Fred Olness
Les Houches
17 May 2005
bbH discussion group

Bottom quark initiated Higgs production at hadron colliders

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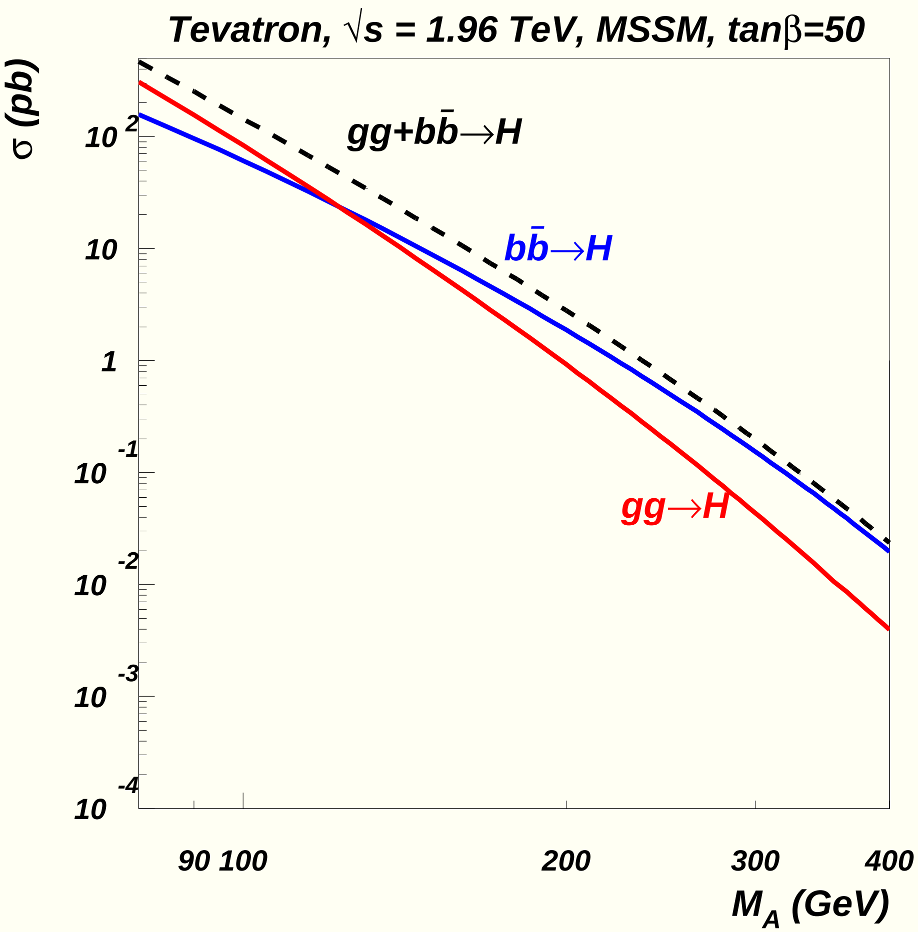
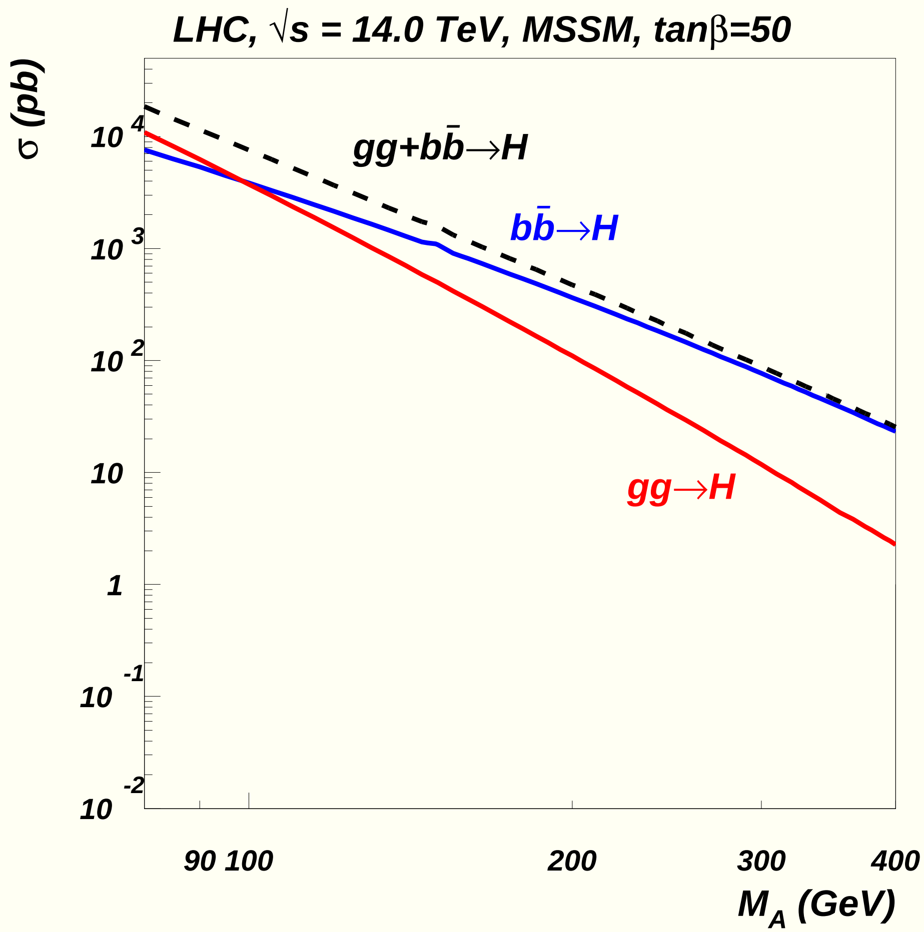
Why $b\bar{b} \to H$?





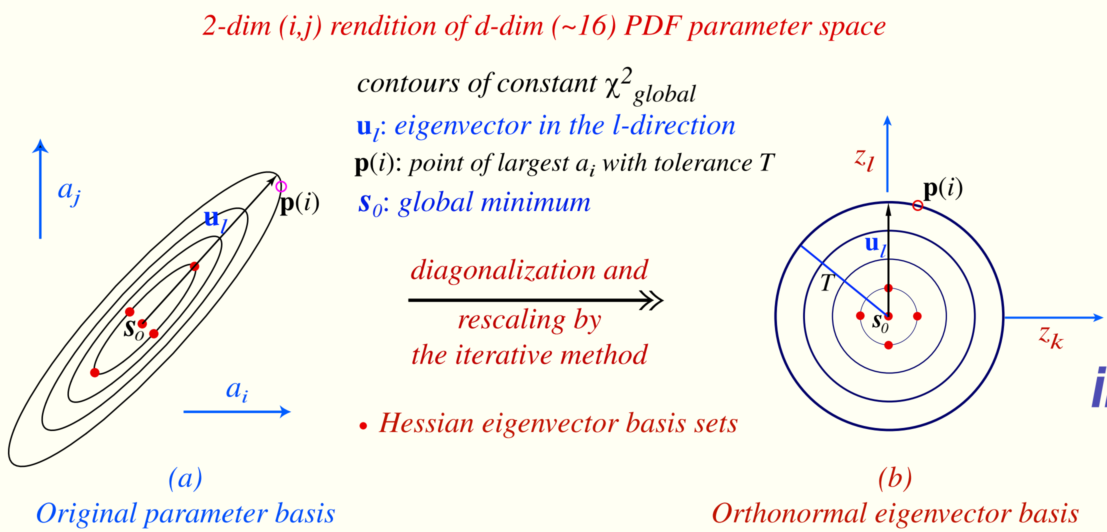
$\Longrightarrow$ cross sections are highly enhanced, the process could serve as a tool to measure Yuakawa coupling and bottom-quark distributions, therefore, the understanding of theoretical uncertaintiy is crucial

Estimation of PDF uncertainties: Hessian method

- **Hessian method**

involves the Hessian matrix $H_{ij} = \frac{1}{2}\frac{\partial^2 \chi_0^2}{\partial a_i \partial a_j}$ calculated at the minimum of χ_0^2. The next step is to diagonolize H_{ij} and to find eigenvectors of Hessian. Then for each eigenvector we have two displacements from $\{a_0\}$ (in the $+$ and $-$ directions along the vector)

At these points, $\chi_\pm^2 = \chi_0^2 + T^2$, where T parametrizes the tolerance.



δX for any quantaty X, which depends on PDF, can be expressed as

$$(\delta X)^2 = T^2 \sum_{i,j} (H^{-1})_{ij} \frac{\partial X}{\partial a_i}\frac{\partial X}{\partial a_j}$$

in terms of the eigenvector basis one has

master equation for 41 CTEQ6.1 PDF set: $(\delta X)^2 = \frac{1}{4}\sum_{k=1}^{n}\left[X(a_i^+) - X(a_i^-)\right]^2$

based on a linear approximation: $\chi^2(a)$ is assumed to be a quadratic function of the parameters $\{a\}$, and $X(a)$ is assumed to be linear

Estimation of PDF uncertainties: LM method

- **Method of Lagrange multiplier (LM): introduction of the Lagrange multiplier variable λ and minimizing the function**

$$\chi^2_\lambda(\lambda, a) = \chi^2(a) + \lambda X(a)$$

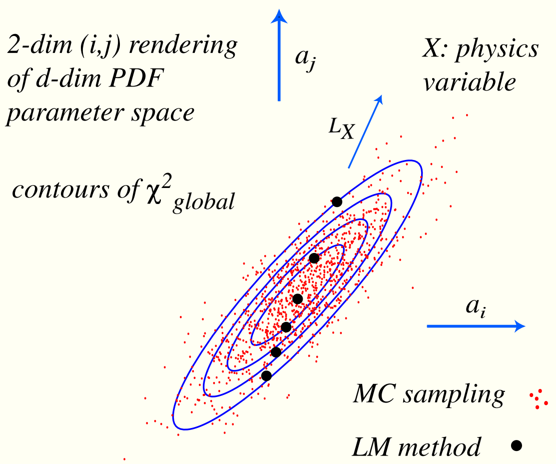
with respect to the original n parameters $\{a\}$ for fixed (many) values of λ

