$K \to 3\pi$: unveiling ε'/ε and π-π scattering

I. Scimemi
CSIC-IFIC, Valencia

in collaboration with E. Gamiz (U. Glasgow), J. Prades (U. Granada), JHEP 0310:042,2003, Moriond 2004 (hep-ph/0405204) and work in progress

Contents of the talk

- A brief introduction to $|\Delta S| = 1$ processes

Contents of the talk

- A brief introduction to $|\Delta S| = 1$ processes
- ε'/ε: status and problems

Contents of the talk

- A brief introduction to $|\Delta S| = 1$ processes
- ε'/ε: status and problems
- Why to study CPV in $K \to 3\pi$
- Newest results on $K \to 3\pi$
- Conclusions for CPV
- Cabibbo proposal for the measure of π-π scattering length
- Summary

SM and Eff. theories

En. scale	Fields	Eff. Theory

SM and Eff. theories

En. scale	Fields	Eff. Theory
M_z	W,Z,γ,g,l,ν_l,q_u,q_d	SM + . . .
	$\downarrow$ $\mathbb{OPE}$	
$\lesssim m_c$	$\gamma, g, \mu, e, \nu_\mu, \nu_e, s, d, u$	$\mathcal{L}_{QCD}^{(n_f=3)}$
	$\downarrow$ Large N_C, Lattice,. . .	
M_K	$\gamma, \mu, e, \nu_\mu, \nu_e, \pi, K, \eta$	ChPT

SM and Eff. theories

En. scale	Fields	Eff. Theory
M_z	W,Z,γ,g,l,ν_l,q_u,q_d	SM + . . .
	$\downarrow$ $\mathbb{OPE}$	
$\lesssim m_c$	$\gamma, g, \mu, e, \nu_\mu, \nu_e, s, d, u$	$\mathcal{L}_{QCD}^{(n_f=3)}$
	$\downarrow$ Large N_C, Lattice,. . .	
M_K	$\gamma, \mu, e, \nu_\mu, \nu_e, \pi, K, \eta$	ChPT

$$\mathcal{L}_{eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu)$$

SM and Eff. theories

En. scale	Fields	Eff. Theory
M_z	W,Z,γ,g,l,ν_l,q_u,q_d	SM + ...
	$\downarrow$ OPE	
$\lesssim m_c$	$\gamma, g, \mu, e, \nu_\mu, \nu_e, s, d, u$	$\mathcal{L}_{QCD}^{(n_f=3)}$
	$\downarrow$ Large N_C, Lattice,...	
M_K	$\gamma, \mu, e, \nu_\mu, \nu_e, \pi, K, \eta$	ChPT

$$\mathcal{L}_{eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu)$$

ChPT for $|\Delta S| = 1$

The e.m. and octet part at lowest order for $|\Delta S| = 1$

$$\mathcal{L}^{(2)} = C' \left\{ e^2 F_0^2 G_E \left(u^\dagger Q u \right) + G_8 \left(u_\mu u^\mu \right) + G_8' \left(\chi_+ \right) + G_{27} (..) \right\}_{32}$$

At order p^4 other operators appear. The octet combinations are

$\tilde{K}_1$	$G_8 \left(N_5^r - 2N_7^r + 2N_8^r + N_9^r \right) + G_{27} \left(-\frac{1}{2} D_6^r \right)$
$\tilde{K}_2$	$G_8 \left(N_1^r + N_2^r \right) + G_{27} \left(\frac{1}{3} D_{26}^r - \frac{4}{3} D_{28}^r \right)$
$\tilde{K}_3$	$G_8 \left(N_3^r \right) + G_{27} \left(\frac{2}{3} D_{27}^r + \frac{2}{3} D_{28}^r \right)$
$\tilde{K}_8$	$G_8 \left(2N_5^r + 4N_7^r + N_8^r - 2N_{10}^r - 4N_{11}^r - 2N_{12}^r \right) - \frac{2}{3} G_{27} \left(D_1^r - D_6^r \right)$
$\tilde{K}_9$	$G_8 \left(N_5^r + N_8^r + N_9^r \right) + G_{27} \left(-\frac{1}{6} D_6^r \right)$

Bijnens, Dhonte, Persson, N.P.B648:317,2003.

