Bottom Fragmentation in Higgs and Top Decay

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CERN TH

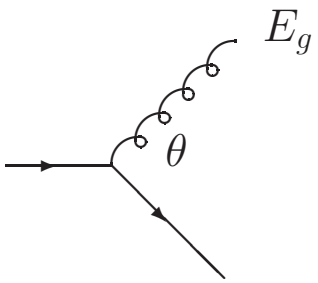
1. $H \to b\bar{b}$ **at NLO**
2. **Collinear and soft resummation**
3. **Bottom quark spectrum in Higgs and top decay**
4. B**-hadron production**
5. **Conclusions**

G.C. and A.D. Mitov, NPB 623 (2002) 247;
M. Cacciari, G.C. and A.D. Mitov, JHEP 0212 (2002) 015;
G.C., Nucl. Phys. B705 (2005) 363

For Higgs searches at Tevatron and LHC, precise QCD calculations are necessary

Fixed-order calculations (NLO) are reliable to predict inclusive observables

Differential distributions present large terms $\alpha_S^n L^m$ corresponding to collinear $(\theta \to 0)$ or soft $(E_g \to 0)$ parton radiation, which need to be resummed



Recent calculation implemented soft and collinear resummation in $H \to b\bar{b}$ processes

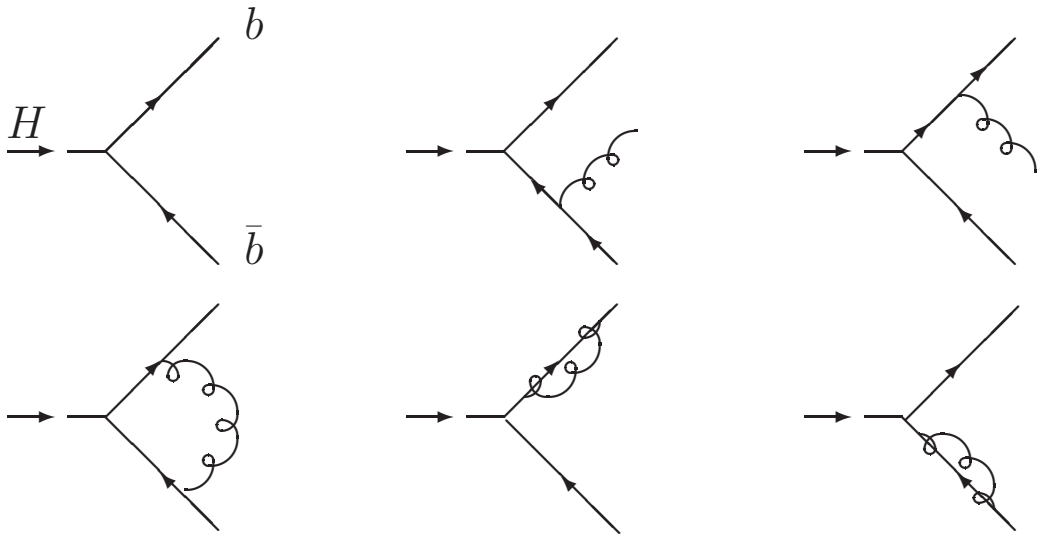
At the Tevatron ($m_H \leq 130$ GeV), the favourite Higgs discovery channel is: $p\bar{p} \to VH$, $H \to b\bar{b}$, $V \to \ell_1\ell_2$

At the LHC, larger QCD backgrounds

$H \to b\bar{b}$ is still relevant for $m_H \leq 140$ GeV and $pp \to t\bar{t}H$, $pp \to WH$, vector boson fusion $W^+W^-(ZZ) \to H$

Bottom quark production in Higgs decay at NLO

$$H(q) \to b(p_b)\bar{b}(p_{\bar{b}})(g(p_g)) \qquad x_b = \frac{2p_b \cdot q}{q^2} = \frac{2E_b}{m_H}$$



$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx_b} = \delta(1-x_b) + \frac{\alpha_S(\mu)}{2\pi}\left[P_{qq}(x_b)\ln\frac{m_H^2}{m_b^2} + A(x_b)\right] + \mathcal{O}\left[\left(\frac{m_b}{m_H}\right)^p\right]$$

Altarelli–Parisi splitting function:

$$P_{qq}(x_b) = C_F\left[\frac{1+x_b^2}{(1-x_b)_+} + \frac{3}{2}\delta(1-x_b)\right] = C_F\left(\frac{1+x_b^2}{1-x_b}\right)_+$$

Plus prescription:

$$\int_0^1 dx_b \frac{f(x_b)}{(1-x_b)_+} = \int_0^1 dx_b \frac{f(x_b)-f(1)}{1-x_b}$$

$$\alpha_S(\mu)\ln\frac{m_H^2}{m_b^2} \simeq \mathcal{O}(1)$$

Perturbative fragmentation functions

B. Mele and P. Nason, NPB 361 (1991) 626

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx_b}(x_b, m_b \neq 0) = \frac{1}{\Gamma_0}\sum_i \int_{x_b}^1 \frac{dz}{z}\frac{d\hat{\Gamma}_i}{dz}(z, m_i = 0, \mu_F) D_i\left(\frac{x_b}{z}, \mu_F, m_b\right) + \mathcal{O}\left[\left(\frac{m_b}{m_H}\right)^p\right]$$

$D_i(x_b, \mu_F, m_b)$**: perturbative fragmentation function (PFF)**

$$\frac{1}{\Gamma_0}\frac{d\hat{\Gamma}_b}{dz} = \delta(1-z) + \frac{\alpha_S(\mu)}{2\pi}\left[P_{qq}(z)\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right) + \hat{A}(z)\right]$$

$$\begin{aligned}\hat{A}(z) &= C_F\left[\left(\frac{1+z^2}{1-z}\right)_+ \ln\frac{m_H^2}{\mu_F^2} + \left(\frac{2}{3}\pi^2 - \frac{5}{2}\right)\delta(1-z) + 5 - z - \frac{3}{2}\frac{z^2}{(1-z)_+}\right.\\ &- (1+z)[\ln(1-z) + 2\ln z] + 6\frac{\ln z}{(1-z)_+}\\ &\left. - 2\frac{\ln z}{1-z} + 2\left(\frac{\ln(1-z)}{1-z}\right)_+\right]\end{aligned}$$

$z \to 1$**: soft-gluon emission**

$\overline{\rm MS}$ **coefficient function:**

$$\left(\frac{1}{\Gamma_0}\frac{d\hat{\Gamma}_b}{dz}\right)^{\overline{\rm MS}} = \delta(1-z) + \frac{\alpha_S(\mu)}{2\pi}\hat{A}_1(z)$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx_b}(m_b) = \left(\frac{1}{\Gamma_0}\frac{d\hat{\Gamma}_b}{dx_b}(m_b = 0)\right)^{\overline{\rm MS}} \otimes D_b^{\overline{\rm MS}}(m_b)$$

