

$$K \rightarrow \pi^+ \nu \bar{\nu} \text{ and } K_L \rightarrow \pi^0 \nu \bar{\nu}$$

Minimal Flavour Violation and Beyond

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(Technical University Munich)

Flavour at the LHC
CERN, November 6-10, 2005

Declaration



$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

Master Formula for Weak Decays

AJB (2001)
hep-ph/0101336
hep-ph/0109197

Non-Perturbative
Factors in the SM

QCD RG
Factors

Short Distance Loop
Functions (Penguins, Boxes)

New Flavour-
Changing Parameters

Represent different
Dirac and Colour
Structures



$$A(\text{Decay}) = B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[F_{\text{SM}}^i + F_{\text{New}}^i \right] + B_i^{\text{New}} \left[\eta_{\text{QCD}}^i \right]^{\text{New}} V_{\text{New}}^i \left[G_{\text{New}}^i \right]$$

(Summation over i)

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(Summation over i)

$$A(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = B_+ \left[\lambda_c \tilde{P}_c + \lambda_t X(\nu) \right]$$

$$A(K_L \rightarrow \pi^0 \nu \bar{\nu}) = B_L \text{Im}(\lambda_t X(\nu))$$

$$\lambda_c = V_{cs}^* V_{cd}$$

$$\lambda_t = V_{ts}^* V_{td}$$

B_+, B_L from $K^+ \rightarrow \pi^0 e^+ \nu$

$$X(\nu) = |X(\nu)| e^{i\theta_x}$$

$\nu =$ parameters (m_ν, \dots)

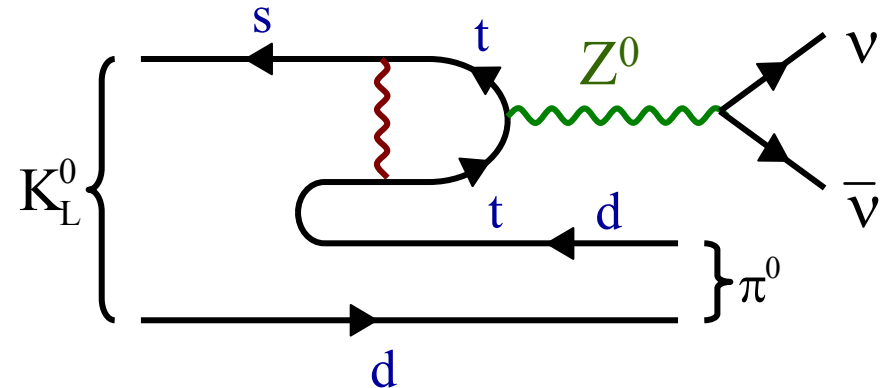
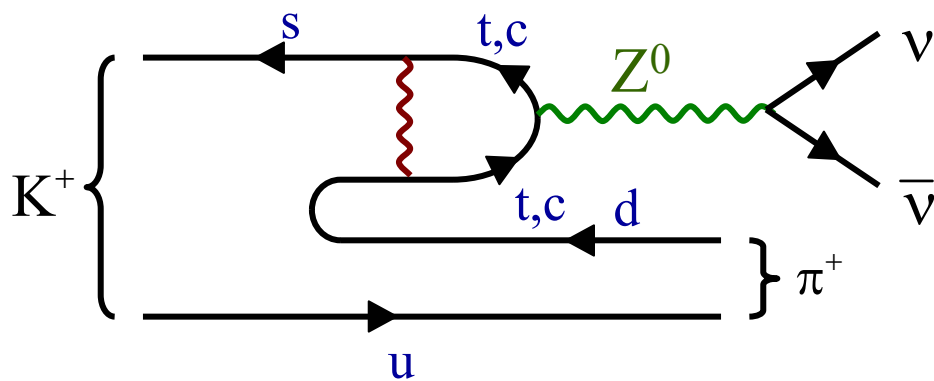
Pure
 Short
 Distance
 Dynamics

AJB, Romanino, Silvestrini (98)

$$\theta_x = \begin{cases} 0 & \text{SM} \\ 0, \pi & \text{MFV} \end{cases}$$

(AJB, Fleischer)

Decays $K \rightarrow \pi \nu \bar{\nu}$



Isospin Symmetry

$$\langle \pi^+ | (\bar{s}d)_{V-A} | K^+ \rangle = \sqrt{2} \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$

$$\langle \pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle = \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$

Leading Decay:

$$K^+ \rightarrow \pi^0 e^+ \nu$$

Isospin Breaking:

Marciano, Parsa (Suppression)

K^+ (10%) K_L (5%)

Long Distance:

K^+ : $+(6 \pm 2)\%$ Isidori, Mescia,
Smith (2005)

K_L : $\leq 1\%$ Buchalla, Isidori

The Rest of this Talk

1. Express Review of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ (NNLO)
2. UT from $K \rightarrow \pi \nu \bar{\nu}$ ($\beta, \gamma, |V_{td}|$)
3. $K \rightarrow \pi \nu \bar{\nu}$ and MFV (upper bounds)
4. $K \rightarrow \pi \nu \bar{\nu}$ beyond MFV
5. $\Delta M_{s,d}$ and $B_{s,d} \rightarrow \mu \bar{\mu}$ (MFV and beyond)
6. Two final Messages

Express Review of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$

AJB
Schwab
Uhlig

hep-ph/0405132

NLO: Buchalla + AJB (94); NNLO: AJB, Gorbahn, Haisch, Nierste (05)

SM: $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (8.0 \pm 1.1) \cdot 10^{-11}$ $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (2.8 \pm 0.6) \cdot 10^{-11}$

Exp: $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \left(14.7^{+13.0}_{-8.9}\right) \cdot 10^{-11}$ $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 5.9 \cdot 10^{-7} \text{ (KTeV)}$

Brookhaven: E787, E949
(CKM, NA48, JPARC, ..)

Soon improved by E391a !!!
(J-PARC, ...)

$2.9 \cdot 10^{-7}$
90% C.L.

TH very clean

• $\left(\begin{array}{l} \text{With improved} \\ \text{CKM parameters} \\ \sim 2008 \end{array} \right) \rightarrow$

$\sigma(\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})) < 5\%$
 $\sigma(\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})) < 5\%$

Very clean
determination
of Unitarity
Triangle

$\sigma(\text{Br}) \cong 10\%$

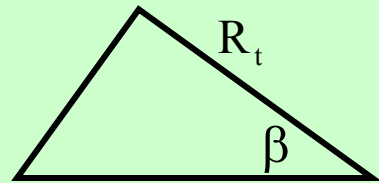
$\sigma(\text{Br}) \cong 5\%$

$\sigma(\sin 2\beta \cong 0.04) \mid \sigma(\gamma) = 9^\circ \mid \sigma(|V_{td}|) = 7\%$
 $\sigma(\sin 2\beta \cong 0.025) \mid \sigma(\gamma) = 5^\circ \mid \sigma(|V_{td}|) = 4\%$

Basic Formulae for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ (SM)

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 4.8 \cdot 10^{-11} \left[A^4 R_t^2 X^2 + 2P_c A^2 R_t X \cos \beta + P_c^2 \right]$$

$$X \equiv X(m_t)$$



$$= 10^{-11} \left[4.2 + 3.1 + 0.7 \right]$$

(top) (top-charm) (charm)

$$A = \frac{|V_{cb}|}{\lambda^2} \cong 0.83$$

Buchalla
AJB (94)
NLO)

$$P_c = 0.367 \pm \underbrace{0.033}_{\Delta m_c = 50 \text{ MeV}} \pm \underbrace{0.037}_{\text{theory}} \pm \underbrace{0.009}_{\alpha_s} \cong 0.37 \pm 0.07$$

BGHN
NNLO (05)

$$P_c = 0.371 \pm 0.031_{m_c} \pm \underbrace{0.009}_{\text{theory}} \pm \underbrace{0.009}_{\alpha_s} = 0.37 \pm 0.04$$

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = \left[8.0 \pm \underbrace{0.5}_{P_c} \pm \underbrace{0.8}_{\text{CKM}} \right] 10^{-11} \cong (8.0 \pm 1.1) 10^{-11}$$

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \left[14.7 \begin{array}{c} +13.0 \\ -8.9 \end{array} \right] 10^{-11}$$

E787 (2)
E949 (1)
3 Events

QCD Corrections to $K \rightarrow \pi \nu \bar{\nu}$

LO

NLO

NNLO

Charm
Part

$$P_c = \frac{4\pi}{\alpha_s(\mu_c)} P_c^{(0)} + P_c^{(1)} + \frac{\alpha_s(\mu_c)}{4\pi} P_c^{(3)}$$

$$\mu_c = 0(m_c)$$

Vainshtein, Zakharov, Novikov
Shifman (1977)
Ellis, Hagelin (1983)
Dib, Dunietz, Gilman (1991)

Buchalla
AJB
(1994)

AJB
Gorbahn
Haisch
Nierste
(2005)