ε'/ε: status and unsolved problems

$$\text{Re}\ \frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2}\ |\varepsilon|} \left[\frac{\text{Im}\ A_2}{\text{Re}\ A_2} - (1 - \Omega_{\text{eff}}) \frac{\text{Im}\ A_0}{\text{Re}\ A_0} \right]$$

Experimental Status:

$$\text{Re}\ \varepsilon'/\varepsilon = (1.7 \pm 0.2) \cdot 10^{-3} \qquad \text{W.A.}$$

Theoretical Status:

★ General agreement on the OPE part (Munich, Rome).

★ Matrix elements and input parameters

★ FSI

Hadronic Matrix Elements

- ☆ Lattice calculations: CP-PACS, SPQ$_{CD}$R, UKQCD
- ☆ QCD Sum Rules: Pich, de Rafael
- ☆ Large N_c: within different treatments of the low-energy physics
  - Vacuum Sat. and improvements: Bardeen et al.; Hambye et al.
  - Nambu–Jona-Lasinio like models: Bijnens and Prades
  - Minimal Hadronic Approximation: Knecht et al.
  - Ladder Resummation Approximation: Bijnens,Gámiz,Lipartia,Prades
- ☆ *Dispersive Methods:* Cirigliano et al.;Narison; Bijnens et al.

LO Chiral couplings

Authors,method	Im G_8/Im τ	e^2Im G_E/Im τ
Large N_c	1.9	-2.9
Bijnens, Gamiz, Lipartia, Prades	4.4 ± 2.2	
Hambye, Peris, de Rafael	5.0 ± 1.7	$-(6.7 \pm 2.0)$
Bijnens, Gamiz, Prades;Narison; Cirigliano, Donoghue, Golowich, Maltman(τ decays)		$-(4.0 \pm 0.9)$
Lattice	..	$-(3.2 \pm 0.3)$

Matrix elements and input parameters: news and old problems

- $\Omega_{IB}^{\pi_0\eta} = 0.163 \pm 0.045$ (Ecker, Neufeld, Pich) updated with e.m. corrections (Cirigliano, Ecker, Neufeld, Pich) $\Omega_{\rm eff} = 0.06 \pm 0.077$
- Im $\tau \equiv -$Im $(V_{td}V_{ts}^*/(V_{ud}V_{us}^*)) \sim -(6.05 \pm 0.50)10^{-4}$. Note: if ε_{th} is used in the formula for ε'/ε the dependence of the final result on Im τ is almost canceled. In this case the final result depends on the value of B_K (This is better in Large N_c).
- Strange quark mass. A big source of error in Large N_c. $m_s(2{\rm GeV}) \sim (110 \pm 25){\rm MeV}$.$\leftrightarrow$ This dependence traded with quark condensates via GMOR relation.

NLO chiral couplings, $\tilde{K}_i$?

Not much is known. Using factorization one needs the counterterms from strong chiral Lagrangian of order p^6...
A naive assumption

$$\frac{\text{Im } \widetilde{K}_i}{\text{Re } \widetilde{K}_i} \simeq \frac{\text{Im } G_8}{\text{Re } G_8} \simeq \frac{\text{Im } G_8'}{\text{Re } G_8'} \simeq (0.9 \pm 0.3)\, \text{Im } \tau \,,$$

		Re $\widetilde{K}_i(M_\rho)$	Im $\widetilde{K}_i(M_\rho)$
8-et	$\widetilde{K}_2(M_\rho)$	0.35 ± 0.02	$[0.31 \pm 0.11]$ Im τ
8-et	$\widetilde{K}_3(M_\rho)$	0.03 ± 0.01	$[0.023 \pm 0.011]$ Im τ
27-et	$\widetilde{K}_5(M_\rho)$	$-(0.02 \pm 0.01)$	0
27-et	$\widetilde{K}_6(M_\rho)$	$-(0.08 \pm 0.05)$	0
27-et	$\widetilde{K}_7(M_\rho)$	0.06 ± 0.02	0

Re $\widetilde{K}_i(M_\rho)$ from Bijnens, Dhonte, Persson

Final State interaction

- FSI have been shown to be an important ingredient for ε'/ε (Pallante, Pich, S.).
- The degeneracy of $I = 0$ and $I = 2$ amplitude is removed by FSI and Ω_{IB}.
- PPS have included FSI using an Omnés dispersion relation.