DGLAP equations for PFF's:

$$\frac{d}{d\ln\mu_F^2} D_i(x_b,\mu_F,m_b) = \sum_j \int_{x_b}^1 \frac{dz}{z} P_{ij}\left(\frac{x_b}{z},\alpha_S(\mu_F)\right) D_j(z,\mu_F,m_b)$$

Initial condition $D(x_b,\mu_{0F})$**:**

$$D_b(x_b,\mu_{0F},m_b) = \delta(1-x_b) + \frac{\alpha_S(\mu_0)C_F}{2\pi}\left[\frac{1+x_b^2}{1-x_b}\left(\ln\frac{\mu_{0F}^2}{m_b^2} - 2\ln(1-x_b) - 1\right)\right]_+$$

$$D_N(\mu_F,m_b) = \int_0^1 dx\; x_b^{N-1} D(x_b,\mu_F,m_b)$$

$$\frac{dD_N(\mu_F,m_b)}{d\ln\mu_F^2} = \frac{\alpha_S(\mu_F)}{2\pi}\left[P_N^{(0)} + \frac{\alpha_S(\mu_F)}{2\pi}P_N^{(1)}\right] D_N(\mu_F,m_b)$$

$$\begin{aligned} D_N(\mu_F,m_b) &= D_N(\mu_{0F},m_b)\exp\left\{\frac{P_N^{(0)}}{2\pi b_0}\ln\frac{\alpha_S(\mu_{0F})}{\alpha_S(\mu_F)}\right. \\ &+ \left.\frac{\alpha_S(\mu_{0F})-\alpha_S(\mu_F)}{4\pi^2 b_0}\left[P_N^{(1)} - \frac{2\pi b_1}{b_0}P_N^{(0)}\right]\right\} \end{aligned}$$

$$\begin{aligned} D_N(\mu_F,m_b) &= D_N(\mu_{0F},m_b)\exp\left\{C_{1,0}\alpha_S(\mu_F) + C_{1,1}\alpha_S(\mu_F)\ln(\mu_F^2/\mu_{0F}^2)\ldots\right. \\ &+ \left. C_{n,n-1}\alpha_S^n(\mu_F)\ln^{n-1}(\mu_F^2/\mu_{0F}^2) + C_{n,n}\alpha_S^n(\mu_F)\ln^n(\mu_F^2/\mu_{0F}^2) + \ldots\right\} \end{aligned}$$

Resummation of leading logarithms $\alpha_S^n\ln^n(\mu_F^2/\mu_{0F}^2)$
and next-to-leading logarithms $\alpha_S^n\ln^{n-1}(\mu_F^2/\mu_{0F}^2)$

$\mu_{0F} \simeq m_b$ **and** $\mu_F \simeq m_H$

Resummation of NLL $\ln(m_H^2/m_b^2)$
(Collinear resummation)

Soft-gluon radiation

Region $x_b \to 1$ corresponds to soft-gluon radiation

Bottom quark spectrum presents terms behaving like $1/(1-x_b)_+$ or $[\ln(1-x_b)/(1-x_b)]_+$ for $x_b \to 1$

$$\frac{1}{(1-x_b)_+} \to \ln N \qquad \left[\frac{1}{1-x_b}\ln(1-x_b)\right]_+ \to \ln^2 N$$

$$\hat{\Gamma}_N(m_H,\mu,\mu_F) = 1+\frac{\alpha_S(\mu)C_F}{2\pi}\left[\ln^2 N+\left(\frac{3}{2}+2\gamma-2\ln\frac{m_H^2}{\mu_F^2}\right)\ln N + K(m_H,\mu_F)+\mathcal{O}\left(\frac{1}{N}\right)\right]$$

$z=1-x_b, \quad k^2=(p_b+p_g)^2(1-z)=2E_g^2(1-\cos\theta_{bg})\simeq E_g^2\sin^2\theta_{bg}$

$$\Delta_N = \exp\left\{\int_0^1 dz\frac{z^{N-1}-1}{1-z}\int_{\mu_F^2}^{m_H^2(1-z)}\left[\frac{dk^2}{k^2}A\left[\alpha_S(k^2)\right] + \frac{1}{2}B\left[\alpha_S\left(m_H^2(1-z)\right)\right]\right]\right\} = \exp\left[\ln N g_1+g_2\right]$$

S. Catani and L. Trentadue, NPB 327 (1989) 323

$$A(\alpha_S)=\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n A^{(n)} \;\;;\;\; B(\alpha_S)=\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n B^{(n)}$$

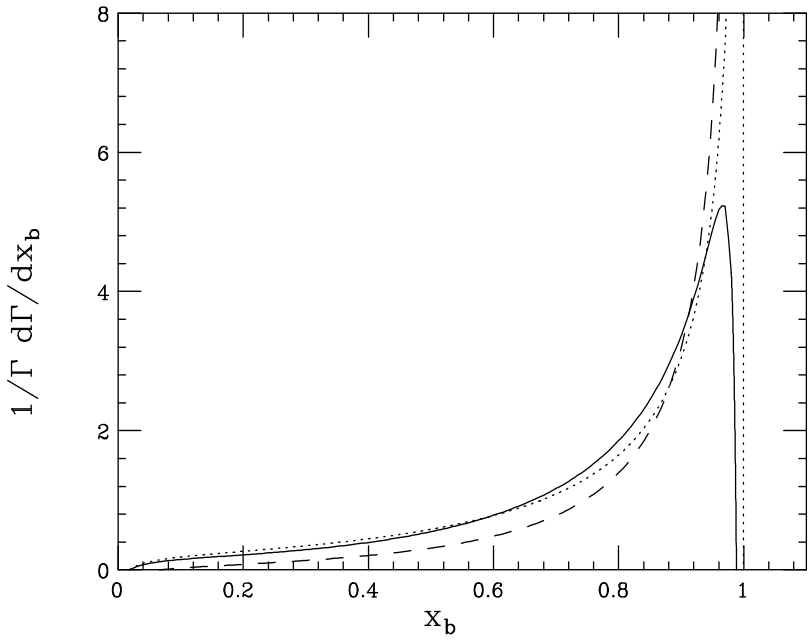
$g_1\ln N$ **resums LL** $A^{(1)}$ **:** $\alpha_S\ln^2 N, \alpha_S^2\ln^4 N\ldots\alpha_S^n\ln^{n+1} N$