Top
Part

$$X^{\text{SM}}(\mathbf{x}_t) = X_0(\mathbf{x}_t) + \frac{\alpha_s(\mu_t)}{4\pi} X_1(\mathbf{x}_t)$$

$$x_t = \frac{m_t^2(\mu_t)}{M_W^2}$$

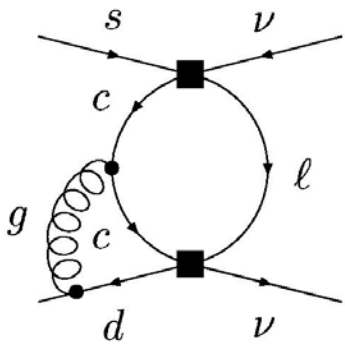
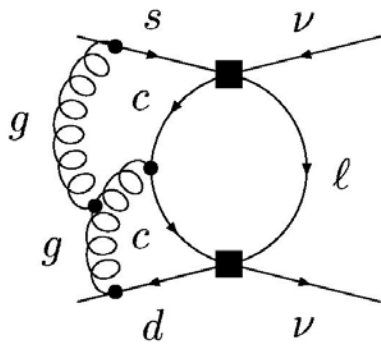
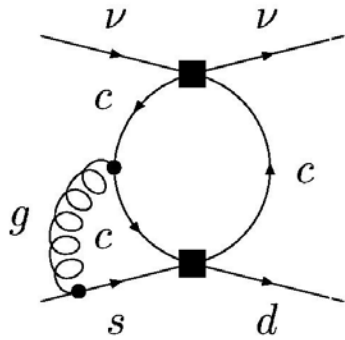
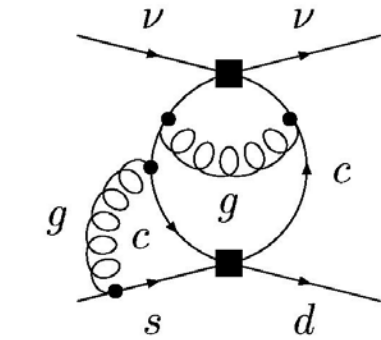
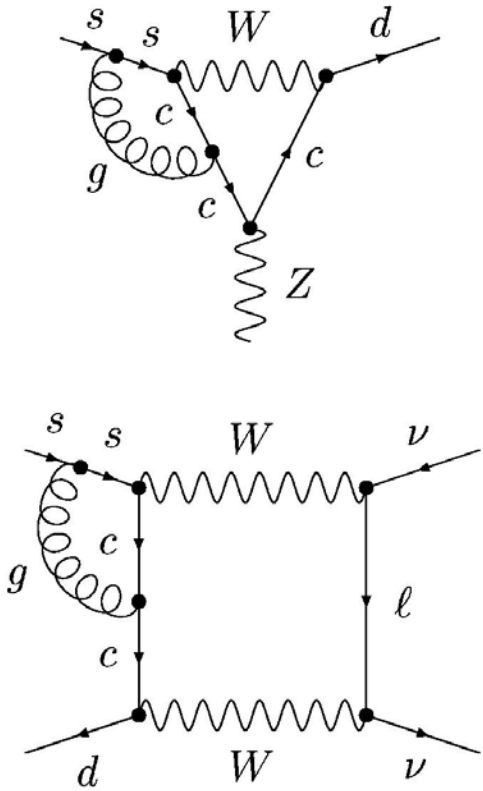
Inami, Lim (81)
AJB (81)

Buchalla, AJB (93)
Misiak, Urban (98)

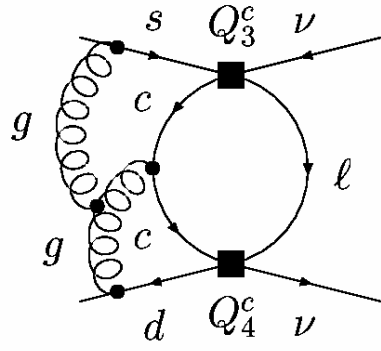
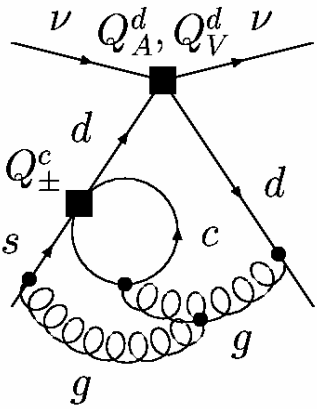
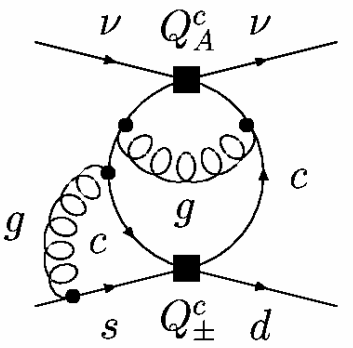
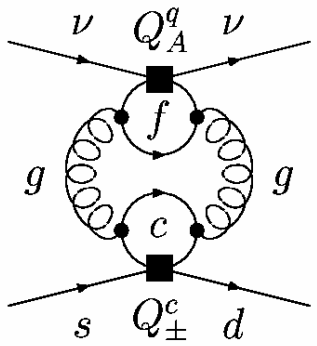
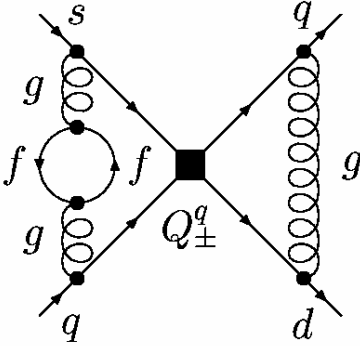
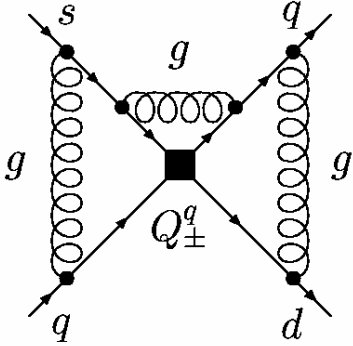
$C_i(\mu_w)$

