$b\bar{b}h$ at NNLO

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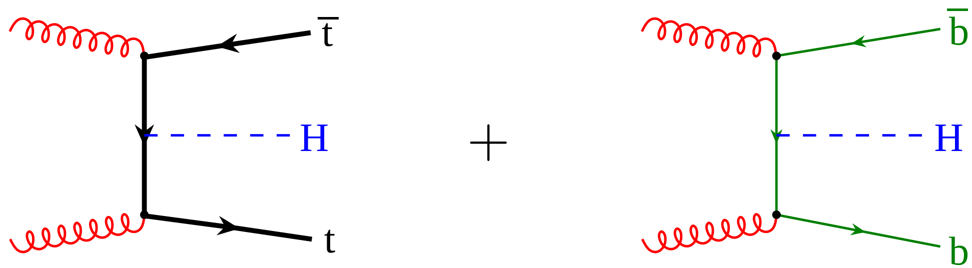
Les Houches, May 2005

$b\bar{b} \to H$ in SUSY

- modified Yukawa couplings in SUSY:

$$\frac{\lambda_b}{\lambda_t} = \frac{m_b}{m_t} \cdot \frac{v_u}{v_d} = \frac{m_b}{m_t} \cdot \tan\beta$$



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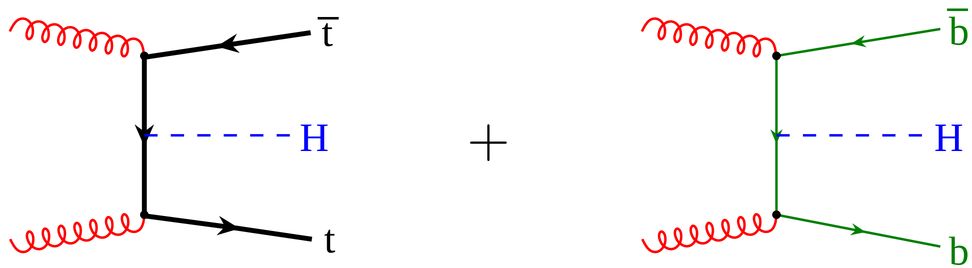

- collinear logarithms: $\sim \alpha_s \ln(m_b/M_H) \sim \alpha_s \ln(5/200)$

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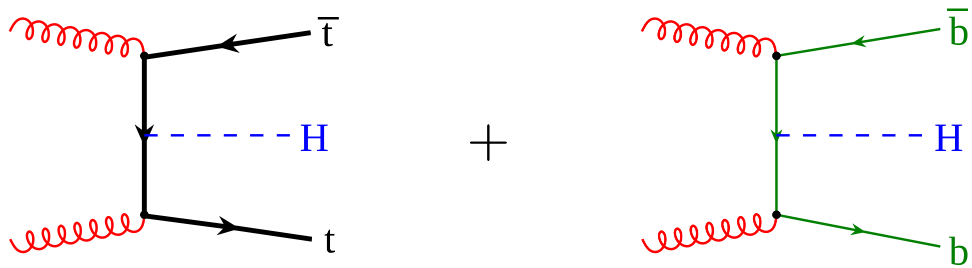

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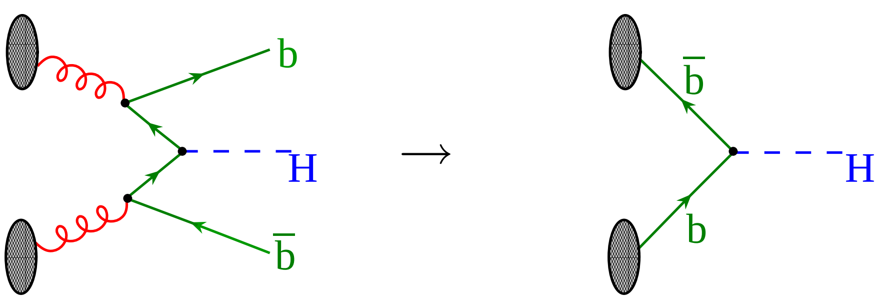
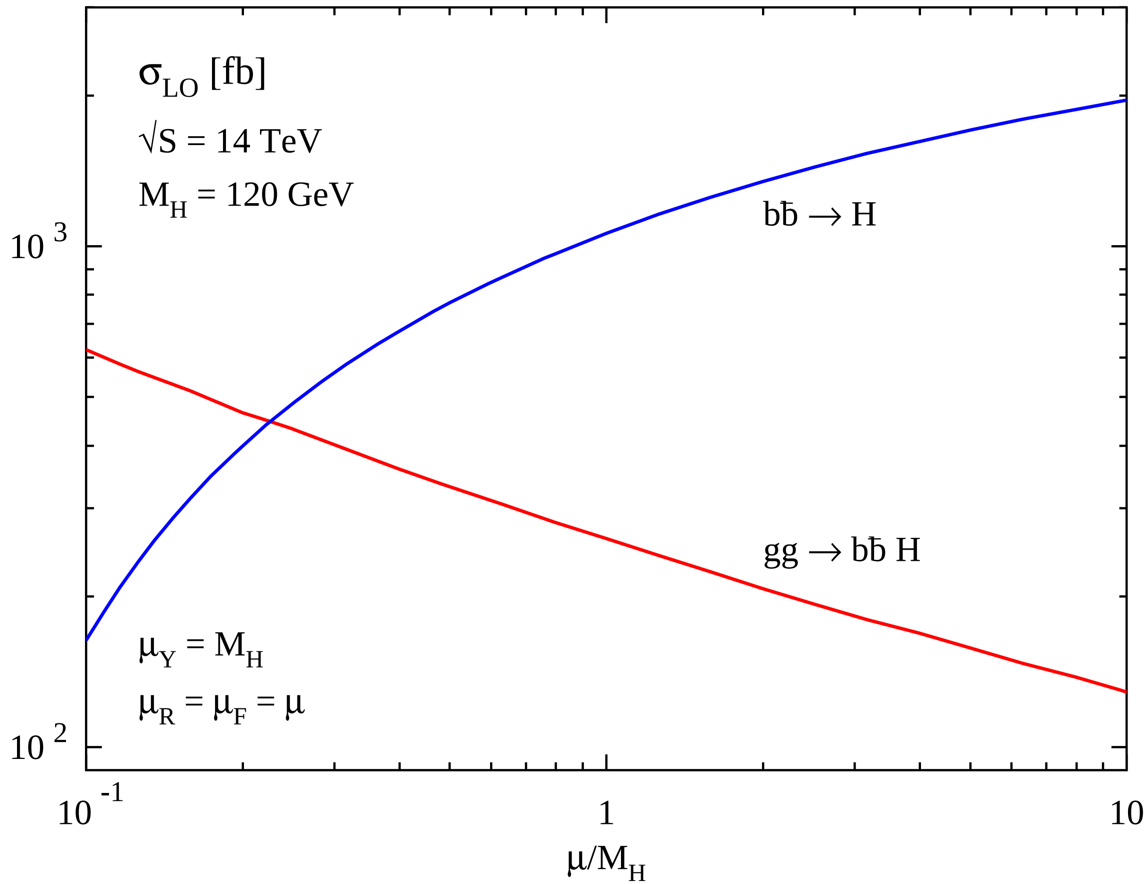
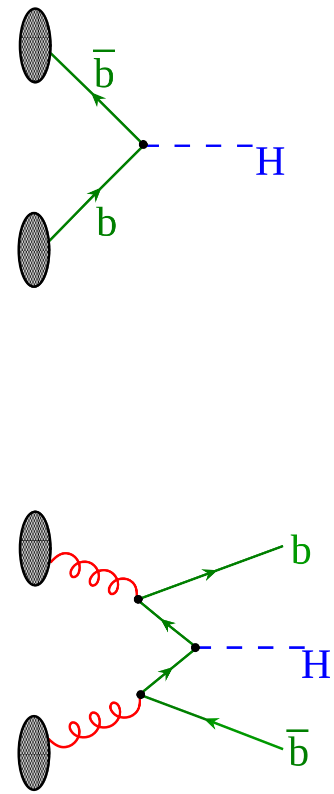


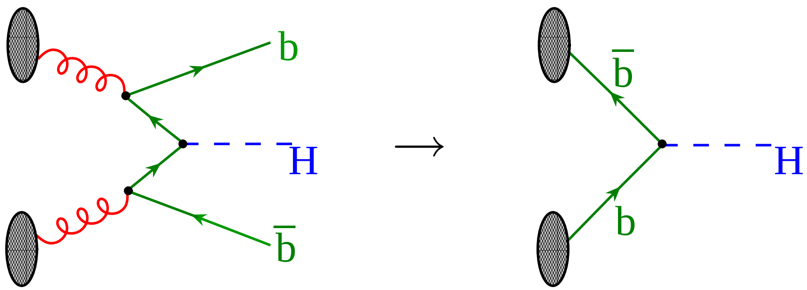
$b\bar{b} \to h$ vs. $gg \to b\bar{b}h$





[Krämer '04]

Higher orders: NLO



$$\sum_{n=2}^{\infty} (\alpha_s l_b)^n$$

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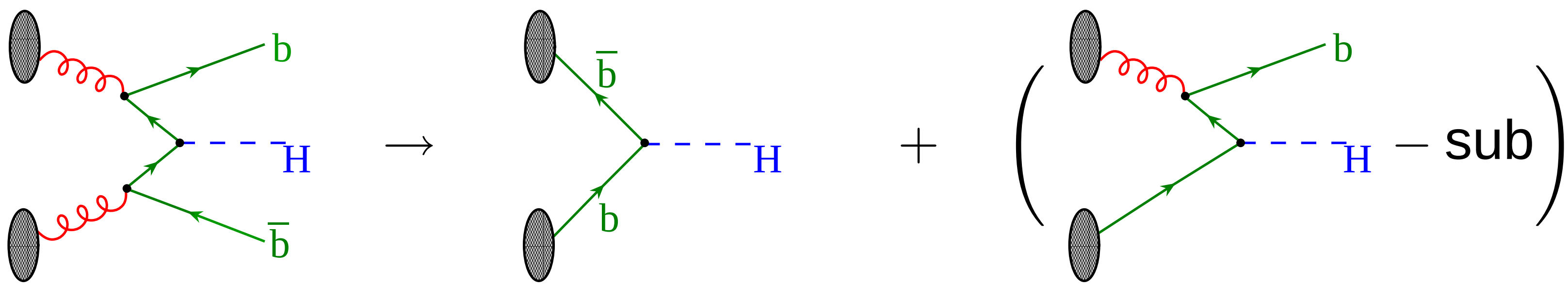

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$$\sum_{n=2}^{\infty}(\alpha_s l_b)^n \cdot \frac{1}{l_b}$$

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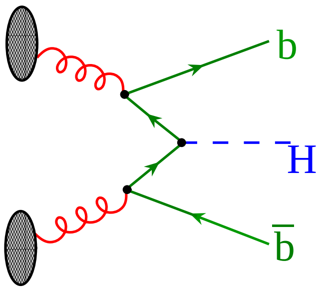

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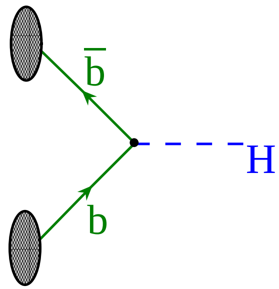

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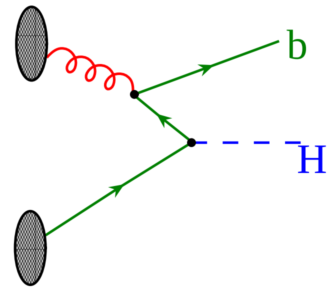

$- \text{sub} \Bigg)$

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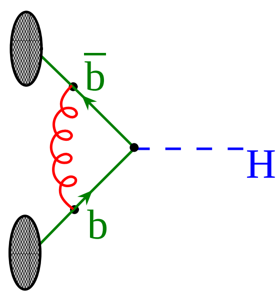
$+$



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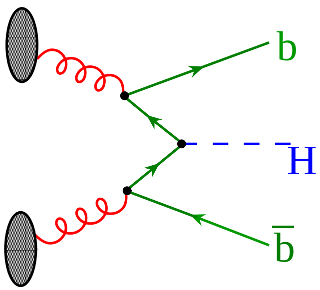

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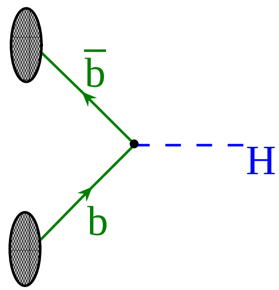

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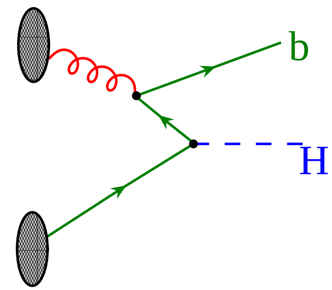

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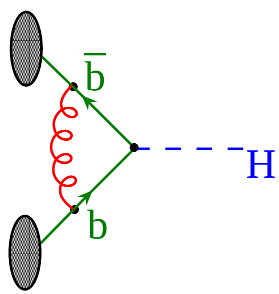

$$\sum_{n=2}^{\infty} (\alpha_s l_b)^n \cdot \alpha_s$$

NLO: $$\sigma(b\bar{b} \to H) = \sum_{n=0}^{\infty} (\alpha_s l_b)^n \; \alpha_s^2 \Big[\; c_{n0} \, l_b{}^2 \; + \; c_{n1} \, l_b \Big]$$

[Maltoni, Sullivan, Willenbrock ('03)]

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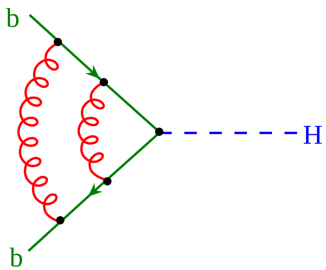

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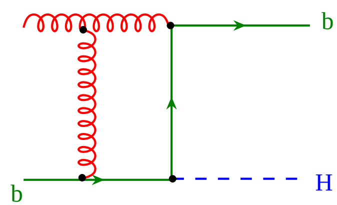

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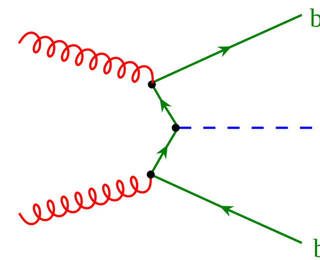

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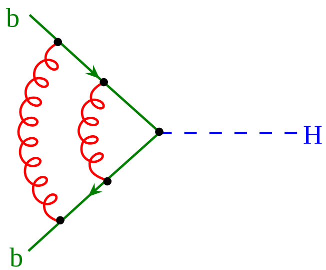

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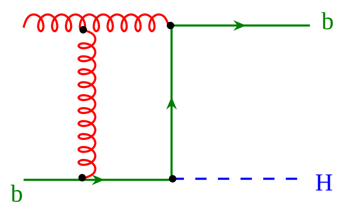

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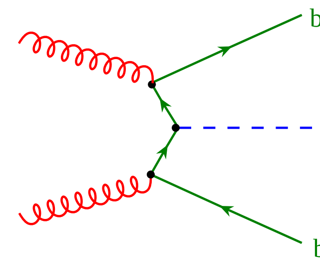

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[R.H., Kilgore ('03)]

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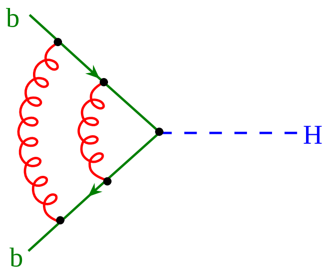

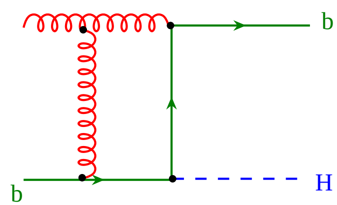

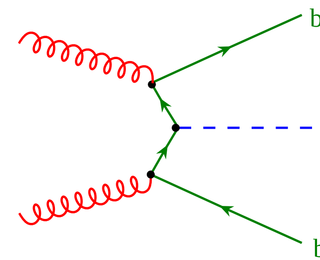

$$\sum_{n=2}^{\infty}(\alpha_s l_b)^n \cdot \alpha_s^2 \qquad \sum_{n=2}^{\infty}(\alpha_s l_b)^n \cdot \alpha_s \frac{1}{l_b} \qquad \sum_{n=2}^{\infty}(\alpha_s l_b)^n \cdot \frac{1}{{l_b}^2}$$

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[R.H., Kilgore ('03)]

higher orders: $$+\, d_{n3}\alpha_s^3 + d_{n4}\alpha_s^4 + \ldots \Big\}$$

Algorithms

- expansion + inversion for phase space integrals [R.H., Kilgore ('02)].