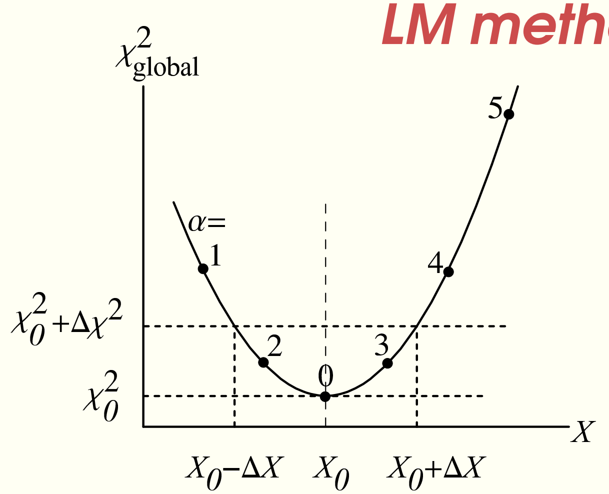
$\Longrightarrow$ **parametric relationship between $\chi^2(a)$ and $X(a)$:**

$\chi^2_\lambda(\lambda, a_0) = \chi^2(a_0) + \lambda X(a_0) \Longrightarrow X = X[\chi^2(a_0, \lambda)]$

For given $\Delta\chi^2 = \chi^2(a_0, \lambda^\Delta_\pm) - \chi^2(a_0, 0) = 100$ one finds two values $\lambda^\Delta_\pm$

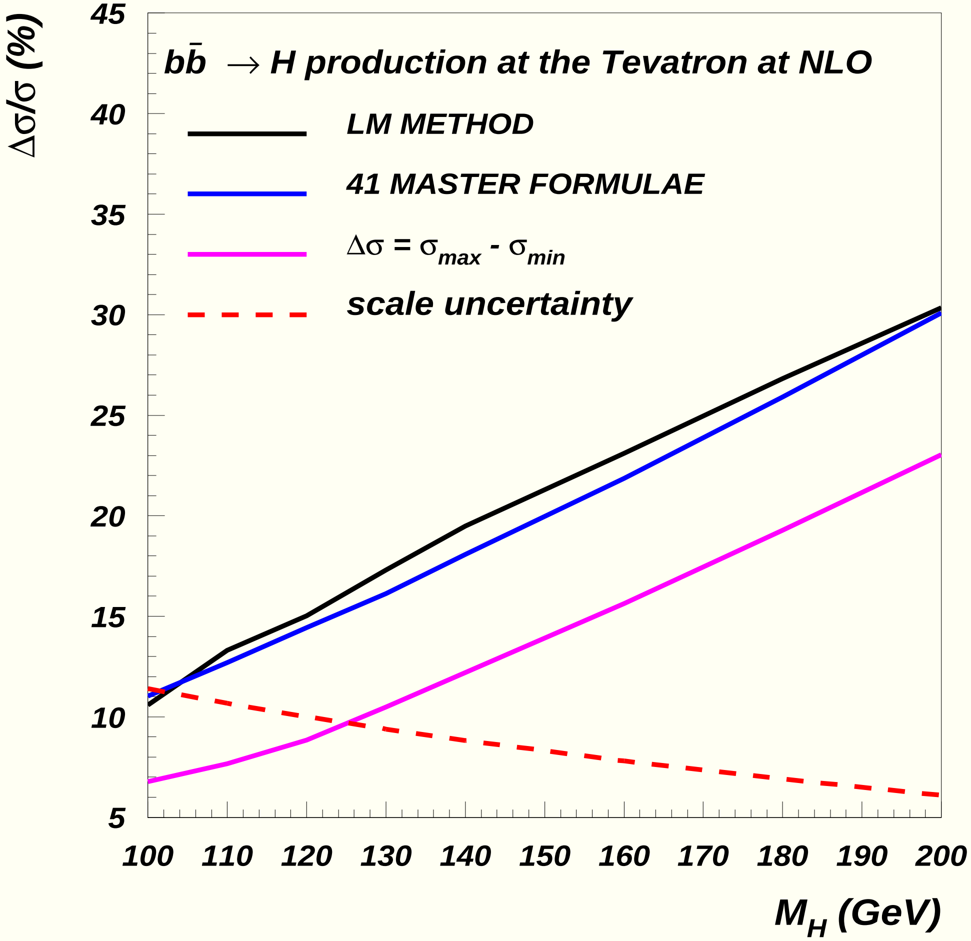
$\Longrightarrow \delta X_\pm = X[\chi^2(a_0, \lambda^\Delta_\pm)] - X[\chi^2(a_0, 0)]$

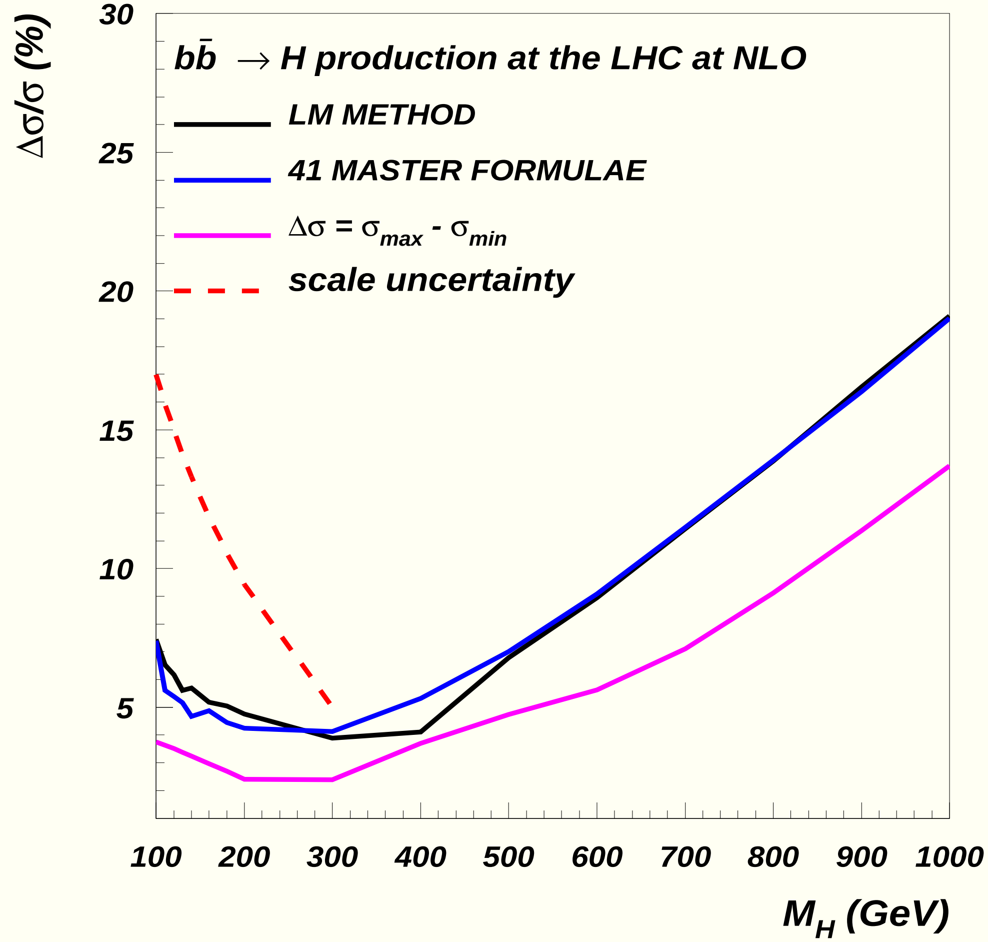




LM method is more robust in general since it does not approximate $X(a)$ and $\chi^2(a)$ by linear and quadratic dependence on $\{a\}$, respectively