**3-Loop
Anomalous
Dimensions**

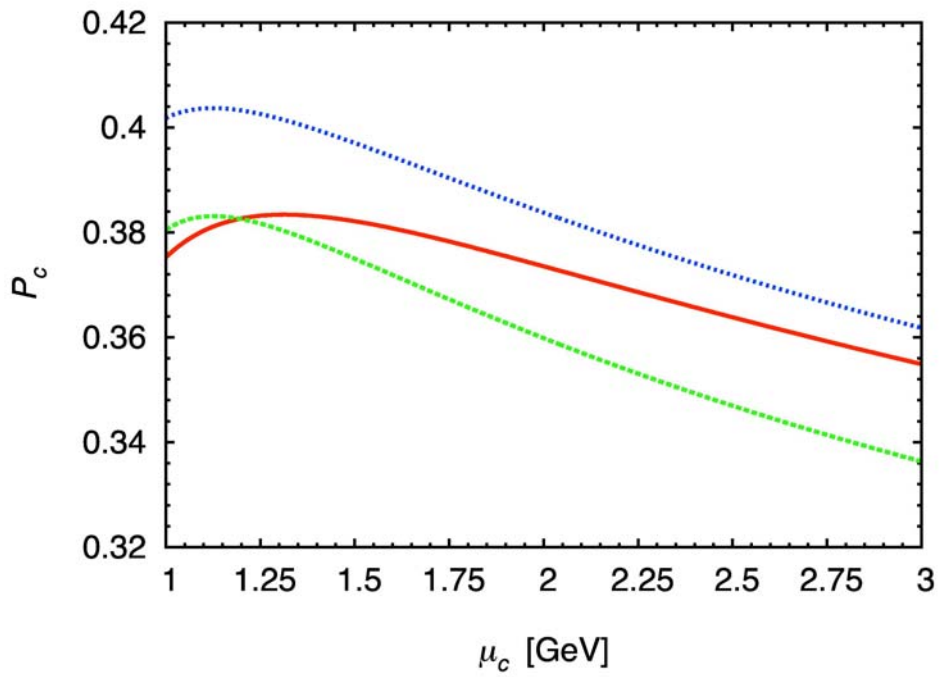
**Matrix
Elements**



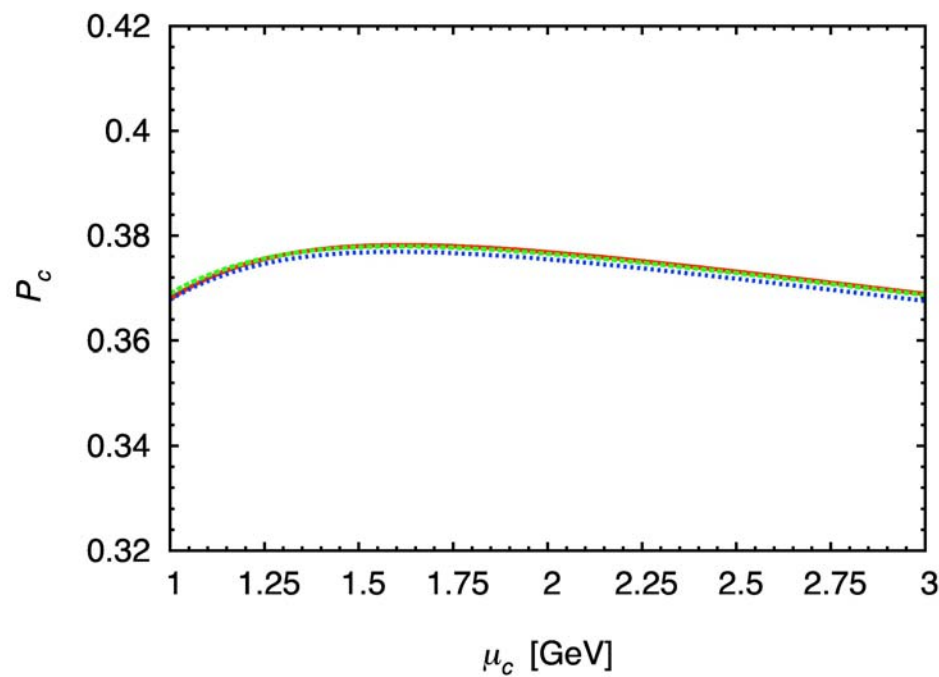
No Comments



$P_c(\mu_c)$ for various calculations
of $\alpha_s(\mu_c)$ from $\alpha_s(M_Z)$

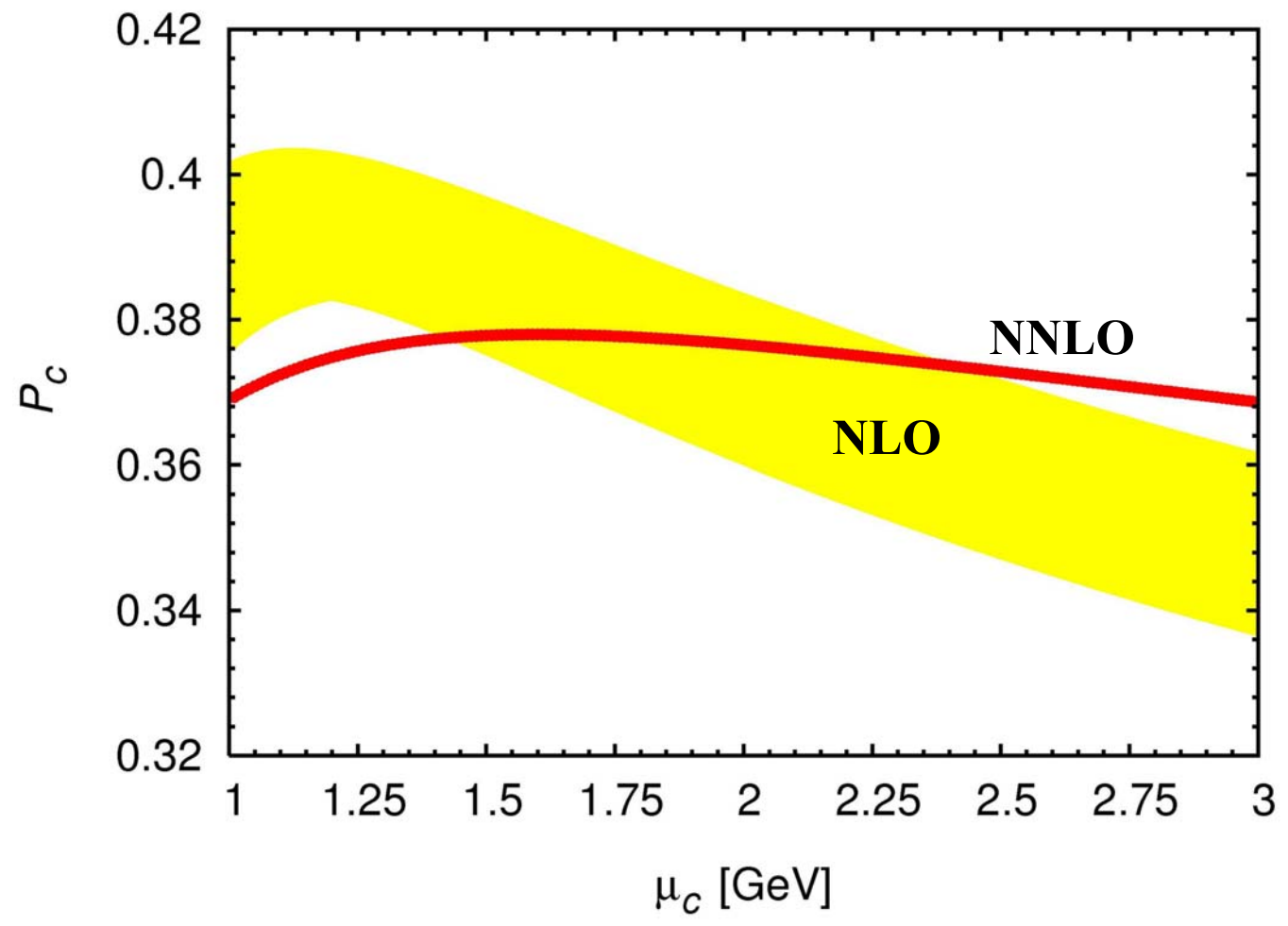


NLO



NNLO

Reduction of TH Error in P_c



Four Musketeers



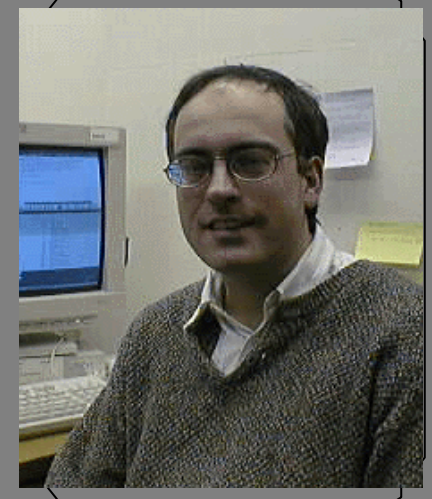
AJB



**Martin
Gorbahn**



Ulrich Haisch



Ulrich Nierste

Anatomy of $|V_{td}|$ from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

AJB
Schwab
Uhlig

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = 0.39 \frac{\sigma(P_c)}{P_c} + 0.70 \frac{\sigma(\text{Br}(K^+))}{\text{Br}(K^+)} + \frac{\sigma(|V_{cb}|)}{|V_{cb}|}$$

Present: $\pm 4\%$ \pm (Very Large) $\pm 2\%$

$\left. \begin{array}{l} \sigma(\text{Br}(K^+)) = 10\% \\ \sigma(P_c) = 0.03 \end{array} \right\} \pm 3\%$ $\pm 7\%$ $\pm 1.4\%$ (Scenario I)

$\left. \begin{array}{l} \sigma(\text{Br}(K^+)) = 5\% \\ \sigma(P_c) = 0.02 \end{array} \right\} \pm 2\%$ $\pm 3.5\%$ $\pm 1\%$ (Scenario II)

Determination
at 4-5% possible

Theoretically clean Relations

D'Ambrosio + Isidori (02)

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \bar{\kappa}_+ |V_{cb}|^4 X^2 \left[R_t^2 \sin^2 \beta + \left(R_t \cos^2 \beta + \frac{\lambda^4 P_c}{|V_{cb}|^2 X} \right)^2 \right]$$

$$R_t \sim \xi \frac{\sqrt{\Delta M_d}}{\sqrt{\Delta M_s}}$$

$$\bar{\kappa}_+ = 7.64 \cdot 10^{-6}$$

$$P_c = 0.37 \pm 0.04$$

AJB, Schwab, Uhlig (04)

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \bar{\kappa}_+ |V_{cb}|^4 X^2 \left[T_1^2 + \left(T_2 + \frac{\lambda^4 P_c}{|V_{cb}|^2 X} \right)^2 \right]$$

$$T_1 = \frac{\sin \beta \sin \gamma}{\sin(\beta + \gamma)}$$

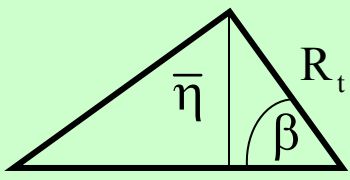
$$T_2 = \frac{\cos \beta \sin \gamma}{\sin(\beta + \gamma)}$$

(Direct CP)

Basic Formulae for $K_L \rightarrow \pi^0 \nu \bar{\nu}$

(SM)

Buchalla
AJB (NLO)

$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 2.8 \cdot 10^{-11} \left[\frac{\bar{\eta}}{0.35} \right]^2 \left[\frac{|V_{cb}|}{41.5 \cdot 10^{-3}} \right]^4 \left[\frac{X}{1.48} \right]^2$$


$$= (2.8 \pm \overbrace{0.6}^{\text{CKM}}) \cdot 10^{-11}$$

(AJB
Schwab
Uhlig)

E391a :

$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 2.9 \cdot 10^{-7}$

Future: E391a, JHF

Model independent bound (Grossman, Nir)

$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \leq 4.4 \text{ Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \leq 1.4 \cdot 10^{-9} (90\% \text{ C.L.})$

