W & Z boson production: theory update

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- Total cross sections
  upcoming CTEQ contribution to Tev4LHC workshop

- Rapidity distributions, charge asymmetry

- Nonperturbative contributions to q_T resummation
  A. Konychev, P. N., hep-ph/0505xxx

For additional details, see also *hep-ph/0412146*

Precision computation of Tevatron W and Z cross sections relies on understanding of

- ❒ NNLO QCD and NLO EW perturbative corrections
- ❒ multiple correlated factors of diverse nature:
  - ❍ theoretical and experimental
  - ❍ perturbative and nonperturbative
  - ❍ rigorous and practical
  - ❍ objective and subjective
  - ❍ ...

Total W and Z cross sections

- Monitors of the beam and parton luminosity at future colliders
  (Dittmar, Pauss, Zurcher; Khoze, Martin, Orava, Ryskin; Giele, Keller)

Total cross sections: NNLO QCD corrections

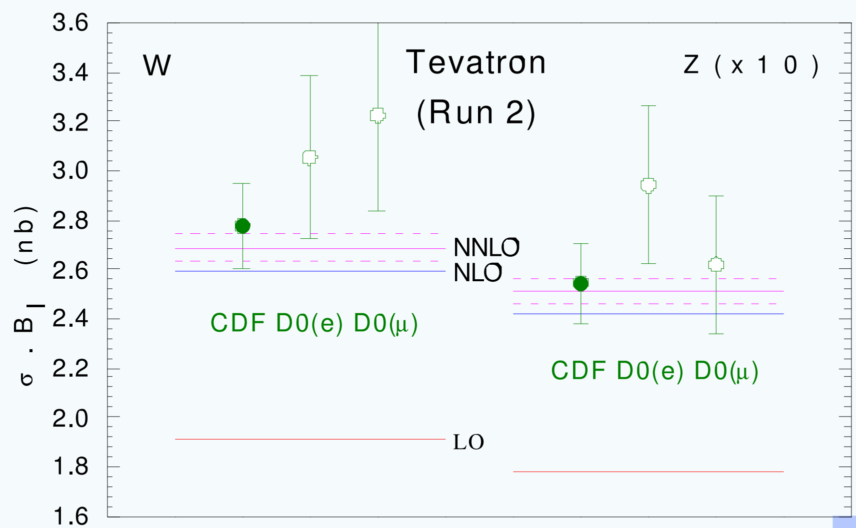
$$\sigma_{tot}(p\bar{p} \to V) = \sum_{partons} \int dx_1 dx_2 f_{a/p}(x_1) f_{b/\bar{p}}(x_2) \hat{\sigma}_{tot}(ab \to V)$$

- NNLO hard cross section $\hat{\sigma}_{tot}(ab \to V)$
  (Hamberg, van Neerven, Matsuura, 1991; Harlander and Kilgore, 2002)

- Partial NNLO results for parton distributions $f_{a/p}(x)$

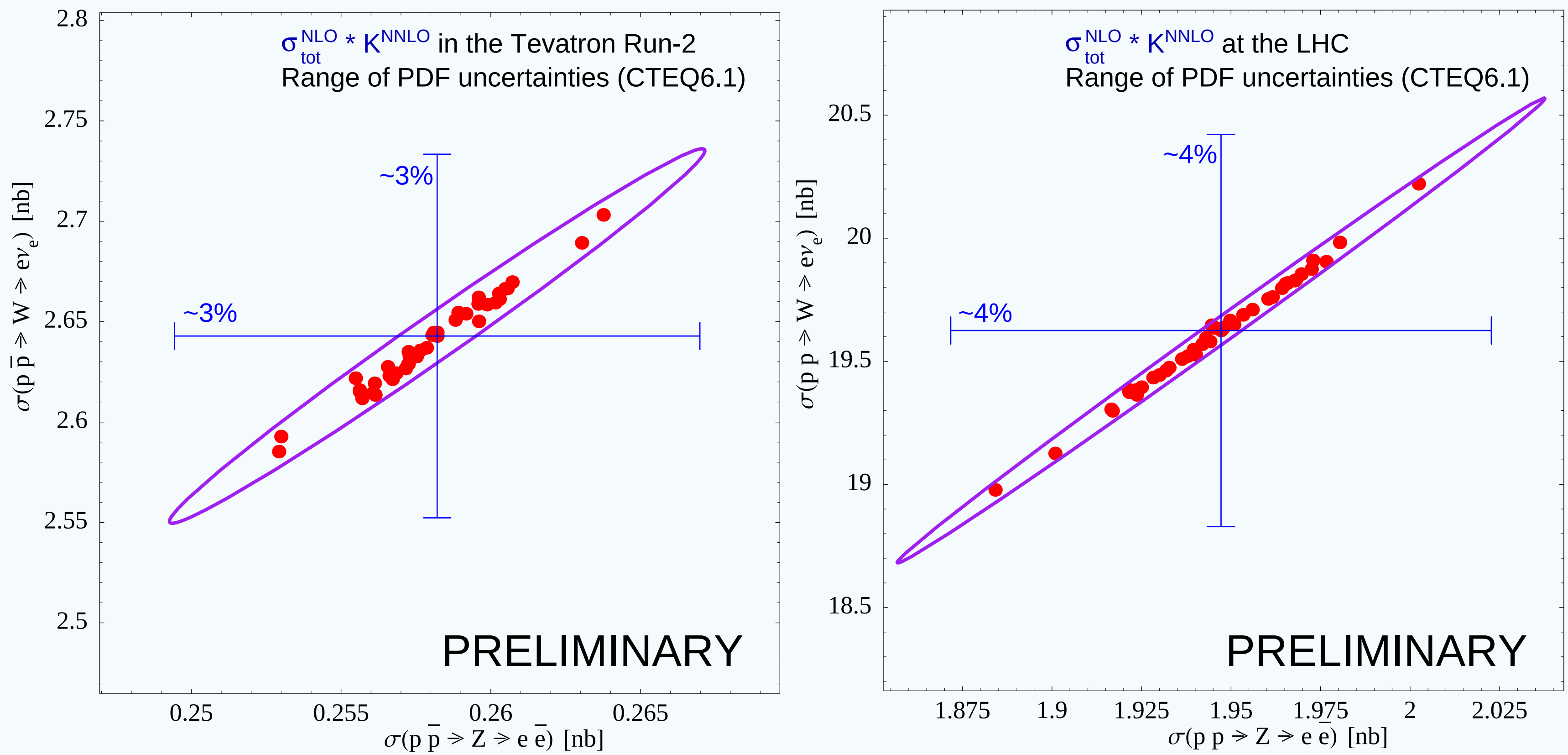
- Scale dependence of order 1%

- NNLO K-factor is about 1.04 at the Tevatron and 0.98 at the LHC
  (MRST'03)



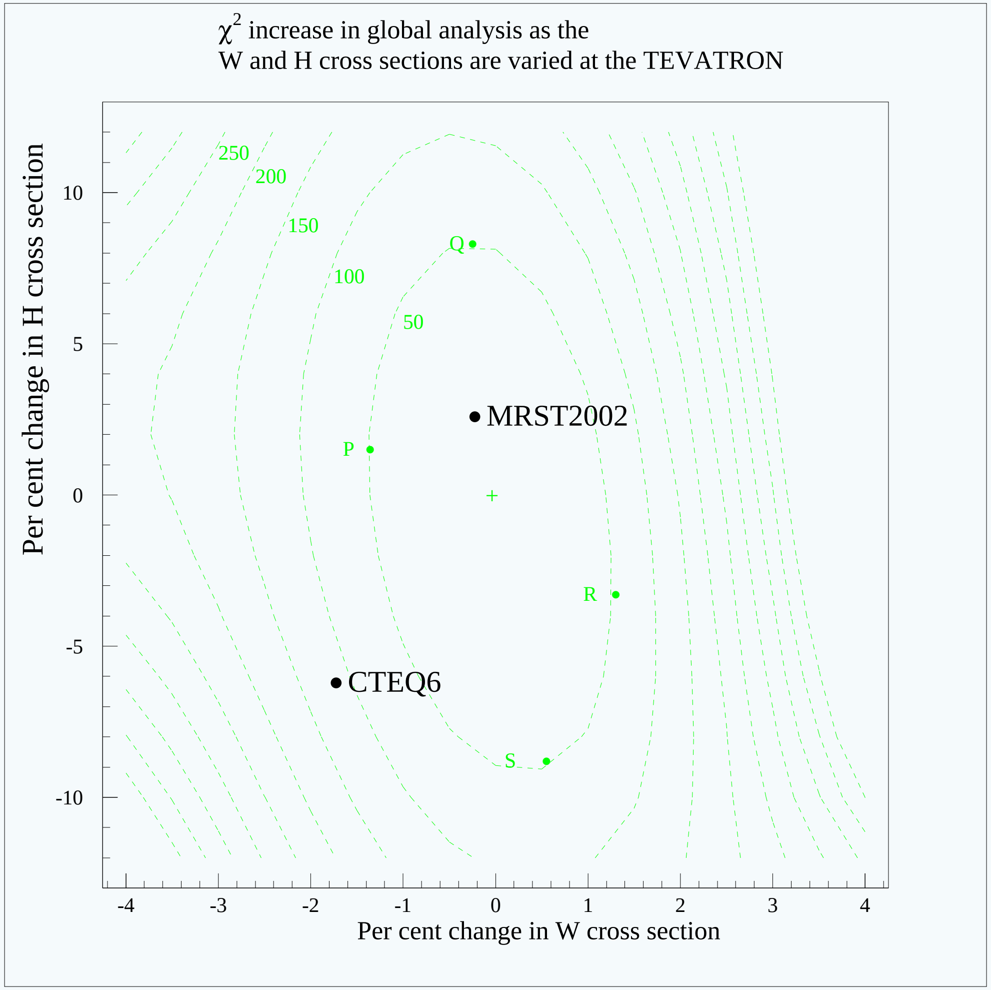
Cancellation of PDF uncertainties in $\sigma_{tot}(Z)/\sigma_{tot}(W)$