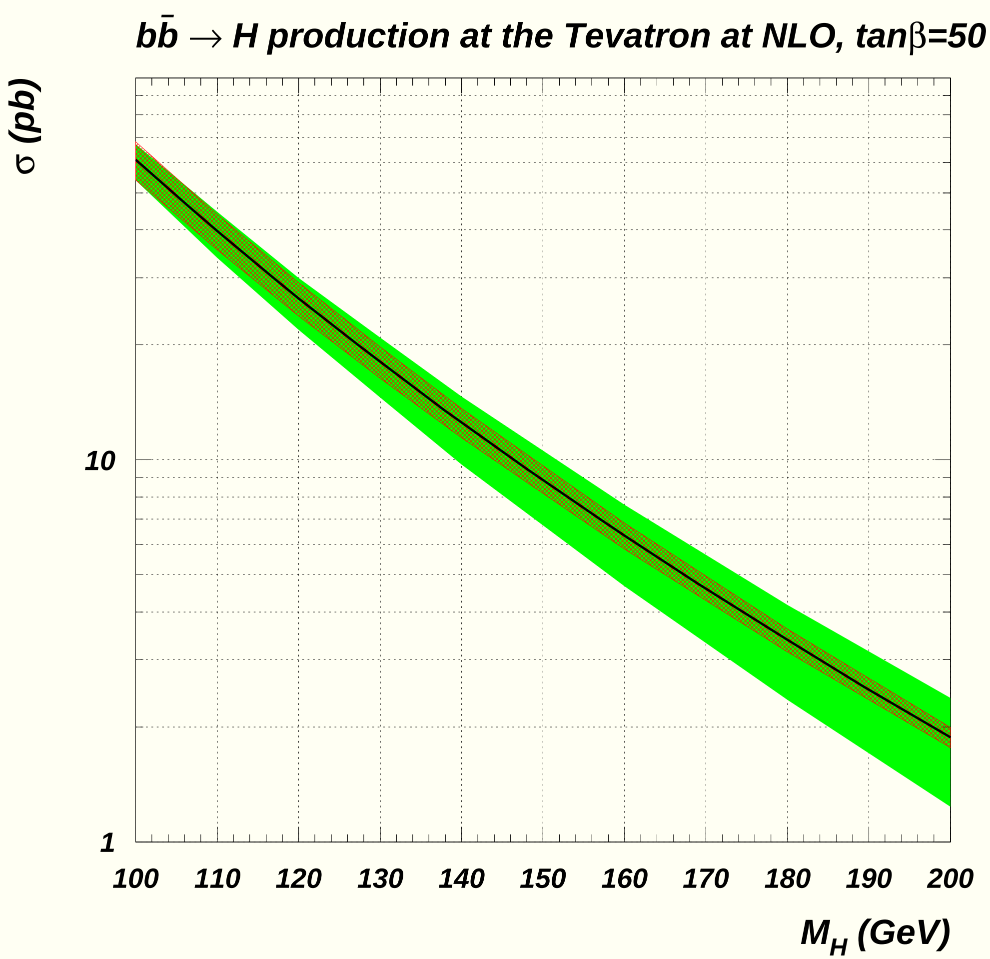
LM versus Hessian method

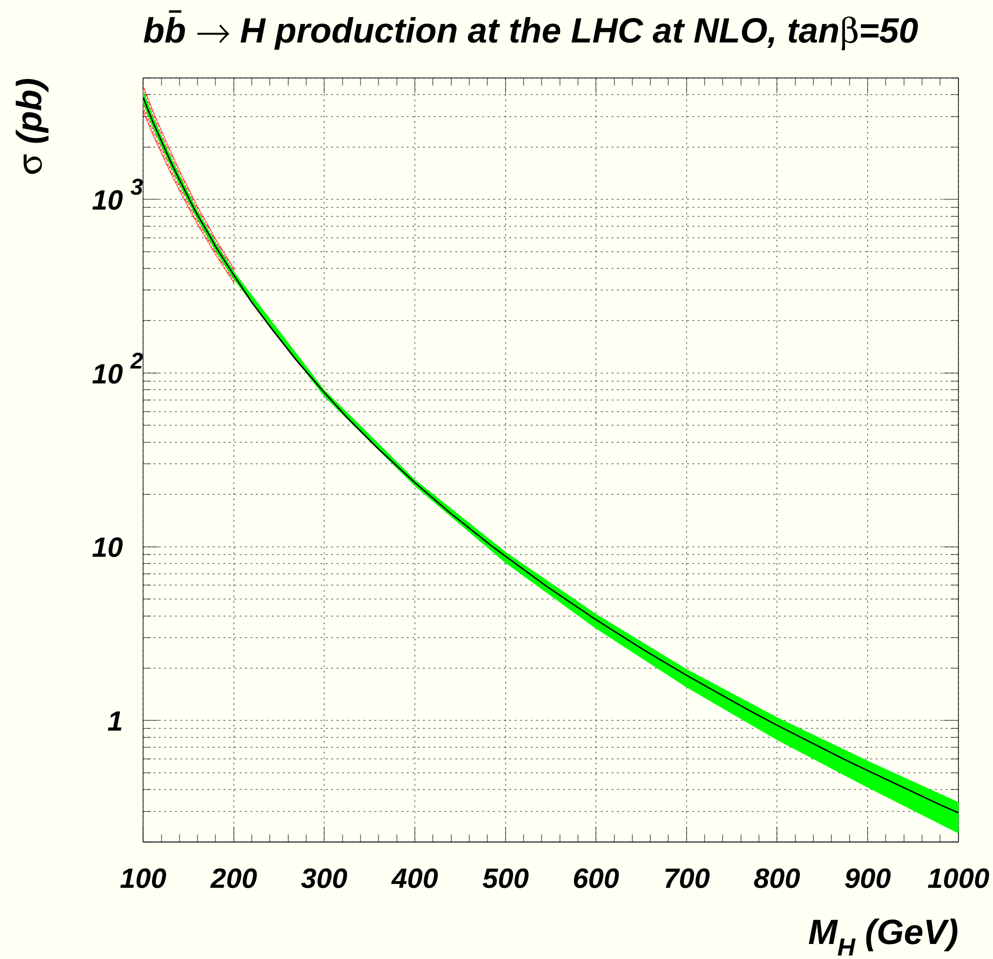




- LM and Hessian results are in a good agreement
- "$\sigma_{max} - \sigma_{min}$" method underestimates PDF uncertainty by about factor 2
- qualitative agreement with $gg \rightarrow H$ (Djouadi,Ferrag) and $gb \rightarrow Hb$ (Dawson,Jackson,Reina,Wakeroth(Tev4LHC)) PDF (41 set) uncert results

Cross section uncertainty band





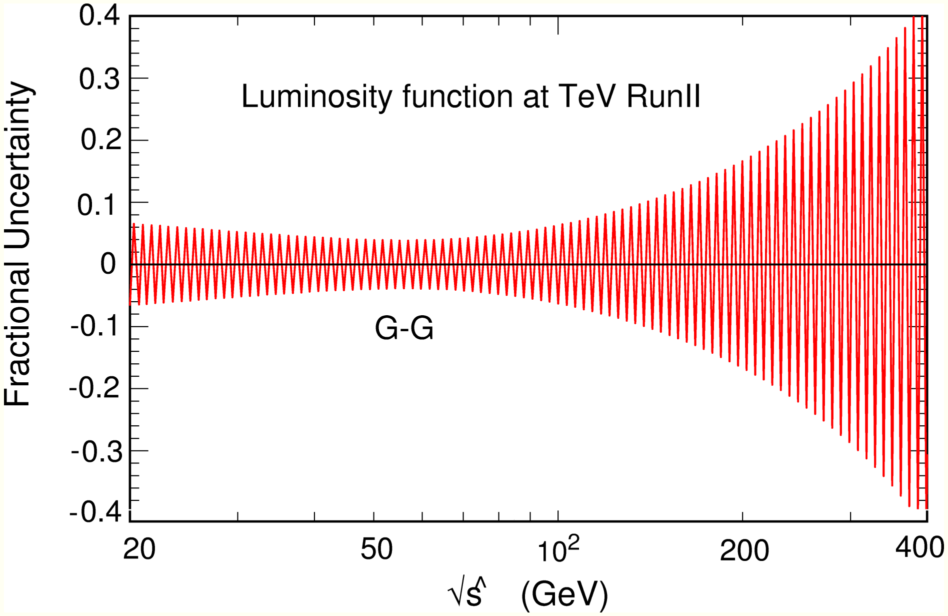
- *PDF uncertainties dominate the scale ones at Tevatron*
- *Scale uncertainties dominate the PDF ones for $M_H < 300$ GeV at LHC – where one could expect high CS and possible precise measurements*
- *the overall uncertainty $25\% \to 10\%$ for M_H=$100 \to 300$ GeV at LHC*

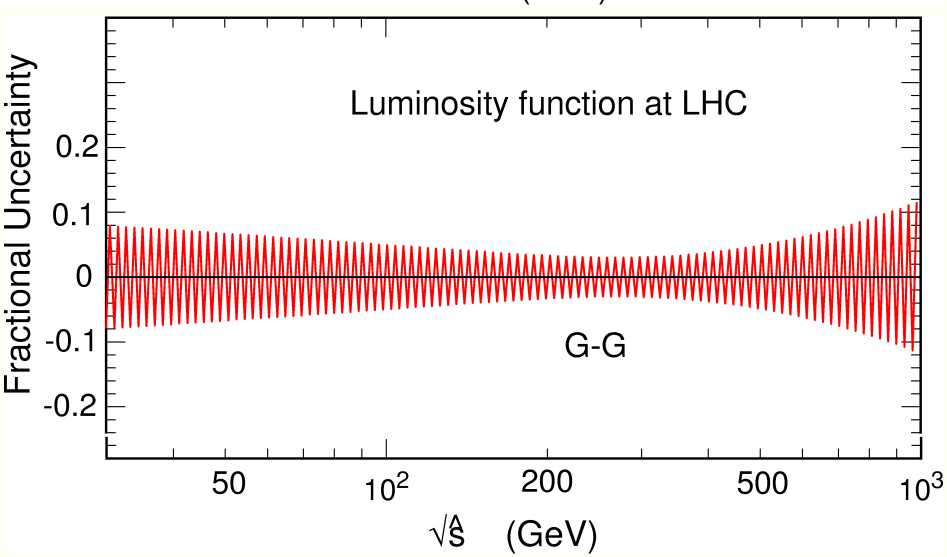
Conclusions and outlook

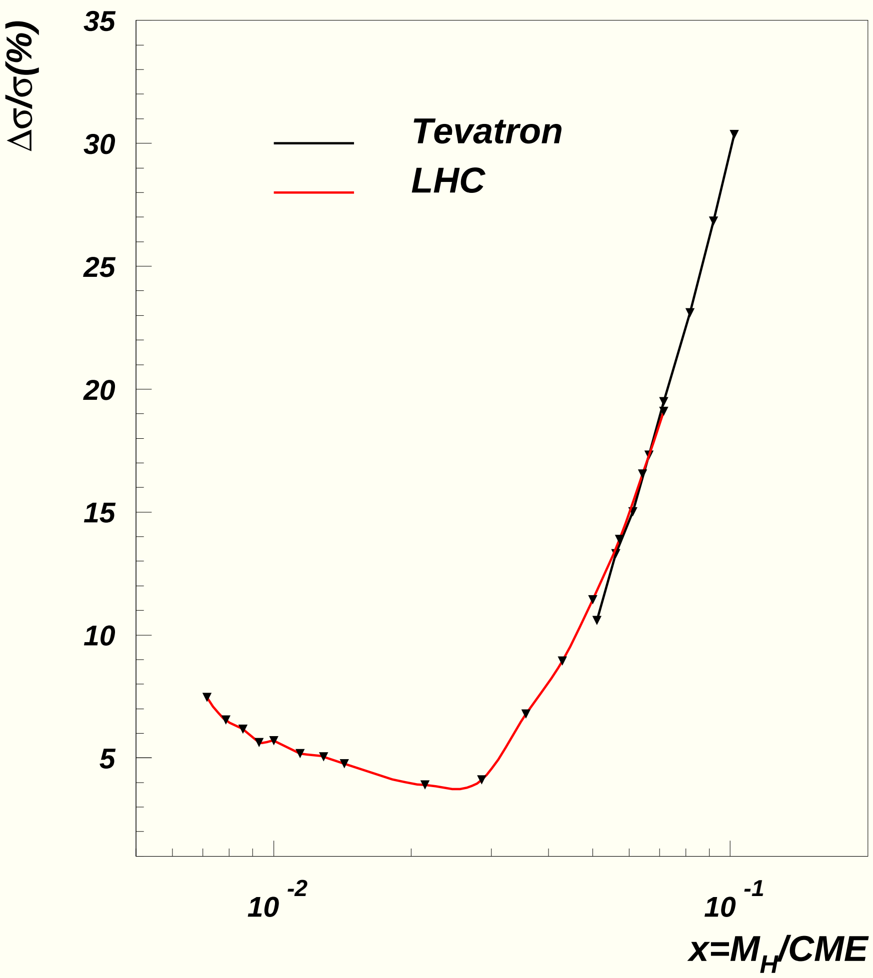
- $b\bar{b} \to H$ process could be the central one for Higgs boson search, (SUSY, 2HDM, Technicolor) $\Rightarrow$ the understanding of this process is crucial
- At Tevatron, PDF uncertainty is dominant, therefore it has a crucial effect on the total uncertainty bringing it to the level of $\sim 20-30\%$ (recent $D\O$ paper on $Hb \to bb\bar{b}$ hep-ex/0504018)
- At LHC, the scale uncertainty is dominant: up to 15% for $M_H < 300$ GeV. In this region one could expect high CS and possible precise measurements – we need better theoretical control of the scale uncertainty in this region
- For $M_H > 300$ GeV, PDF uncertainty becomes dominant at LHC
- Lagrange Multiplier and Hessian methods are in a good agreement, while the method of "two extreeme values" underestimates PDF uncertainty by factor 2
- it is important to apply similar technique for PDF uncertainty of principal shape distributions