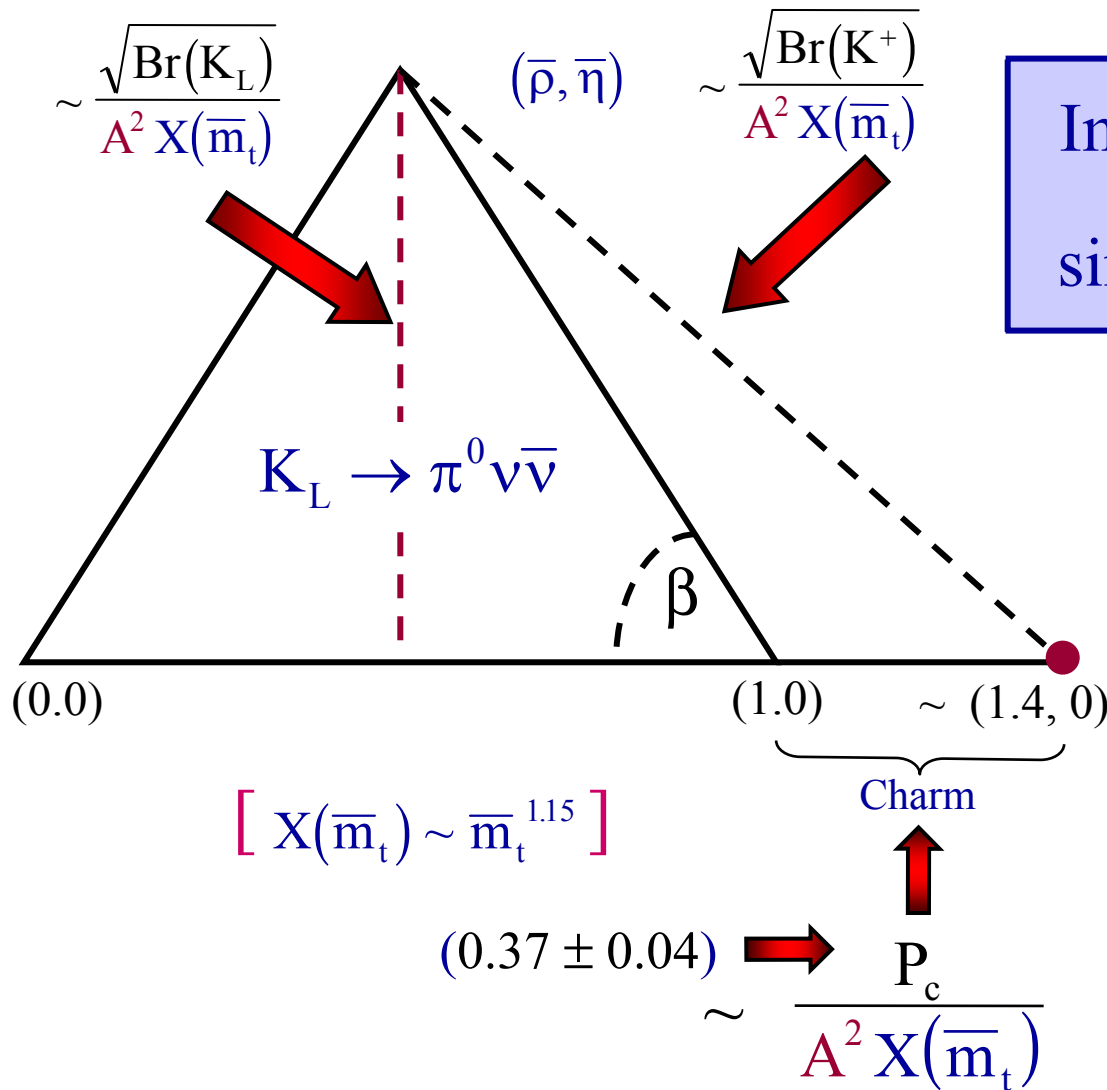
E391a could get the first non-trivial upper bound.

$\bar{\eta} = R_t \sin \beta$

E391 (JHF): ~ 1000 Events

UT from $K \rightarrow \pi \nu \bar{\nu}$

Buchalla
AJB



$\text{Im } \lambda_t = F_1(\bar{m}_t, \text{Br}(K_L))$
 $\sin 2\beta = F_2(P_c, \text{Br}(K_L), \text{Br}(K^+))$

$\lambda_t = V_{ts}^* V_{td}$

$\sin 2\beta$ \longleftrightarrow $\sin 2\beta$
 $(K \rightarrow \pi \nu \bar{\nu})$ $(B \rightarrow J/\psi, K_s) \rightarrow \phi K_s$

K-Physics \longleftrightarrow B - Physics

Test of SM

and

Beyond

Golden Relations

(All involving $K_L \rightarrow \pi^0 \nu \bar{\nu}$)

Buchalla, AJB (94)

$$\cot \beta = \frac{\sqrt{B_1 - B_2} - P_c}{\sqrt{B_2}}$$



$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_S}$$

$$B_1 \sim \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$$

$$B_2 \sim \text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$$

$$J_{\text{CP}} \sim \text{triangle} \sim \frac{\sqrt{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}}{X(m_t)}$$

AJB, Schwab, Uhlig (04)

$$\frac{\sin \beta \sin \gamma}{\sin(\beta + \gamma)} = 0.35 \left[\frac{1.53}{X(m_t)} \right] \left[\frac{0.0415}{|V_{cb}|} \right]^2 \sqrt{\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{3 \cdot 10^{-11}}}$$

β from $a_{\psi K_S}$

γ from $B_s^0 \rightarrow D_s^\pm K^\mp$
 $B_d^0 \rightarrow D^\pm \pi^\mp$

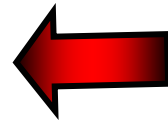


X

Reduction of TH Error: NLO \rightarrow NNLO

AJB, Gorbahn, Haisch, Nierste (2005)

$$\begin{aligned}\sigma(|V_{td}|) &: \pm 4.1\% \rightarrow \pm 1\% \\ \sigma(\sin 2\beta) &: \pm 0.025 \rightarrow \pm 0.006 \\ \sigma(\gamma) &: \pm 4.9^\circ \rightarrow \pm 1.2^\circ\end{aligned}$$



$$\begin{aligned}\text{CKM from} \\ K^+ &\rightarrow \pi^+ \nu \bar{\nu} \\ K_L &\rightarrow \pi^0 \nu \bar{\nu}\end{aligned}$$

Main parametric uncertainties :

$$\begin{aligned}m_c(m_c) \\ |V_{cb}| \end{aligned}$$

$$\sigma(P_c)_{m_c} = \left[\frac{0.67}{\text{GeV}} \right] \sigma(m_c(m_c))$$

The Angle β from $K \rightarrow \pi\nu\bar{\nu}$

Buchalla, AJB (94)
 AJB, Schwab, Uhlig (04)

BSU:
$$\frac{\sigma(\sin 2\beta)}{\sin 2\beta} = 0.31 \frac{\sigma(P_c)}{P_c} + 0.55 \frac{\sigma(\text{Br}(K^+))}{\text{Br}(K^+)} \pm 0.39 \frac{\sigma(\text{Br}(K_L))}{\text{Br}(K_L)}$$

$\sigma(\sin 2\beta) = \pm 0.034 \quad \pm ? \quad \pm ? \quad (\text{Present})$

$\sigma(\sin 2\beta) = 0.017 \quad \pm 0.039 \quad \pm 0.028 \quad (\text{Scenario I})$
 Br's at 10%

$\sigma(\sin 2\beta) = 0.011 \quad \pm 0.020 \quad \pm 0.014 \quad (\text{Scenario II})$
 Br's at 5%

TH
 very
 clean

$\sigma(\sin 2\beta) \approx 0.02 - 0.03 \quad \text{requires } \sigma(\text{Br's}) \leq 5\%$

The Angle γ from $K \rightarrow \pi\nu\bar{\nu}$

AJB, Schwab, Uhlig (04)

$$\frac{\sigma(\gamma)}{\gamma} = 0.75 \frac{\sigma(P_c)}{P_c} + 1.32 \frac{\sigma(\text{Br}(K^+))}{\text{Br}(K^+)} + 0.07 \frac{\sigma(\text{Br}(K_L))}{\text{Br}(K_L)} + 4.1 \frac{\sigma(|V_{cb}|)}{|V_{cb}|}$$

$$\sigma(\gamma) = \quad \pm 4.9^\circ \quad \pm ? \quad \pm ? \quad \pm 4.9^\circ \quad (\text{Present})$$

$$\sigma(\gamma) = \quad \pm 3.7^\circ \quad \pm 8.5^\circ \quad \pm 0.4^\circ \quad \pm 3.8^\circ \quad (\text{Scenario I})$$

Br's at 10%

$$\sigma(\gamma) = \quad \pm 2.5^\circ \quad \pm 4.2^\circ \quad \pm 0.2^\circ \quad \pm 2.5^\circ \quad (\text{Scenario II})$$

Br's at 5%

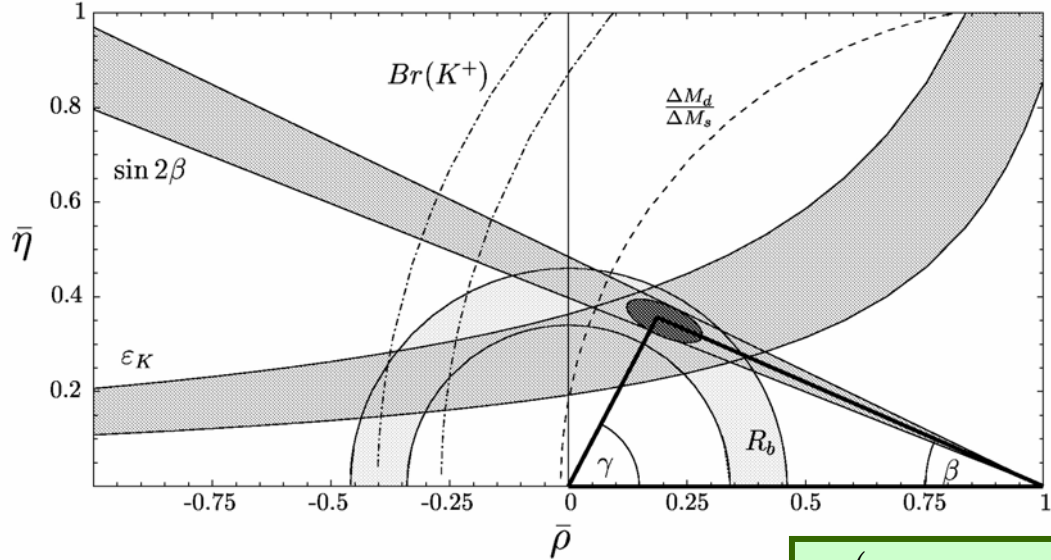
TH
very
clean

$$\sigma(\gamma) \approx \pm 5^\circ \quad \text{requires} \quad \sigma(\text{Br}(K^+)) \leq 5\%$$

Unitarity Triangle 2004

(AJB, Schwab, Uhlig)

$$\text{Br}(K^+) \equiv \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 14.7 \cdot 10^{-11}$$