Some conclusion from ε'/ε and $K \to 3\pi$

- ♢ FSI and IB effects are under control and/or are better checked
- ♢ The main uncertainty of ε'/ε come from the determination of the imaginary part of the couplings of the chiral Lagrangian.
- ♢ The same chiral Lagrangian describes CPV also in $K \to 3\pi$. Recent proposal by NA48 (CERN), KLOE(Frascati), OKA (Protvino). New precision $2\ 10^{-4}$ (Improvement of 2 orders of magnitude). Why not to check better?
- ✍ Conflicting results in the literature $(10^{-3} - 10^{-6})$

The Target

We have provided a complete (8-et, 27-et, ew-octet) one–loop (NLO)evaluation of chiral correction for both CP conserving and CPV observables in $K \to 3\pi$.

$$
\begin{aligned}
K^+ &\to \pi^+\pi^+\pi^- \qquad & K_{1,2} \to \pi^+\pi^-\pi^0 \\
K^+ &\to \pi^0\pi^0\pi^+ \qquad & K_{1,2} \to \pi^0\pi^0\pi^0
\end{aligned}
$$

Observables: Decay rates, Γ, and

$$\frac{|A_{K^+\to 3\pi}(s_1,s_2,s_3)|^2}{|A_{K^+\to 3\pi}(s_0,s_0,s_0)|^2} = 1 + g\,y + h\,y^2 + k\,x^2 + \mathcal{O}(yx^2, y^3)$$

$$x \equiv \frac{s_1 - s_2}{m^2_{\pi^+}}\; y \equiv \frac{s_3 - s_0}{m^2_{\pi^+}} \text{ and } s_i \equiv (k - p_i)^2,\; 3s_0 \equiv m_K^2 + \sum_{i=1,2,3} m^2_{\pi(i)}\,.$$

Status of 1-loop in ChPT for $K \to 3\pi$

CP conserving part

- The 8-et and 27-et done also by Bijnens, Dhonte, Persson. We fully agree. They provide also a fit of the Re $\tilde{K}_i$. We checked Γ, *g, h, k*
  Re $G_8 = 6.6 \pm 0.6$ and Re $G_{27} = 0.44 \pm 0.06$
- Isospin breaking effects included by Bijnens, Borg.

CP violating part

- We included e.m. penguin contribution (all decays, orders e^2p^0 and e^2p^2) and 2-loop imaginary part of the amplitudes, say FSI, using the optical theorem (for charged decays only, neutral decays are in progress)
- All results are analytical

CP violating asymmetries

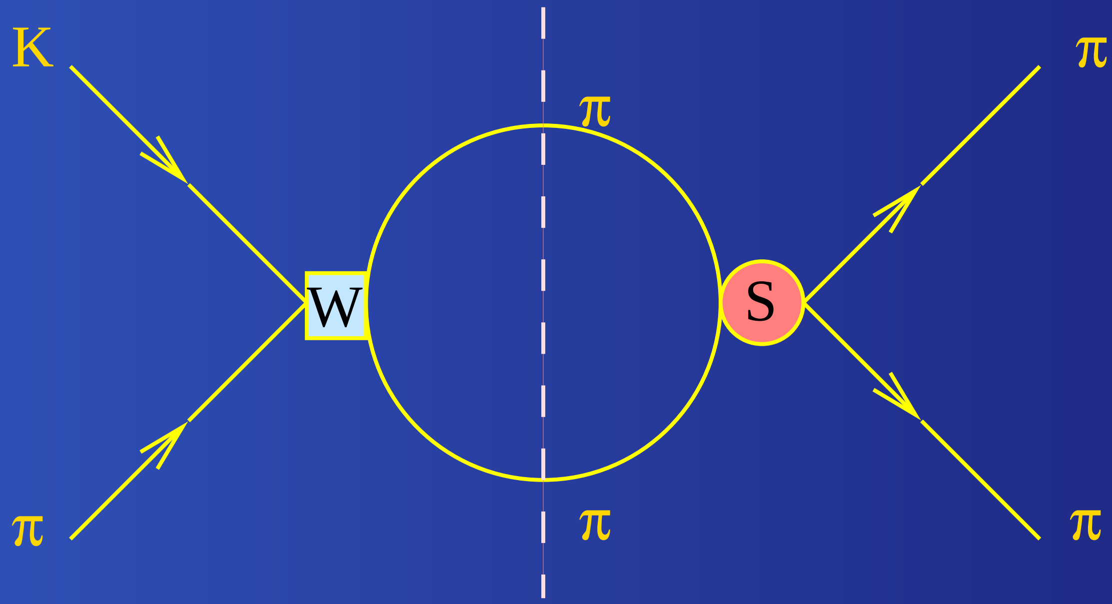
Definitions: Slope Asymmetries

$$\Delta g_C \equiv \frac{g[K^+ \to \pi^+\pi^+\pi^-] - g[K^- \to \pi^-\pi^-\pi^+]}{g[K^+ \to \pi^+\pi^+\pi^-] + g[K^- \to \pi^-\pi^-\pi^+]}$$