(Huston, P. N., Pumplin, Stump, Tung, Yuan, 2004)



- In spite of different quark flavors, a measurement of $\sigma(Z)$ will constrain $\sigma(W)$ (and possibly other quark-dominated cross sections)!

Differences between various NLO predictions for σ_{tot} arise not only from higher-order corrections



- ❒ Selection & weighting of data in the fit
- ❒ Parametric form of PDFs at $\mu = \mu_0$
- ❒ Definition of α_s at (N)NLO
- ❒ Assumptions about sea flavor symmetries
- ❒ Treatment of heavy flavors
- ❒ Implementation of electroweak corrections
- ❒ acceptance, lepton ID

$\sigma_{tot}(W)$ and $\sigma_{tot}(Z)$: standardization of theory predictions (in progress)

Collider/ program	Cross section (pb)	CTEQ6M	MRST 2002(NLO)
Tevatron ($\sqrt{s}=1.96$TeV) wttot	$\sigma(W\to\ell\nu)$(SigmaTot1)	2526	2548
	$\sigma(W)$ at Q=80.423 GeV	23773	23988
	$\sigma(W)\cdot 0.1068$	2539	2562
	$\sigma(W)\cdot 0.1084$	2577	2601
ResBos	$\sigma(W\to\ell\nu)$	2588 ± 6	2606 ± 6
MRST'02 paper	$\sigma(W)\cdot 0.1068$		2600 (1.4% above WTTOT)
LHC ($\sqrt{s}=14$TeV) wttot	$\sigma(W^+\to\ell\nu)$(SigmaTot1)	11525	11444
	$\sigma(W^-\to\ell\nu)$(SigmaTot1)	8497	8500
	$\sigma(W\to\ell\nu)$(SigmaTot1)	20022	19944
	$\sigma(W)$ at Q=80.423 GeV	188549	187885
	$\sigma(W)\cdot 0.1068$	20137	20066
	$\sigma(W)\cdot 0.1084$	20439	20367
ResBos	$\sigma(W^+\to\ell\nu)$	11899 ± 43	11891 ± 43
	$\sigma(W^-\to\ell\nu)$	8717 ± 29	8799 ± 29
	$\sigma(W\to\ell\nu)$	20616 ± 52	20690 ± 52
MRST'02 paper	$\sigma(W)\cdot 0.1068$		20400 (1.6% above WTTOT)

The “correct” standard candle observable

“$\sigma_{tot}(Z)$” is a theoretical construct to be derived from experimental data for $p\bar{p} \to (\gamma^*, Z \to e^+e^-)X$ and $p\bar{p} \to (\gamma^*, Z \to \mu^+\mu^-)X$

Z boson decay can be described at various levels of sophistication

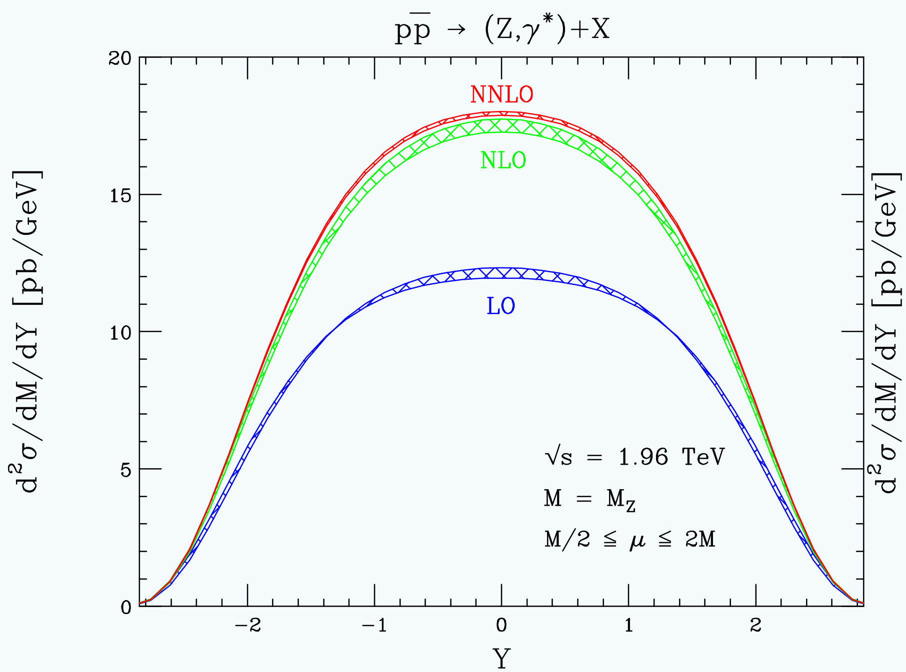
- narrow Z width approximation (MRST)
- effective Born approximation (ResBos, MCFM,...)
- final-state NLO QED corrections (ResBos-A)
- inclusive NLO-EW total cross section
  - + γ^*, Z interference
- NLO-EW + acceptance and lepton ID cuts (ZGRAD)
  - + dependence on m_e and m_μ
- + effects of detection, triggering, ...

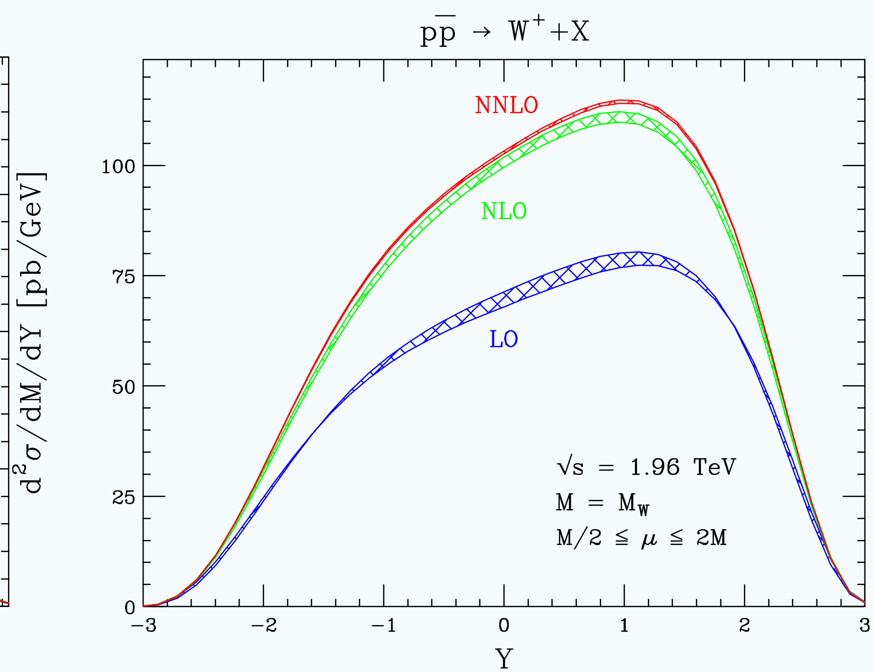
Which level is the most suitable for presentation of a universally used standard-candle quantity?