  Idea: $$f(x,a) = \frac{1}{x}\log(1-ax) + \frac{1}{ax}\text{Li}_2(ax)\,, \qquad f_{\text{exp}}(x,a) = 1 - a + \frac{ax}{4} - \frac{a^2x}{2} + \cdots$$

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$$= -\mathrm{Li}_2(a) + \frac{1}{a}\,\mathrm{Li}_3(a)$$

$$\int_0^1 \mathrm{d}x\, f_{\exp}(x,a) = 1 + a\,\frac{13}{36} + a^2\,\frac{809}{4050} + a^3\,\frac{1927}{14700} + a^4\,\frac{234314}{2480625} + a^5\,\frac{7803574}{108056025} + \ldots$$

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&+ a^{10}\,\frac{1056398775221248}{35860111300528515} + \ldots \\
&+ a^{20}\,\frac{221770658235183345519262960923443 2}{1970200070322193965696541892718176 25} + \ldots
\end{aligned}
$$

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&+ a^{30}\,\frac{349236466671635422491277237990399242846765692175253504}{5548433718772234654307047647946923757399614308955410812 5} + \ldots
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$$

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$$+ a^{40}\,\frac{6111345605632231174487017506450424419259516771994603512726507861363916 8}{14745493454562605394456699787099536537401020068836289098341590591777721875}$$

$$+ \ldots$$

$$
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&+ a^{40}\,\frac{61113456056322311744870175064504244192595167719946035127265078613639168}{1474549345456260539445669978709953653740102006883628909834159059177772187 5} \\
&+ \ldots
\end{aligned}
$$

$\Rightarrow$

$$
\ldots\,\mathrm{Li}_3\left(1-a^2\right)\,,\quad a^n\,\mathrm{Li}_3\,(1-a)\,,\quad \frac{\mathrm{Li}_3\,(1-a)}{1+a}\,,\quad \mathrm{Li}_3\left(\frac{1-a}{1+a}\right)\,,\quad \mathrm{Li}_2(1-a)\,,\quad \mathrm{Li}_2(1-a)\ln(a)\,,\quad \ldots
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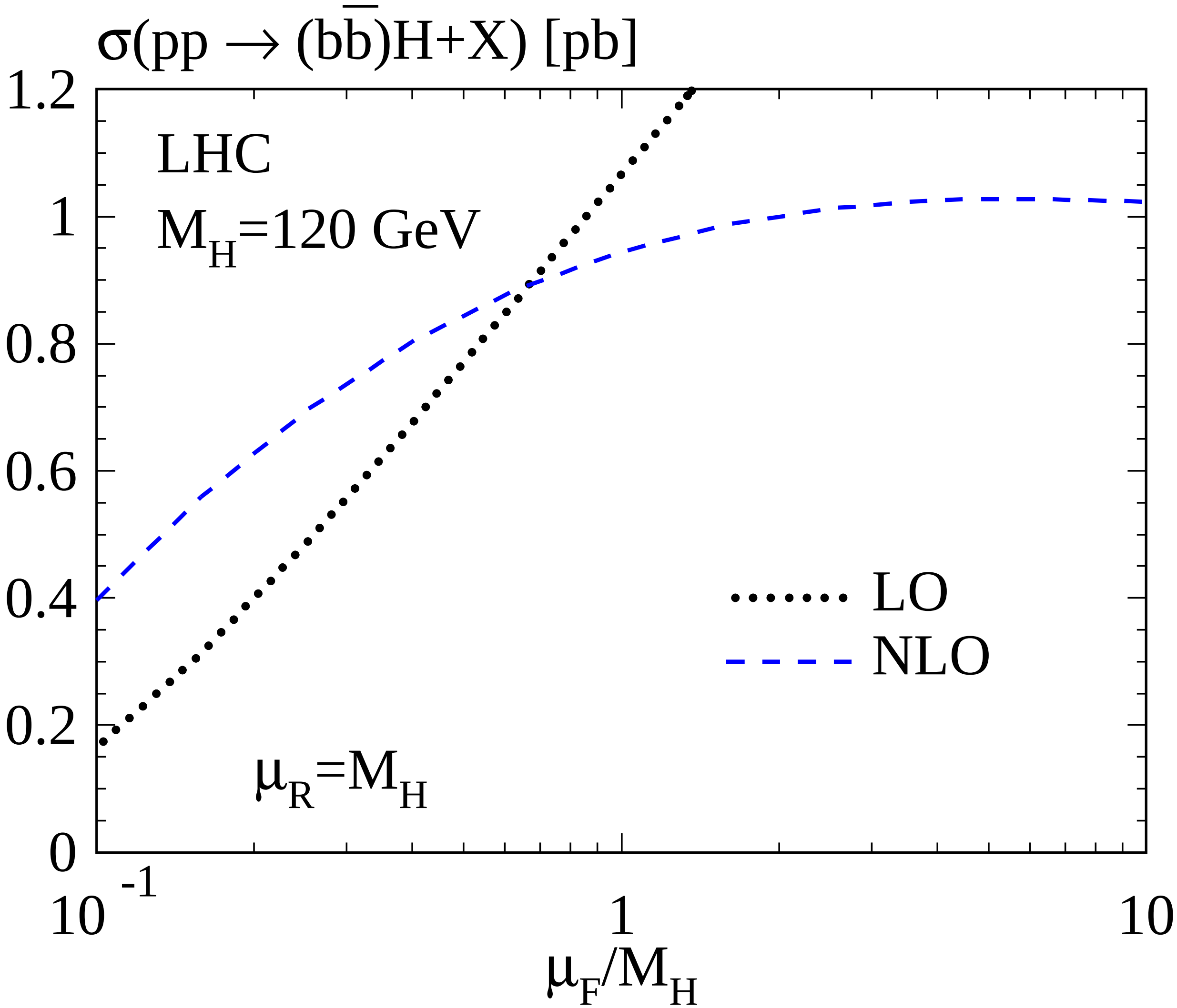
$$

$\sim$ 100 functions

$b\bar{b} \longrightarrow H$



[Maltoni, Sullivan, Willenbrock ('03)]

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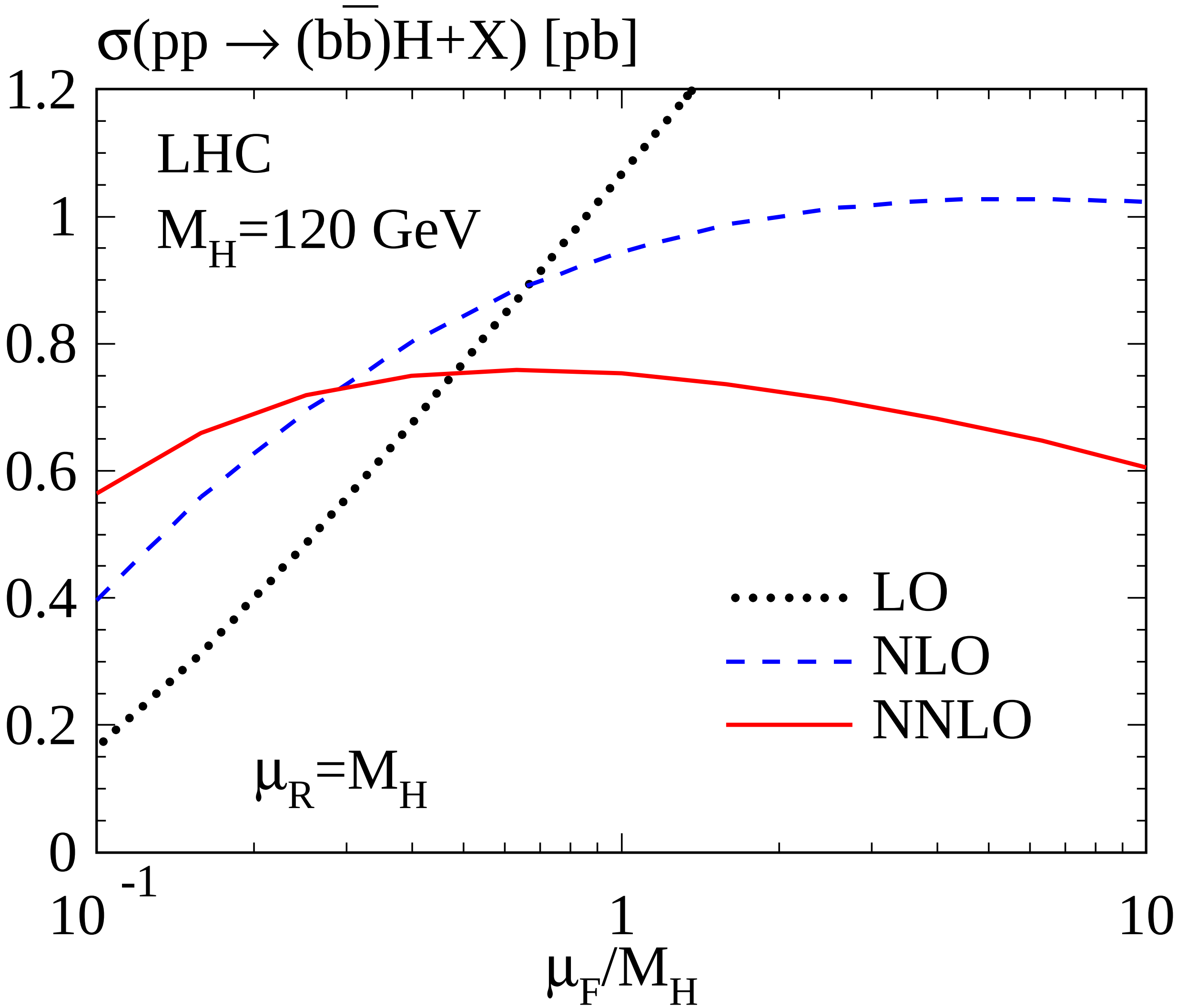
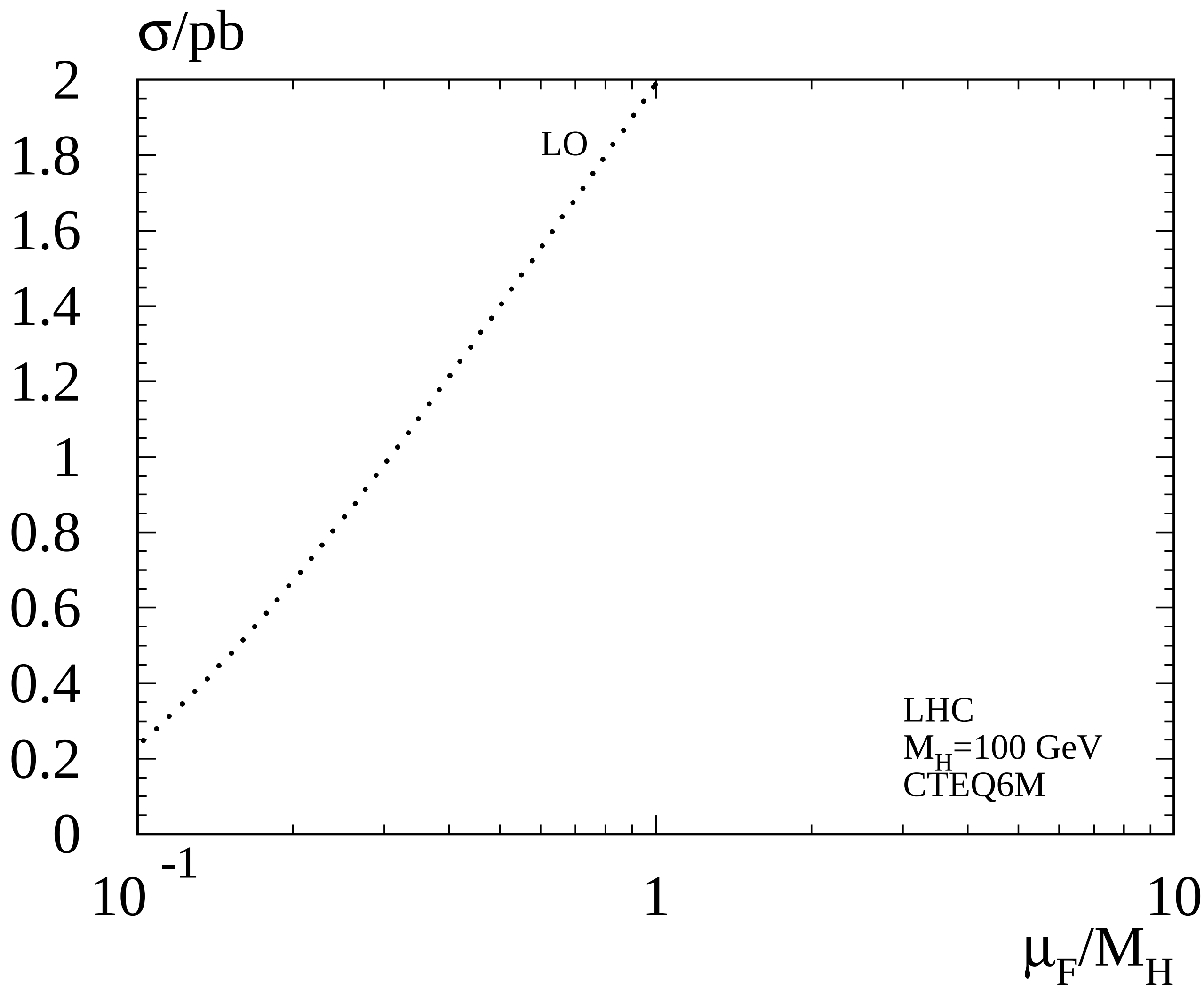