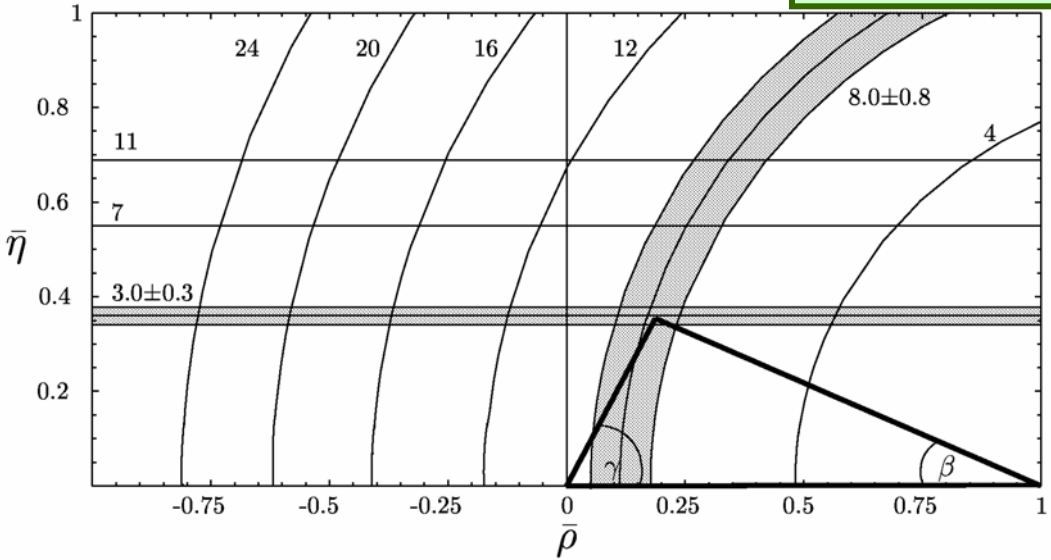
$$P_c = \underline{0.37 \pm 0.04}$$

m_c, V_{cb}, μ_c

$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

Unitarity Triangle from $K \rightarrow \pi \nu \bar{\nu}$

(2012)



$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$

Minimal Flavour Violation (MFV)

$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \underbrace{\left[F_{\text{SM}}^i + F_{\text{New}}^i \right]}_{\text{real}} F_i(\mathbf{v})$$

K and B
Physics
related
to each
other

AJB, Gambino, Gorbahn, Jäger, Silvestrini
D'Ambrosio, Giudice, Isidori, Strumia

1.

All flavour changing processes governed by V_{CKM}^i .

2.

Only SM Operators are relevant.

3.

New Physics enters only through 7 Master Functions

$$F_i(\mathbf{v}) = S(\mathbf{v}), X(\mathbf{v}), Y(\mathbf{v}), Z(\mathbf{v}), D'(\mathbf{v}), E'(\mathbf{v}), E(\mathbf{v})$$

\mathbf{v} = collects parameters specific to a given MFV model.

SM:

$$\mathbf{v} = \mathbf{x}_t$$

Review: AJB hep-ph/0310208

MFV "Sum Rules"

Relations that do not involve the Master Functions X, Y, Z, S, etc.

Violation of these relations signals new flavour (CP) violating interactions beyond CKM or new operators that are strongly suppressed in SM

Examples

$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_s}$$

$$\frac{\text{Br}(B_s \rightarrow \mu^+ \mu^-)}{\text{Br}(B_d \rightarrow \mu^+ \mu^-)} = \frac{\tau(B_s) m_{B_s}}{\tau(B_d) m_{B_d}} \left[\frac{F_{B_s}}{F_{B_d}} \right]^2 \left[\frac{|V_{ts}|}{|V_{td}|} \right]^2$$

$$\frac{\Delta M_d}{\Delta M_s} = \frac{m_{B_d}}{m_{B_s}} \frac{\hat{B}_d}{\hat{B}_s} \frac{F_{B_d}^2}{F_{B_s}^2} \frac{|V_{td}|^2}{|V_{ts}|^2}$$

$$\frac{\text{Br}(B \rightarrow X_d \nu\bar{\nu})}{\text{Br}(B \rightarrow X_s \nu\bar{\nu})} = \frac{|V_{td}|^2}{|V_{ts}|^2}$$

Universal Unitarity Triangle (UUT) can be constructed

Intriguing Property of Models with Minimal Flavour Violation

AJB, Fleischer (01)

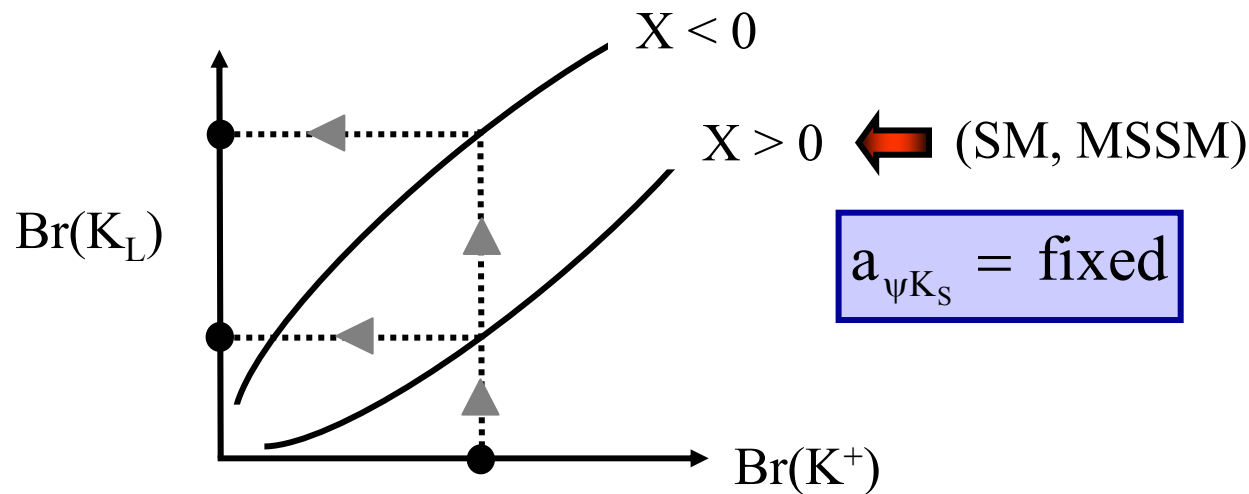
$$\text{Br}(K_L) = F(\text{Br}(K^+), a_{\psi K_S}, \text{sgn}(X))$$

TH very clean

Independently of any parameters, for given $\text{Br}(K^+)$ and $a_{\psi K_S}$ only two values of $\text{Br}(K_L)$ possible.



$X < 0$
very unlikely

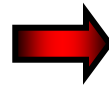


MFVfit Collaboration

(BBBEPSW)
hep-ph/0505110

Use the existing results for

1. UUTfit
2. $B \rightarrow X_s \gamma$
3. $B \rightarrow X_s l^+ l^-$
4. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$



$$X_{\max}(\nu) = 1.95 \quad Y_{\max}(\nu) = 1.43$$
$$(X_{\text{SM}} \cong 1.48) \quad (Y_{\text{SM}} \cong 0.98)$$



Model Independent
Upper Bounds
within MFV Scenario

Conclusion

:

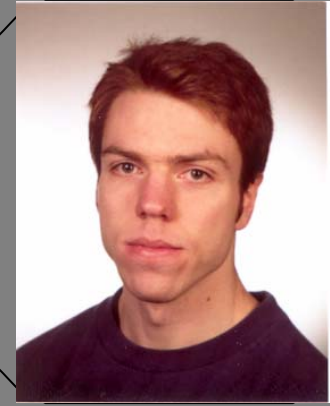
Large Departures from
SM within MFV not
possible

Upper Bounds on Rare K and B Decays from MFV

Bobeth, Bona, AJB, Ewerth, Pierini, Silvestrini, Weiler hep-ph/0505110

Branching Ratios	MFV (95%)	SM (95%)	SM (68%)	Exp
★ $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \cdot 10^{11}$	<11.9	<10.9	8.3 ± 1.2	$14.7^{+13.0}_{-8.9}$
★ $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \cdot 10^{11}$	<4.6	<4.2	3.1 ± 0.6	$<5.9 \cdot 10^4$
$\text{Br}(B \rightarrow X_s \nu \bar{\nu}) \cdot 10^5$	<5.2	<4.1	3.7 ± 0.2	<64
$\text{Br}(B_s \rightarrow \mu^+ \mu^-) \cdot 10^9$	<7.4	<5.9	3.7 ± 1.0	$<5.0 \cdot 10^2$
$\text{Br}(B_d \rightarrow \mu^+ \mu^-) \cdot 10^{10}$	<2.2	<1.8	1.1 ± 0.4	$<1.6 \cdot 10^3$

Magnificent Seven



$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ beyond MFV

- 1.** $\{\text{Real } X\} \rightarrow |X|e^{i\theta_x} \quad \theta_x \neq 0$
- 2.** Relation to other K decays and in particular to B decays not necessarily present.
- 3.** CP conserving contributions to $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in models with Lepton mixing (Grossman, Nir, Perez, Isidori, Murayama)

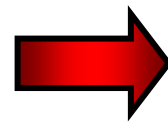
Scenario with New Physics Contributions dominated by enhanced EW-Penguins with a new complex weak phase

Colangelo, Isidori (98); AJB, Silvestrini (99); AJB, Romanino Silvestrini (99); AJB, Colangelo, Isidori, Romanino, Silvestrini (00); Buchalla, Hiller, Isidori (01); Atwood, Hiller (02)

AJB, Fleischer, Recksiegel, Schwab (03, 04, 05)



Enhanced EW-Penguins with $\theta_x \approx -90^\circ$ signaled by $B \rightarrow \pi K$ anomalies



Spectacular Modifications of $K \rightarrow \pi \nu \bar{\nu}$ relative to SM and MFV

Implications for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$

(BFRS)

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{SM} = (7.8 \pm 1.2) \cdot 10^{-11} \Rightarrow (7.5 \pm 2.1) \cdot 10^{-11}$$