Rapidity distributions and charge asymmetry

NNLO rapidity distributions at the Tevatron

(Anastasiou, Dixon, Melnikov, Petriello, 2004)





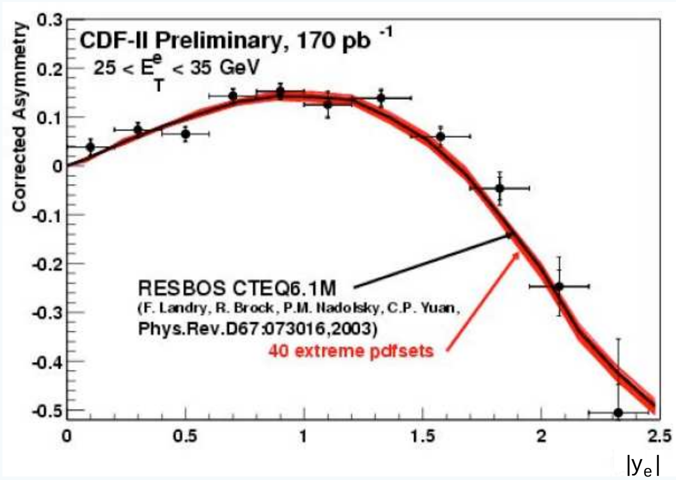
- Tiny scale dependence $(< 1\%)$
- For $|y| < 2$, NNLO leads to a uniform enhancement

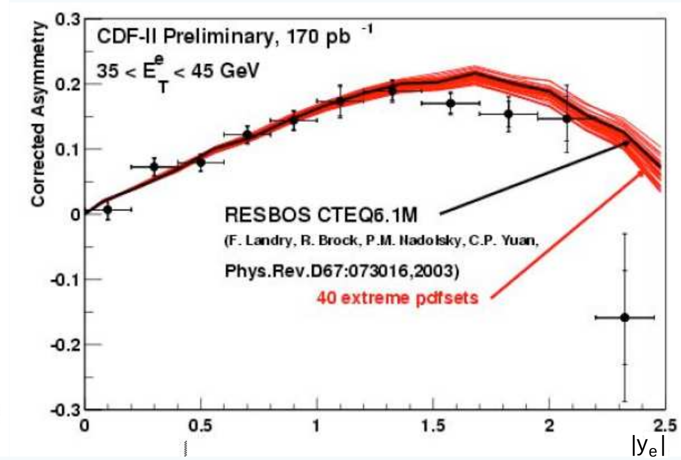
$$\sigma_{NNLO} \approx K \cdot \sigma_{NLO}$$

$K(Z) \sim 3-5\%$, $K(W) \sim 2.5-4\%$

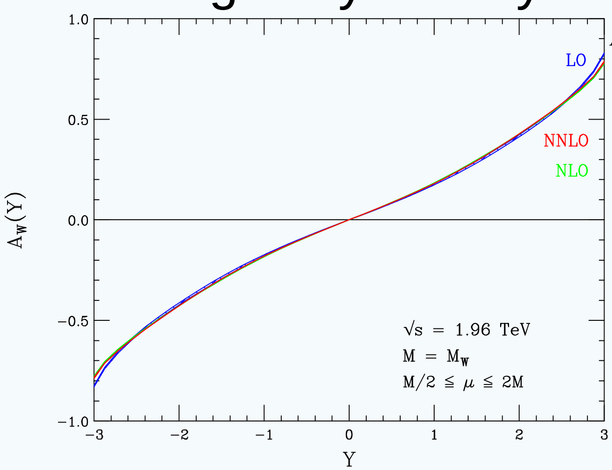
- Larger corrections in forward regions

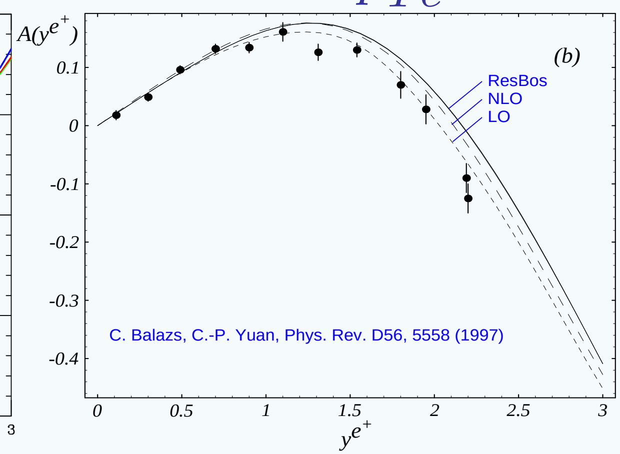
Charge asymmetry: CDF Run-2 vs. CTEQ6.1 and ResBos





Charge asymmetry without and with p_{Te} cut





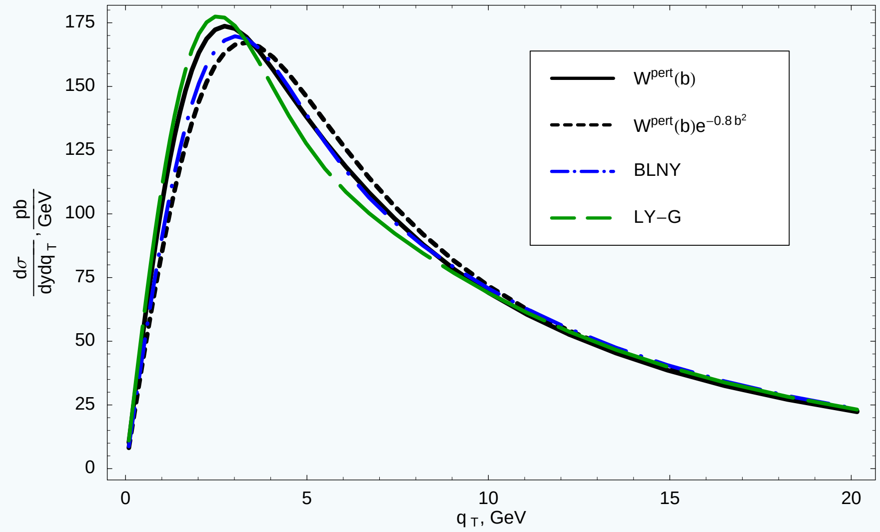
- p_{Te} cut introduces dependence of $A_{ch}(y_e)$ on QCD corrections

Nonperturbative contributions in transverse momentum resummation

Anton Konychev, P. N., hep-ph/0505xxx

The largest theory uncertainties in the measured M_W arise from

- ❒ the model of W boson's recoil in the transverse plane
- ❒ parton densities



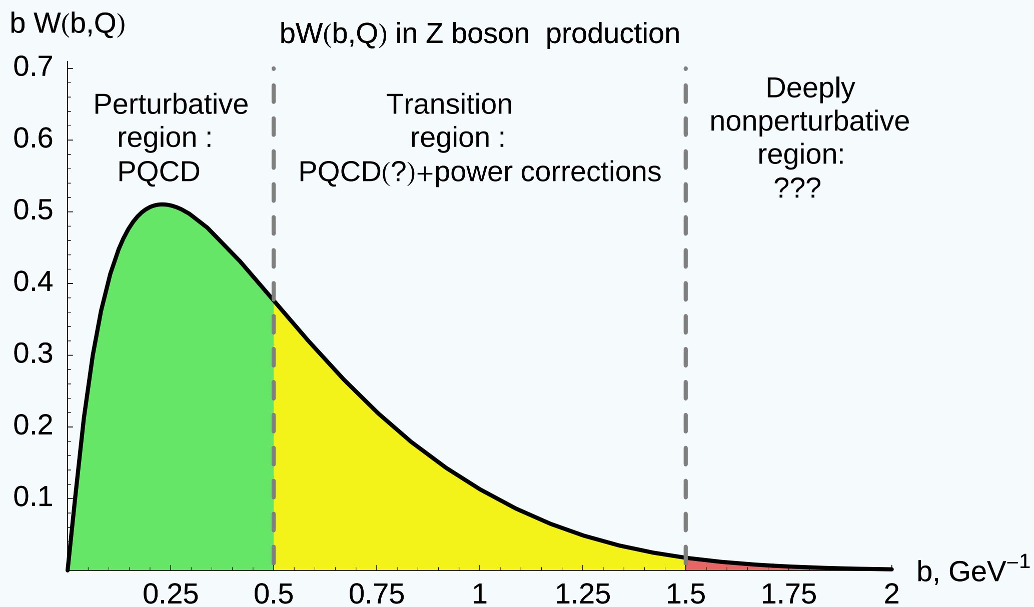
A W boson acquires $q_T \neq 0$ by recoiling against perturbative or nonperturbative QCD radiation

The peak of $d\sigma/dq_T$ moves by up to $\sim$ 500 MeV depending on the nonperturbative model (large effect compared to the targeted $\delta M_W \sim$ 30 MeV)

Behavior of nonpert. contributions and their uncertainties are studied within a global analysis of q_T distributions of Drell-Yan pairs and Z bosons in the Collins-Soper-Sterman resummation formalism $\Rightarrow$ today's talk;
$\Rightarrow$ non-trivial variations in $d\sigma/dq_T$ at $x < 10^{-2}$ will be neglected