$$\text{and} \quad \Delta g_N \equiv \frac{g[K^+ \to \pi^0\pi^0\pi^+] - g[K^- \to \pi^0\pi^0\pi^-]}{g[K^+ \to \pi^0\pi^0\pi^+] + g[K^- \to \pi^0\pi^0\pi^-]}.$$

and the same for Decay Rates As. with $g \to \Gamma$.

NLO results: graphics for FSI



For Im$A \sim \mathcal{O}(p^4)$ (LO) one needs W, S$\sim \mathcal{O}(p^2)$.
For Im$A \sim \mathcal{O}(p^6)$ (NLO) one needs W$\sim \mathcal{O}(p^2)$ and S$\sim \mathcal{O}(p^4)$ and viceversa.

NLO results: FSI in the asymmetries

$$|A(K^\pm \to 3\,\pi)|^2 \;=\; A_0^\pm + y\,A_y^\pm + \mathcal{O}(x, y^2)$$

$$\Delta g \;=\; \frac{A_y^+ A_0^- - A_0^+ A_y^-}{A_y^+ A_0^- + A_0^+ A_y^-}\,.$$

- ✗ The sum $A_y^+ A_0^- + A_0^+ A_y^-$ does NOT contain FSI (i.e. $\mathcal{O}(p^6)$) at NLO (they would be part of the NNLO)
- ✗ The difference $A_y^+ A_0^- - A_0^+ A_y^- \sim$ Im A: to have it at NLO we must take into account FSI phases $\to$ FSI at NLO only in imaginary parts (in other words: Re $A \sim \mathcal{O}(p^2) + \mathcal{O}(p^4) + ..$ while Im $A \sim \mathcal{O}(p^4) + \mathcal{O}(p^6) + ..$)
- ✗ The calculation of the imaginary part can be done in ChPT using the optical theorem

Results for the asymmetries:NLO

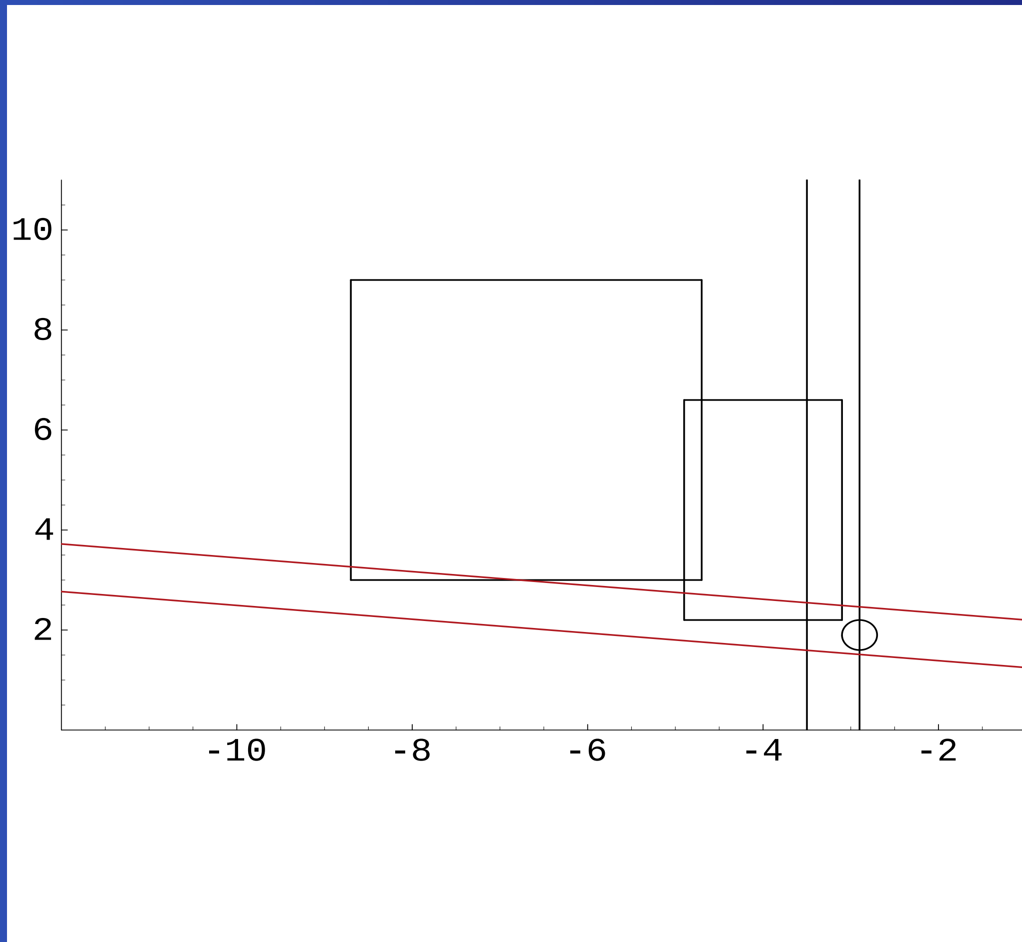
$$\frac{\Delta g_C^{\rm NLO}}{10^{-2}} \simeq (0.66 \pm 0.13){\rm Im}G_8 + (4.33 \pm 1.6){\rm Im}\widetilde{K}_2$$