Enhancement of $|X|$ compensated by destructive "top-charm" interference

★ $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{SM} = (3.0 \pm 0.6) \cdot 10^{-11} \Rightarrow (3.1 \pm 1.0) \cdot 10^{-10}$

★ Strong Violation of the "Golden" Relation

Buchalla, AJB (94)

SM: $(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_S}$
 Here: $-\begin{pmatrix} 0.69^{+0.23} \\ -0.41 \end{pmatrix} \neq 0.72 \pm 0.03$
 $\sin 2\beta_X$

$\beta_X = \beta - \theta_X$
 $X = |X| e^{i\theta_X}$
 $\beta_X \cong 110^\circ$

★ Saturation of the model-independent Grossman-Nir bound

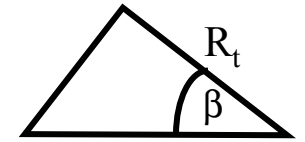
FIG

$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \leq 4.4 \text{ Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

Here:

$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})} \approx 4.4 \quad \sin^2(\beta_X) \approx 4.2 \pm 0.2$

Impact of $X = |X|e^{i\theta_X}$ on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$



(A ≈ 0.83)

(P_c ≈ 0.39)

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 4.8 \cdot 10^{-11} \left[A^4 R_t^2 |X|^2 + 2P_c A^2 R_t |X| \overbrace{\cos(\beta - \theta_X)}^{\beta_X} + P_c^2 \right]$$

SM: = $10^{-11} [4.1 + 3.0 + 0.7]$ X = 1.53

Here: = $10^{-11} [8.5 - 1.7 + 0.7]$ X = $2.2e^{-i86^\circ}$

(top)

(charm-top)

(charm)

$\beta - \theta_X \approx 110^\circ$

$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (7.8 \pm 1.2) \cdot 10^{-11} \rightarrow (7.5 \pm 2.1) \cdot 10^{-11}$

(SM)

Impact of $X = |X|e^{i\theta_X}$ on $K_L \rightarrow \pi^0 \nu \bar{\nu}$

(BFRS)

$$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}}} = \underbrace{\left| \frac{X}{X_{\text{SM}}} \right|^2}_{\approx 2} \underbrace{\left[\frac{\sin(\beta - \theta_X)}{\sin \beta} \right]^2}_{\approx 5}$$

$\beta - \theta_X \approx 110^\circ$
 $\beta \approx 24^\circ$

$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.6) \cdot 10^{-11} \quad \Rightarrow \quad (3.1 \pm 1.0) \cdot 10^{-10}$$

(SM)

(Here)

$$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}}} = 0.4 \quad \Rightarrow \quad 4.2$$

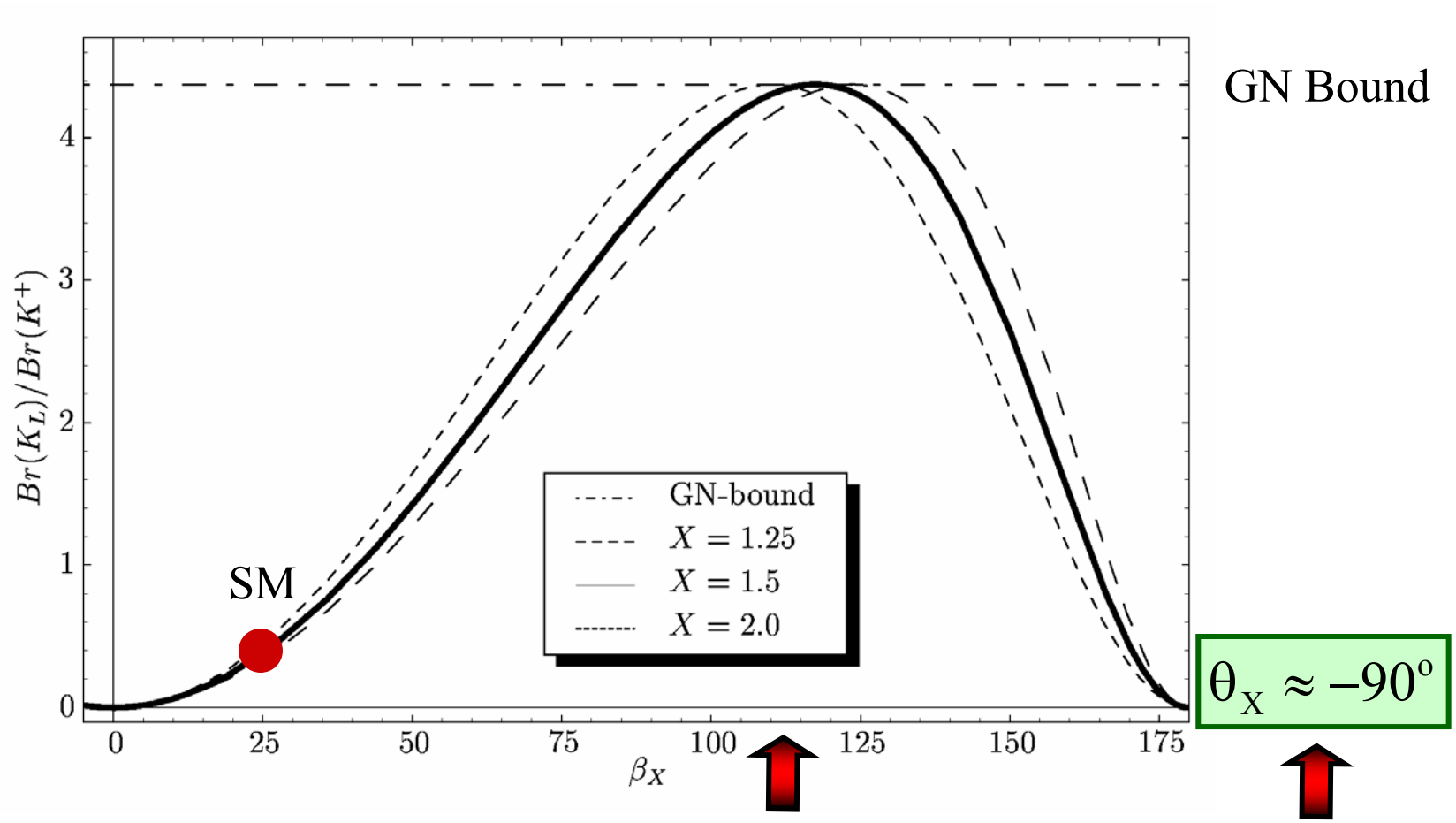
< 4.4
 Grossman-Nir bound

Order of magnitude enhancement !!

$$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})} \text{ versus } \beta_X$$

BSU (04)

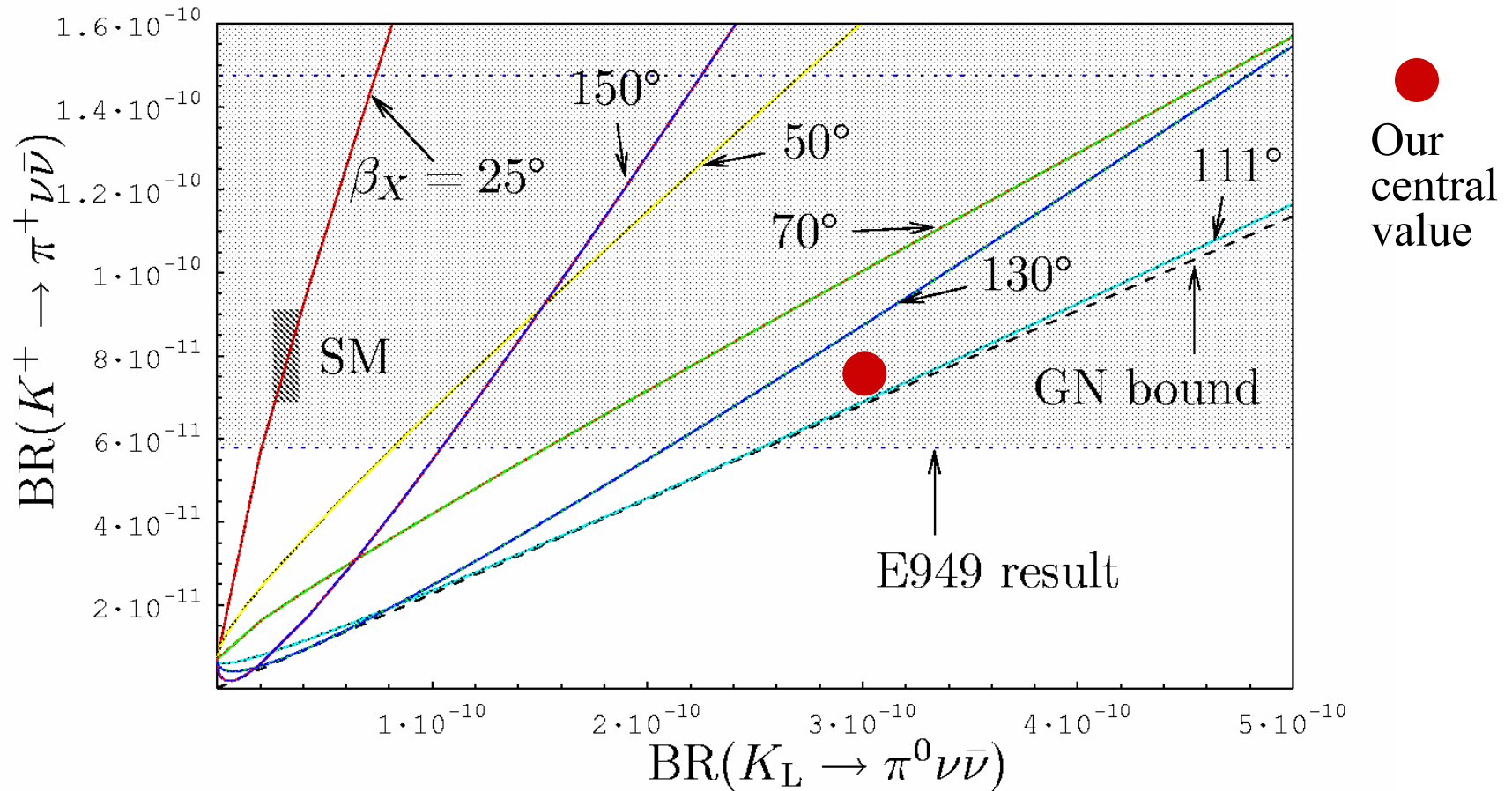
$\beta_x = \beta - \theta_X$ ← New Phase in EWP