$bW(b,Q)$ in Z boson production

In the CSS formalism, the small-q_T cross section is given by a Fourier-Bessel transform of an impact parameter $b\widetilde{W}(b,Q)$ in impact parameter (b) space



- ❐ $b \lesssim 0.5$ GeV^{-1} : $\widetilde{W}(b,Q) \approx \widetilde{W}_{pert}(b,Q)$

contributes most of the rate at the Tevatron

- ❐ $0.5 \lesssim b \lesssim 1.5-2$ GeV^{-1} : higher-order terms in α_s and b^p important; contributes some variations in $d\sigma/dq_T$ at $q_T \lesssim 10$ GeV
- ❐ $b \gtrsim 1.5-2$ GeV^{-1} : terra incognita; tiny contributions

The puzzling behavior of nonperturbative contributions

On one hand:

- The nonperturbative "k_T-smearing" function $\mathcal{S}_{NP}(b,Q)$ is universal in Drell-Yan-like processes and SIDIS *(Collins, Soper, 1981; CSS, 1985; Collins, Metz, 2004)*

- Renormalon analysis *(Korchemsky, Sterman)* predicts that the "genuine" $\mathcal{S}_{NP}(b,Q)$ is approximately quadratic in b and linear in $\ln Q$:

  $$\mathcal{S}_{NP}(b,Q) \approx b^2\{a_1 + a_2 \ln Q\} \oplus \text{ smaller corrections}$$

  A lattice QCD estimate gives $a_2 = 0.19^{+0.11}_{-0.1}$ GeV^2 *(Tafat)*

On the other hand:

- A previous global q_T fit *(Brock, Landry, P. N., Yuan, 2002)* finds

$$\mathcal{S}_{NP}(b,Q) = b^2\left[g_1 + g_2 \ln\left(\frac{Q}{3.2\ \text{GeV}}\right) + g_1 g_3 \ln\left(100 x_A x_B\right)\right],$$

  with $g_1 = 0.21^{+0.01}_{-0.01}$ GeV^2, $g_2 = 0.68^{+0.01}_{-0.02}$ GeV^2, $g_3 = -0.6^{+0.05}_{-0.04}$

- parametrizations with linear terms in b or $g_3 = 0$ fail spectacularly ($\chi^2/d.o.f. > 3$)
- some tension between experiments ($\chi^2/d.o.f. = 176/119 \sim 1.48$)
- $g_2 = 0.68^{+0.01}_{-0.02}$ GeV^2 does not agree with $a_2 = 0.19^{+0.11}_{-0.1}$ GeV^2
- the fit suggests intrinsic $\langle k_T^2\rangle = 2g(Q) \approx 5.4$ GeV^2 at $Q = M_Z$
- $\langle k_T^2\rangle \approx 1.6$ GeV^2 at $Q = M_Z$ in other models for large-b continuation of perturbative terms *(Qiu, Zhang; Kulezsa, Sterman, Vogelsang)*

Does $\mathcal{S}_{NP}^{BLNY}$ inadvertently include a sizable perturbative component?

$\widetilde{W}_{pert}(b,Q)$ at large b: the b_* prescription *(Collins, Soper, 1982; CSS, 1985)*

$$\widetilde{W}(b,Q) = \widetilde{W}_{pert}(b_*,Q)e^{-\mathcal{S}_{NP}(b,Q;b_{max})}$$

$$b_*(b,b_{max}) \equiv \frac{b}{(1+b^2/b_{max}^2)^{1/2}} = \begin{cases} b & \text{at} \quad b \ll b_{max} \\ b_{max} & \text{at} \quad b \gg b_{max} \end{cases}$$

$$\widetilde{W}_{pert}(b_*,Q) = \sum_j \sigma_0 e^{-\mathcal{S}_{pert}(b_*,Q)}$$
$$\times \quad \left[\mathcal{C}_{j/a}\otimes f_{a/A}\right](x_A,b_*,\mu_F(b_*)) \left[\mathcal{C}_{\bar{j}/b}\otimes f_{b/B}\right](x_B,b_*,\mu_F(b_*))$$

The arbitrary scale μ_F in the PDF's $f_{a/A}(x_A,\mu_F)$ is usually set equal to b_0/b_* to avoid $|\ln(\mu_F b_*/b_0)| \gg 1$ in $\mathcal{C}_{j/a}(x_A,b_*,\mu_F)$ (here $b_0 = const \approx 1.12$)

- b_{max} cannot exceed $b_0/Q_{ini} \approx 1$ GeV^{-1} ($Q_{ini} \approx 1$ GeV is the initial PDF scale); $b_{max} = 0.5$ GeV^{-1} in the BLNY fit
- b_* anzatz modifies $\widetilde{W}_{pert}$ in the transition region $b \sim 1$ GeV^{-1}
- compensated in part by phenomenological $\mathcal{S}_{NP}(b,Q)$

- We would like to increase b_{max} above 1 GeV^{-1} to reduce impact of b_* anzatz on $\widetilde{W}_{pert}$ in the transition region

- The PDF parametrization requires that $\mu_F \sim 1/b_* > 1$ GeV
  - unless the GRV PDFs are used

- Other parts of $\widetilde{W}_{pert}(b, Q)$ can be continued to $b > 1$ GeV^{-1} by using their fixed-order expressions

- Solution: decouple μ_F from b_*

The "modified b_* prescription"

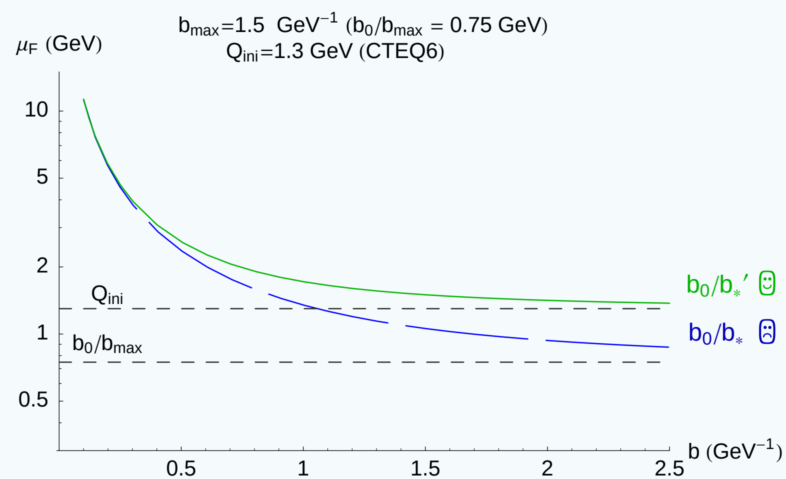
1. Take the original b_* prescription

$$\widetilde{W}(b,Q) = \widetilde{W}_{pert}(b_*,Q)e^{-\mathcal{S}_{NP}(b,Q;b_{max})}$$

2. If $b_{max} < b_0/Q_{ini}$, choose $\mu_F = b_0/b_*(b, b_{max})$ (original b_* anzatz)

3. If $b_{max} > b_0/Q_{ini}$, choose $\mu_F = b_0/b_*(b, b_0/Q_{ini})$

$$\mu_F = \begin{cases} \sim 1/b & \text{for} \quad b \ll b_0/Q_{ini} \\ Q_{ini} & \text{for} \quad b \gtrsim b_0/Q_{ini} \end{cases}$$



b_{max} can be safely varied between $0.5 - 2$ GeV^{-1} in both low-Q Drell-Yan and Z boson production,

but the scale μ_F in $f_{a/A}(x, \mu_F)$ never goes below Q_{ini}

Modified b_* prescription: factorization scale dependence

❒ If $\mu_F \sim Q_{ini}$, large non-resummed logarithms appear at $b_* \gg b_0/Q_{ini}$

$$\mathcal{C}_{j/a}\left(x, \frac{b_* \mu_F}{b_0}\right) = \sum_{k,m} \left(\frac{\alpha_s}{\pi}\right)^k \left[P_{j/a}(x) \ln^m\left(\frac{b_* \mu_F}{b_0}\right) + \ldots\right]$$