Understanding PDF uncertainties

Uncertainty of the gluon-gluon luminosity functions at Tevatron and LHC







History of $Q\bar{Q} \to H$ and its present status

- Eichten,Hinchliffe,Lane,Quigg(84): $t\bar{t} \to H$ at the SSC

  Olness and Tung (87): "When is a heavy quark not a parton?"

  elaborated teqnique for combining $Q\bar{Q} \to H$ with higher order corrections

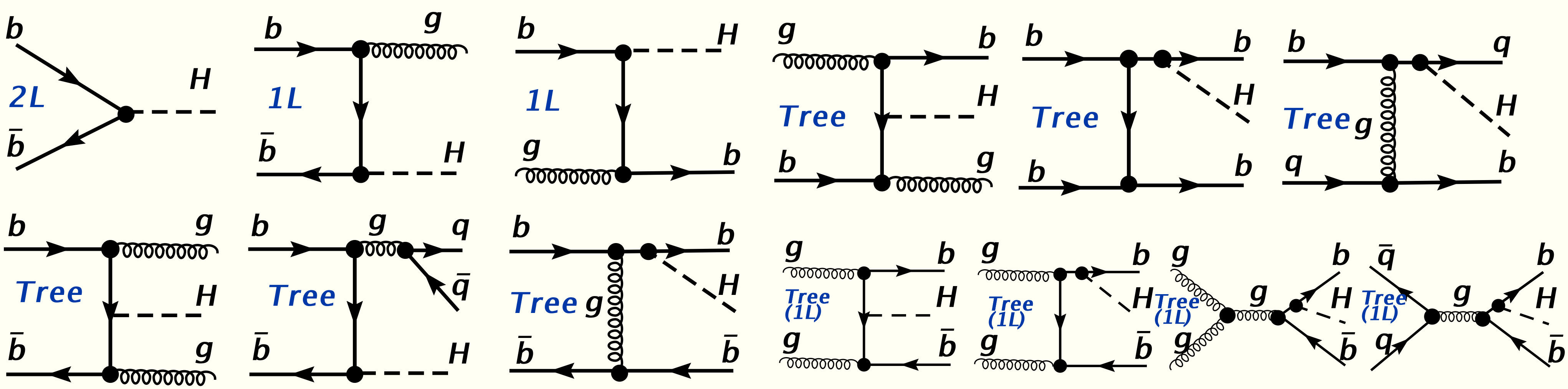
Dicus and Willenbrock(89); Dicus,Stelzer,Sullivan,Willenbrock(99); Balazs,He,Yuan(99);

Campbell,Ellis,Maltoni,Willenbrock(02); Cao,Gao,Oakes,Yang(02)(SUSY); Maltoni,Sullivan,Willenbrock(03);

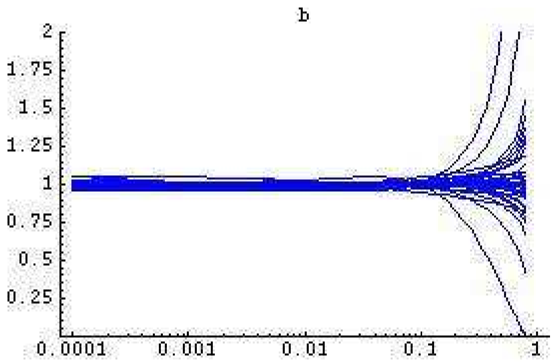
Hou,Ma,Lei,Zhang(03)(SUSY); Ditmaer,Kramer,Spira(03); Hou,Ma,Zhang,Sun,Wu(03);

Dawson,Jackson,Reina,Wackerot(03,04); Boos,Plehn(03); Harlander,Kilgore(03); Kramer(04); Field(04)

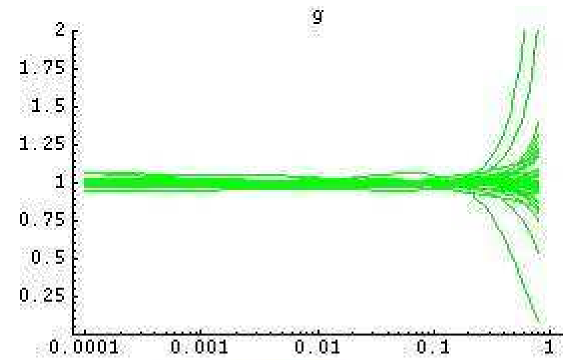
- big progress in understanding $b\bar{b}(gg) \to H(b\bar{b})$ process and reduction of scale uncertainties!

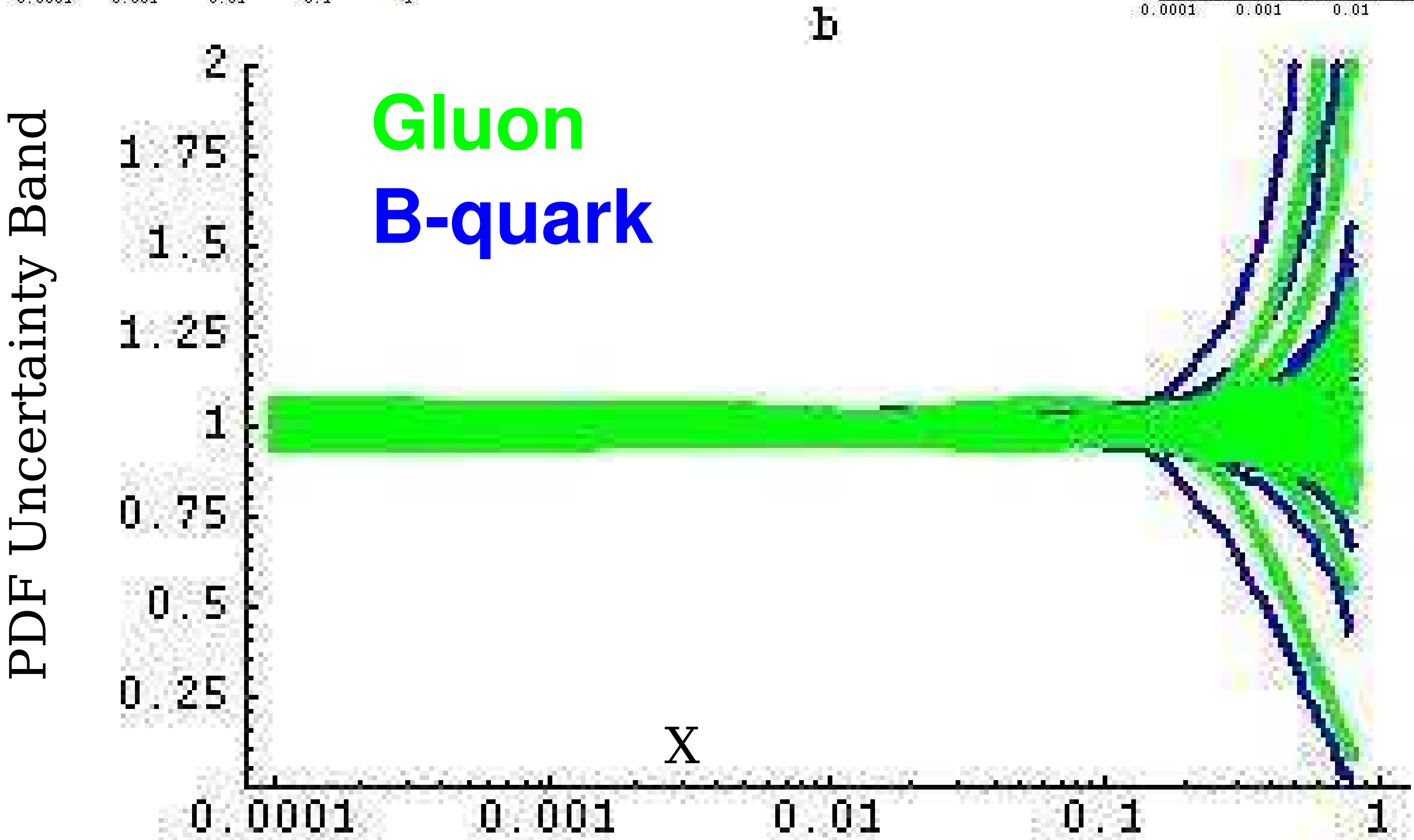


What is PDF Uncertainty???