$$-(18.11 \pm 2.2){\rm Im}\widetilde{K}_3 - (0.07 \pm 0.03){\rm Im}(e^2 G_E)\,,$$

	$\Delta g_C^{\rm NLO}$ (10^{-5})	$\Delta\Gamma_C^{\rm NLO}$ (10^{-6})	$\Delta g_N^{\rm NLO}$ (10^{-5})	$\Delta\Gamma_N^{\rm NLO}$ (10^{-6})
$\widetilde{K}_i(M_\rho)$,BDP	-2.4 ± 1.2	$[-11, 9]$	1.1 ± 0.8	$[-9, 11]$
$\widetilde{K}_i(M_\rho) = 0$	-2.4 ± 1.3	1.0 ± 0.7	0.9 ± 0.5	4.0 ± 3.2

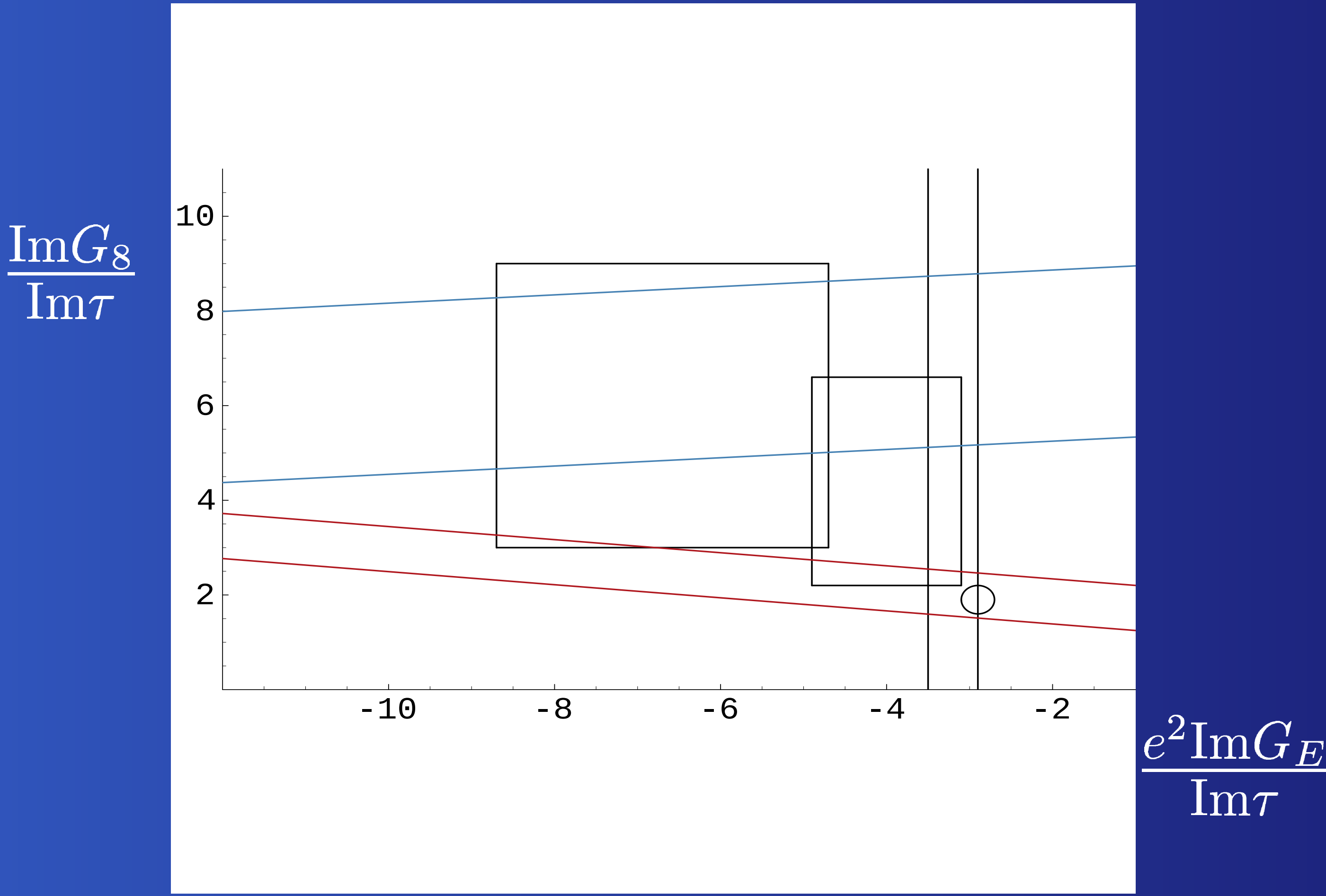
ε'/ε vs Δg_C: Status of ε'/ε

$\frac{\mathrm{Im} G_8}{\mathrm{Im}\tau}$



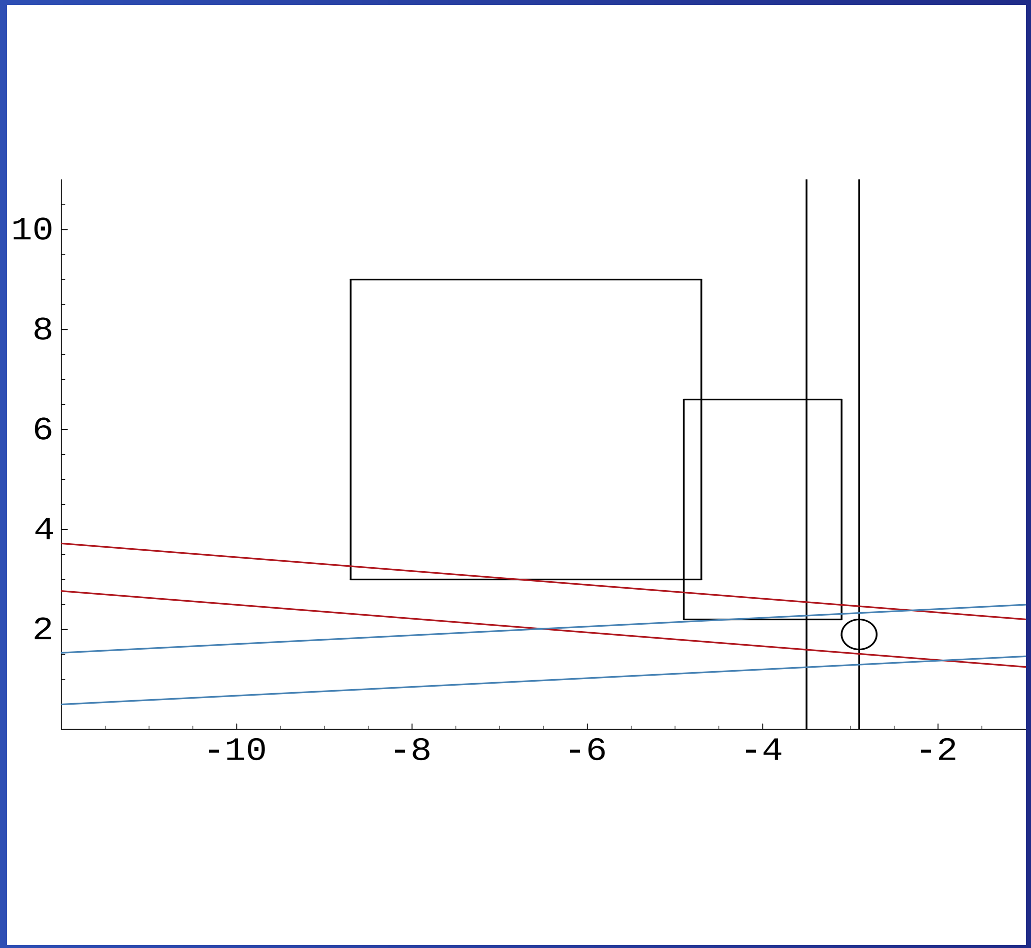
$\frac{e^2 \mathrm{Im} G_E}{\mathrm{Im}\tau}$