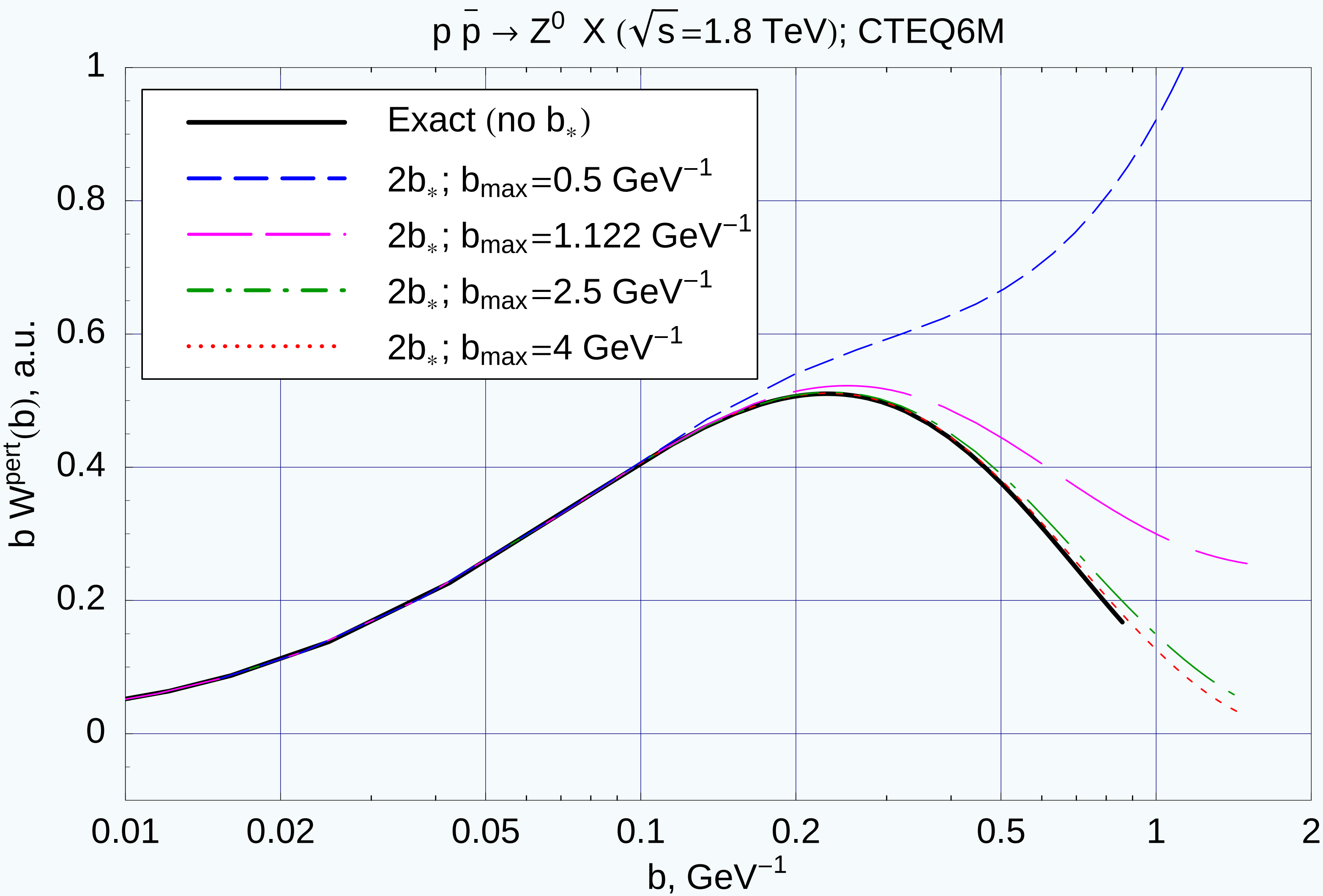
❒ should not create problems, because the region $b_* \gg b_0/Q_{ini}$ is exponentially suppressed by $e^{-\mathcal{S}_{pert}(b_*,Q) - \mathcal{S}_{NP}(b,Q)}$

- ❍ confirmed by a numerical calculation
  - ▷ our fits are made for $\mu_F = C_3/b'_*$, with $C_3 = b_0$ and $C_3 = 2b_0$; scale variations with C_3 are roughly independent from b_{max}

Properties of the modified b_* prescription

- no new parameters (utilizes freedom in the choice of μ_F)
- preserves continuity of $\widetilde{W}(b,Q)$ and its derivatives
- the balance of pert. and nonpert. contributions in $\widetilde{W}(b,Q)$ is smoothly changed by varying b_{max}
- at $b_{max} \leq b_0/Q_{ini}$, reduces to the original b_* prescription
- at $b_{max} \gg b_0/Q_{ini}$, is structurally and numerically close to the leading-log extrapolation of $\widetilde{W}_{pert}(b,Q)$, such as that in the principal value resummation *(Sterman; Kulesza, Sterman, Vogelsang...)*

Perturbative form factors $b\widetilde{W}^{pert}(b,Q)$ and $b\widetilde{W}^{pert}(b_*,Q)$ in the modified b_* prescription for the Tevatron Run-1 Z production



Global fits in the modified b_* prescription New!

98 data points

- Tevatron Run-1 Z boson production (CDF, D0)
  - $Q \approx M_Z$, $\sqrt{s} = 1.8$TeV, $p_T < 10$ GeV
  - sizable errors
- Fixed-target Drell-Yan pair production (E288, E605, R209)
  - $Q = 5 - 18$ GeV, $p_T < 1.4$ GeV
  - small statistical errors, incomplete systematical errors; 2 outlier points in E605 sample contribute $\delta\chi^2 \approx 25$

Nonperturbative function:

$$\mathcal{S}_{NP}(b) = b^{2-\beta}\left[a_1 + a_2 \ln\left(\frac{Q}{3.2\text{ GeV}}\right) + a_3 \ln\left(100 x_A x_B\right)\right],$$

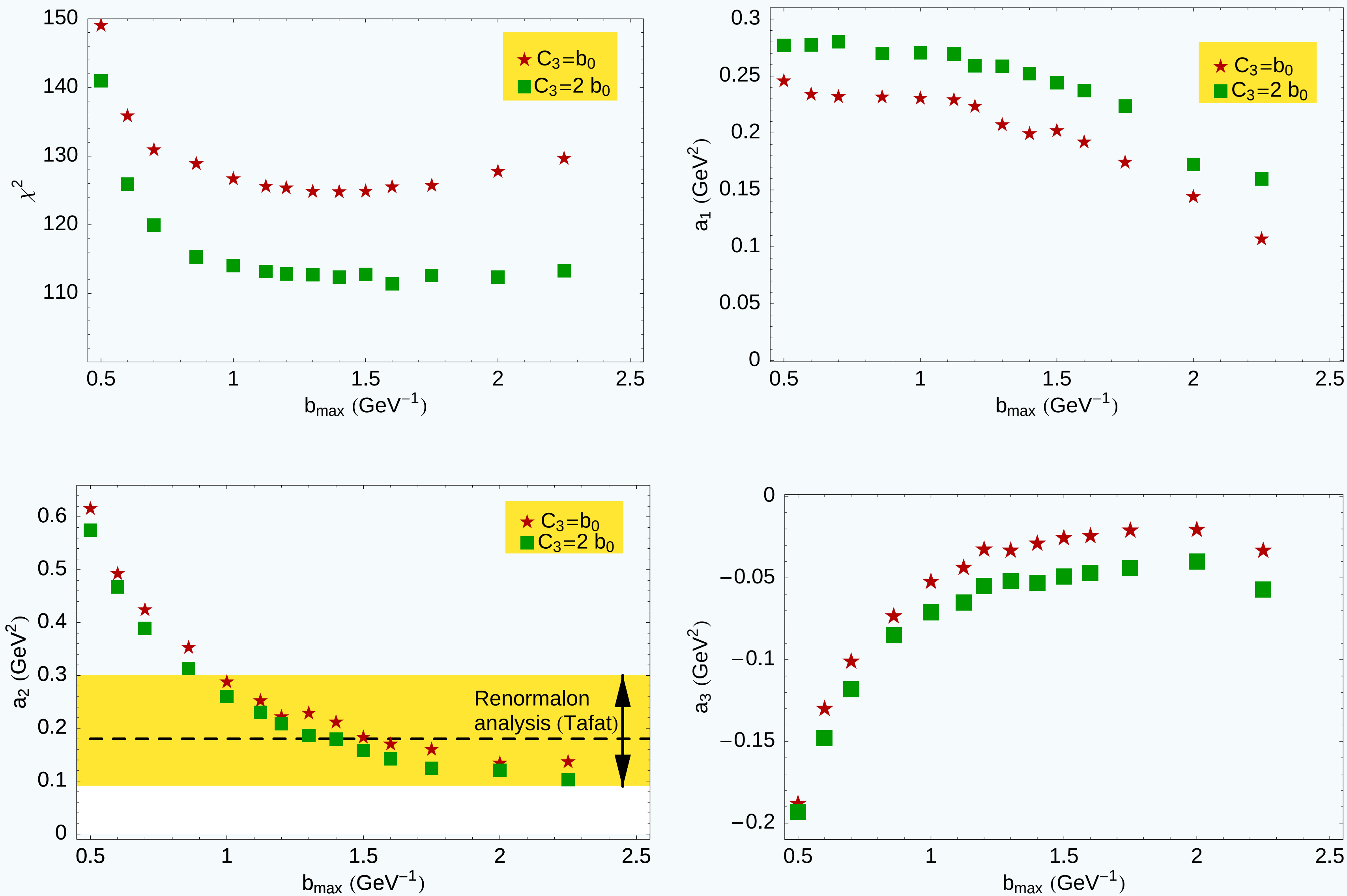
where $\beta = 0$ (Gaussian form) or free; a_3(here)=$g_1 g_3$ (BLNY)