$$f_b(x,\mu)=f_g\otimes P_{g\to q}$$



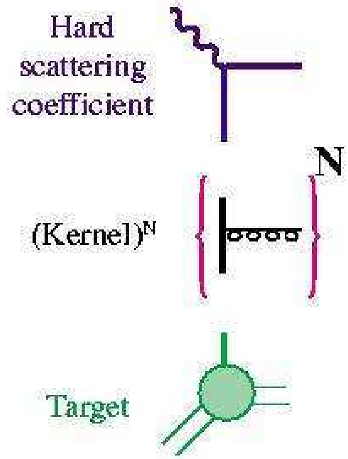


Ingredients of Factorization

Factorization Ingredients

Decompose into (t-channel) 2PI amplitudes:

$$\sigma = \Sigma_{N=1}^{\infty} \; C \; (K)^N \; T \; + O(\Lambda^2_{QCD}/Q^2)$$



New Factorization Proof for Massive Quarks
Collins, PRD58, 1998

After reorganization of the infinite sum:

Parton Model | Remainder

$$\sigma \approx C\,[1 - (1\text{-}Z)\,K\,]^{-1}\,Z\;[1 - K\,]^{-1}\,T + C\,[1 - (1\text{-}Z)\,K\,]^{-1}\,(1\text{-}Z)T$$

Wilson Coefficient (Hard Scatt. $\hat{\sigma}$) — Z collinear projection — Parton Distribution — Power Suppressed

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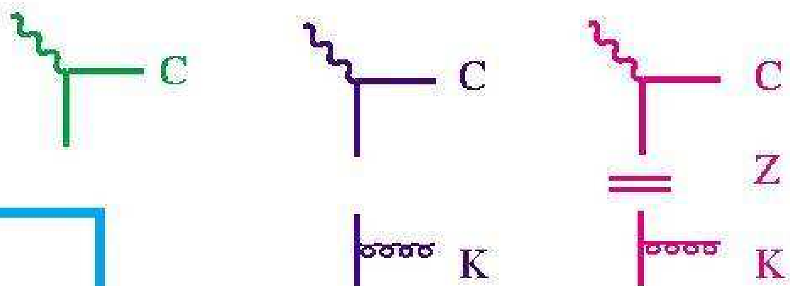
Wilson Coefficient:

Leading Order | Next to Leading Order

$$C\,[1 - (1\text{-}Z)\,K\,]^{-1} \approx C \; + \; C\,K \; - \; C\,Z\,K \; + \ldots$$



- All orders result
- Valid for <u>all</u> masses

C Z K is the subtraction

Unambiguous all-orders definition of Wilson-Coeff and PDFs

Valid for both zero and non-zero mass

(Key is projection operator Z)

Details:

$$f_{i/H}(\xi, \mu^2) = \int_{-\infty}^{\infty} \frac{dy^-}{4\pi} e^{-i\xi p^+ y^-} \langle H(p) | \bar{\psi}_i(0^+, y^-, \vec{0}_\perp) \gamma^+ \mathcal{P} \psi_i(0^+, 0^-, \vec{0}_\perp) | H(p) \rangle$$

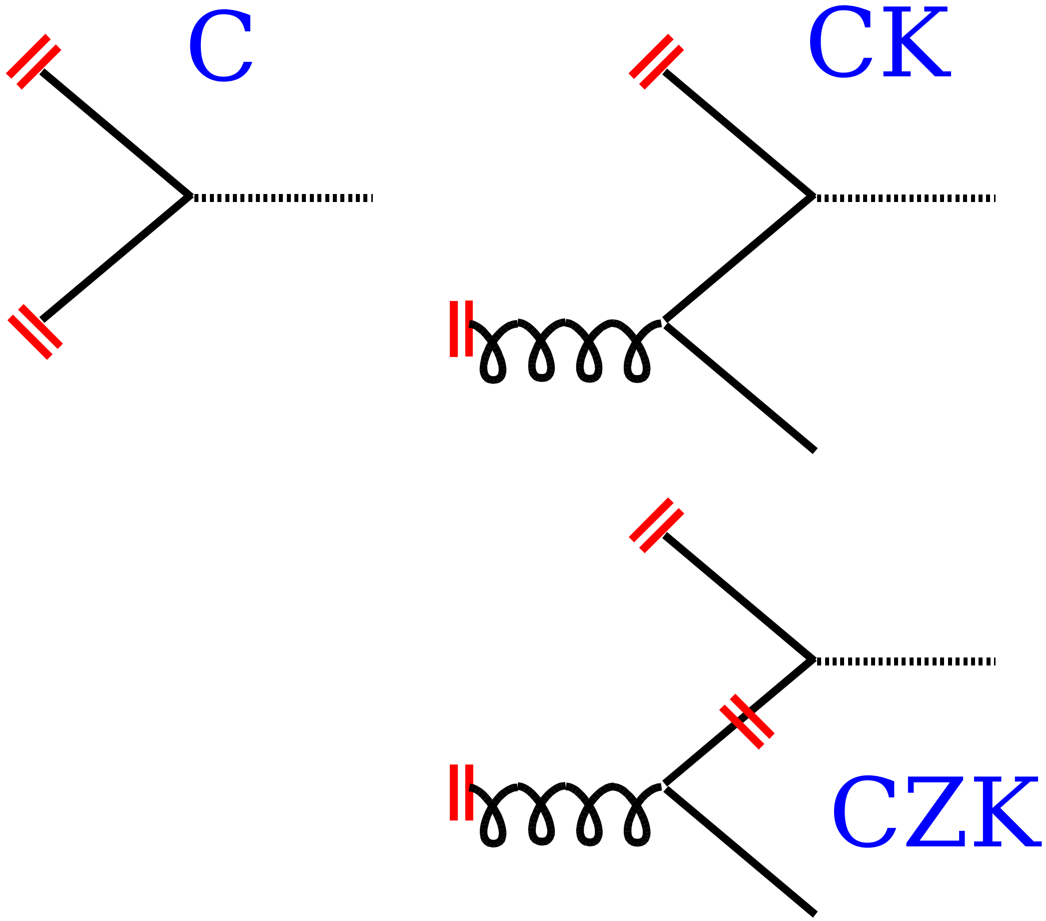
$$\mathcal{P} = P e^{-ig \int_0^{y^-} dy'^- A_a^+(0^+, y'^-, \vec{0}_\perp) t_a}$$

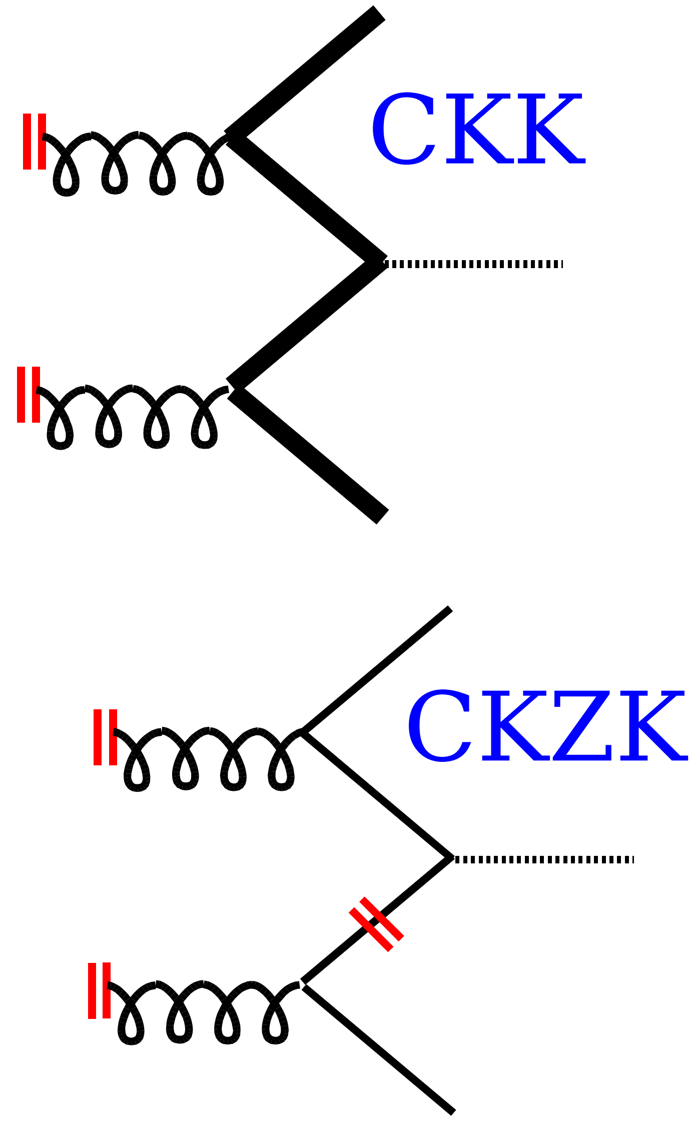
where Z is a collinear projection operator: $Z^2 = Z$, and $Z(1-Z) = 0$,

$$Z = \frac{1}{4} \gamma^-_{\alpha\alpha'} \gamma^+_{\beta\beta} (2\pi)^4 \delta(k^+ - \ell^+) \delta(k^-) \delta^2(\vec{k}_T^2)$$

Extend to massive case:

$$Z = \frac{1}{4} \left(\frac{k \cdot \gamma^-_{\alpha\alpha'} + m}{k^+} \right) \gamma^+_{\beta\beta} (2\pi)^4 \delta(k^+ - \ell^+) \delta(k^- - m^2/2k^+) \delta^2(\vec{k}_T^2)$$