ε'/ε vs Δg_C: $\Delta g_C \sim 3.5\ 10^{-5}$



ε'/ε vs Δg_C: $\Delta g_C \sim 10^{-5}$

$\frac{\mathrm{Im} G_8}{\mathrm{Im} \tau}$



$\frac{e^2 \mathrm{Im} G_E}{\mathrm{Im} \tau}$

ε'/ε vs Δg_C: Summary

- $\Delta g_C > 5 \cdot 10^{-5} \to$ New Physics.
- $3 \cdot 10^{-5} < \Delta g_C < 5 \cdot 10^{-5} \to$ Compatible with high values of ImG_8 but in bad agreement with ε'/ε.
- $\Delta g_C \sim 10^{-5} \to$ Perfectly compatible with SM.
- The experimental errors should be $\sim 10^{-5}$.

Comments on the charged $K \to 3\pi$ as.

- Δg_C is dominated by Im G_8. Ch-NLO on Δg_C give effects of about 20-30$\%$. The final error is due mainly to Im G_8.
  - consistency with ε'/ε
- Δg_N and $\Delta\Gamma_{C,N}$ are dominated by $\mathcal{O}(p^4)$ counterterms, $\tilde{K}_i$
  - New important information on Im $\tilde{K}_i$
- The new exp. limit I. Mikulec, Na48 Coll., hep-ex/0505081, $\Delta g_C = 10^{-4}(0.5 \pm 2.4_{stat} \pm 2.1_{stat(trig)} + 2.1_{syst})$. SM prefers values of $\Delta g_C < 0.4 \times 10^{-4}$. For consistency with ε'/ε $\Delta g_C \simeq 10^{-5}$.

SUSY

Question: Can NP enhance $\Delta g_{C,N}$ respecting all constraints? In generic SUSY models the gluonic penguin operator (D'Ambrosio,Isidori, Martinelli):

$$\begin{aligned} \mathcal{H} &= C_g^+ Og^+ + C_g^- Og^- \\ Og^\pm &= \frac{g}{16\pi^2}\left(\bar{s}_L \sigma_{\mu\nu} G^{\mu\nu} d_R \pm \bar{s}_R \sigma_{\mu\nu} G^{\mu\nu} d_L\right) \\ C_g^\pm &= \frac{\pi \alpha_s(m_{\tilde{g}})}{m_{\tilde{g}}}\left(\delta^D_{LR21} \pm \delta^{D*}_{LR12}\right) G_0(x_{gq}) \end{aligned}$$

SUSY

Question: Can NP enhance $\Delta g_{C,N}$ respecting all constraints? In generic SUSY models the gluonic penguin operator (D'Ambrosio,Isidori, Martinelli):

They find Δg_C can be as big as 10^{-4}.

However also big uncertainties due to the hadronization of the operator.

Conclusions for CPV

- The main problem for a good estimate of ε'/ε are still hadronic matrix elements. It would be extremely helpful to measure other CP-violating channels in hadronic kaon decays. $K \to 3\pi$ offers several chances.

- We have provided the first NLO in ChPT estimate of CP-violating asymmetries in charged $K \to 3\pi$. The results for the 8-et and 27-et part agree with BDP. We have included e.w. penguin contribution (up to $\mathcal{O}(e^2 p^2)$) and imaginary part of the amplitudes up to $\mathcal{O}(p^6)$ (FSI). Neutral channels are in progress.

- Forthcoming experiments on hadronic kaon decays have still the possibility to give many surprises.

