Electroweak Precision Observables in the MSSM: New Results on Two-Loop Yukawa Corrections

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Based on collaboration with J. Haestier, S. Heinemeyer, D. Stöckinger, hep-ph/0508139

1. Introduction
2. Evaluation of the relevant contributions
3. Numerical results
4. Estimate of remaining uncertainties from unknown higher-orders
5. Conclusions

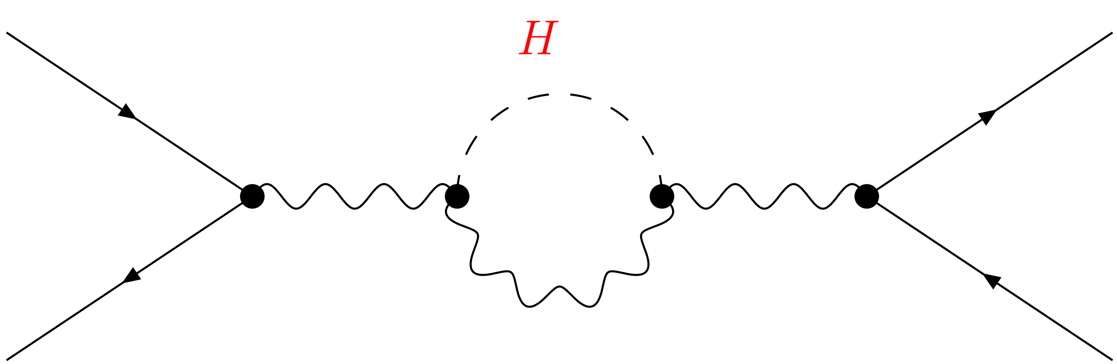
1. Introduction

EW precision data:
$M_Z, M_W, \sin^2\theta_{\text{eff}}^{\text{lept}}, \ldots$

Theory:
SM, MSSM, ...

$\Downarrow$

Test of theory at quantum level: sensitivity to loop corrections



$\Downarrow$

Indirect constraints on unknown parameters: M_H, $m_{\tilde{t}}$, ...

Effects of "new physics"?

Theoretical predictions for M_W, $\sin^2\theta_\mathrm{eff}$:

Comparison of prediction for muon decay with experiment (Fermi constant G_μ)

$$\Rightarrow \quad M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}\left(1 + \underbrace{\Delta r}_{\Updownarrow}\right),$$

loop corrections

$\Rightarrow$ Theo. prediction for M_W in terms of M_Z, α, G_μ, $\Delta r(m_\mathrm{t}, m_{\tilde{\mathrm{t}}}, \ldots)$

Effective couplings at the Z resonance:

$$\Rightarrow \quad \sin^2\theta_\mathrm{eff} = \frac{1}{4}\left(1 - \mathrm{Re}\,\frac{g_V}{g_A}\right) = \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right)\mathrm{Re}\,\kappa_l(s = M_\mathrm{Z}^2)$$

Leading contributions to precison observables

SM result for M_W, one-loop: [*A. Sirlin '80*], [*W. Marciano, A. Sirlin '80*]

$$\begin{array}{ccccccc} \Delta r_{1-\mathrm{loop}} = & \Delta\alpha & - & \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho & + & \Delta r_\mathrm{rem}(M_\mathrm{H}) \\ & \sim \log\frac{M_\mathrm{Z}}{m_f} & & \sim m_\mathrm{t}^2 & & \\ & \sim 6\% & & \sim 3.3\% & & \sim 1\% \end{array}$$

Leading contributions to M_W, $\sin^2\theta_\mathrm{eff}$, ... from mass splitting between isospin doublet fields enter via

$$\Delta\rho = \frac{\Sigma_\mathrm{Z}(0)}{M_\mathrm{Z}^2} - \frac{\Sigma_\mathrm{W}(0)}{M_\mathrm{W}^2}$$

$$\Rightarrow \quad \Delta M_\mathrm{W} \approx \frac{M_\mathrm{W}}{2}\frac{c_\mathrm{W}^2}{c_\mathrm{W}^2 - s_\mathrm{W}^2}\Delta\rho, \qquad \Delta\sin^2\theta_\mathrm{eff} \approx -\frac{c_\mathrm{W}^2 s_\mathrm{W}^2}{c_\mathrm{W}^2 - s_\mathrm{W}^2}\Delta\rho$$

Theoretical uncertainties: current status

- From experimental errors of input parameters

$$\delta m_{\mathrm{t}} = 2.9\ \mathrm{GeV} \;\Rightarrow\; \Delta M_{\mathrm{W}}^{\mathrm{para}} \approx 18\ \mathrm{MeV}, \quad \Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{para}} \approx 9 \times 10^{-5}$$

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- From unknown higher-order corrections (“intrinsic”)