Necessary to fit $B \rightarrow \pi K$ data (BFRS)

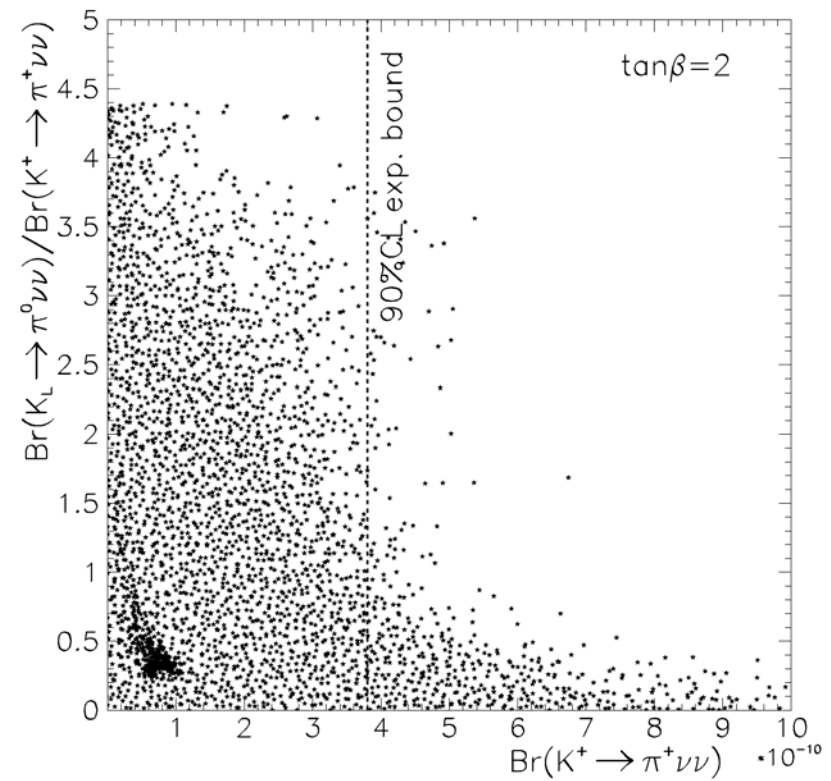
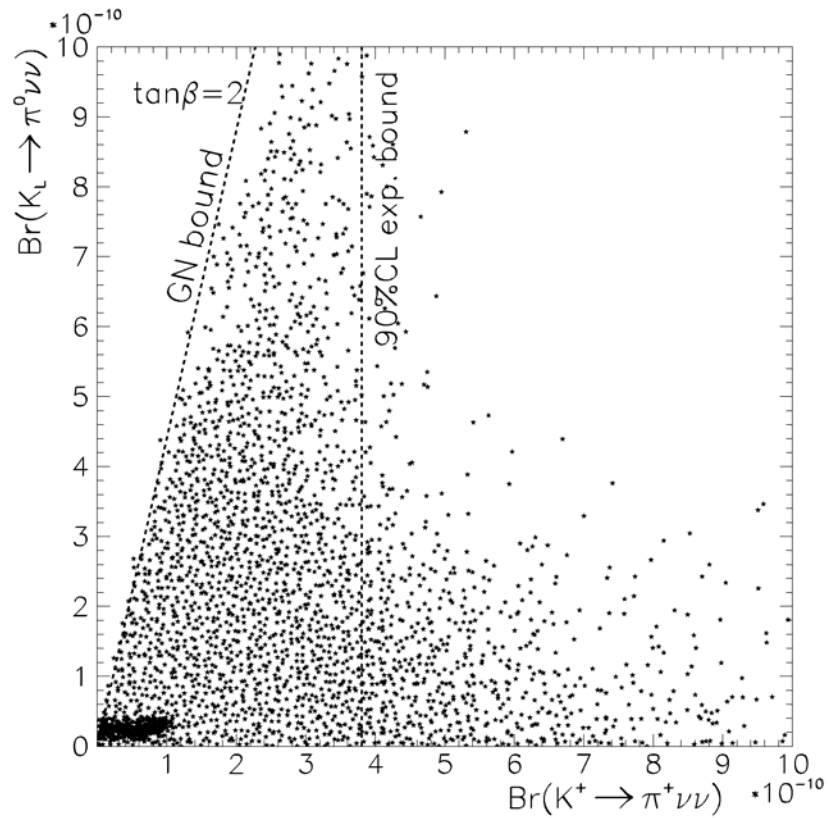
$$K^+ \rightarrow \pi^+ \nu \bar{\nu} \text{ and } K_L \rightarrow \pi^0 \nu \bar{\nu} \text{ with } X = |X| e^{i\theta_x} \quad (\text{BFRS})$$

$$\beta_x = \beta - \theta_x$$



$K_L \rightarrow \pi^0 \nu\bar{\nu}$ and $K^+ \rightarrow \pi^0 \nu\bar{\nu}$ from a general MSSM

AJB, Ewerth, Jäger, Rosiek (04)



Relations between $\Delta M_{s,d}$ and $B_{s,d} \rightarrow \mu\bar{\mu}$ in Models with Minimal Flavour Violation

(AJB, hep-ph/0303060)

$$\Delta M_q \sim \hat{B}_q F_{B_q}^2 |V_{tq}|^2 S(x_t, x_{\text{new}})$$

$$\text{Br}(B_q \rightarrow \mu\bar{\mu}) \sim F_{B_q}^2 |V_{tq}|^2 Y^2(x_t, \bar{x}_{\text{new}})$$

Large hadronic
uncertainties
due to $F_{B_q}^2$

$$F_{B_d} \sqrt{\hat{B}_d} = \begin{pmatrix} 235 \pm 33 & +0 \\ & -24 \end{pmatrix} \text{MeV} \quad F_{B_d} = (189 \pm 27) \text{MeV}$$

$$F_{B_s} \sqrt{\hat{B}_d} = (276 \pm 38) \text{MeV} \quad F_{B_s} = (230 \pm 30) \text{MeV}$$

$$\hat{B}_d = 1.34 \pm 0.12$$

$$\hat{B}_s = 1.34 \pm 0.12$$

$$\frac{\hat{B}_s}{\hat{B}_d} = 1.00 \pm 0.03$$

(No problems with
chiral logs and
quenching)

$$\text{Br}(B_{s,d} \rightarrow \mu\bar{\mu}) \text{ from } \Delta M_{s,d}$$

$$\text{Br}(B_q \rightarrow \mu\bar{\mu}) = 4.36 \cdot 10^{-10} \frac{\tau(B_q)}{\hat{B}_q} \frac{Y^2(x_t, \bar{x}_{\text{new}})}{S(x_t, x_{\text{new}})} \Delta M_q$$

No dependence
on $F_{B_q}^2$

SM:

$$\text{Br}(B_s \rightarrow \mu\bar{\mu}) = 3.42 \cdot 10^{-9} \left[\frac{\tau(B_s)}{1.46 \text{ ps}} \right] \left[\frac{1.34}{\hat{B}_s} \right] \left[\frac{\bar{m}_t(m_t)}{167 \text{ GeV}} \right]^{1.6} \left[\frac{\Delta M_s}{18.0 / \text{ps}} \right]$$

$$\text{Br}(B_d \rightarrow \mu\bar{\mu}) = 1.00 \cdot 10^{-10} \left[\frac{\tau(B_d)}{1.54 \text{ ps}} \right] \left[\frac{1.34}{\hat{B}_d} \right] \left[\frac{\bar{m}_t(m_t)}{167 \text{ GeV}} \right]^{1.6} \left[\frac{\Delta M_d}{0.50 / \text{ps}} \right]$$

(Example)

$$\Delta M_s = (18.0 \pm 0.5 / \text{ps})$$



$$\text{Br}(B_s \rightarrow \mu\bar{\mu}) = (3.42 \pm 0.54) \cdot 10^{-9}$$

$$\Delta M_d = (0.503 \pm 0.006 / \text{ps})$$



$$\text{Br}(B_d \rightarrow \mu\bar{\mu}) = (1.00 \pm 0.14) \cdot 10^{-10}$$

Moreover new Physics Effects can be easier seen



Testing MFV through $B_{s,d} \rightarrow \mu\bar{\mu}$ and $\Delta M_{s,d}$

$$\frac{\text{Br}(B_s \rightarrow \mu\bar{\mu})}{\text{Br}(B_d \rightarrow \mu\bar{\mu})} = \frac{\hat{B}_d}{\hat{B}_s} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d}$$

(1.00 ± 0.03) Experiment

Valid in MFV models in which only SM operators relevant.

Violation of this relation would indicate the presence of new operators and generally of non-minimal flavour violation.