[Maltoni, Sullivan, Willenbrock ('03)]

[R.H., Kilgore ('03)]

LHC, $M_H = 100 GeV$



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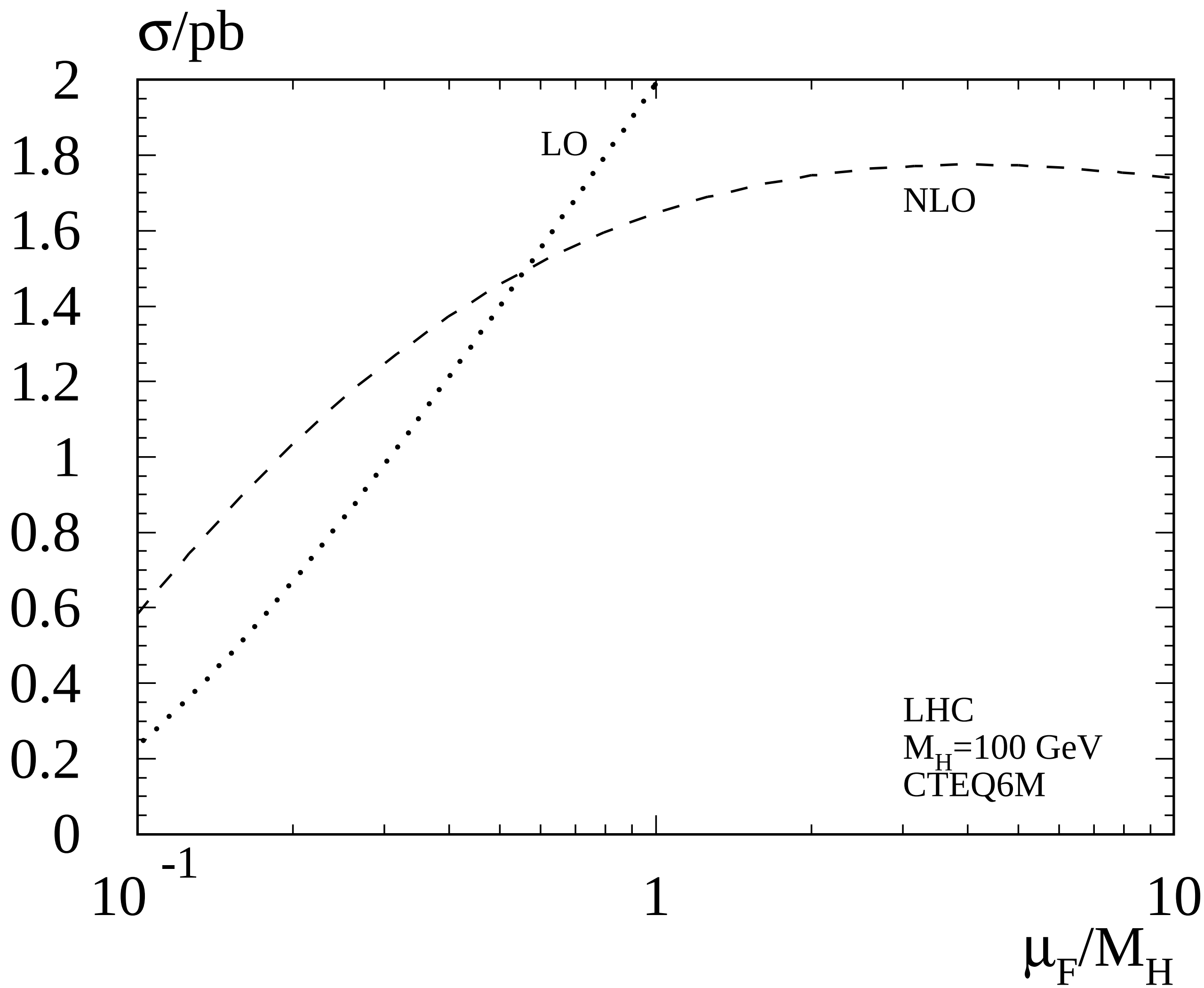

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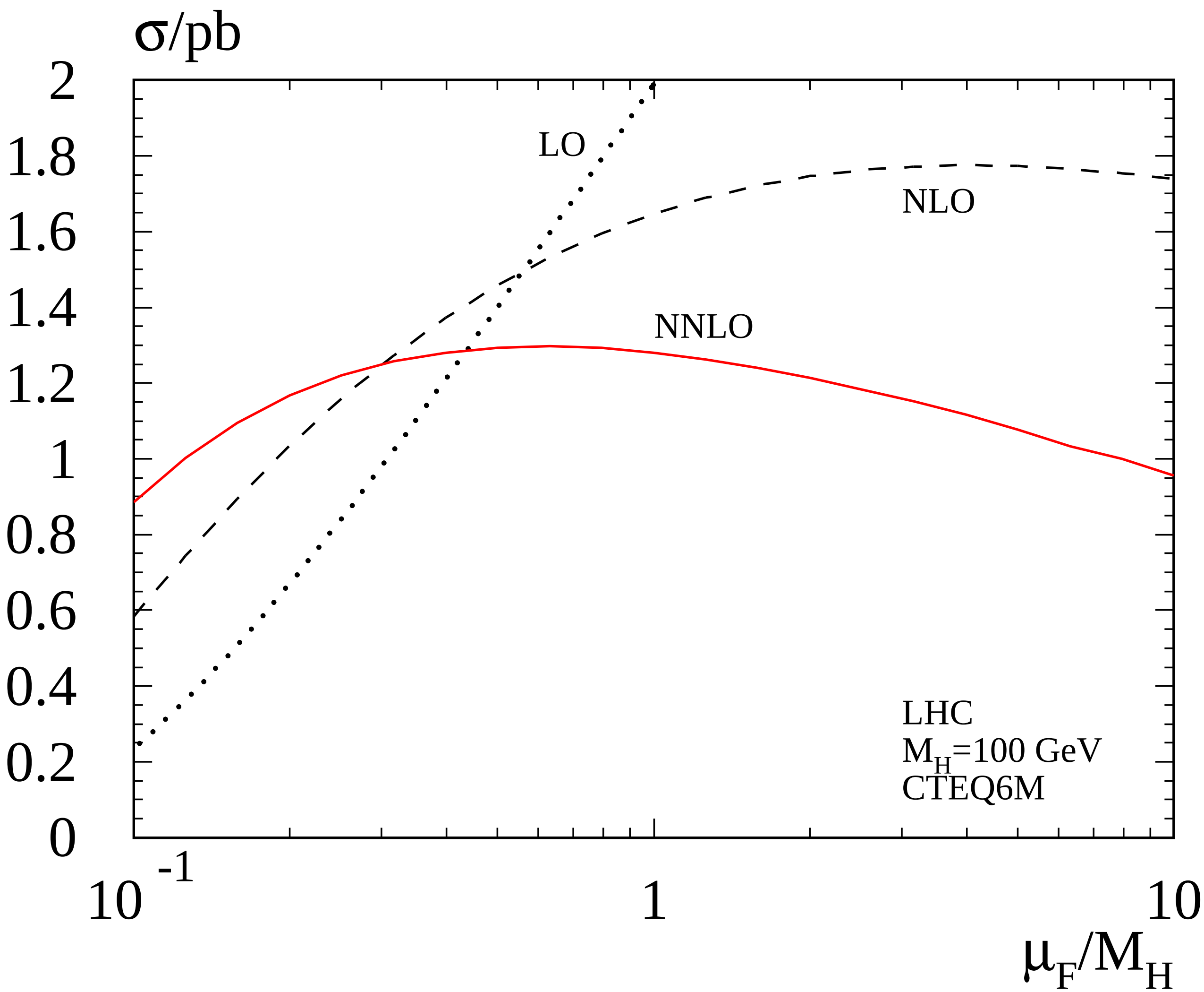

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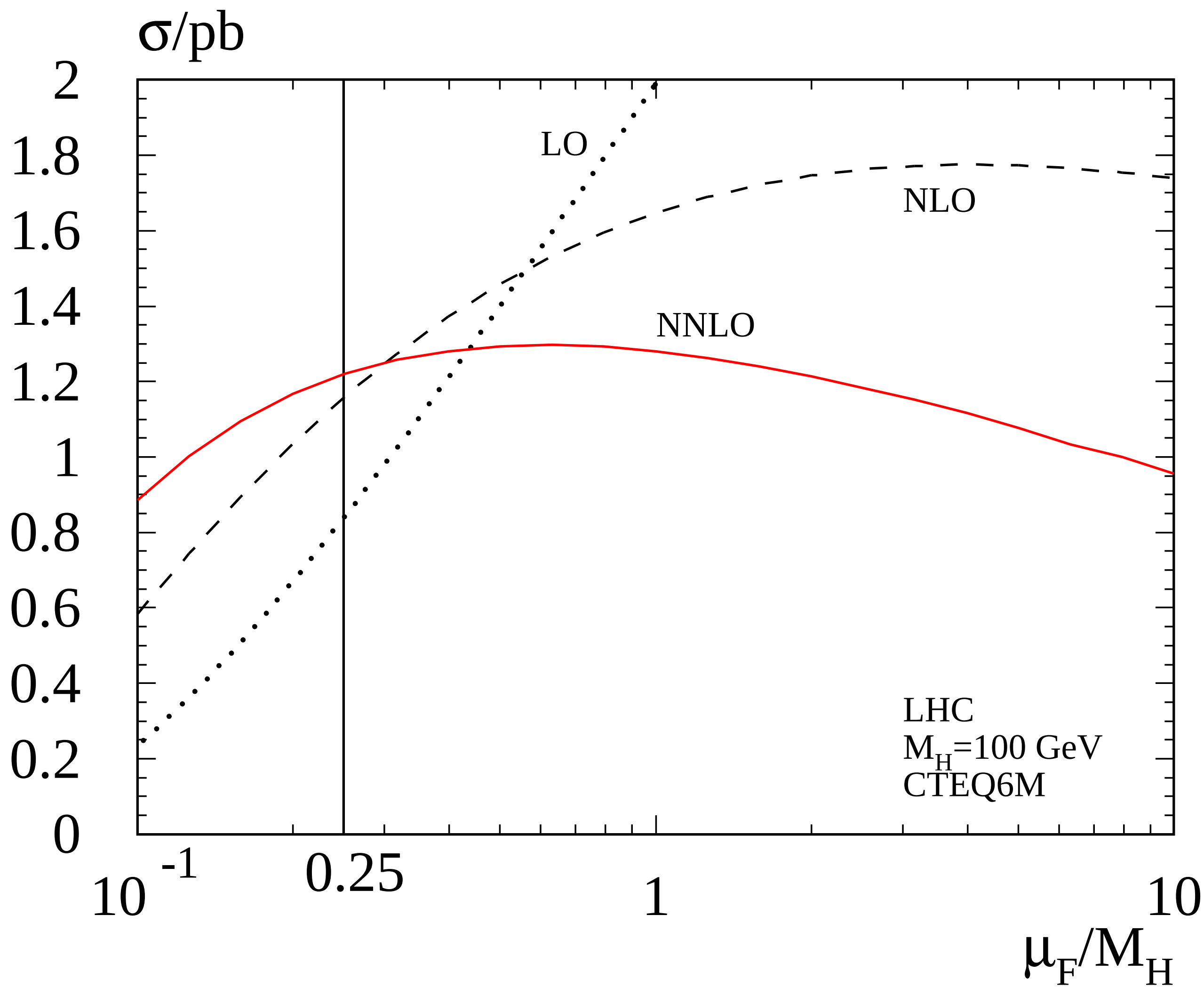

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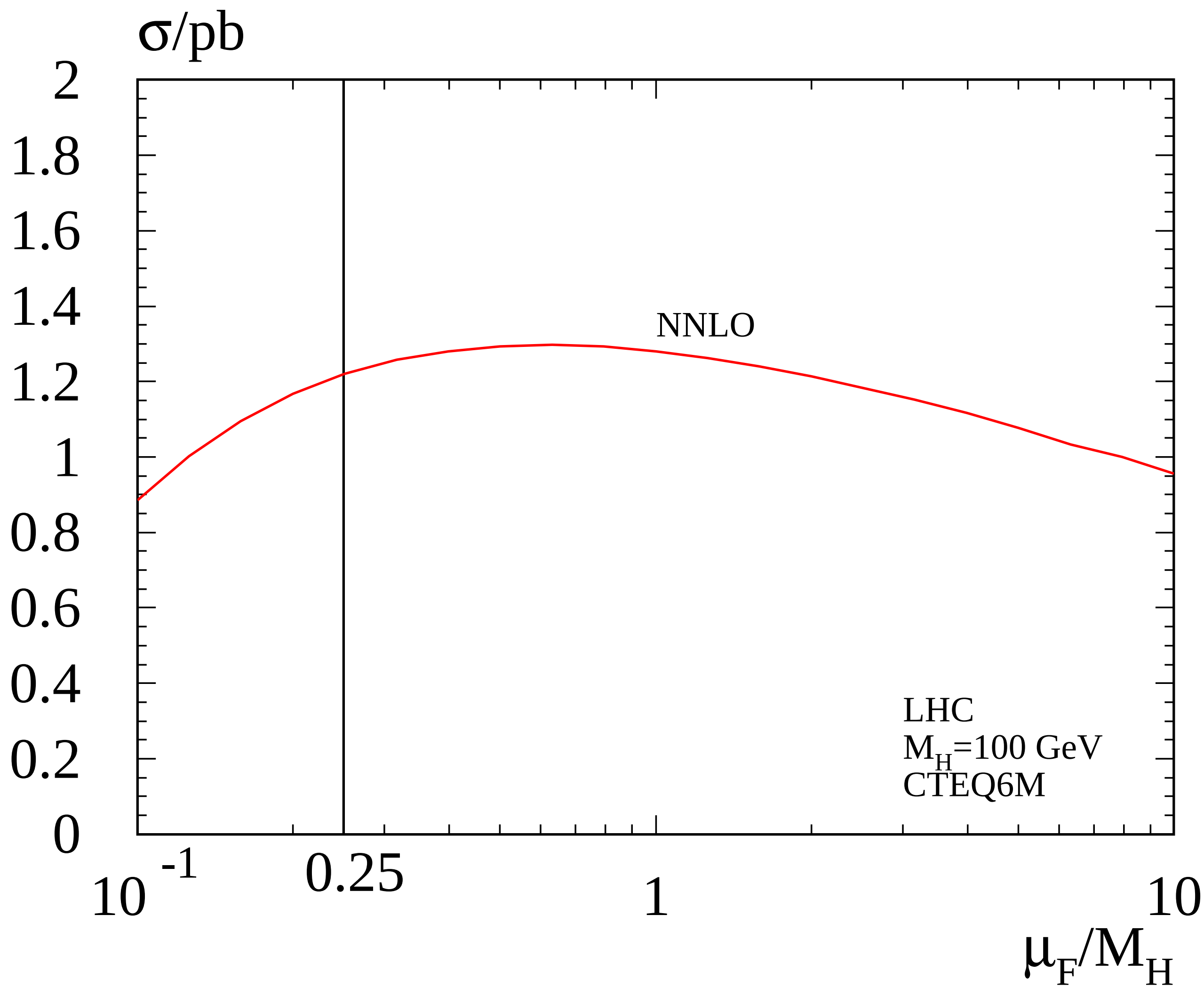