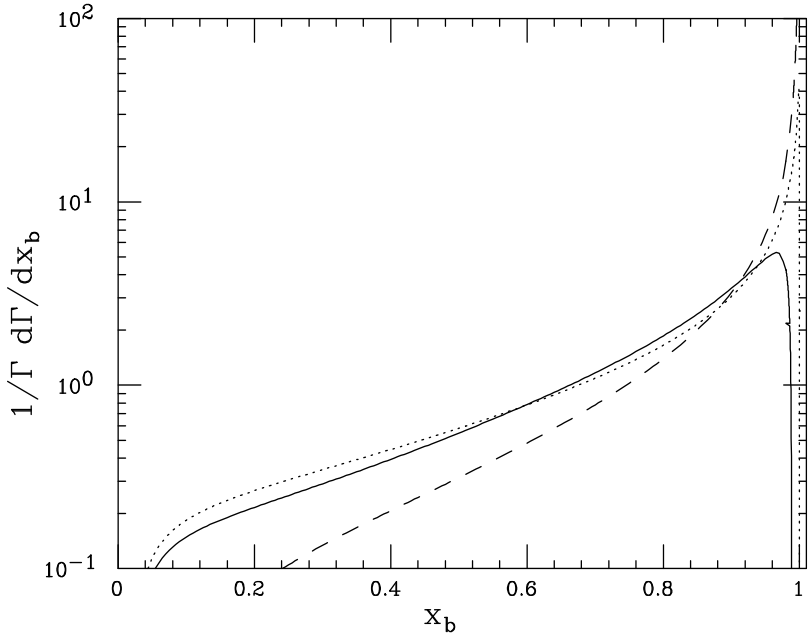
g_2 **resums NLL** $A^{(2)}, B^{(1)}$ **:** $\alpha_S\ln N, \alpha_S^2\ln^2 N\ldots\alpha_S^n\ln^n N$

b-quark spectrum in Higgs decay

$m_H = 120$ GeV $\quad m_b = 5$ GeV $\quad \Lambda_{\overline{\rm MS}} = 200$ MeV

