \pm 0.03$ | PQCD | 0508041 | Li, Mishima, Sanda |
| $+0.08 \pm 0.16$ | $\mathrm{SCET}+\mathrm{SU}(3)$ | 0510241 | Bauer, Rothstein, Stewart |

## SU(3)-based methods

- For $b \rightarrow s$, define: $d_{f}^{q}=A_{f}^{q} /\left(V_{q b}^{*} V_{q s}\right)$

$$
A_{f}=V_{c b}^{*} V_{c s} d_{f}^{c}+V_{u b} V_{u s}^{*} d_{f}^{u}
$$

- For $b \rightarrow d$, define: $h_{f^{\prime}}^{q}=A_{f^{\prime}}^{q} /\left(V_{q b}^{*} V_{q d}\right)$

$$
A_{f^{\prime}}=V_{c b}^{*} V_{c d} h_{f^{\prime}}^{c}+V_{u b} V_{u d}^{*} h_{f^{\prime}}^{u}
$$

The approximate $\mathrm{SU}(3)$ symmetry of the strong interactions
$\Longrightarrow d_{f}^{q}=\Sigma_{f^{\prime}} X_{f^{\prime}} h_{f^{\prime}}^{q}$
where $X_{f^{\prime}}$ are known (CG) coefficients

$$
\left|a_{f}^{u}\right|=\frac{\left|V_{u b} V_{u s} d_{f}^{u}\right|}{\left|V_{c b} V_{c s} d_{f}^{c}\right|} \leq\left|\frac{V_{u s}}{V_{u d}}\right| \frac{\Sigma_{f^{\prime}}\left|X_{f^{\prime}}\right| \sqrt{\mathcal{B}\left(B \rightarrow f^{\prime}\right)}}{\sqrt{\mathcal{B}(B \rightarrow f)}}
$$

## SU(3)-based methods

$\ominus \mathrm{SU}(3)$ breaking effects of $\mathcal{O}(0.3)$
$\Longrightarrow$ The bounds are only approximate
$\ominus$ Adding (conservatively) the amplitudes coherently

+ Dependence on measured $\mathcal{B}$ 's
$\Longrightarrow$ Bounds often much weaker than actual estimates
$\oplus$ Hadronic model independence
$\Longrightarrow$ Complimentary to the factorization-related methods


## SU(3)-based methods

Example: $B \rightarrow K^{0} \pi^{0}$ :

$$
\begin{gathered}
d_{B_{d}^{0} \rightarrow K^{0} \pi^{0}}^{q}=\frac{1}{\sqrt{2}} h_{B_{d}^{0} \rightarrow K^{+}}^{q}-h_{B_{d}^{0} \rightarrow \pi^{0} \pi^{0}}^{q} \\
\left|a_{K^{0} \pi^{0}}^{u}\right| \leq\left|\frac{V_{u s}}{V_{u d}}\right| \frac{\frac{1}{\sqrt{2}} \sqrt{\mathcal{B}\left(B \rightarrow K^{+} K^{-}\right)}+\sqrt{\mathcal{B}\left(B \rightarrow \pi^{0} \pi^{0}\right)}}{\sqrt{\mathcal{B}\left(B \rightarrow K^{0} \pi^{0}\right)}} \\
\downarrow \\
\Longrightarrow\left[\Delta S_{\pi K_{S}} / \cos 2 \beta\right]^{2}+C_{\pi K_{S}}^{2} \leq[0.30 \sin \gamma]^{2}
\end{gathered}
$$

hep-ph/0310020 (Gronau, Grossman, Rosner), hep-ph/0509125 (Raz)