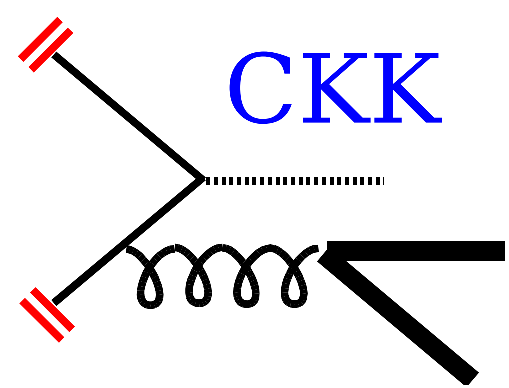


$C[1-(1-Z)K]^{-1} =$

$+ C$

$+ CK - CZK$

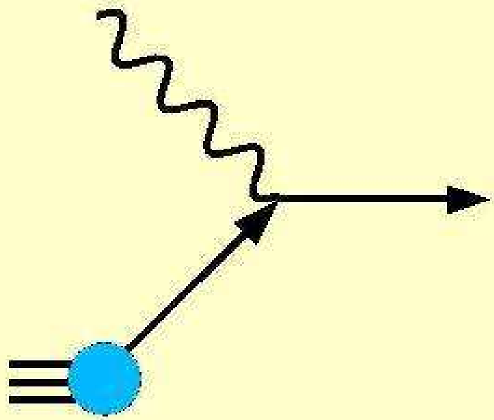
$+ CKK - CKZK - CZKK + CZKZK$

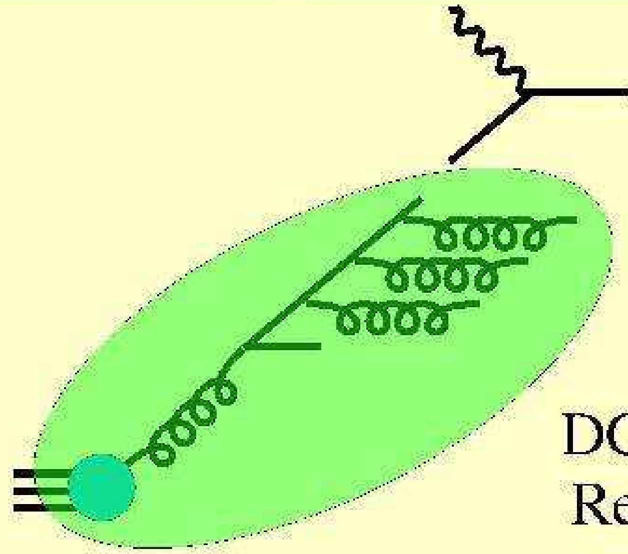


Mass-Independent
vs.
Mass Dependent
PDF's

... or why the mass doesn't matter in the evolution

DGLAP Equation and the Heavy Quark PDF





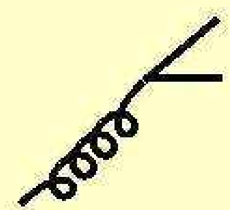
DGLAP equation
Resums iterative
splittings inside
the proton

$$\mathrm{HE} = \int f(P \to a) \otimes \sigma(a \to c)$$

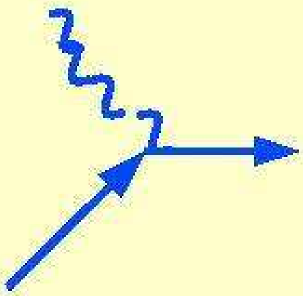
DGLAP Equation
$$\frac{df_i}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\ {}^1P_{j\to i} \otimes f_j + \ldots$$

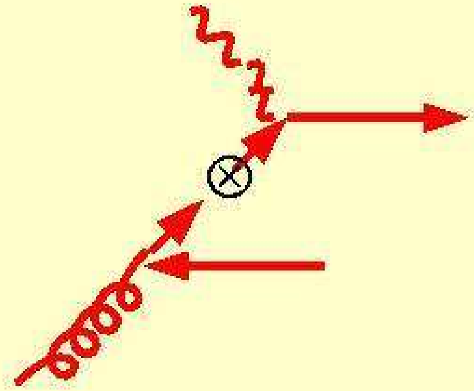
Splitting Function

$${}^1P_{g\to q} = \frac{1}{2}[x^2 + (1-x)^2] + \left(\frac{M_H^2}{\mu^2}\right)[x(1-x)]$$



Effect of Heavy Quark Mass in the Calculation





$$\mathrm{HE} = \int \underbrace{f(P \to a)}\otimes \sigma(a \to c)$$

$$\mathrm{SUB} = \int f(P \to g) \otimes {}^{1}P(g \to a) \otimes \sigma(a \to c)$$

$$\approx \; f(P \to g) \otimes {}^{1}P(g \to a)$$

valid near threshold ($M_H \sim Q$)

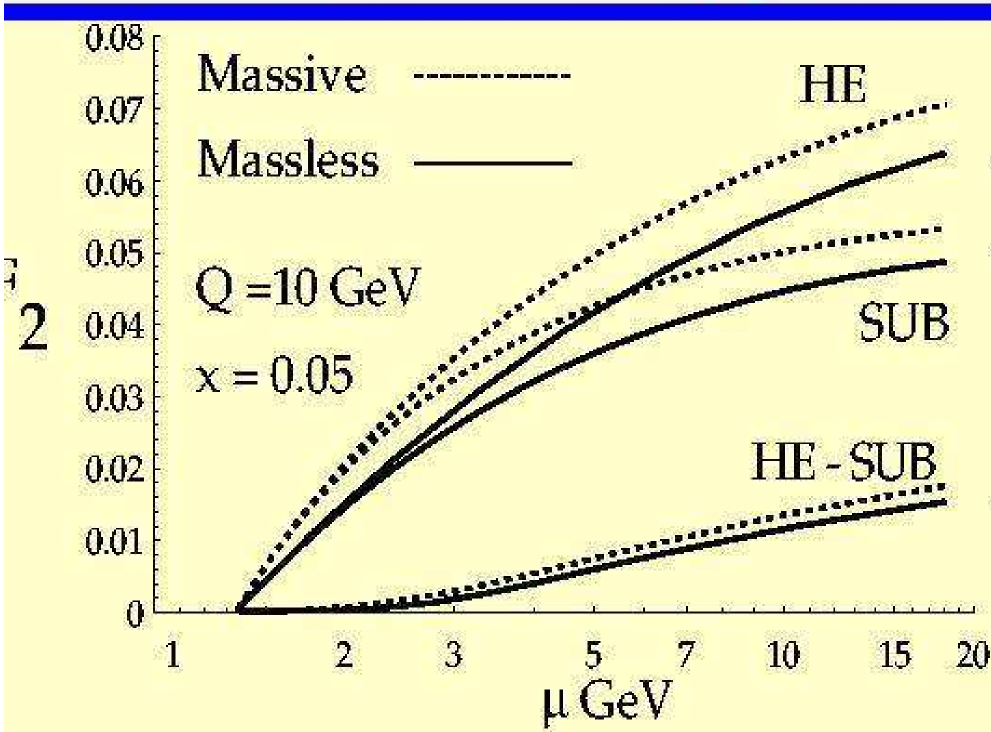
${}^{1}P$ splittings must match

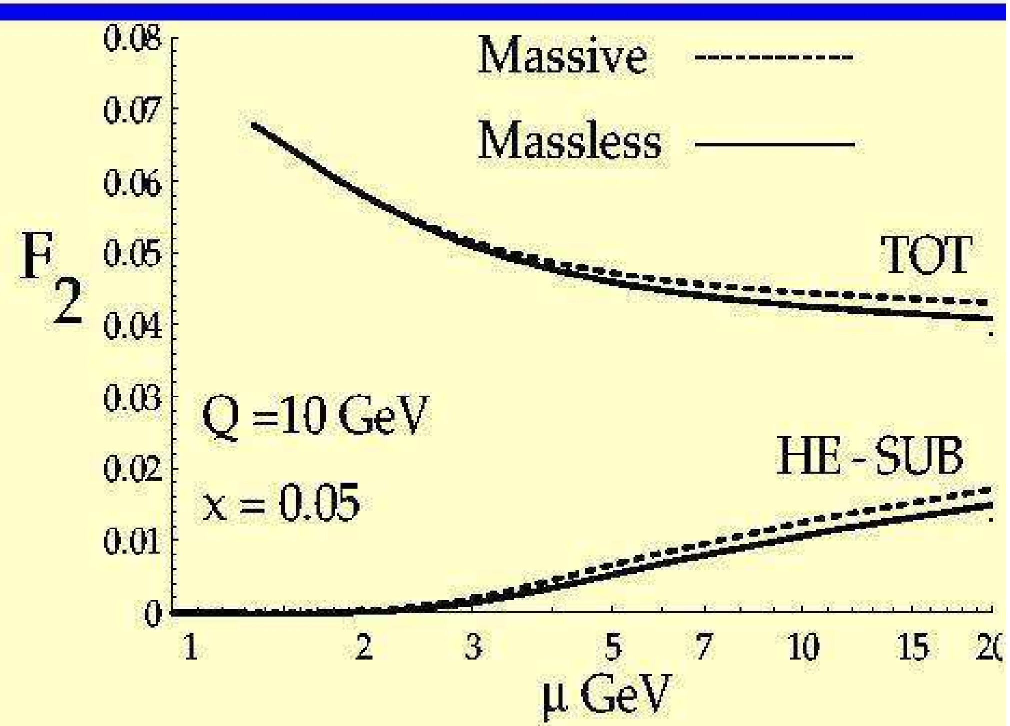
In Summary:

Near threshold($M_H \sim Q$), mass effects cancel between HE and SUB

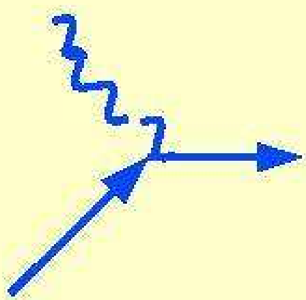
Above threshold($M_H \ll Q$), mass effects can be ignored

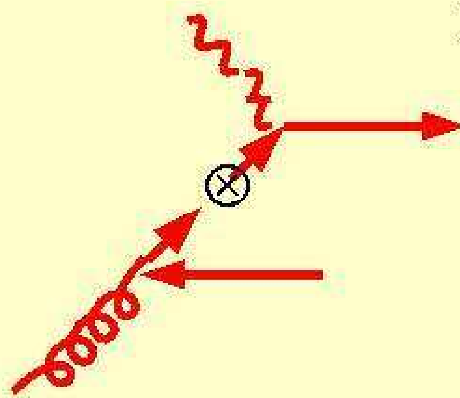
Effect of Heavy Quark Mass in the Calculation is Trivial