$\mu = \mu_F = m_H \qquad \mu_0 = \mu_{0F} = m_b$



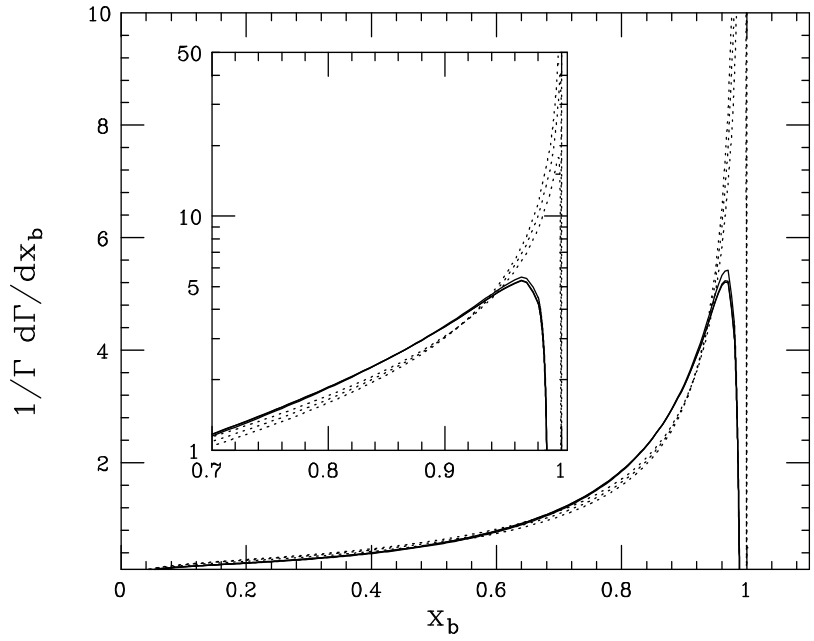


Solid: soft and collinear resummation Dots: only collinear resummation

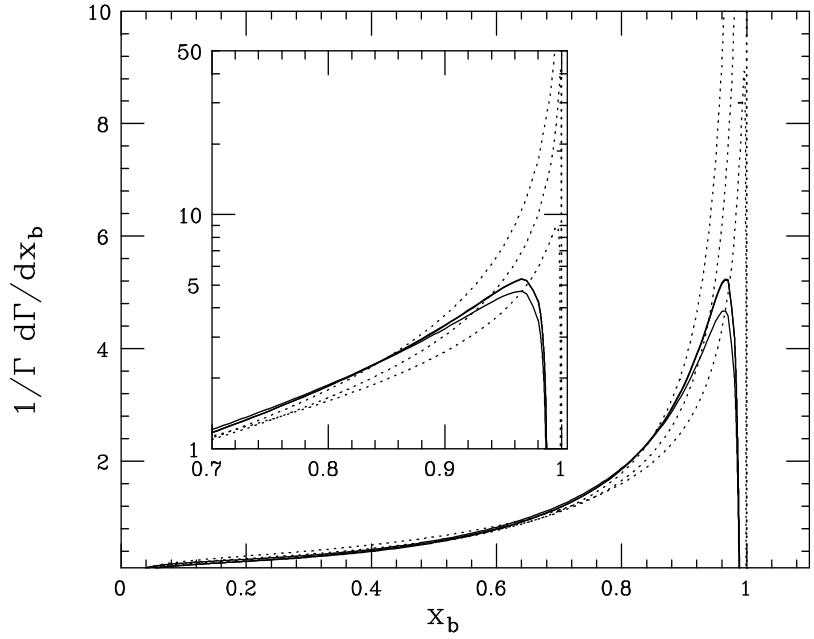
Dashes: massive NLO without resummation

Dependence on the factorization scales

Solid: collinear and soft resummation; dots: only collinear

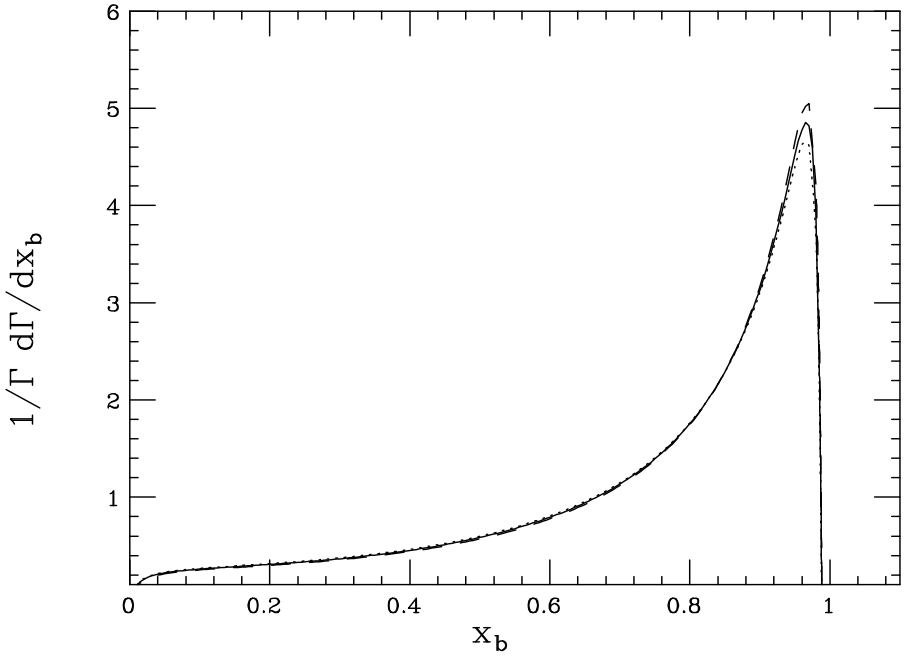


$\mu_F = m_H/2,\ m_H,\ 2m_H \qquad \mu_0 = \mu_{0F} = m_b,\ \mu = m_H$



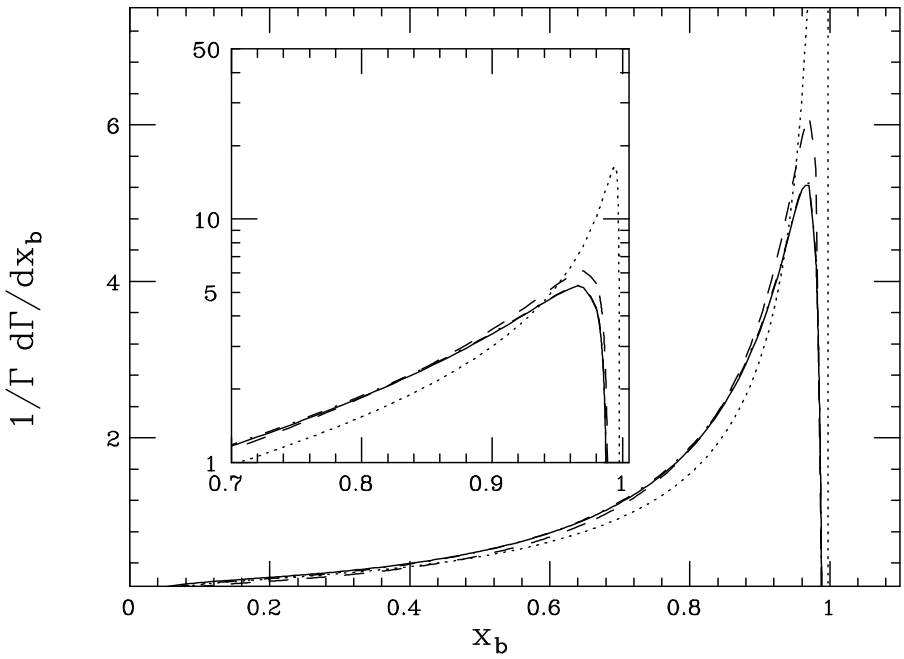
$\mu_{0F} = m_b/2,\ m_b,\ 2m_b \qquad \mu_0 = m_b,\ \mu = \mu_F = m_H$

Dependence on the Higgs mass m_H



Dashes: $m_H = 110$ GeV; solid: $m_H = 120$ GeV; dots: $m_H = 130$ GeV

Comparison of $e^+e^- \to b\bar{b}$, Higgs and top decay



Solid: Higgs decay; dots: top decay; dashes: e^+e^- at $\sqrt{s} = 91.2$ GeV; dot-dashes: e^+e^- at $\sqrt{s} = m_H = 120$ GeV

$e^+e^- \to b\bar{b}$: M. Cacciari and S. Catani, NPB 617 (2001) 167;

top decay: M. Cacciari, G.C. and A.D. Mitov, JHEP 0212 (2002) 015

Hadron-level results

$$\frac{d\Gamma_{\rm had}}{dx_B}(B) = \frac{d\Gamma_{\rm part}}{dx_b}(b) \otimes D_{np}(b \to B)$$

$d\Gamma_{\rm part}/dx_b$ **according to PFF approach**

Non-perturbative fragmentation functions:

Power law with two tunable parameters:

$$D_{np}(x;\alpha,\beta) = \frac{1}{B(\beta+1,\alpha+1)}(1-x)^\alpha x^\beta$$

Model of Kartvelishvili et al.:

$$D_{np}(x;\delta) = (1+\delta)(2+\delta)(1-x)x^\delta$$

Model of Peterson et al.:

$$D_{np}(x;\epsilon) = \frac{A}{x[1-1/x-\epsilon/(1-x)]^2}$$

Parameters α, β, δ, ϵ **from fits to** $e^+e^- \to b\bar{b}$ **data**

$$\frac{d\sigma_{\rm had}}{dx_B}(e^+e^- \to B) = \frac{d\sigma_{\rm part}}{dx_b}(e^+e^- \to b\bar{b}) \otimes D_{np}(b \to B)$$

$d\sigma_{\rm part}/dx_b$ **must be computed within the same framework as** $d\Gamma_{\rm part}/dx_b$

Results of fits of fragmentation models to e^+e^- data

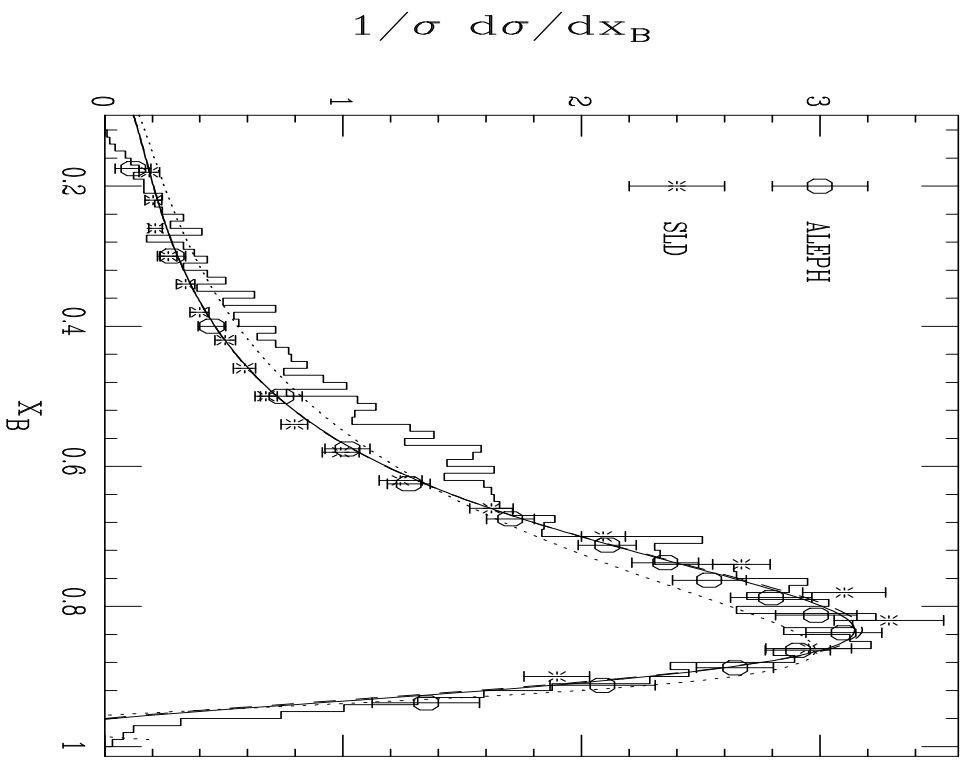
Preliminary: correlations between data points are neglected