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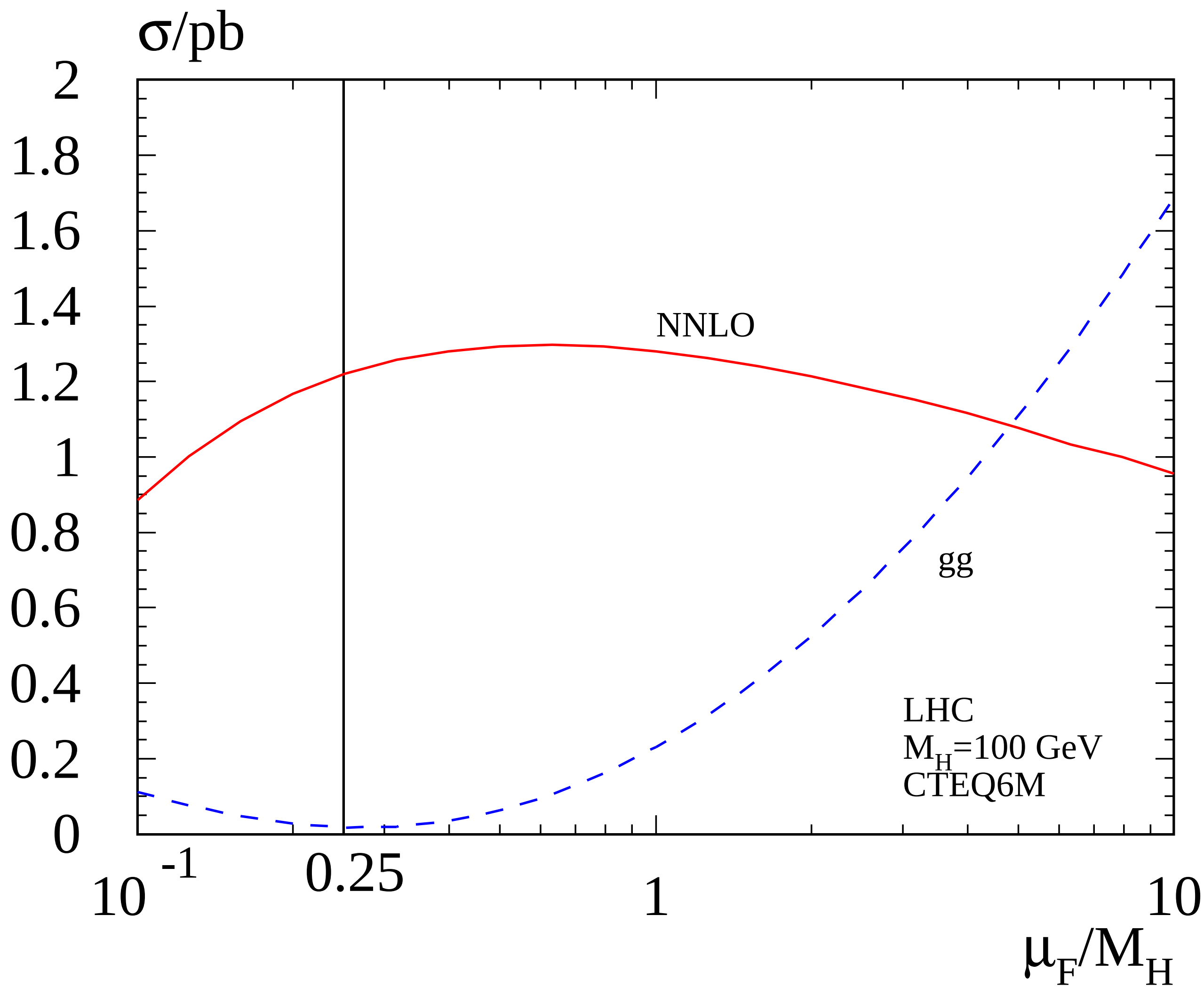
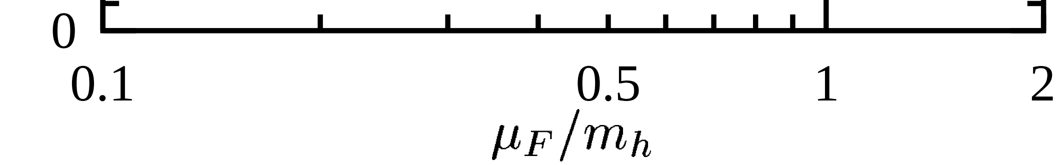
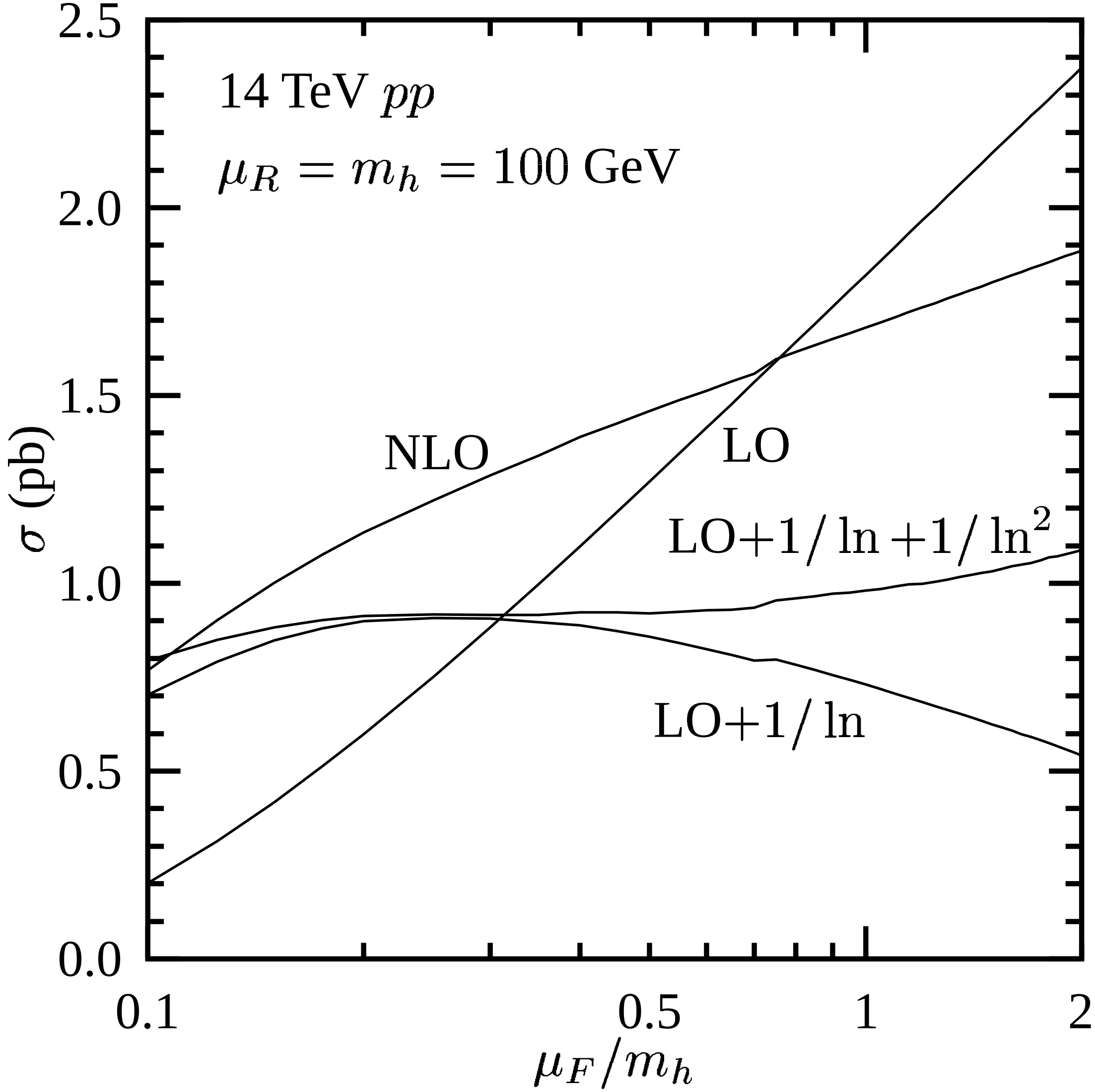
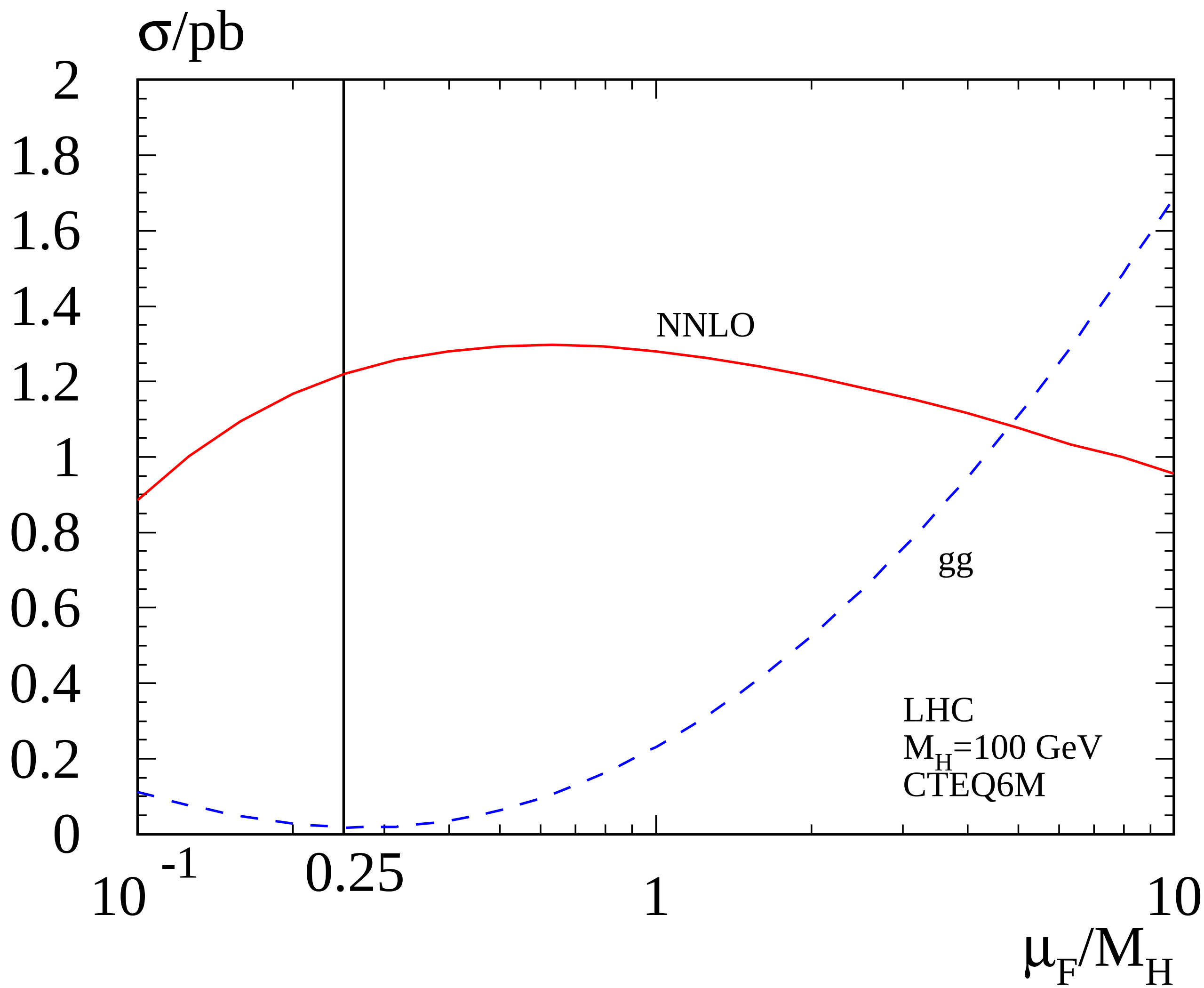