Scan over $b_{max} = 0.5 - 2.5$ GeV^{-1}

Summary of the results

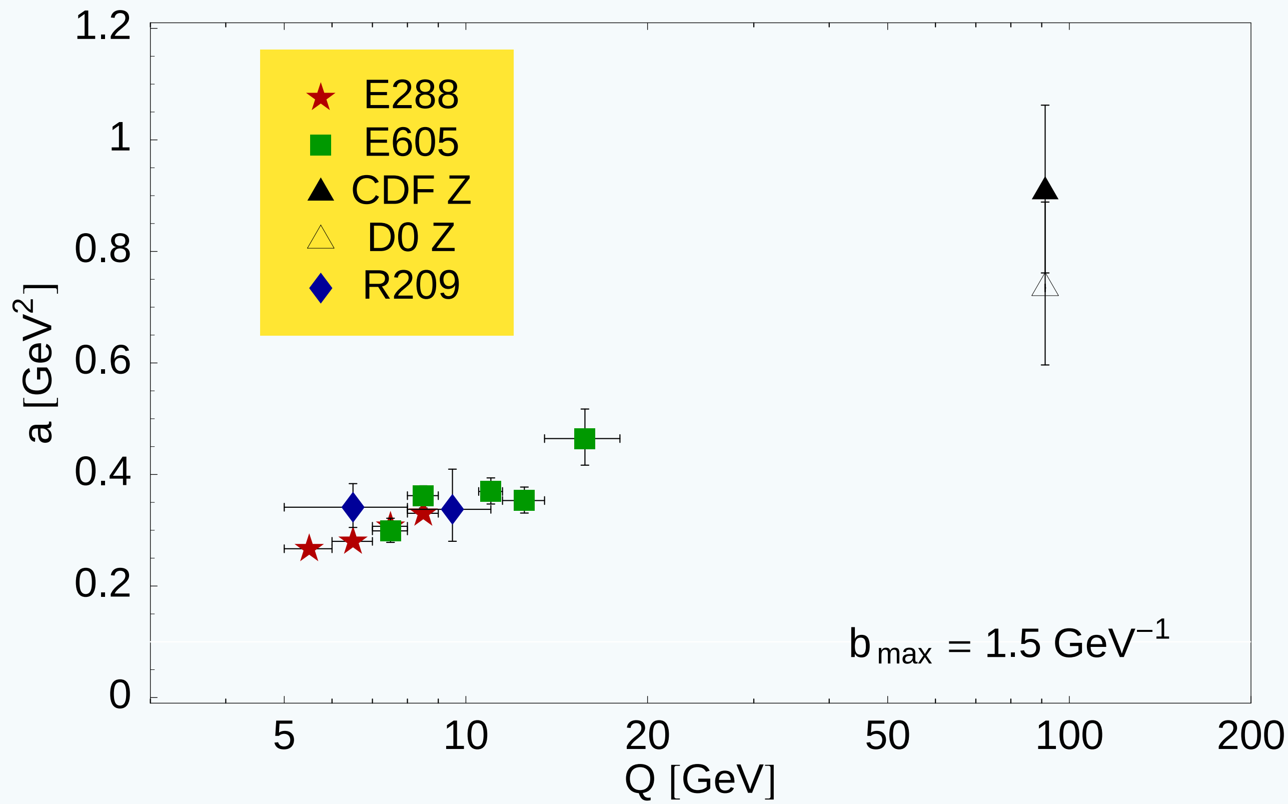
- Increasing b_{max} up to $1 - 1.5$ GeV^{-1} improves the quality of the fit
  - χ^2 and $|\mathcal{S}_{NP}(b,Q)|$ decrease
  - Best-fit $|a_3| \approx 0$
  - Best-fit $\beta = -0.2\ (+0.3)$ in Drell-Yan (Z) experiments; correlated with normalizations of DY data; $\beta = 0$ in the next slides
- The preferred $\mathcal{S}_{NP}(b,Q)$ is close to a two-parameter Gaussian form, $\mathcal{S}_{NP}(b,Q) \approx [a_1 + a_2 \ln(Q/3.2)]\, b^2$, with a_2 in excellent agreement with lattice QCD
- Small, but non-zero, a_3 and β are needed because of high accuracy of E288 and E605 data

Modified b_* prescription: scan over b_{max}



Best fit: $b_{max} = 1.5$ GeV^{-1}

Nonperturbative smearing a: independent scans of 5 experiments



$a_3 \approx 0$, $\beta \approx 0$

$\mathcal{S}_{NP}(b) \approx a(Q)b^2$,

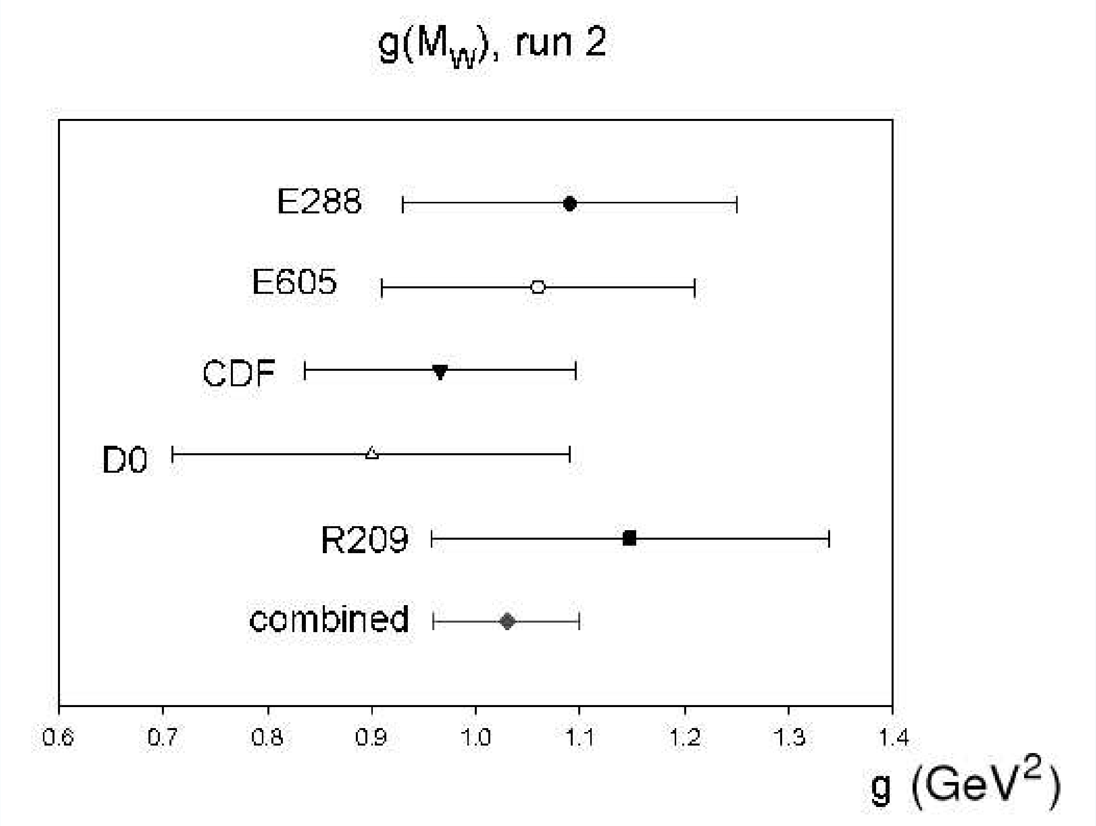
with
$a \sim \langle k_T^2 \rangle / 2$

- The best-fit $a(Q)$ shows quasi-linear dependence on $\ln(Q)$

- Its energy derivative, $a_2 = da/d(\ln Q) \sim 0.18$ GeV^2, agrees well with the lattice QCD estimate, $(a_2)_{lattice} = 0.19^{+0.11}_{-0.1}$ GeV^2