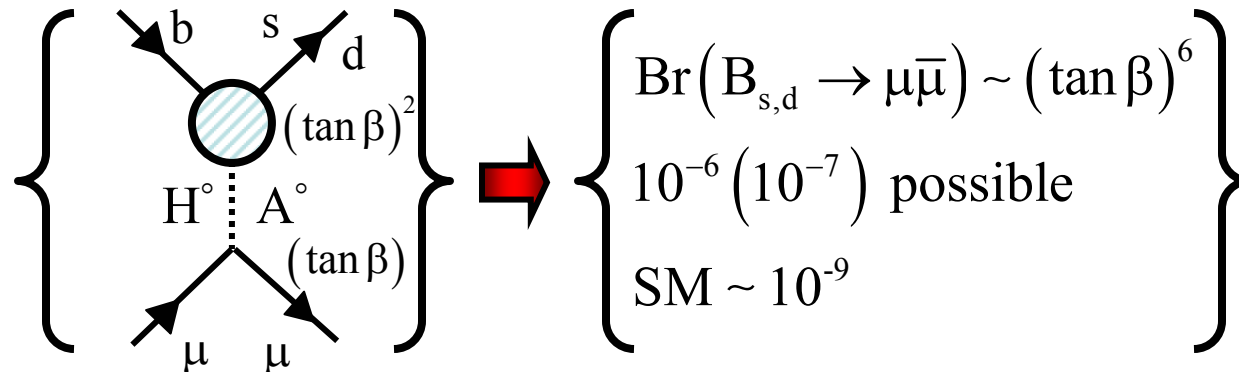
MSSM at large $\tan\beta$

$$(B_{s,d} \rightarrow \mu^+ \mu^-)$$

In MSSM at large $\tan\beta$

(CKM still the only source of Flavour and CP Violation)

Strong Enhancement



$$\left. \begin{aligned} \text{Br}(B_{s,d} \rightarrow \mu \bar{\mu}) &\sim (\tan\beta)^6 \\ 10^{-6} (10^{-7}) &\text{ possible} \\ \text{SM} &\sim 10^{-9} \end{aligned} \right\}$$

Babu, Kolda
 Chankowski, Slawianowska
 Bobeth, Ewerth, Krüger, Urban
 Huang, Liao, Yan, Zhu
 Isidori, Retico
 Dedes, Dreiner, Nierste
 Dedes, Pilaftis
 Chankowski, Rosiek
 Foster, Okumura, Roszkowski

$$\text{Br}(B_s \rightarrow \mu \bar{\mu}) < 1.5 \cdot 10^{-7}$$

$$\text{Br}(B_d \rightarrow \mu \bar{\mu}) < 4.0 \cdot 10^{-8}$$

CDF, DØ

95% C.L.

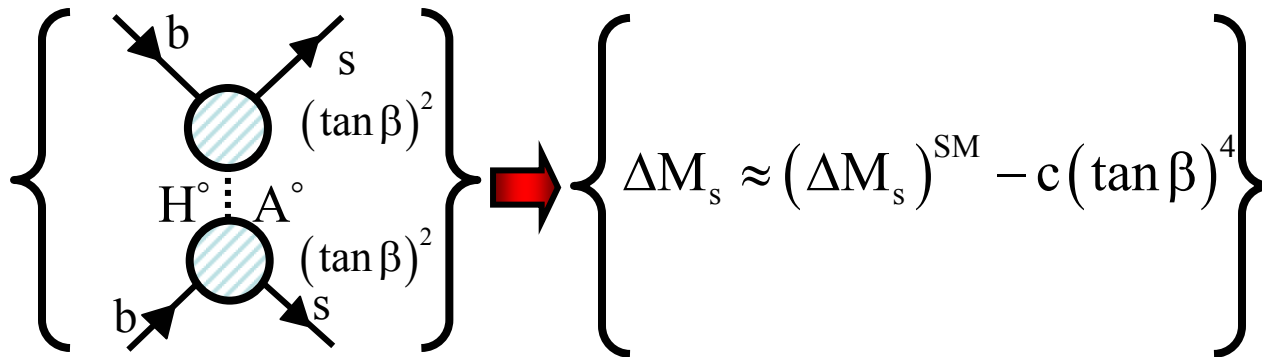
MSSM at large $\tan\beta$ (cont.)

(Double-Higgs Penguin)

(ΔM_s)

$B_s^0 - \bar{B}_s^0$ Mixing

Suppression



AJB, Chankowski, Rosiek
Slawianowska (2001, 2002)

Correlation between
SUSY effects in
 $\text{Br}(B_{s,d} \rightarrow \mu\bar{\mu})$ and ΔM_s

Negligible contributions to ΔM_d , ϵ_K

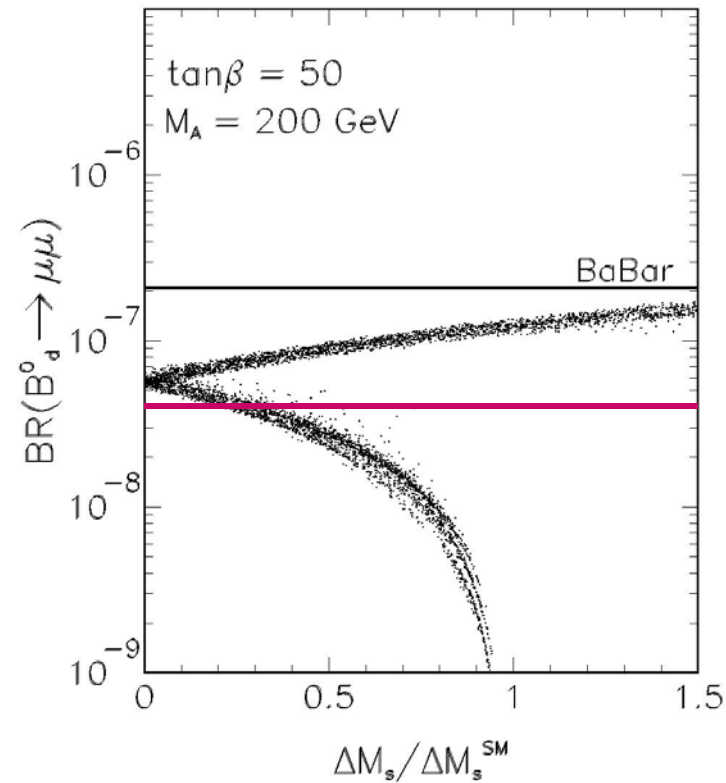
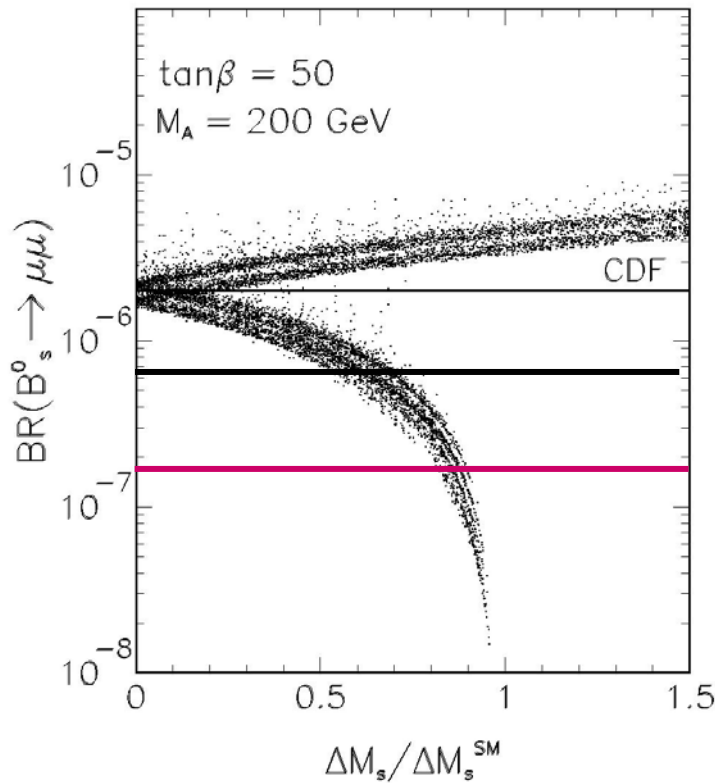
Clear violation
of $K \leftrightarrow B$ MFV
relations

Subsequent analyses: D'Ambrosio, Giudice, Isidori, Stumia
Dedes, Pilaftis

$\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)$ vs $(\Delta M_s)^{\text{exp}} / (\Delta M_s)^{\text{SM}}$ in SUSY at Large $\tan \beta$

AJB, Chankowski, Rosiek, Slawianowska, hep-ph/0207241

2002
2004
2005



2002
2005

Comparison of different Models

	universal extra dimensions	MSSM (CKM) large $\tan\beta$	MSSM (CKM) low $\tan\beta$	Littlest Higgs
$\text{Br}(\text{K}^+ \rightarrow \pi^+ \nu \bar{\nu})$	↑	No effect	↓	↓
$\text{Br}(\text{K}_L \rightarrow \pi^0 \nu \bar{\nu})$	↑	No effect	↓	↓
$\text{Br}(\text{B}_s \rightarrow \mu \bar{\mu})$	↑	↑↑↑	↑↓	↑
ΔM_s	↑	↓	↑	↑

Collaboration with:

Spranger
Weiler (02)

see previous
refs.

Gambino
Gorbahn
Jäger
Silvestrini (01)

Poschenrieder
Uhlig (04)

Please !! Measure !!

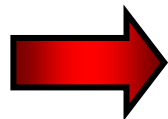
(Before 2012)

$$\text{Br}\left(\text{K}^+ \rightarrow \pi^+ \nu \bar{\nu}\right) \quad \text{Br}\left(\text{K}_L \rightarrow \pi^0 \nu \bar{\nu}\right)$$

$$\text{Br}\left(\text{B}_{s,d} \rightarrow \mu \bar{\mu}\right) \quad \Delta M_S$$

and

γ independently of NP



We will know much more
about flavour physics