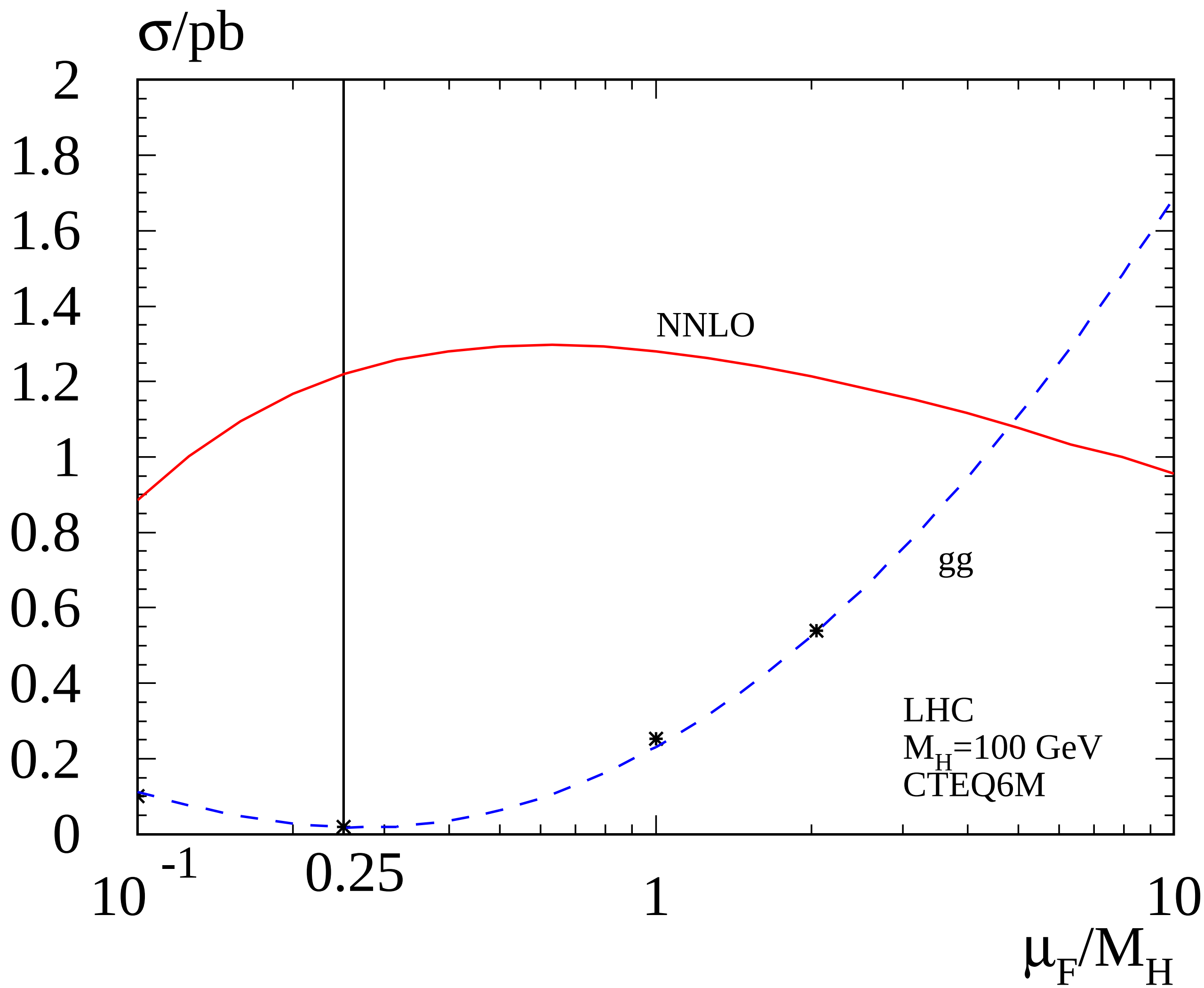

0
0.1
0.5
1
2
μ_F/m_h


14 TeV pp
$\mu_R = m_h = 100$ GeV
NLO
LO
LO+1/ ln +1/ ln^2
LO+1/ ln
σ (pb)
2.5
2.0
1.5
1.0
0.5
0.0
0.1
0.5
1
2
μ_F/m_h

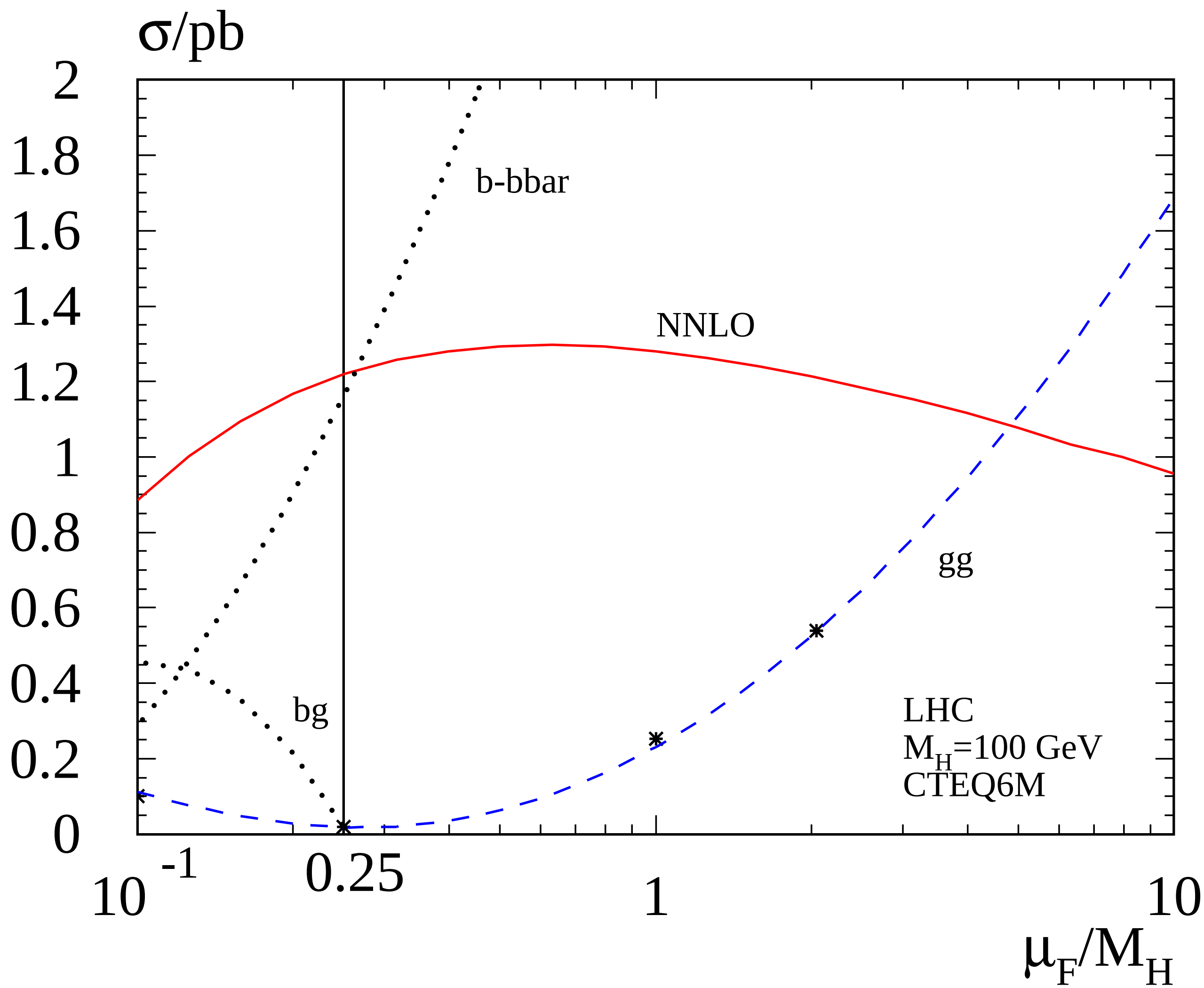
LHC, $M_H = 100$ GeV



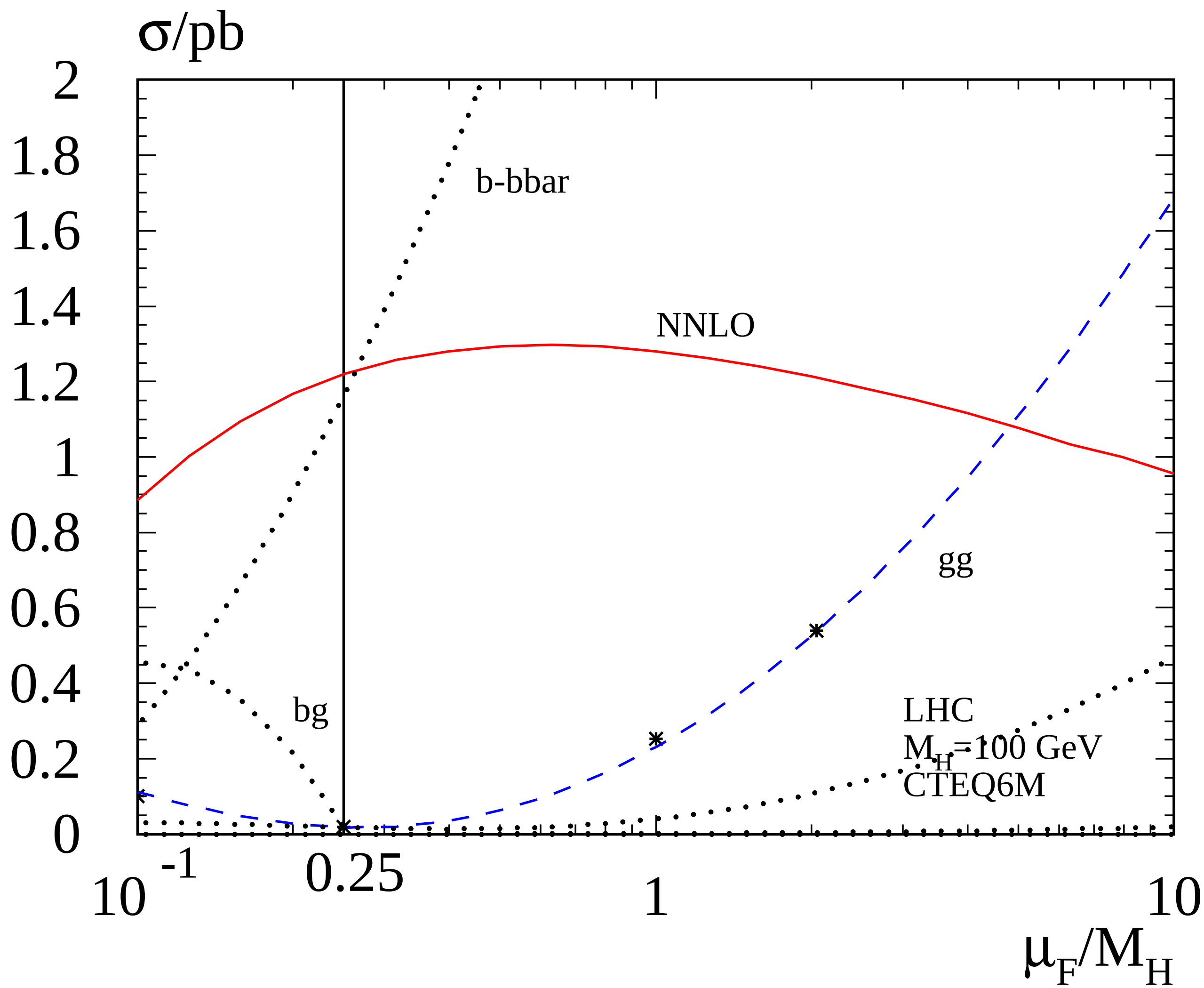
LHC, $M_H = 100$ GeV