ALEPH Collaboration, A. Heister et al., PLB 512 (2001) 30: only B mesons

SLD Collaboration, K. Abe et al., PRL 84 (2000) 4300: both b-flavoured mesons and baryons

$0.18 \lesssim x_B \lesssim 0.94$

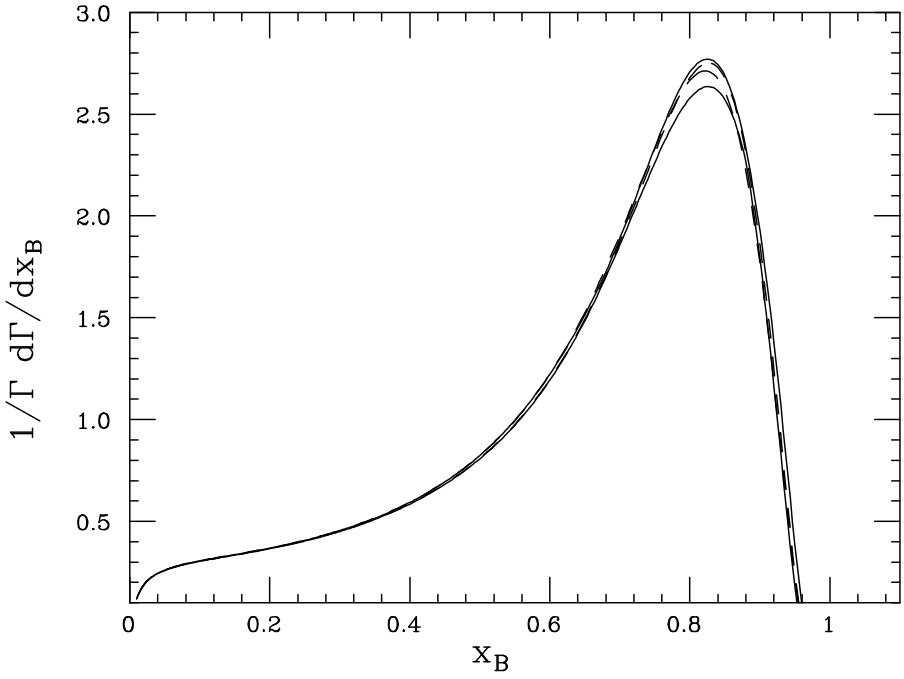
α	0.90 ± 0.15
β	16.23 ± 1.37
$\chi^2(\alpha, \beta)/\mathrm{dof}$	$33.42/32$
δ	17.07 ± 0.39
$\chi^2(\delta)/\mathrm{dof}$	$33.80/32$
ϵ	$(1.71 \pm 0.09) \times 10^{-3}$
$\chi^2(\epsilon)/\mathrm{dof}$	$166.36/32$



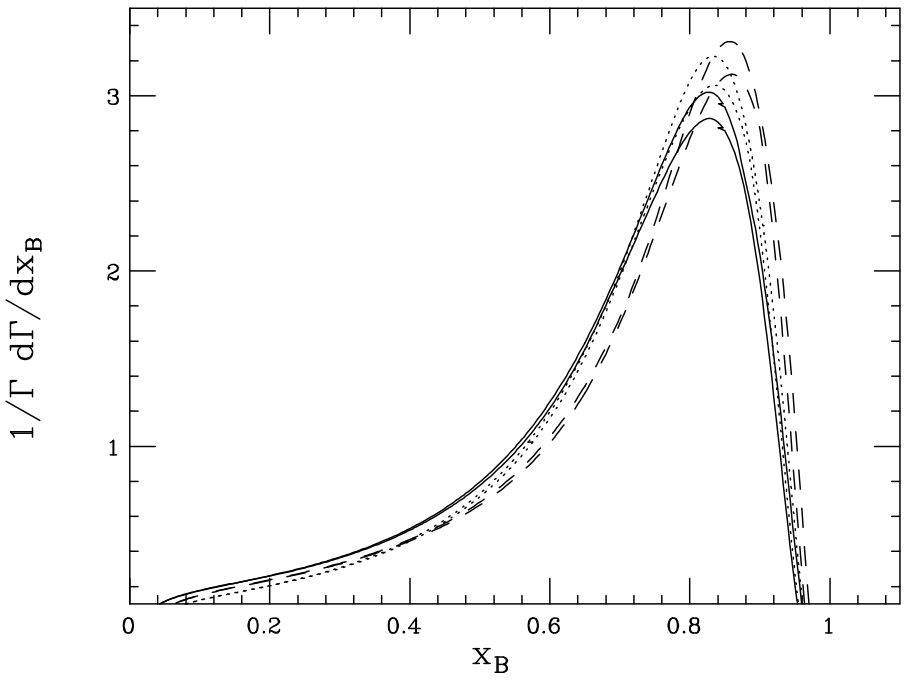
Solid: power law; dashes: Kartvelishvili; dots: Peterson

b-flavoured hadron spectrum in Higgs decay

Solid: power law; Dashes: Kartvelishvili



B spectrum in Higgs decay (solid), top decay (dashes) and e^+e^- annihilation (dots) at $\sqrt{s} = m_Z$



Fits in moment space

e^+e^- annihilation $\sigma_N^B = \sigma_N^b D_N^{np}$

σ_N^B **measured ;** σ_N^b **calculated ;** D_N^{np} **fitted**

Higgs and top decay: $\Gamma_N^B = \Gamma_N^b D_N^{np} = \Gamma_N^b \sigma_N^B / \sigma_N^b$

Fits to DELPHI data
(ICHEP 2002 Note, DELPHI 2002-069 CONF 603)