## Estimating $\Delta S_{f} \equiv-\eta_{f} S_{f}-S_{\psi K_{s}}$

| $f_{\mathrm{CP}}$ | EXP | $\mathrm{NF}^{*}$ | $\mathrm{QCDF}^{* *}$ | $\mathrm{SU}(3)^{* * *}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\phi K_{S}$ | $-0.22 \pm 0.19$ | $+0.02 \pm 0.01$ | $+0.02 \pm 0.01$ | $\left(K^{*} K\right)^{\dagger}$ |
| $\eta^{\prime} K_{S}$ | $-0.19 \pm 0.09$ | $+0.01 \pm 0.02$ | $+0.01 \pm 0.01$ | 0.31 |
| $f_{0} K_{S}$ | $+0.06 \pm 0.24$ |  |  |  |
| $\pi^{0} K_{S}$ | $-0.38 \pm 0.26$ | $+0.06 \pm 0.04$ | $+0.07 \pm 0.04$ | 0.18 |
| $\omega K_{S}$ | $-0.06 \pm 0.30$ | $+0.19_{-0.14}^{+0.06}$ | $0.13 \pm 0.08$ | $\left(K^{*} K\right)^{\dagger}$ |
| $K_{S} K_{S} K_{S}$ | $-0.08 \pm 0.23$ |  |  | $1(0.37)$ |
| $K^{+} K^{-} K_{S}^{\ddagger}$ | $-0.16 \pm 0.17$ |  |  | 0.94 |

* Buchalla, Hiller, Nir, Raz, hep-ph/0503151
** Beneke, hep-ph/0505075
*     *         * Grossman, Ligeti, Quinn, Nir, hep-ph/0303171; Raz et al., hep-ph/0505195,0508046,0509125
$\dagger$ Available once $\mathcal{B}\left(K^{*} K\right)$ is measured


## Supersymmetry: constraints and predictions


$\left(\delta_{23}^{d}\right)_{L L}$

$S_{\pi K}$

$\left(\delta_{23}^{d}\right)_{R R}$

$S_{\phi K}$

$\left(\delta_{23}^{d}\right)_{L R}$

$S_{\eta^{\prime} K}$
$S_{\omega K}$

Silvestrini, hep-ph/0510077

## Supersymmetry: constraints and predictions

| $\left(\delta_{23}^{d}\right)_{M N}$ | Upper Bound $^{*}$ | $\left\|\Delta S_{\phi K_{S}}\right\| \sim 0.1^{* *}$ | Alignment $^{* * *}$ |
| :---: | :---: | :---: | :---: |
| $\left(\delta_{23}^{d}\right)_{L L}$ | $\lambda^{2}(\mathcal{R} e)-\lambda(\mathcal{I} m)$ | $\lambda$ | $\lambda^{2}$ |
| $\left(\delta_{23}^{d}\right)_{R R}$ | 1 | $\lambda$ | $\lambda^{4}-\lambda^{2}$ |
| $\left(\delta_{23}^{d}\right)_{L R}$ | $\lambda^{4}(\mathcal{R} e)-\lambda^{3}(\mathcal{I} m)$ | $\lambda^{4}$ | $\lambda^{2}\left(m_{b} / \tilde{m}\right)$ |
| $\left(\delta_{23}^{d}\right)_{R L}$ | $\lambda^{3}$ | $\lambda^{4}$ | $\lambda^{4}\left(m_{b} / \tilde{m}\right)$ |

* Ciuchini et al., hep-ph/0407073
** Silvestrini, hep-ph/0510077
*     *         * Nir, Raz, hep-ph/0206064


## Conclusions

## Unitarity Triangles 2005



$$
b \rightarrow d
$$

$\Delta m_{B_{d}}, S_{\psi K}, S_{\rho \rho}$

$b \rightarrow s$
$\Delta m_{B_{s}}, S_{b \rightarrow s \bar{s} s}$

There is still a lot to be learned from future measurements

## Conclusions

- The KM phase is different from zero (SM violates CP)
- The KM mechanism is, very likely, the dominant source of the CP violation observed in meson decays
- The size and the phase of NP contributions to $B^{0}-\bar{B}^{0}$ mixing are severely constrained
- Complete alternatives to the KM mechanism are excluded (Superweak, Approximate CP)
- Corrections to KM are possible, particularly for $b \rightarrow s$; No evidence for such corrections at present
- There is still a lot to be learned from flavor/CP physics