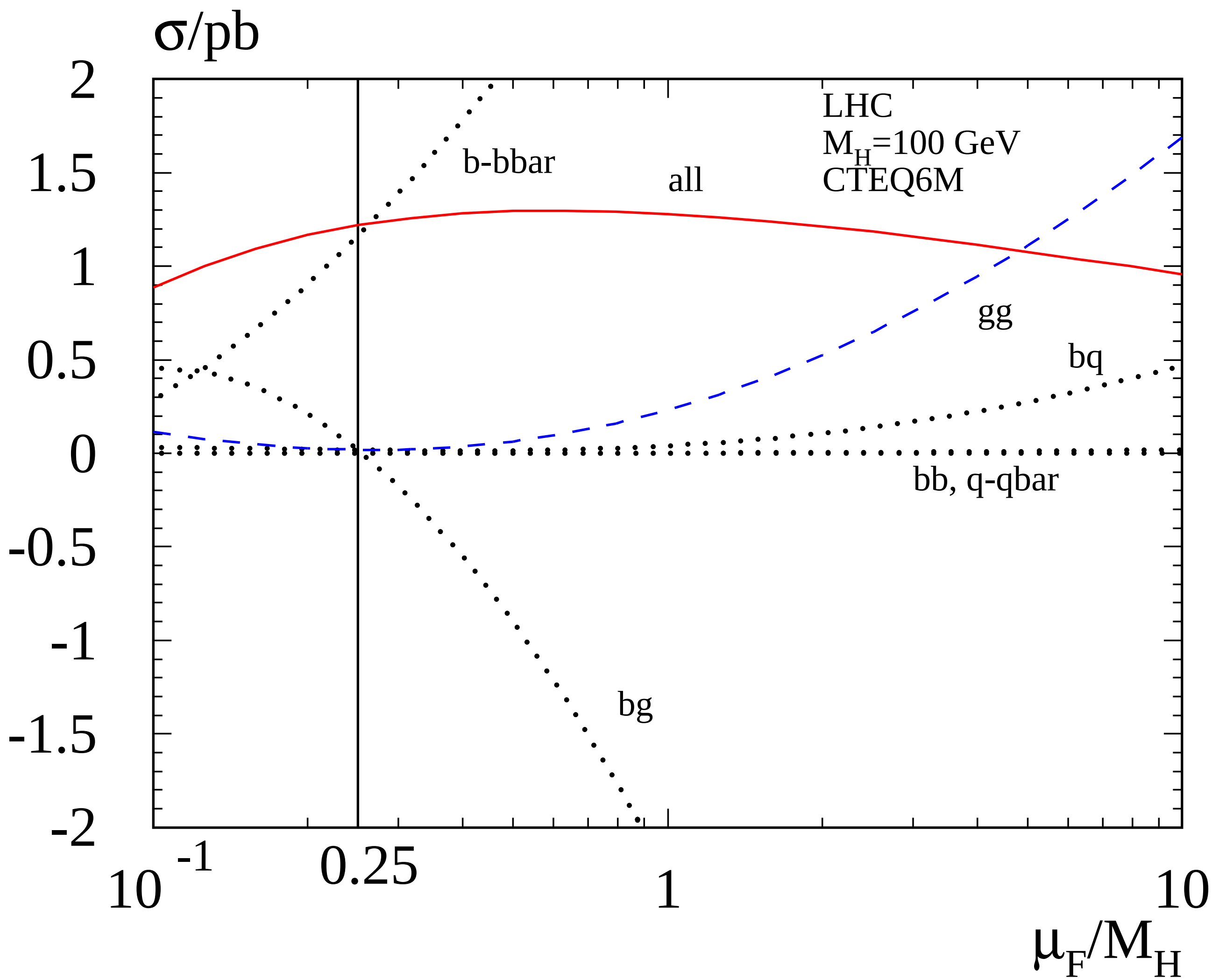
LHC, $M_H = 100$ GeV



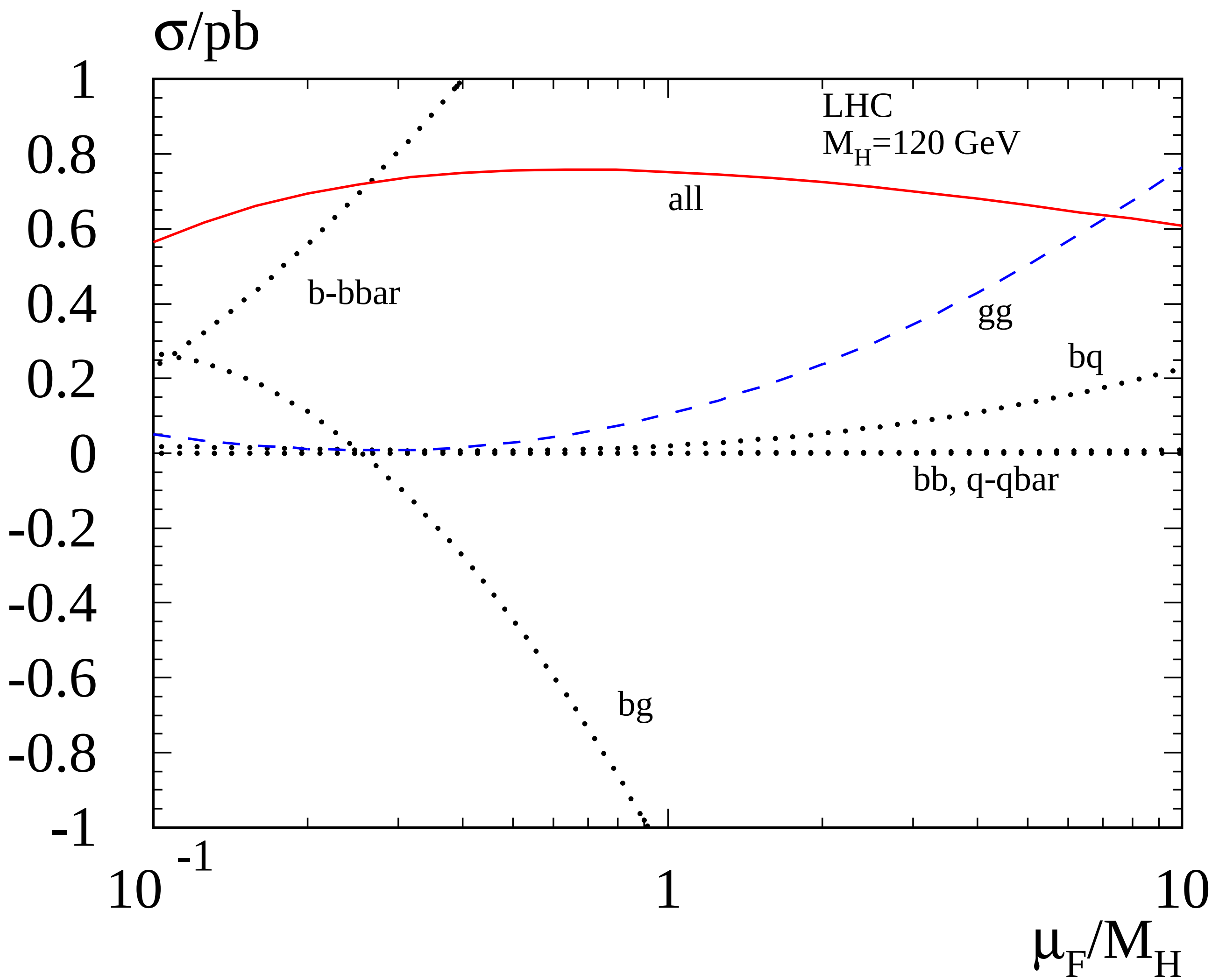
LHC, $M_H = 100$ GeV



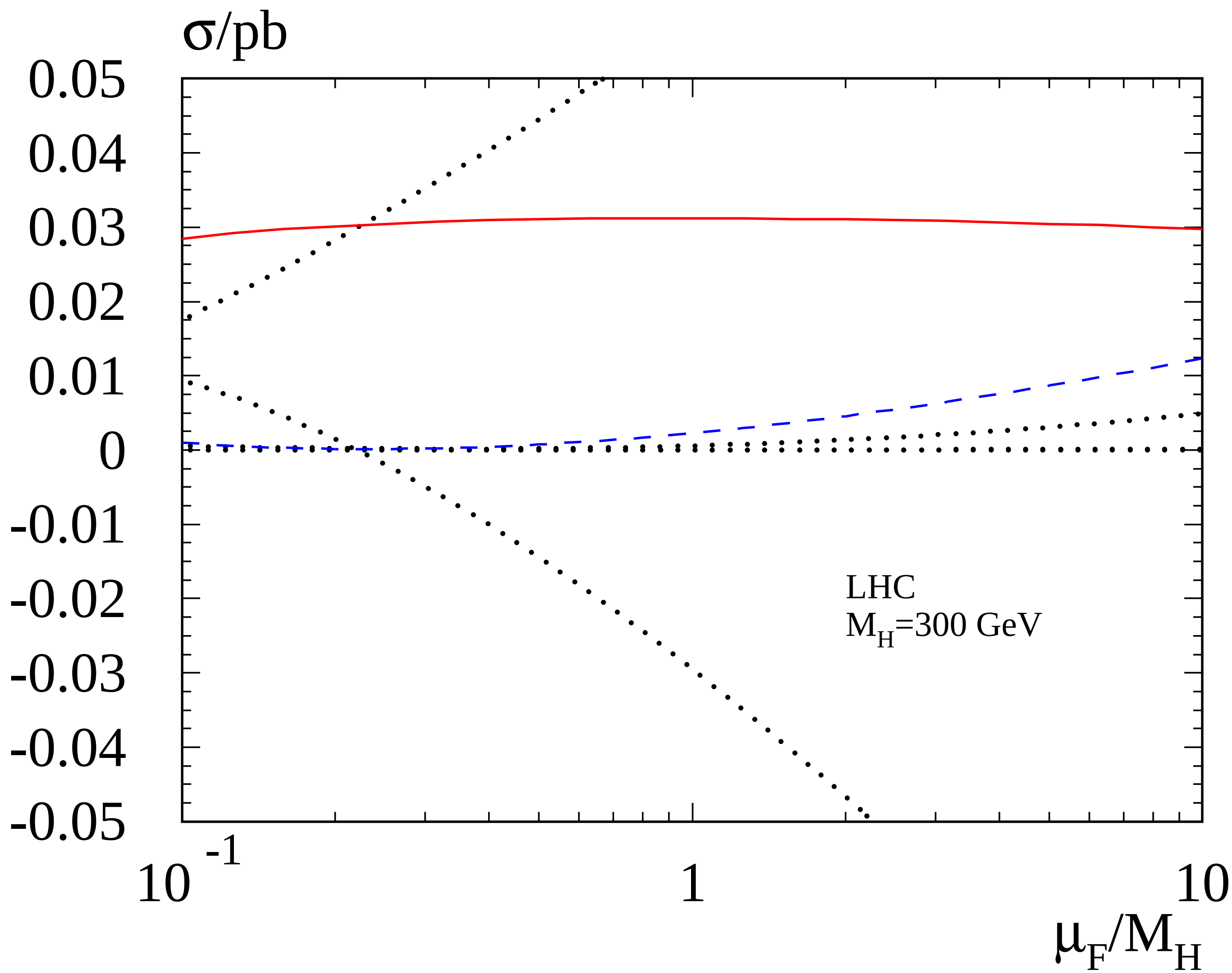
LHC, $M_H = 100$ GeV



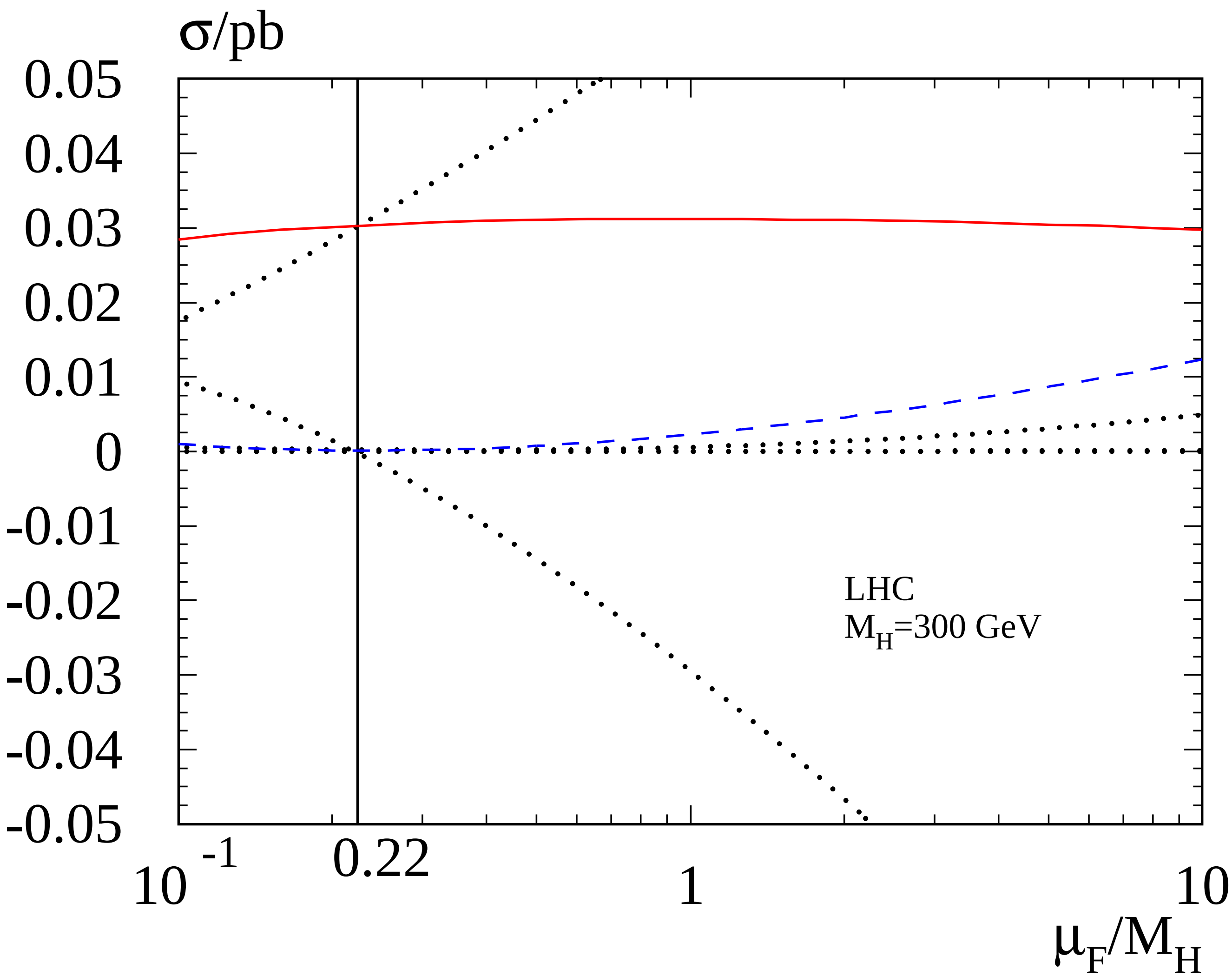
LHC, $M_H = 120$ GeV

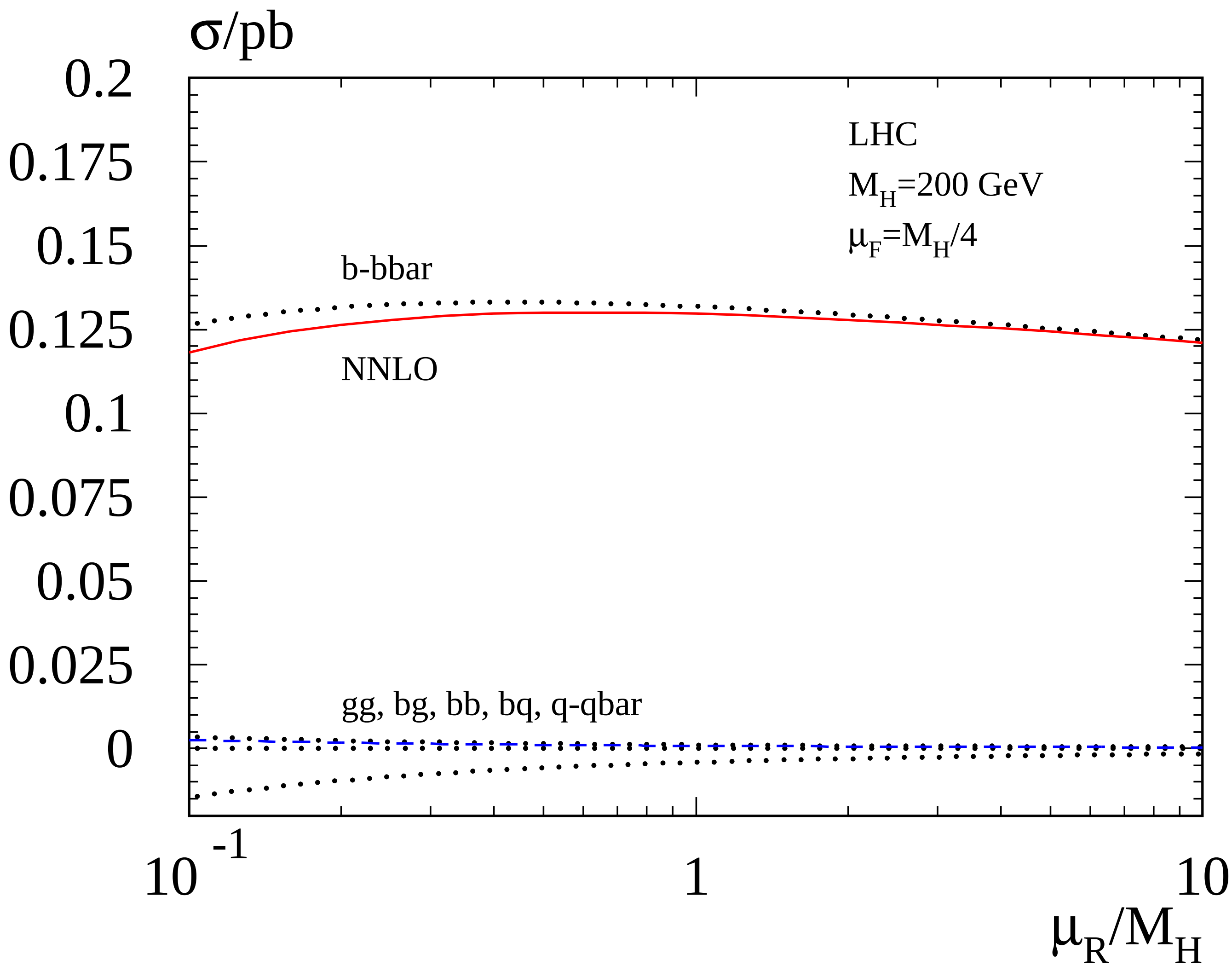


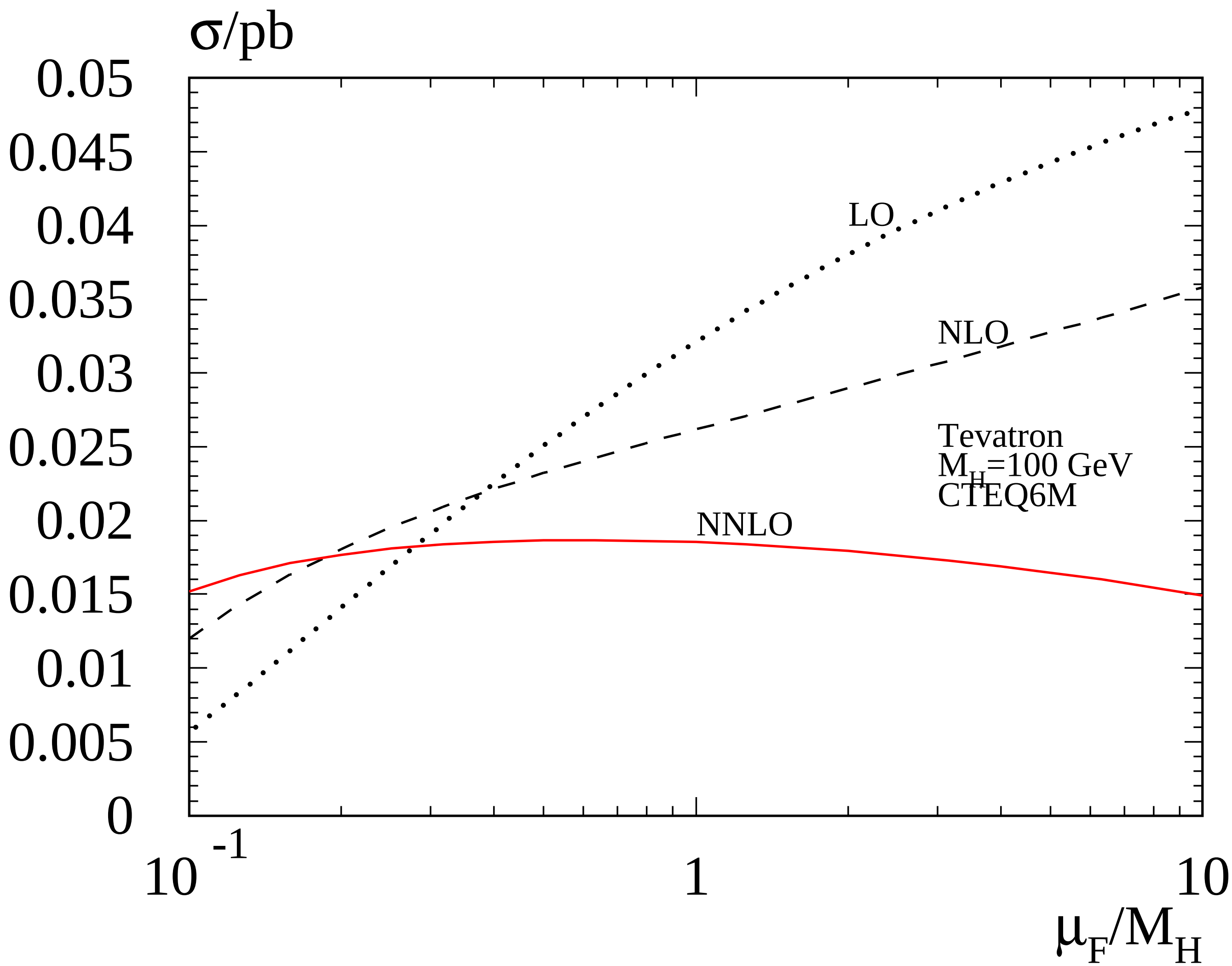