π-π scattering length

$$
\begin{aligned}
\mathbf{S} &= 1 + i \sum_I T^I(s,t) \\
T^I(s,t) &= 32\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta) t_\ell^I(s) \\
t_\ell^I(s) &= \sqrt{\frac{1}{1-4m_\pi^2/s}} \frac{1}{2i} \left\{ \exp^{2i\delta_\ell^I(s)} - 1 \right\} \\
\text{Re } t_\ell^I(s) &= (s-4m_\pi^2)^{2l} \left\{ a_\ell^I + (s-4m_\pi^2) a_\ell^I + \ldots \right\}
\end{aligned}
$$

π-π scattering length

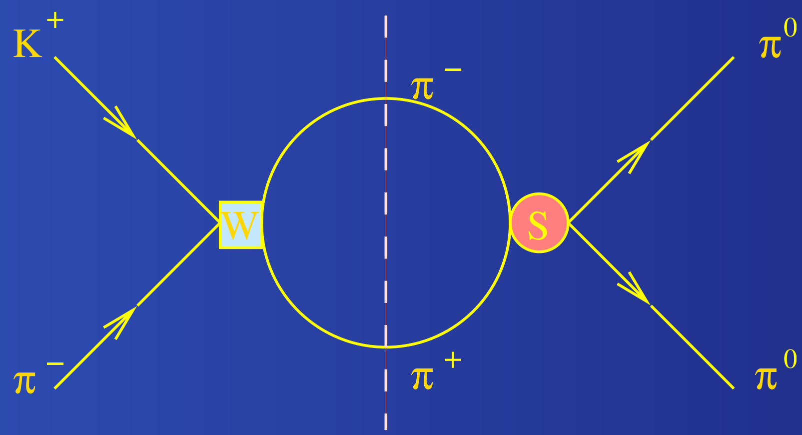
Theoretical status (Colangelo, Gasser, Leutwyler, Ananthanarayan)

$$(a_0 - a_2) m_{\pi^+} = 0.265 \pm 0.004$$

Experimentally from K_{e4} BNL-E865 with 6% statistical error.
Cabibbo,Isidori propose to measure $(a_0 - a_2) m_{\pi^+}$ from $K \to 3\pi$ with a precision of 1-2%

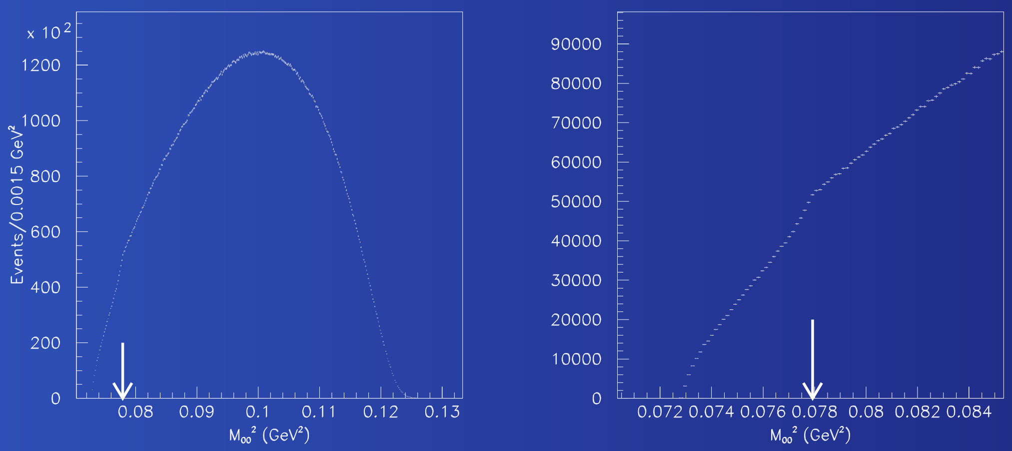
π-π scattering length

Cabibbo proposed to do the measure using this effect



The imaginary part of the amplitude generated by this contribution is discontinous in the phase space due to the mass difference $\pi^{\pm}$–π^0

Experimental results



S. Giudici, NA48 Coll. hep-ex/0505032

The experiment and summary

Most recent result $(a_0 - a_2)m_+ = 0.265 \pm 0.004$
NA48 preliminary result

$$(a_0 - a_2)m_+ = 0.281 \pm 0.007(stat.) \pm 0.014(syst.) \pm 0.014(theo.)$$

The theoretical error is under discussion (E.Gamiz, J. Prades, I.S., and also the Bern group, both in progress).

The Cabibbo proposal is interesting but.. improving the precision above 5% may require a big theoretical effort. (More about this in the near future)