	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$
e^+e^- data σ_N^B	0.7153±0.0052	0.5401±0.0064	0.4236±0.0065	0.3406±0.0064
e^+e^- NLL σ_N^b	0.7801	0.6436	0.5479	0.4755
D_N^{np} [B]	0.9169	0.8392	0.7731	0.7163
t-decay NLL Γ_N^b	0.7884	0.6617	0.5737	0.5072
t-decay Γ_N^B	0.7228	0.5553	0.4435	0.3633
H-decay NLL Γ_N^b	0.7578	0.6162	0.5193	0.4473
H-decay Γ_N^B	0.6948	0.5171	0.4015	0.3204

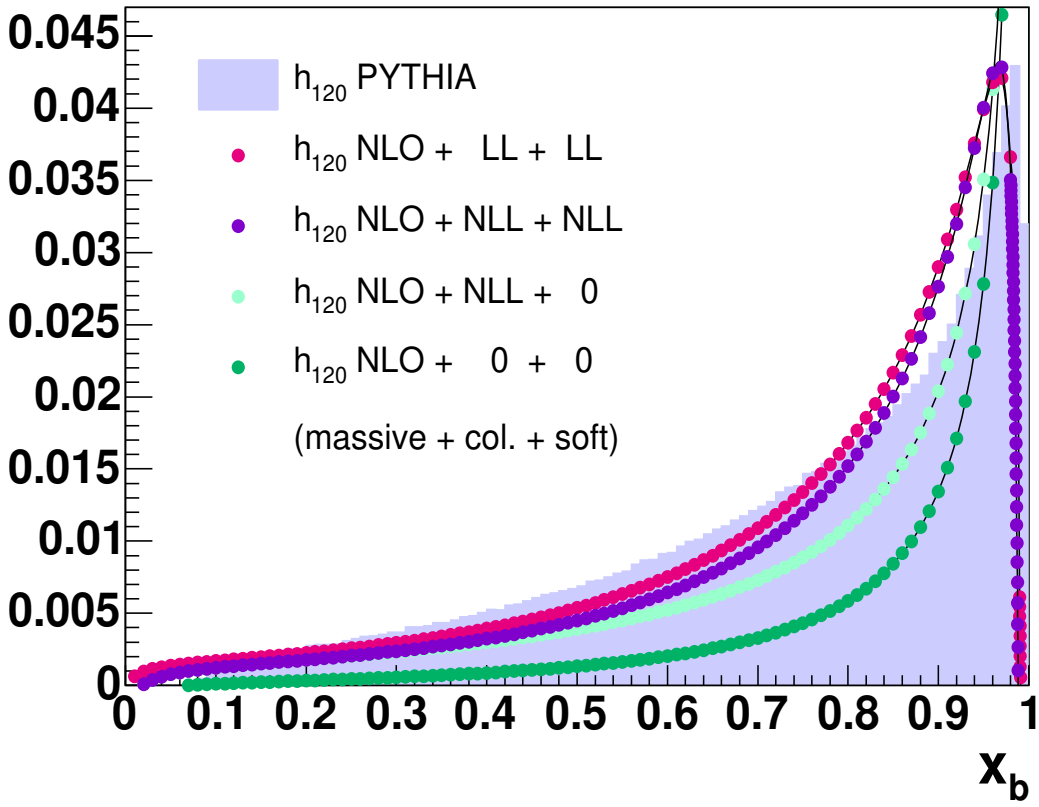
Comparison with Monte Carlo event generators (with V.Drollinger, CMS)

MC generators like HERWIG or PYTHIA are LO+LL programs, with some NLL terms and, after matrix-element corrections, hard real-gluon radiation

No matrix-element corrections for $H \to b\bar{b}$

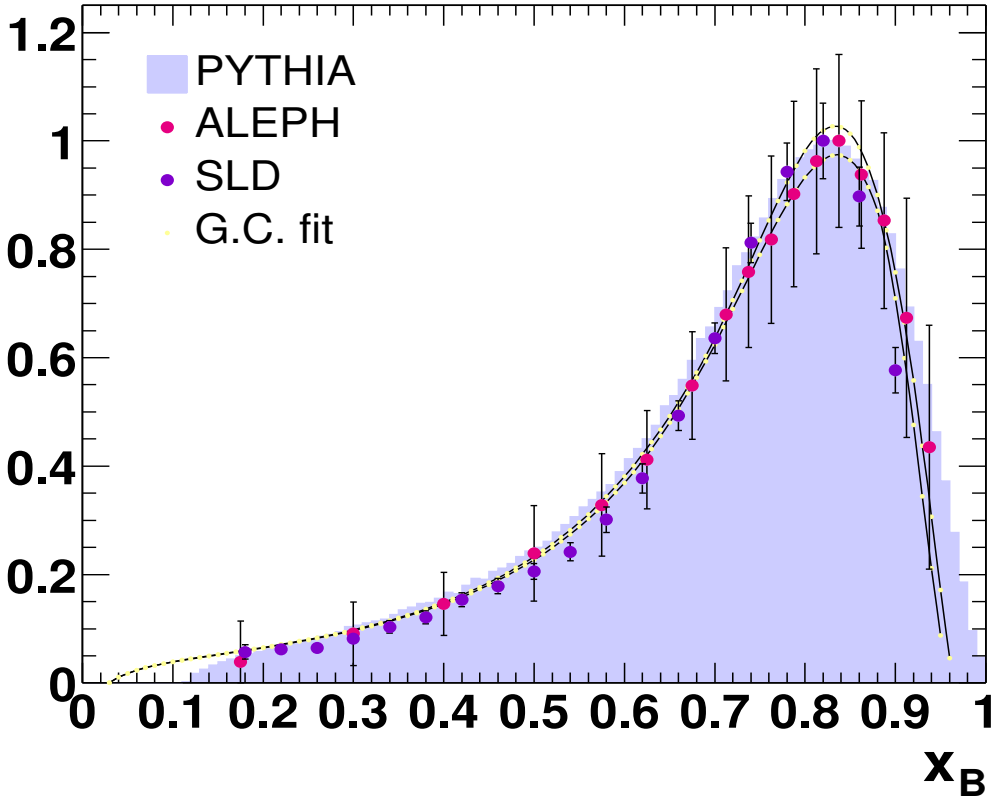
Input parameters need to be tuned: gluon effective mass, $\Lambda_{\rm QCD}$, cutoff Q_0 for parton shower evolution, hadronization parameters, etc.