## Is there NP in $b \rightarrow s$ transitions?



Kirkby and Nir, PDG
No evidence at present

## Experimental status of CP asymmetries

| $f_{\mathrm{CP}}$ | $-\eta_{\mathrm{CP}} S$ | $C$ |
| :---: | :---: | :---: |
| $\psi \pi^{0}$ | $+0.69 \pm 0.25$ | $-0.11 \pm 0.20$ |
| $D^{+} D^{-}$ | $+0.29 \pm 0.63$ | $+0.11 \pm 0.35$ |
| $D^{*+} D^{*-}$ | $+0.75 \pm 0.23$ | $-0.04 \pm 0.14$ |
| $\pi^{+} \pi^{-}$ | $+0.50 \pm 0.12(0.18)$ | $-0.37 \pm 0.10(0.23)$ |
| $\pi^{0} \pi^{0}$ |  | $-0.28 \pm 0.39$ |
| $\rho^{+} \rho^{-}$ | $+0.22 \pm 0.22$ | $-0.02 \pm 0.17$ |

## The NP CP /Flavor Problem

- $m_{H}^{2} \sim\left(m_{H}^{2}\right)_{\text {tree }}+\frac{1}{16 \pi^{2}} \Lambda_{\mathrm{NP}}^{2}$

To avoid fine-tuning of the Higgs mass,

$$
\Lambda_{\mathrm{NP}} \lesssim 4 \pi m_{W} \sim 1 T e V
$$

- $\mathcal{L}_{\mathrm{NP}} \sim \frac{1}{\Lambda_{\mathrm{NP}}^{2}} s \bar{d} s \bar{d}$

To avoid too large contributions to $\varepsilon_{K}$ and to $\Delta m_{K, D, B}$, $\Lambda_{\mathrm{NP}} \gtrsim 10^{3-4} \mathrm{TeV}$.

> New Physics at the TeV scale must have a very non-generic flavor and CP structure

## SU(3) Relations

$$
\left.\begin{array}{l}
A_{f}=V_{c b}^{*} V_{c s} a_{f}^{c}+V_{u b}^{*} V_{u s} a_{f}^{u}, \quad \xi_{f} \equiv\left|V_{u b} V_{u s} / V_{c b} V_{c s}\right|\left(a_{f}^{c} / a_{f}^{u}\right) \\
--\eta_{f} S_{f}-S_{\psi K}=2 \cos 2 \phi_{1} \sin \phi_{3} \operatorname{Re}\left(\xi_{f}\right) \\
\hline \hline C_{f}=-2 \sin \phi_{3} \operatorname{Im}\left(\xi_{f}\right)
\end{array} \quad \begin{array}{c}
\text { Grossman, Ligeti, Nir, Quinn (03) } \\
\text { Engelhard, Nir, Raz (05) }
\end{array}\right)
$$

- Example: $\left|\xi_{\eta^{\prime} K_{S}}\right| \leq \sqrt{\frac{3 \mathcal{B}\left(\eta^{\prime} \eta\right)}{2 \mathcal{B}\left(\eta^{\prime} K_{S}\right)}}+\sqrt{\frac{\mathcal{B}\left(\eta^{\prime} \pi^{0}\right)}{2 \mathcal{B}\left(\eta^{\prime} K_{S}\right)}}$

| mode | $\eta^{\prime} K_{S}$ | $\pi^{0} K_{S}$ | $K^{-} \pi^{+}$ | $\eta^{\prime} K^{+}$ | $\phi K^{+}$ | $K_{S} K_{S} K_{S}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\|\xi\|<$ | 0.25 | 0.18 | 0.23 | 0.07 | 0.22 | $0.31^{\dagger}$ |

$\dagger$ Extra (mild) dynamical assumptions