LHC, $M_H = 300$ GeV

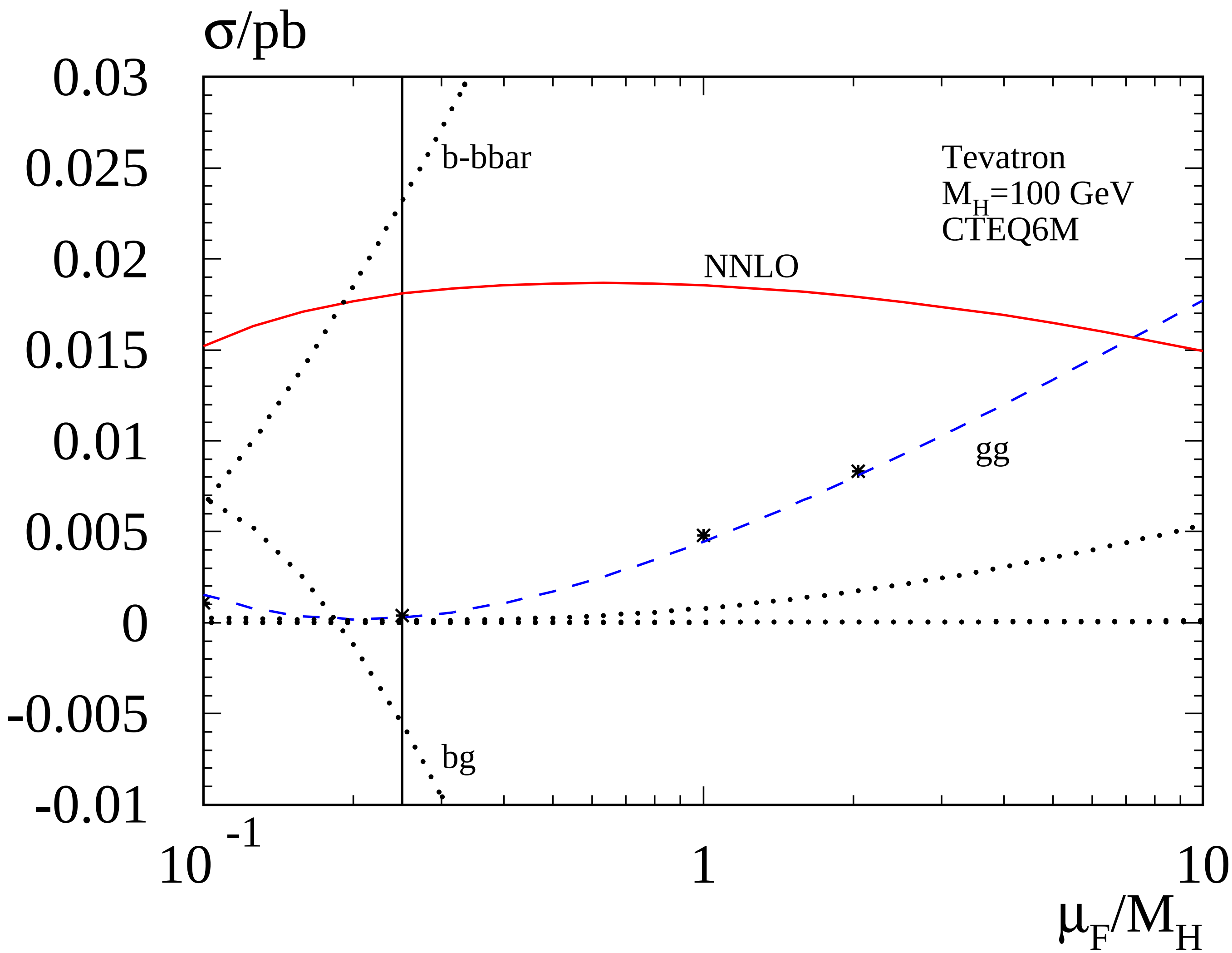


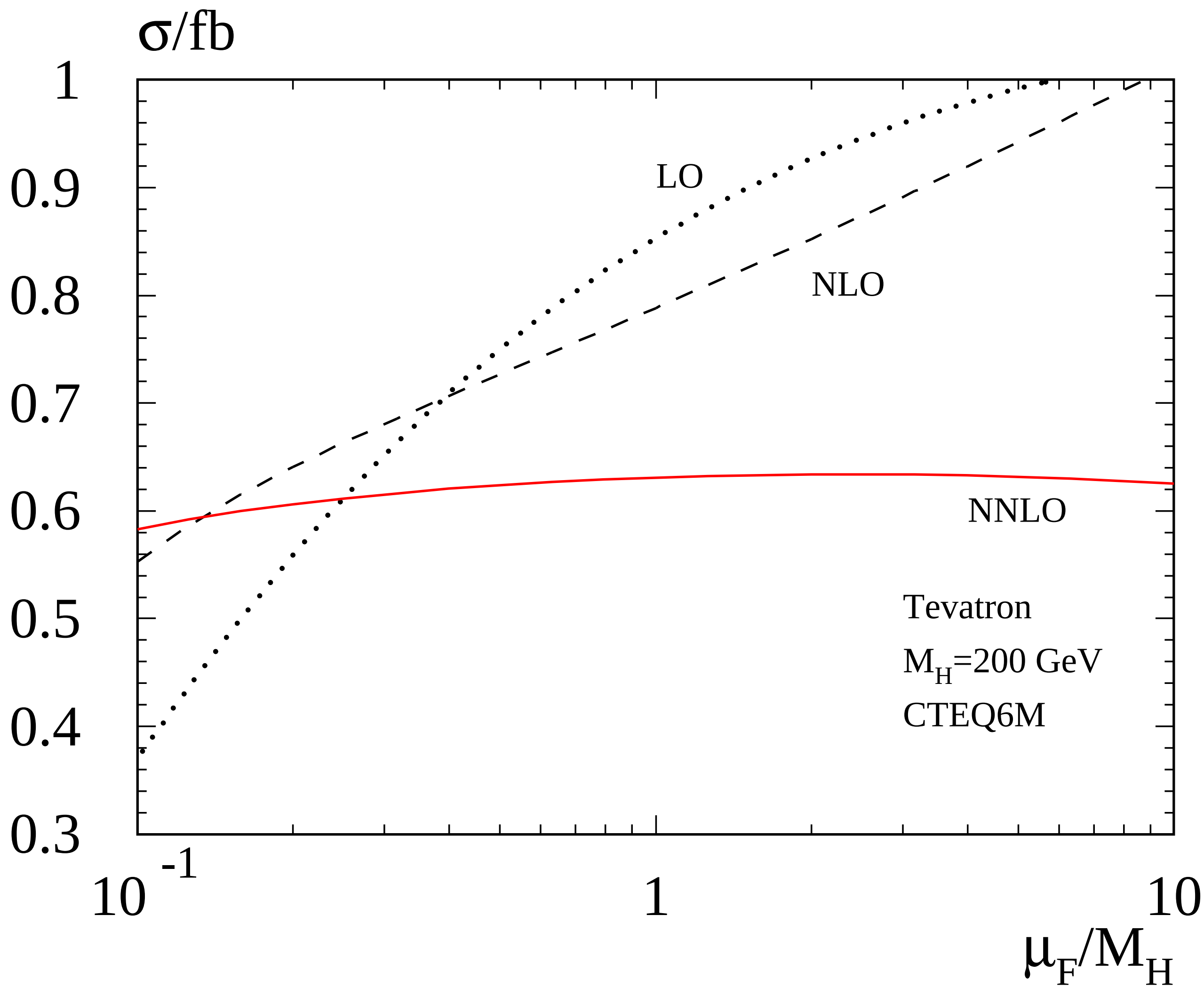
LHC, $M_H = 300$ GeV

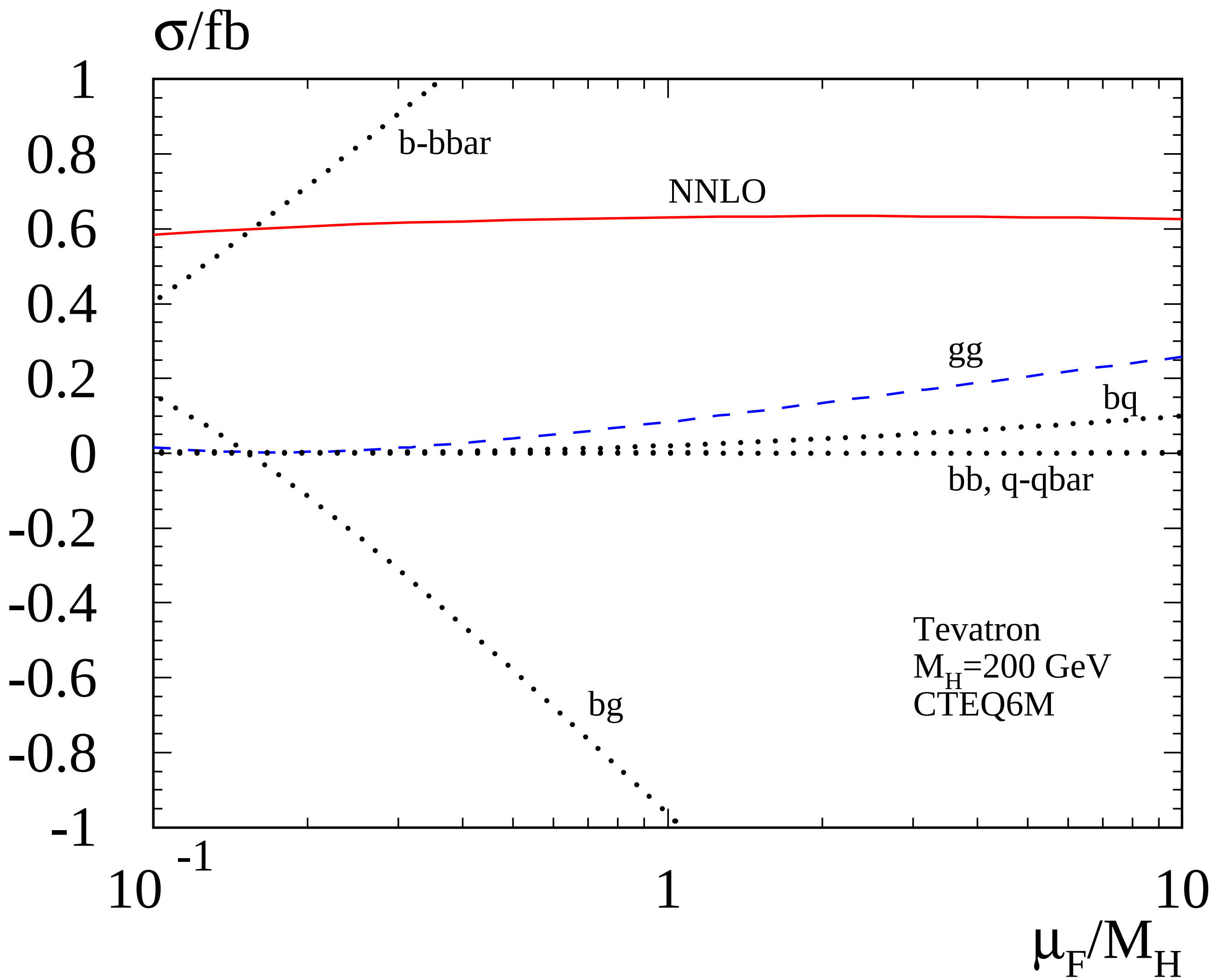



σ/pb
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0
LHC
M_H=200 GeV
μ_F=M_H/4
b-bbar
NNLO
gg, bg, bb, bq, q-qbar
10^-1
1
10
μ_R/M_H