Comparison with default PYTHIA



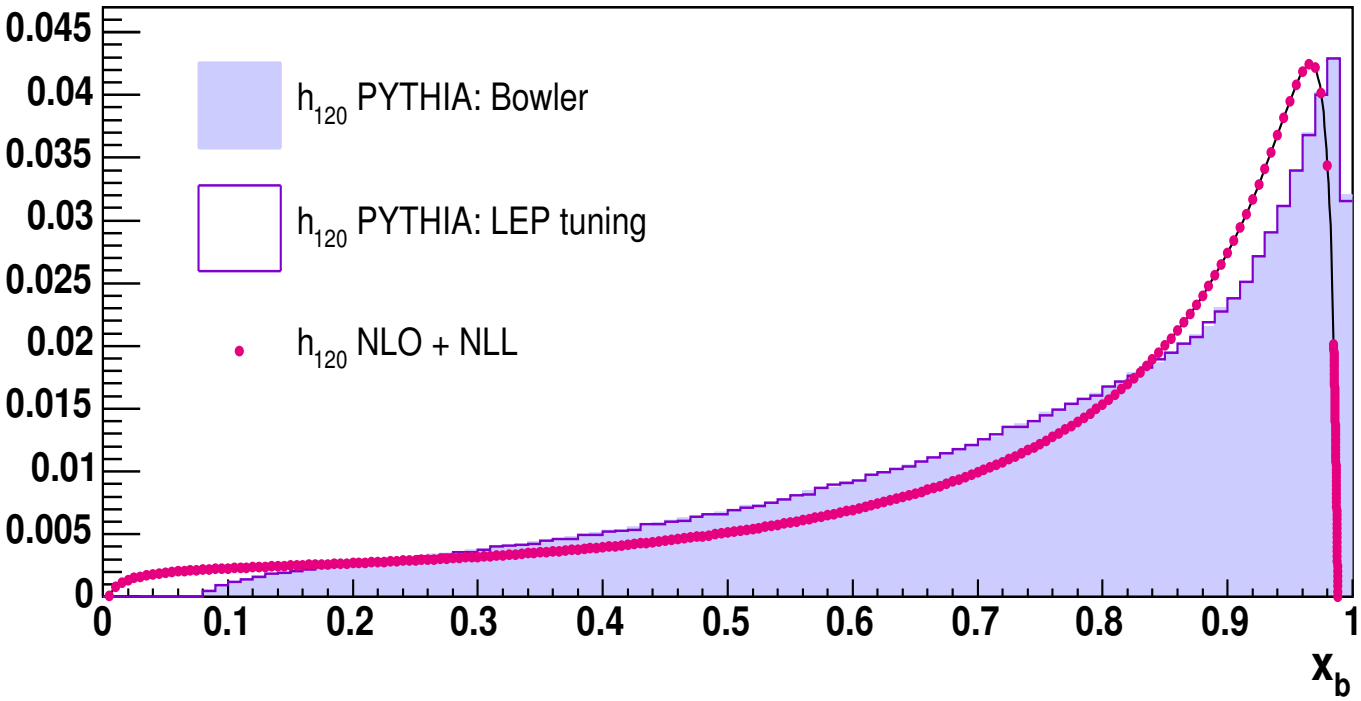
Tuning PYTHIA to ALEPH and SLD data

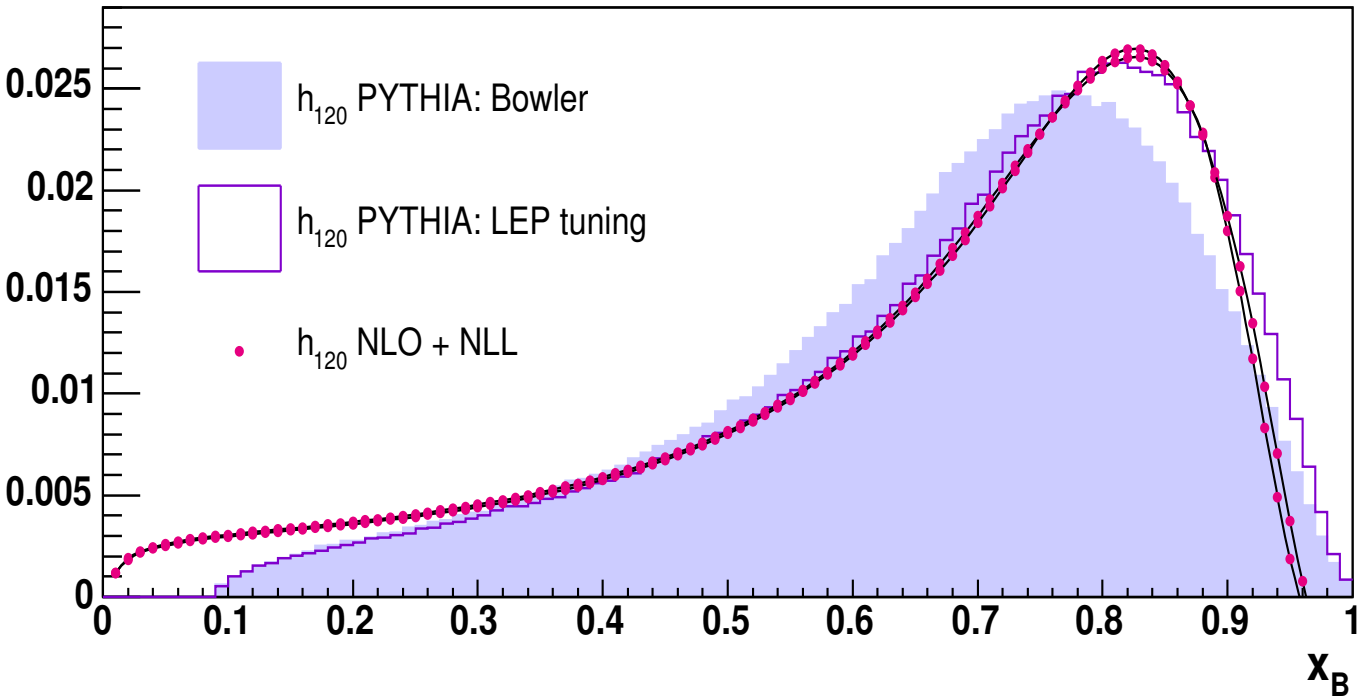
MSTJ(11)=4; PARJ(41)=0.43
PARJ(42)=0.63; PARJ(46)=0.75