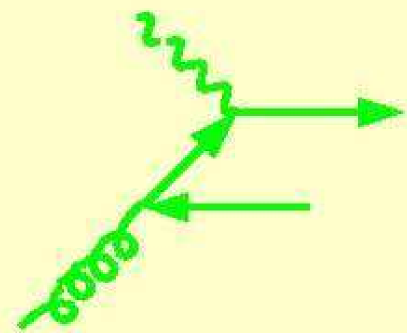


$$\text{HE} = \int f(P \to a) \otimes \sigma(a \to c)$$





$$\text{HC} = \int f(P \to g) \otimes \sigma(g \to c)$$

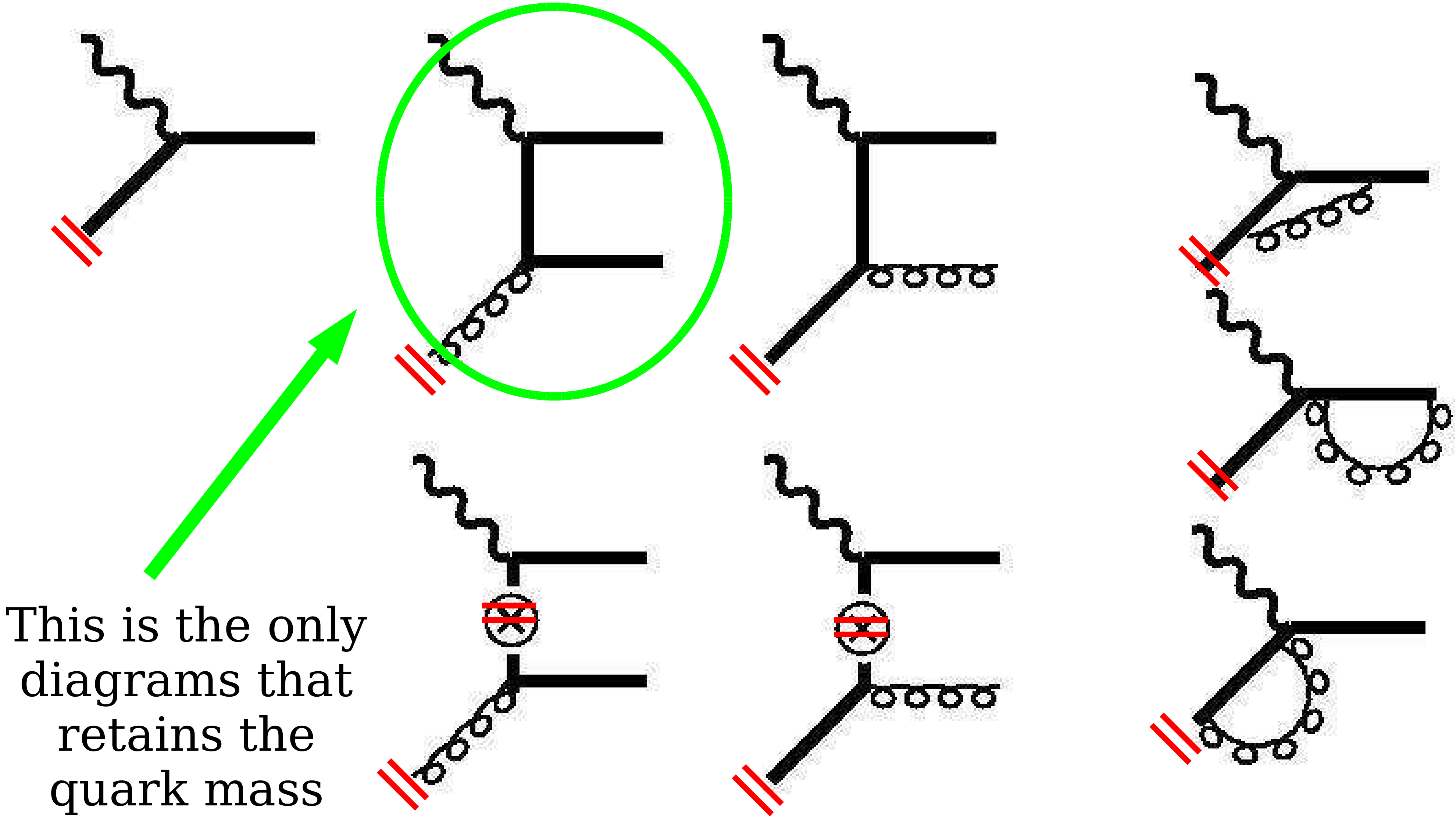


$$\text{SUB} = \int f(P \to g) \otimes {}^{1}P(g \to a) \otimes \sigma(a \to c)$$

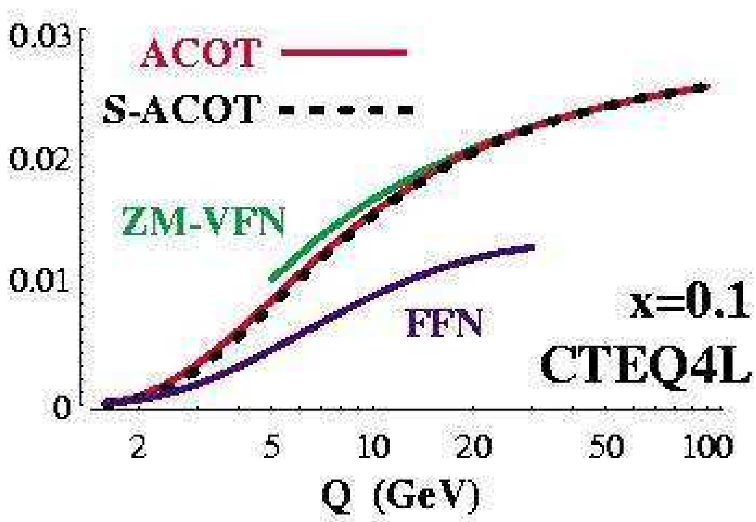
Simplified ACOT

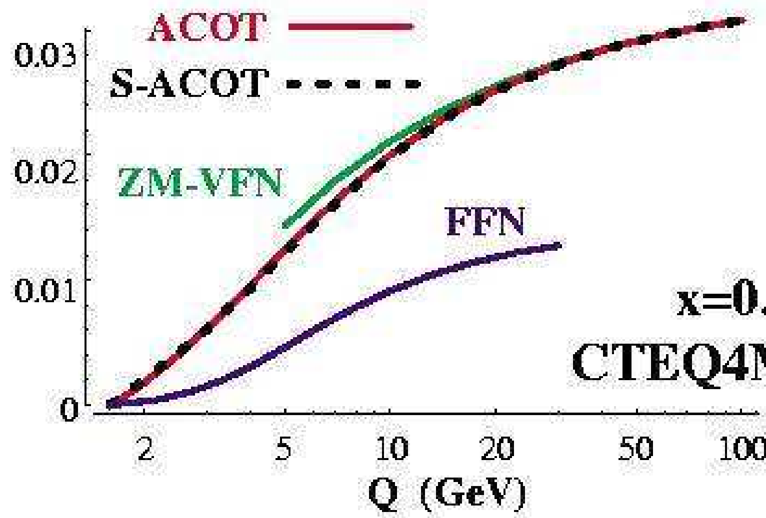
... or why the mass doesn't matter in the matrix element

Simplified-ACOT Scheme for Heavy Quarks

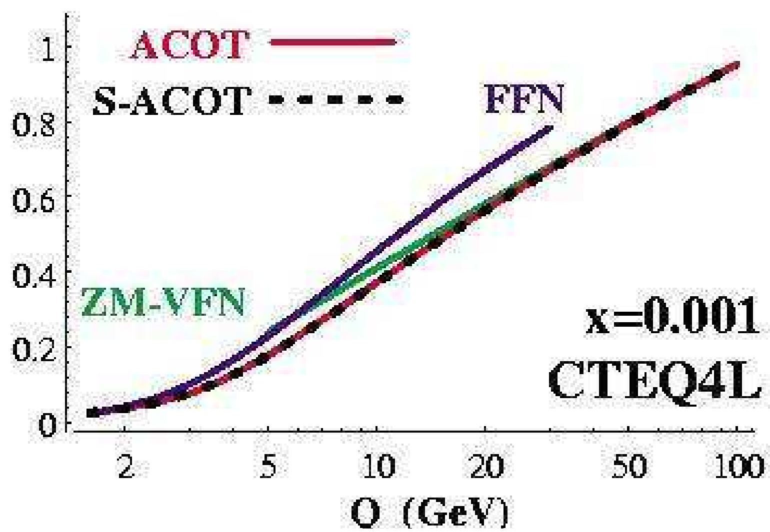


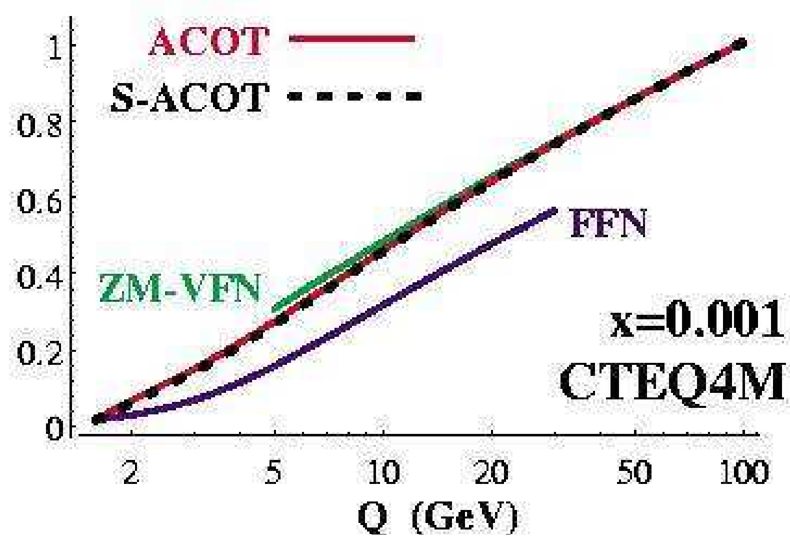
How does S-ACOT Compare???