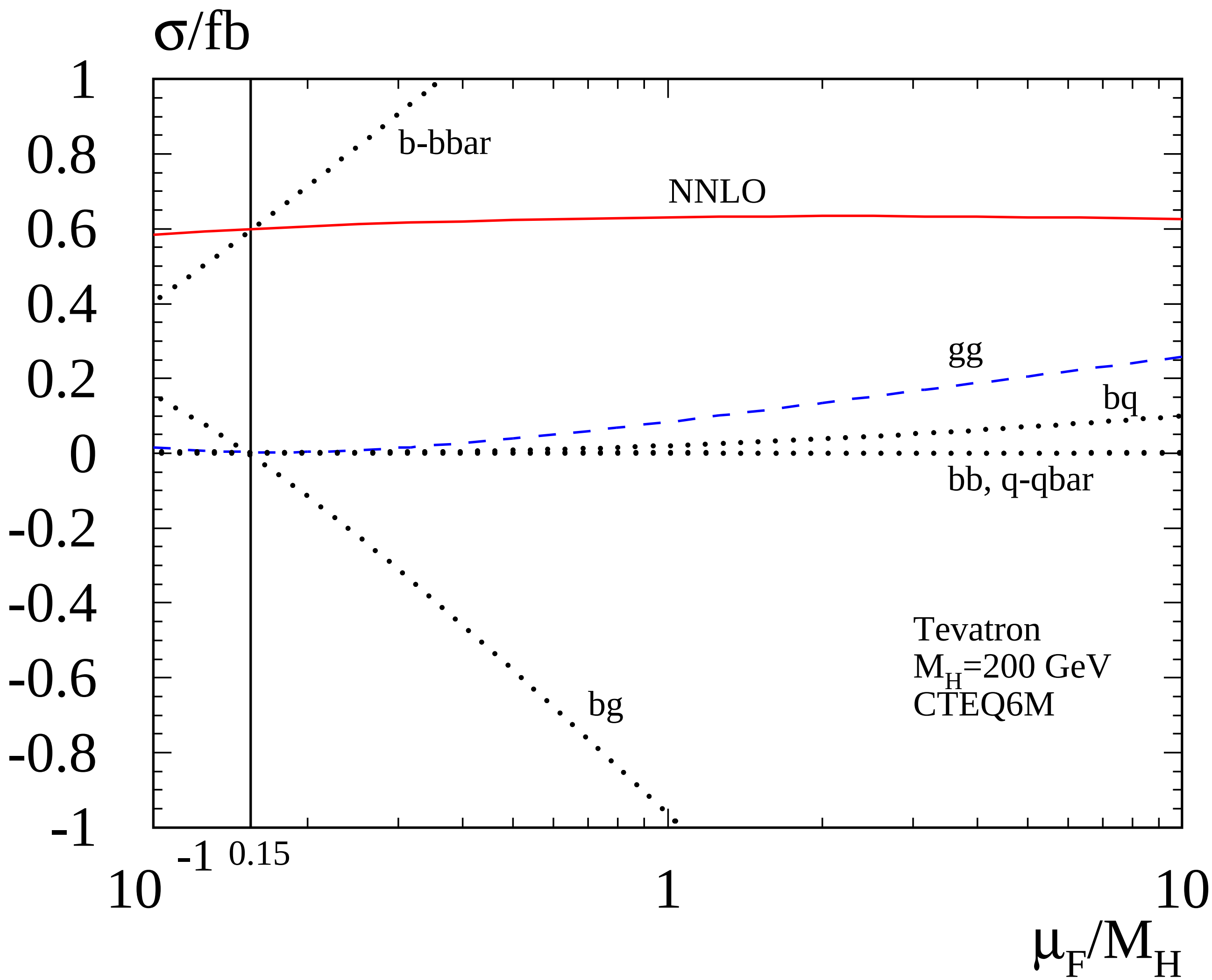

σ/pb
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
LO
NLO
NNLO
Tevatron
M_H=100 GeV
CTEQ6M
10^-1
1
10
μ_F/M_H


σ/pb
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
b-bbar
Tevatron
M_H=100 GeV
CTEQ6M
NNLO
gg
bg
10^-1
1
10
μ_F/M_H

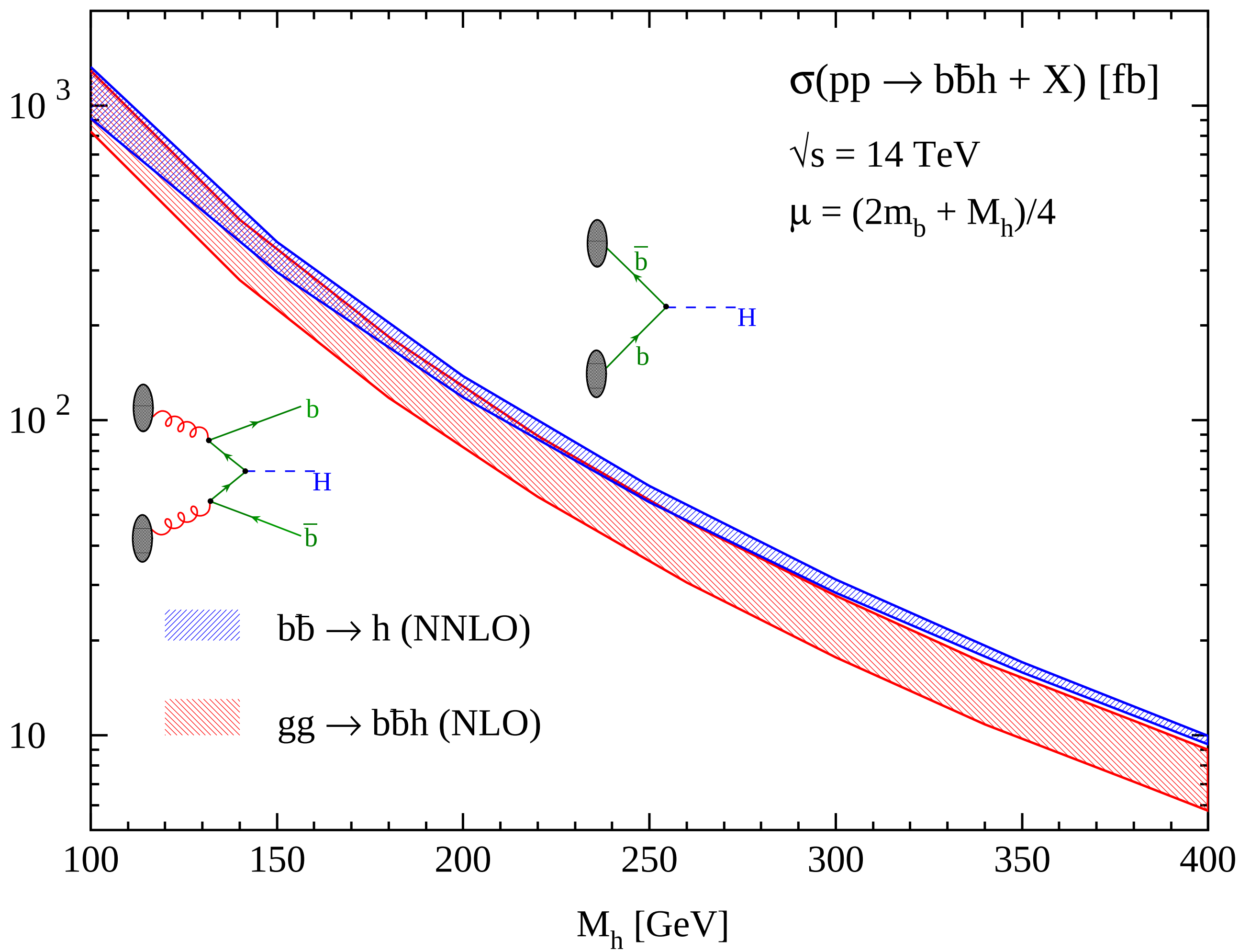

σ/fb
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
LO
NLO
NNLO
Tevatron
M_H=200 GeV
CTEQ6M
10^-1
1
10
μ_F/M_H



σ/fb
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
b-bbar
NNLO
gg
bq
bb, q-qbar
bg
Tevatron
M_H=200 GeV
CTEQ6M
10^-1
1
10
μ_F/M_H

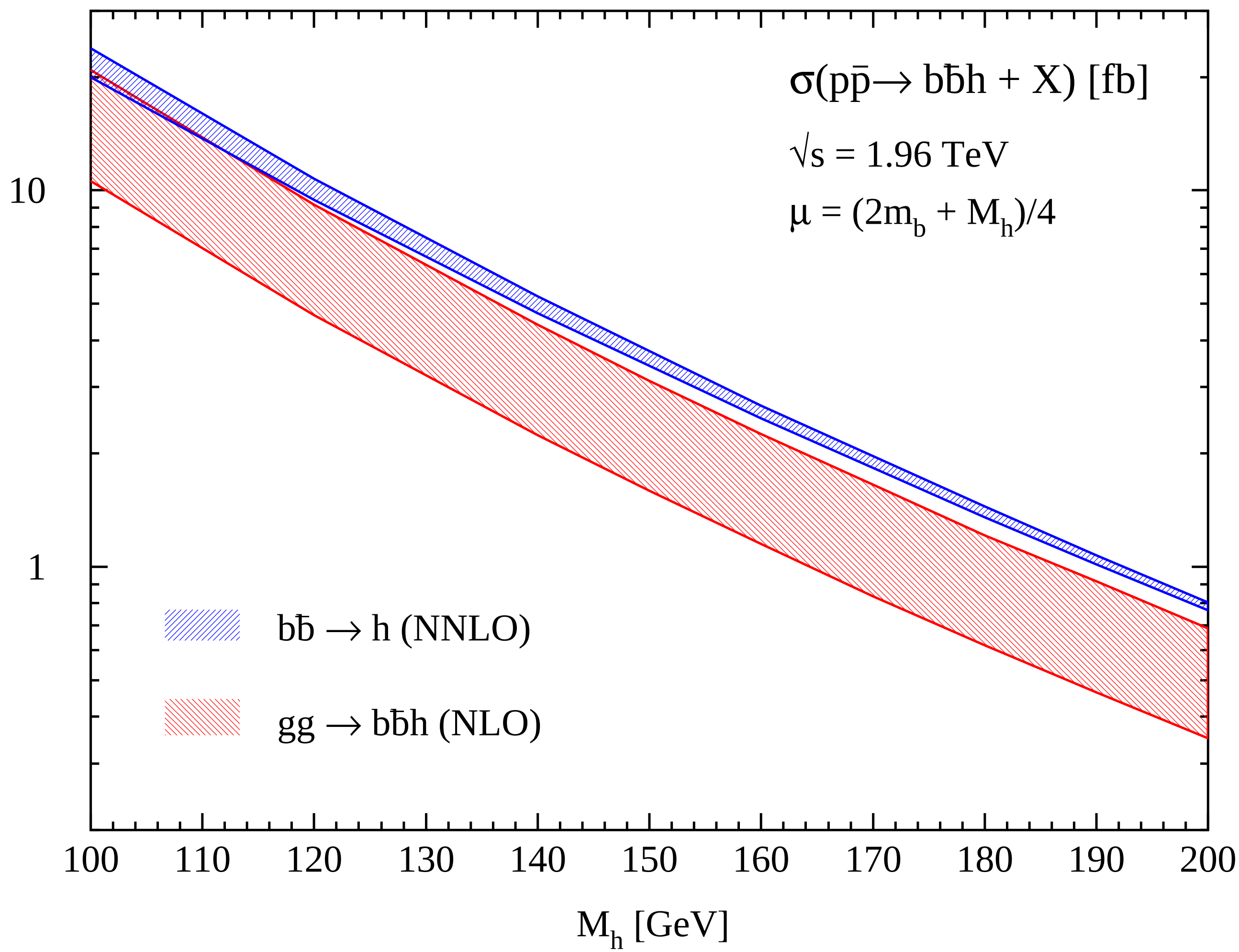

σ/fb
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
b-bbar
NNLO
gg
bq
bb, q-qbar
bg
Tevatron
M_H=200 GeV
CTEQ6M
0.15
10^-1
1
10
μ_F/M_H

$pp \longrightarrow H + b\bar{b}$



- $b\bar{b} \to H$: [R.H., Kilgore '03]
- $gg \to b\bar{b}H$: [Dawson *et al.* '04], [Dittmaier *et al.* '04]

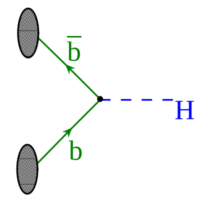
$p\bar{p} \longrightarrow H + b\bar{b}$



- $b\bar{b} \to H$: [R.H., Kilgore '03]
- $gg \to b\bar{b}H$: [Dawson *et al.* '04], [Dittmaier *et al.* '04]

$b\bar{b} + h$

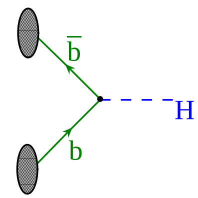
- $b\bar{b} \to h$ known through NNLO
  [R.H., Kilgore '03]



$b\bar{b} + h$

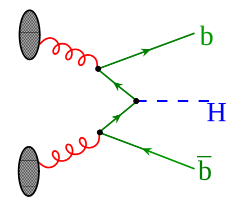
- $b\bar{b} \to h$ known through NNLO

  [R.H., Kilgore '03]



- $gg \to b\bar{b}h$ known through NLO

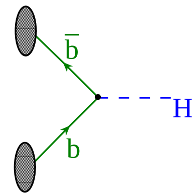
  [Dittmaier, Krämer, Spira '03]
  [Dawson, Jackson, Reina, Wackeroth '04]



$b\bar{b} + h$

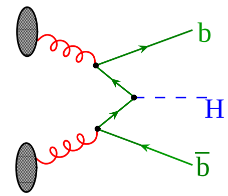
- $b\bar{b} \to h$ known through NNLO

  [R.H., Kilgore '03]



- $gg \to b\bar{b}h$ known through NLO

  [Dittmaier, Krämer, Spira '03]
  [Dawson, Jackson, Reina, Wackeroth '04]



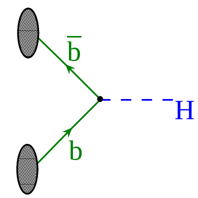
- discrepancy resolved: proper factorization scale important

  [Plehn '03], [Maltoni, Sullivan, Willenbrock '03], [Boos, Plehn '04]

$b\bar{b} + h$

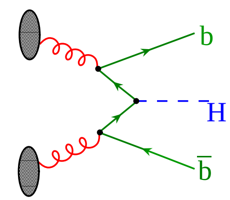
- $b\bar{b} \to h$ known through NNLO

  [R.H., Kilgore '03]



- $gg \to b\bar{b}h$ known through NLO

  [Dittmaier, Krämer, Spira '03]
  [Dawson, Jackson, Reina, Wackeroth '04]



- discrepancy resolved: proper factorization scale important

  [Plehn '03], [Maltoni, Sullivan, Willenbrock '03], [Boos, Plehn '04]

- bottom density approach is viable

  [Barnett, Haber, Soper '88], [Dicus, Willenbrock '89]