Good agreement with NLO+NLL calculation

Using tuned PYTHIA to predict $H \to b\bar{b}$



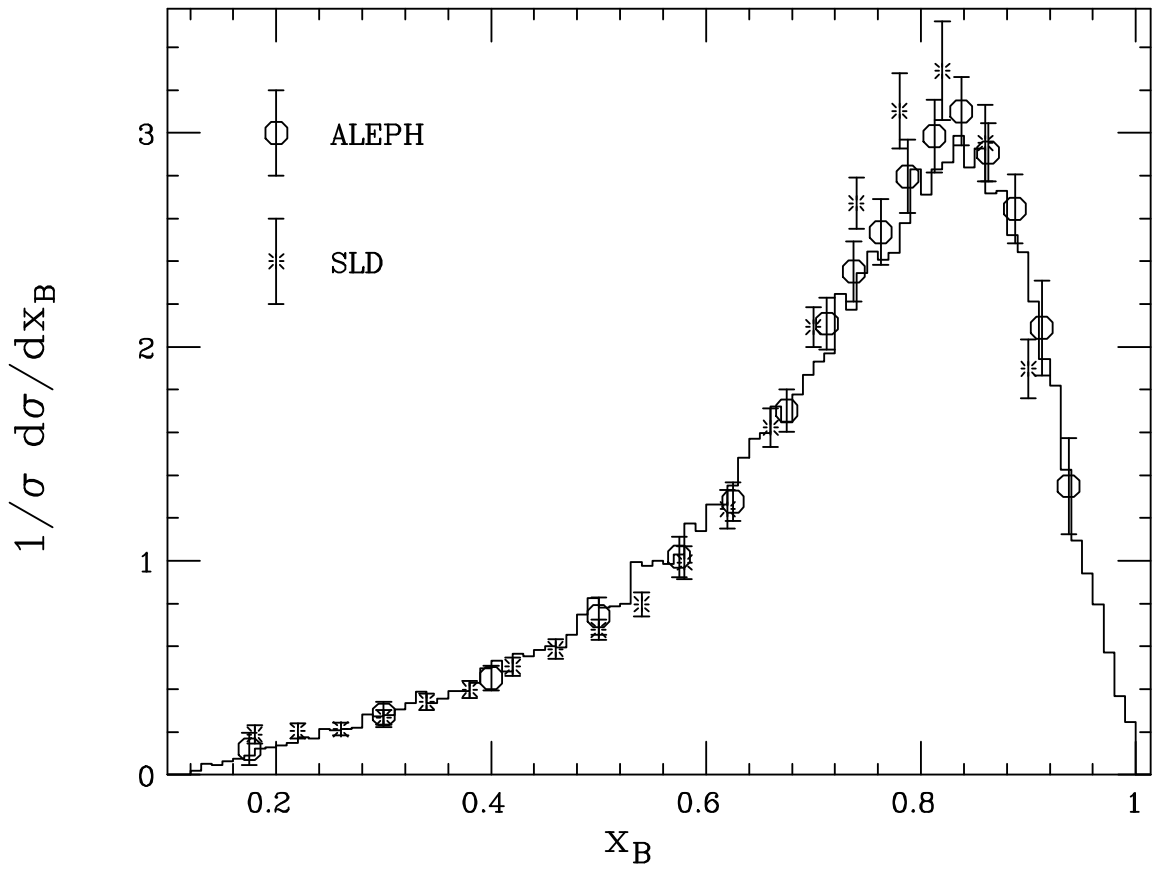


Better agreement at hadron level (x_B); still problems at parton level (x_b): tuning PYTHIA perturbative parameters?

Comparison with HERWIG

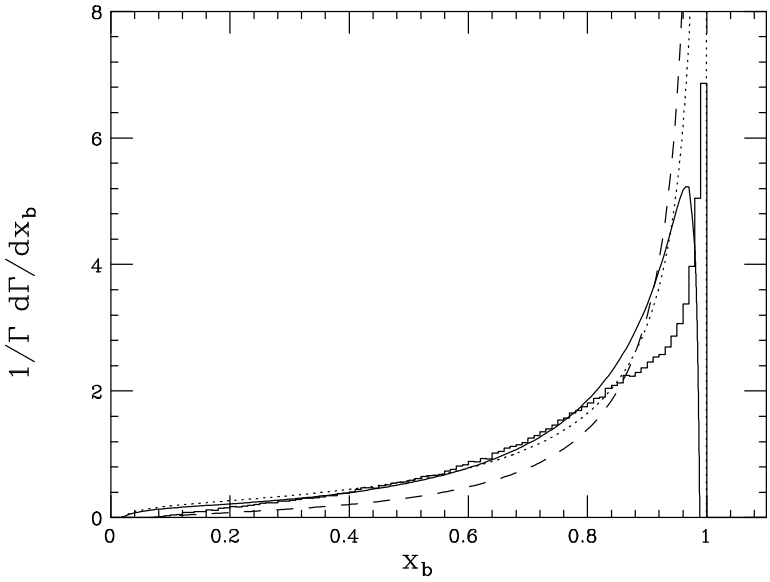
Most recent tuning: R. Hemingway, OPAL TN652 (2000)

Comparison with B data from ALEPH and SLD



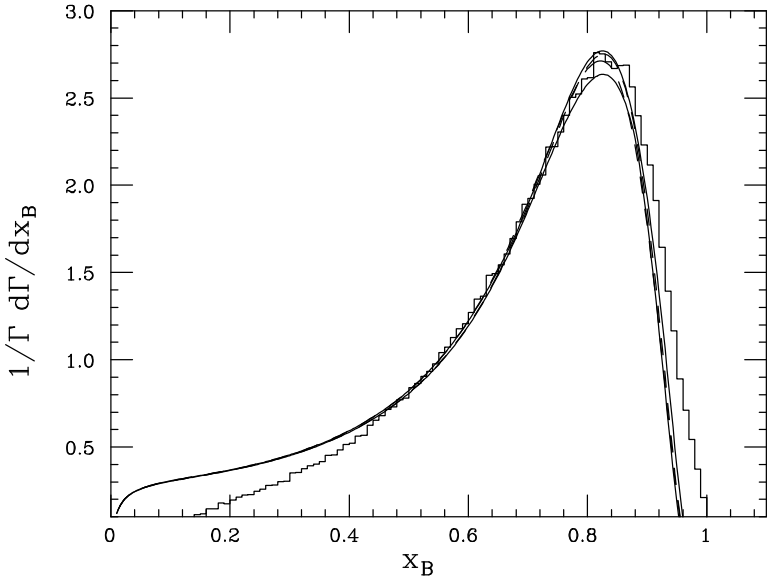
Comparison of HERWIG with NLO+NLL calculation on $H \to b\bar{b}$

Parton level:



Solid: NLL soft and collinear; dashes: only collinear; dots: NLO; histogram: HERWIG

Hadron level:



Solid lines: NLO+NLL+power law; histogram: HERWIG

Conclusions and outlook

NLO b spectrum in $H \to b\bar{b}$ presents large collinear- and soft-enhanced terms

Collinear and soft resummation

Big effect of resummation on b energy distribution

Fits of hadronization models to ALEPH and SLD data in x_B space and to DELPHI in N space

B-hadron spectrum in x_B and N spaces

Comparison of $e^+e^- \to b\bar{b}$, Higgs and top decay

In progress:

Application to Higgs and top physics at Tevatron and LHC

Comparison with Monte Carlo event generators

Matrix-element corrections to $H \to b\bar{b}$ in HERWIG