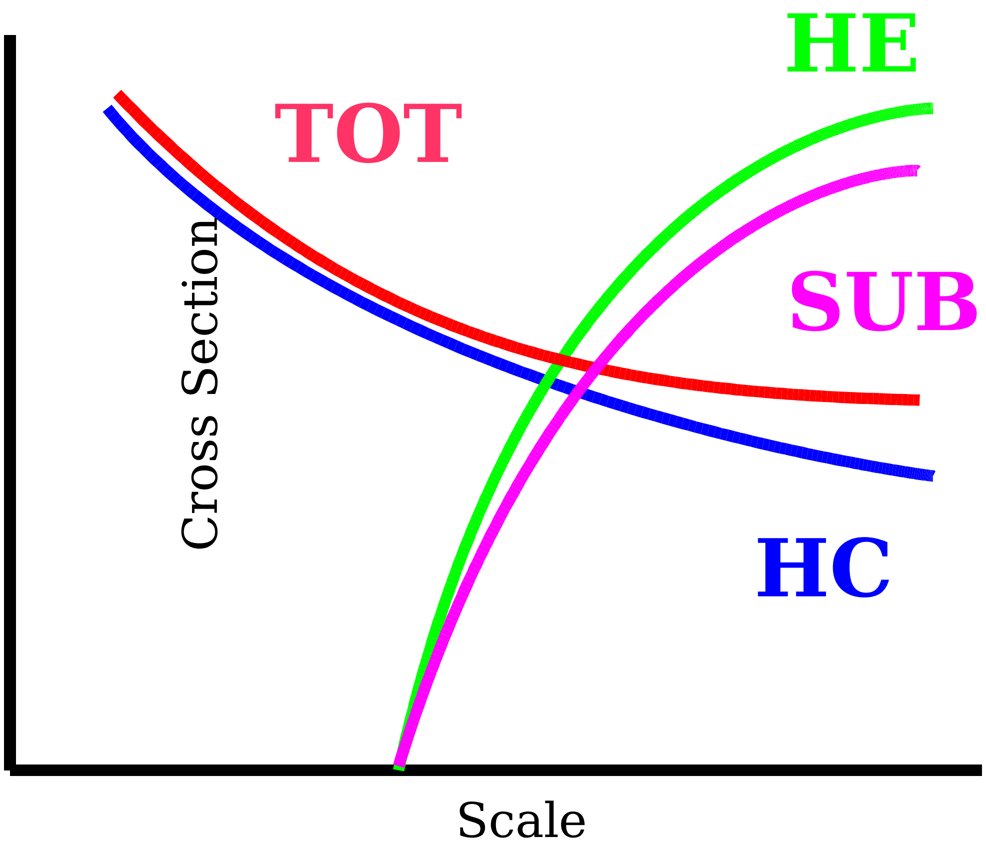




Retention of mass terms provides no additional information

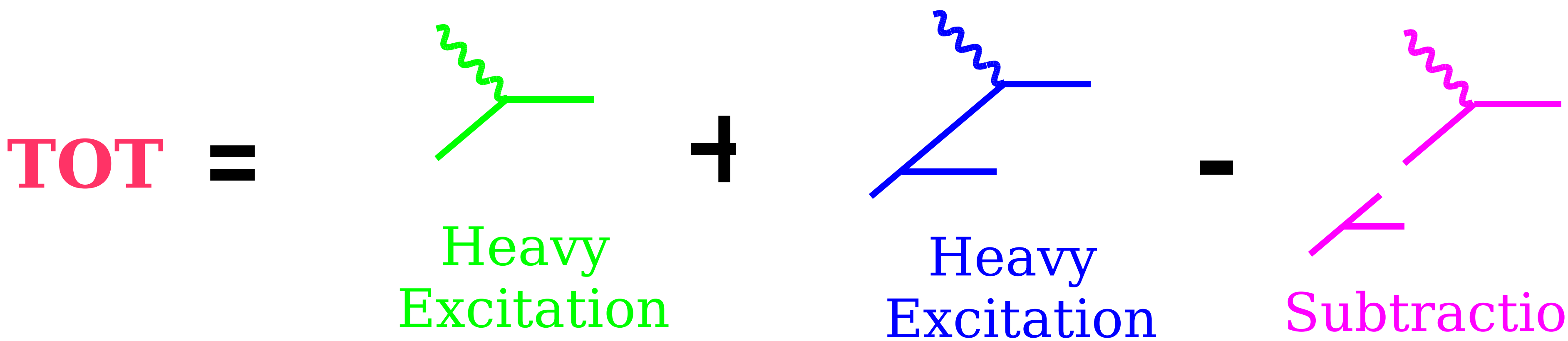




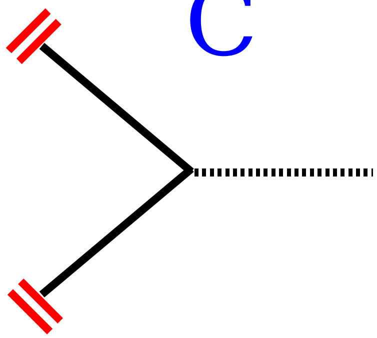


Benefit of using the heavy quark PDF:

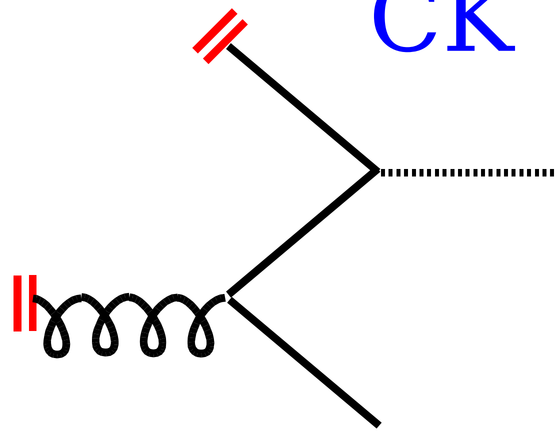
Typically the gluon and heavy quark have opposite dependence



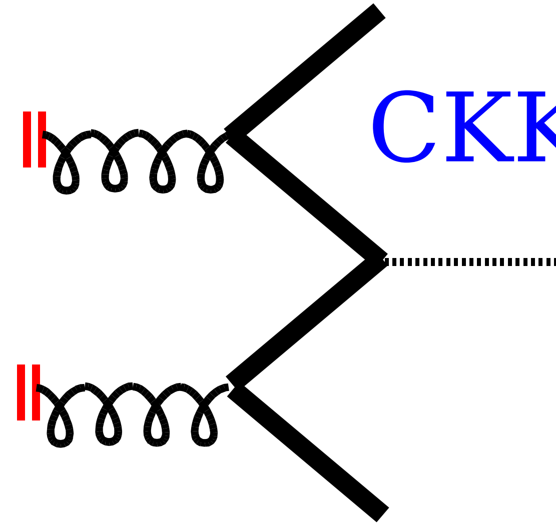
Issues for $bb \rightarrow H$



C

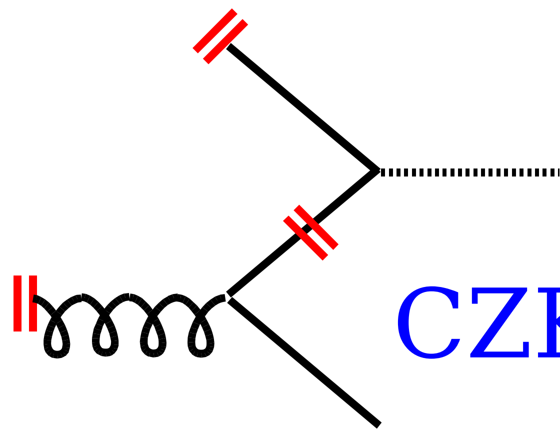


CK

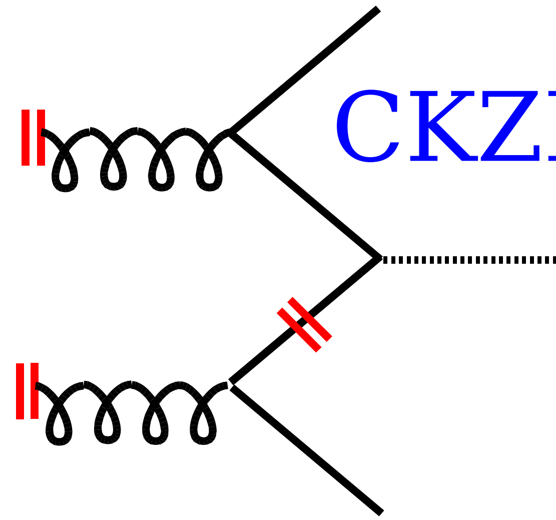


CKK

In S-ACOT scheme, mass is only retained in some $O(a_s^2)$ diagrams



CZK



CKZK

Errors will be $O(a_s^3)$ and $O(L^2/Q^2)$

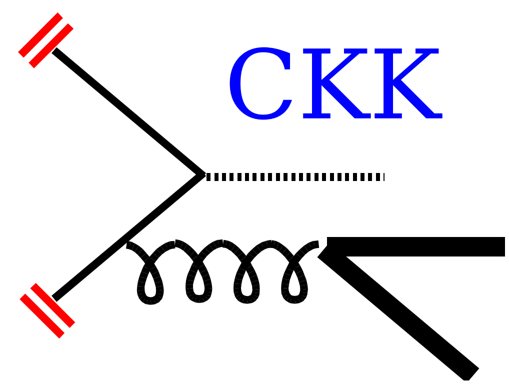
$$C[1-(1-Z)K]^{-1} =$$

$+ C$

$+ CK - CZK$

$+ CKK - CKZK - CZKK + CZKZK$

Note: massless incoming particles satisfies Bloch-Nordsieck



CKK

Conclusions:

* PDF uncertainties can be dominant

  gluon & b PDF's closely related: $f_b = f_g \otimes P_{g \to b}$

* Mass-Independent PDF Evolution

  No benefit by including masses

* ACOT vs. Simplified-ACOT

  No benefit by including masses

* S-ACOT calculation of bb→ H:

  Errrrrors of order: $O(a_s^3)$ and $O(L^2/Q^2)$

  No errors of order: $O(m^2/Q^2)$

  Massless initial state ⇒ Block-Nordsieck satisfied