$a(M_W)$: constraints from individual experiments

- Obtained using a Lagrange multiplier method

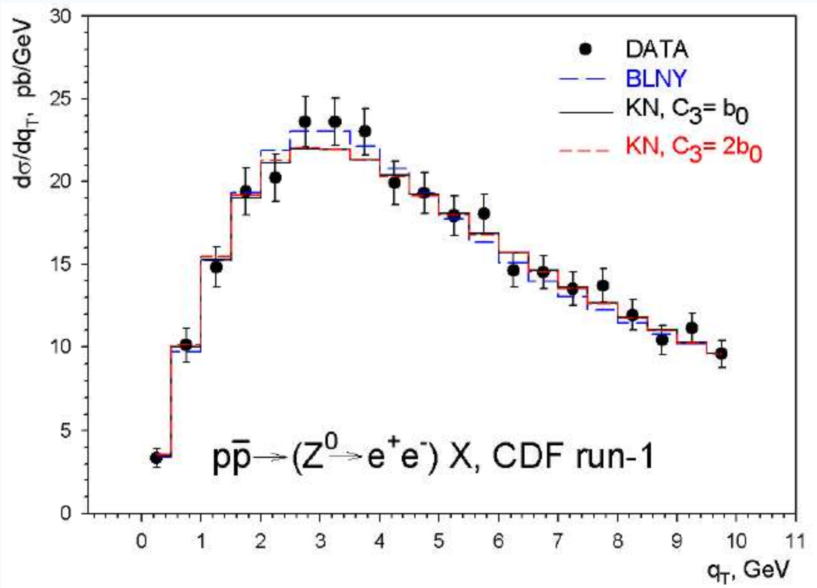


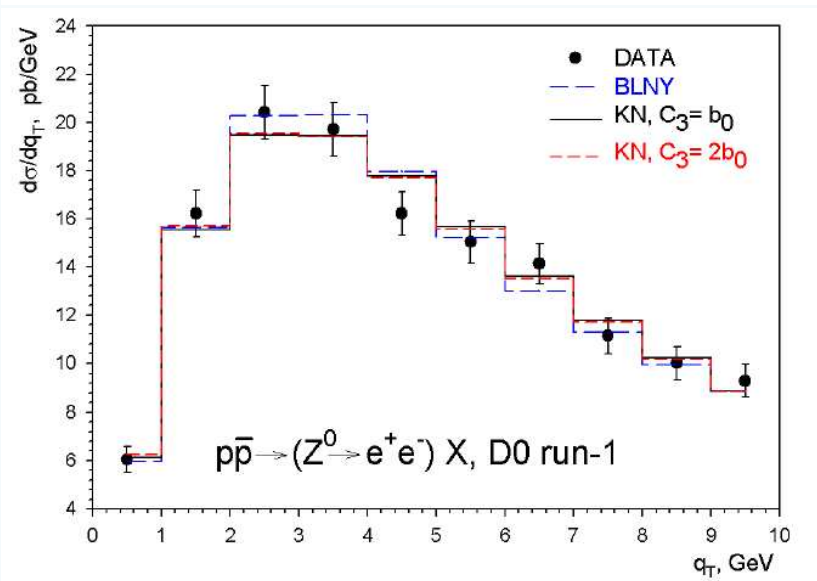
- Errors are for $\delta\chi^2_{tot} = 1$

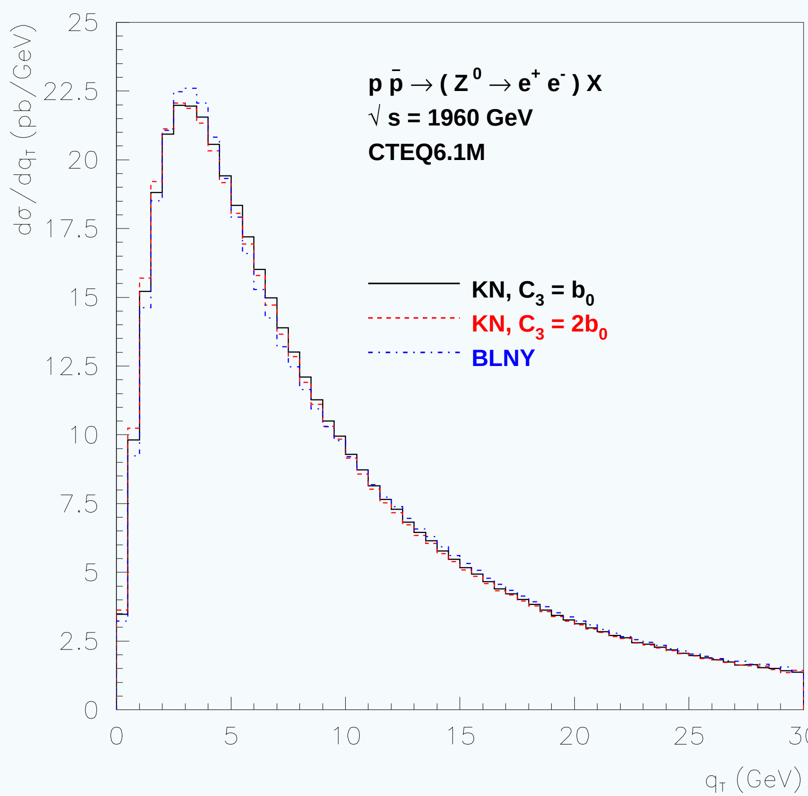
A preliminary fit for $b_{max} = 1.2$ GeV^{-1}: $g(Q) = a(Q)$

All data sets agree within errors; constraints from low-Q DY and Z Run-1 data are comparable

Z boson production in the Tevatron Runs 1 and 2







Remaining theory uncertainties may exceed experimental errors and are not fully understood

- Low-Q DY process
  - substantial dependence on factorization scales (e.g., C_3/b)
  - poor large-p_T matching
  - correlation between $\mathcal{S}_{NP}(b,Q)$ and X-sec. normalizations
    - correlations between $\mathcal{S}_{NP}(b,Q)$ and PDF's
- rapidity dependence, especially at $x < 10^{-2}$

Further improvements in accuracy may require

- NNLO resummed corrections
- simultaneous fit of $\mathcal{S}_{NP}(b,Q)$ and PDF's
  - a "proof-of-principle" fitting package is finished in CTEQ

Conclusions

- Modifications in b_* prescription lead to better agreement with the data
- High quality of the obtained global q_T fits supports universality of k_T-dependent factorization in Drell-Yan-like processes
- Combination of 5 Drell-Yan and Tevatron experiments places stronger constraints on $\mathcal{S}_{NP}(b, M_Z)$ than Run-1 Z boson production alone
- For $b_{max} \sim 1 - 1.5$ GeV^{-1}, the data prefer a nearly Gaussian $\mathcal{S}_{NP}(b, Q)$ with quasi-linear universal dependence on $\ln Q$ ($a_3 \approx 0$)
- The best-fit $a_2 \equiv d\mathcal{S}_{NP}(b, Q)/d(\ln Q)$ agrees well with the renormalon analysis & lattice QCD
- Experimental uncertainties in $\mathcal{S}_{NP}(b, Q)$ are estimated by applying Lagrange multiplier and Hessian matrix methods
- $\mathcal{S}_{NP}(b, Q = M_W) \approx (0.85 \pm 0.09)b^2$ for $b_{max} = 1.5$ GeV^{-1} in Run-2

Backup slides

Charge lepton asymmetry

$$A_{ch}(y_e) \equiv \frac{\frac{d\sigma^{W^+}}{dy_e} - \frac{d\sigma^{W^-}}{dy_e}}{\frac{d\sigma^{W^+}}{dy_e} + \frac{d\sigma^{W^-}}{dy_e}}$$

- related to the boson Born-level asymmetry (y_W=rapidity of W)

$$A_{ch}(y_W) \stackrel{y_W \to y_{max}}{\longrightarrow} \frac{r(x_b) - r(x_a)}{r(x_b) + r(x_a)},\ r(x) \equiv \frac{d(x, M_W)}{u(x, M_W)}$$

- constrains the PDF ratio $d(x, M_W)/u(x, M_W)$ at $x \to 1$
- In experimental analyses, a selection cut $p_{Te} > p_{Te}^{min}$ is imposed

The resummed cross section in theory

$$\left.\frac{d\sigma_{AB\to VX}}{dQ^2 dy dq_T^2}\right|_{q_T^2\ll Q^2} = \sum_{a,b=g,\overset{(-)}{u},\overset{(-)}{d},\dots} \int_0^\infty \frac{bdb}{2\pi} J_0(q_T b)\widetilde{W}_{ab}(b,Q,x_A,x_B)$$

$$\begin{aligned}\widetilde{W}_{ab}(b,Q,x_A,x_B) &= |\mathcal{H}_{ab}|^2\, e^{-\mathcal{S}(b,Q)}\overline{\mathcal{P}}_a(x_A,b)\overline{\mathcal{P}}_b(x_B,b)\\ &= \widetilde{W}_{LP}(b,Q,x_A,x_B)\otimes\widetilde{W}_{PS}(b,Q,x_A,x_B)\end{aligned}$$

$S(b,Q), \overline{\mathcal{P}}_a(x,b)$ are universal in Drell-Yan-like processes

Leading-power (LP) terms: do not vanish at $b\to 0$

$$\widetilde{W}_{LP}(b,Q) = \sum_{k=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^k \sum_{m=0}^{2k} w_{km}\ln^m(Qb)$$

Power-suppressed (PS) terms are proportional to even powers of b

(Korchemsky, Sterman; Tafat)

$$\widetilde{W}_{PS}(b,Q)\approx \exp\left[-\sum_{p=1}^{\infty} b^{2p} f_p(\ln Q)\right];\quad f_p\sim\Lambda_{QCD}^{2p}$$

The resummed cross section in a global fit

$$\widetilde{W}_{ab}(b,Q) \equiv \widetilde{W}_{pert}(b,Q)e^{-\mathcal{S}_{NP}(b,Q)},$$

where

- at $b \lesssim 1$ GeV^{-1},

$$\widetilde{W}_{pert}(b,Q) = \sum_{k=0}^{N}\left(\frac{\alpha_s}{\pi}\right)^k \sum_{m=0}^{2k} w_{km}\ln^m(Qb)$$

- $\widetilde{W}_{pert}(b,Q)$ is continued in some fashion to $b > 1$ GeV^{-1};
- $e^{-\mathcal{S}_{NP}}$ is the universal effective nonperturbative exponent to be found from the fit:

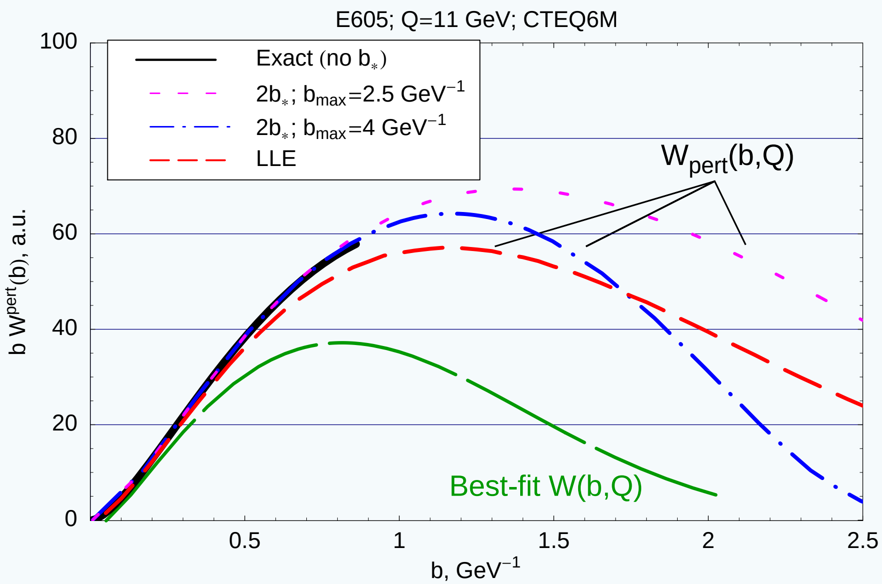
$$e^{-\mathcal{S}_{NP}(b,Q)} \equiv \frac{\widetilde{W}}{\widetilde{W}_{pert}} = \frac{\widetilde{W}_{LP}\otimes\widetilde{W}_{PS}}{\widetilde{W}_{pert}}$$

- if $\widetilde{W}_{pert} \approx \widetilde{W}_{LP}$ at all b, the fit should prefer

$$\mathcal{S}_{NP}(b,Q) \approx -\ln\left[\widetilde{W}_{PS}(b,Q)\right] \approx b^2 f(\ln Q) \oplus \text{ small corrections}$$

Choosing $b_{max} > 1.5$ GeV^{-1}

- Z production is described well for b_{max} up to $3-4$ GeV^{-1}
- Description of low-Q Drell-Yan data worsens for $b_{max} > 1.5$ GeV^{-1} because of rapid variations in $\widetilde{W}_{pert}(b,Q)$ at $b = 1.5-3$ GeV^{-1}

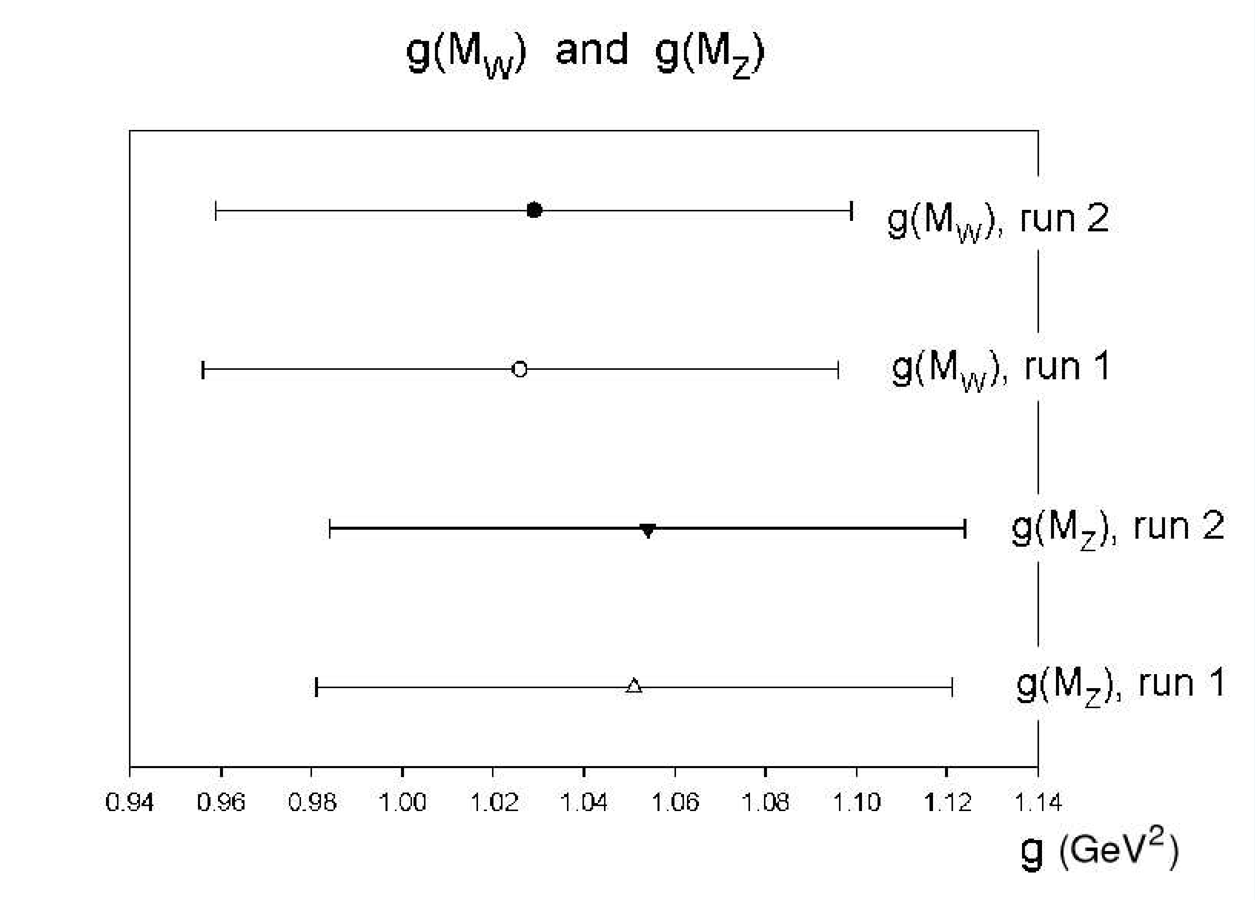


- The variations reflect absence of important higher-order logs $\sum \alpha_s^k \sum_m w_{km} \ln^m(Qb_{(*)})$
- are not easily compensated by adjustments in $\mathcal{S}_{NP}(b,Q)$

- $b_{max} \sim 1-1.5$ GeV^{-1} is the optimal range

Experimental uncertainties: $a(Q)$ at $Q = M_W$ and $Q = M_Z$ for $b_{max} = 1.2$ GeV^{-1}

- Obtained using a Lagrange multiplier method



- Errors are for $\delta\chi^2_{tot} = 1$

A preliminary fit: $g(Q) = a(Q)$

- Translates into a variation $\approx \pm 50$ MeV in the peak of $d\sigma(W)/dq_T$