SM: Complete 2-loop result $+$ leading higher-order corrections known for $M_{\rm W}$, complete 2-loop fermionic corr. $+$ leading higher-order corrections known for $\sin^2\theta_{\rm eff}$

$\Rightarrow$ Remaining uncertainties:
[*M. Awramik, M. Czakon, A. Freitas, G.W. ’03, ’04*]

$$\Delta M_{\rm W}^{\rm intr} \approx 4 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff}^{\rm intr} \approx 5 \times 10^{-5}$$

Higher-order corrections in the MSSM

Only known higher-order SUSY corrections to M_W, $\sin^2\theta_\mathrm{eff}$:

- $\mathcal{O}(\alpha\alpha_\mathrm{s})$ corrections to $\Delta\rho$ ($+$ gluonic corrections to Δr)
  [*A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97*]
  [*S. Heinemeyer, W. Hollik, G. W. '98*]

- $\mathcal{O}(\alpha_\mathrm{t}^2)$, $\mathcal{O}(\alpha_\mathrm{t}\alpha_\mathrm{b})$, $\mathcal{O}(\alpha_\mathrm{b}^2)$ Yukawa corrections to $\Delta\rho$ in limit $M_\mathrm{SUSY} \to \infty$ (SUSY loop contributions decouple)
  [*S. Heinemeyer, G. W. '02*]

  $\Rightarrow$ well approximated by SM contribution

  $\Rightarrow$ SUSY loop contribution potentially larger (no SM counterpart)

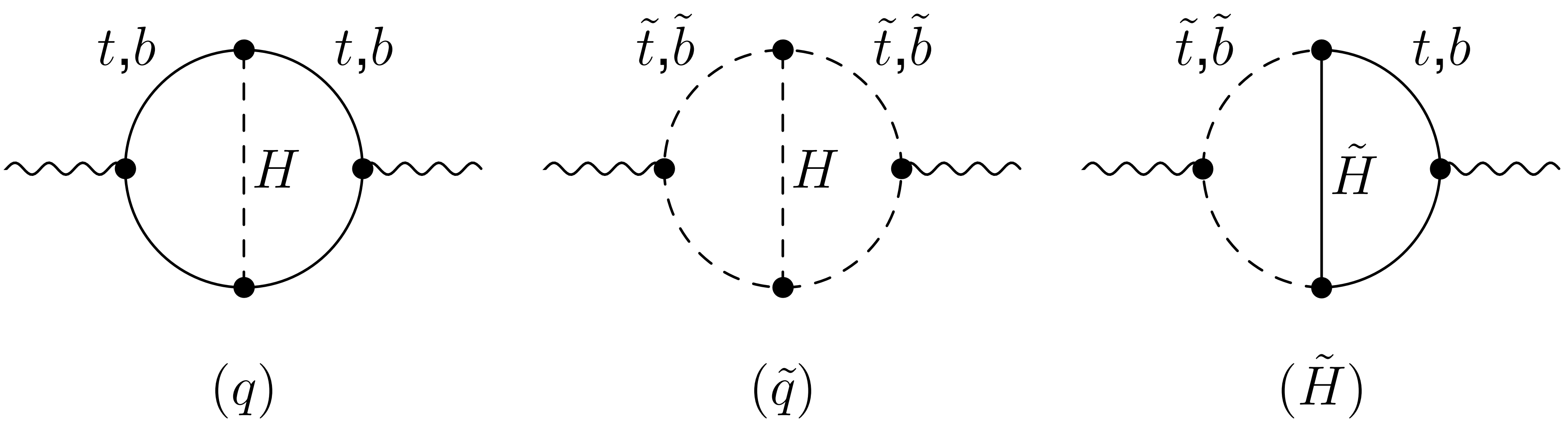
$\Rightarrow$ Intrinsic theoretical uncertainties can be much larger than in the SM

Two-loop Yukawa corrections to electroweak precision observables in the MSSM

Calculation of complete $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t \alpha_b)$, $\mathcal{O}(\alpha_b^2)$ Yukawa corrections to $\Delta\rho$ in the MSSM:

[J. Haestier, S. Heinemeyer, D. Stöckinger, G. W. '05]

– quark loops with Higgs exchange,
– squark loops with Higgs exchange,
– quark/squark loops with Higgsino exchange



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Counter example: one-loop bosonic contributions in the SM; contribution to $\Delta\rho$ is neither UV-finite nor gauge-independent

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Counter example: one-loop bosonic contributions in the SM; contribution to $\Delta\rho$ is neither UV-finite nor gauge-independent

$\Rightarrow$ Need a consistent prescription for extracting leading higher-order contributions

The gauge-less limit

Yukawa couplings of top and bottom quarks:

$$y_t = \frac{\sqrt{2}\, m_\mathrm{t}}{v\ \sin\beta}, \qquad y_b = \frac{\sqrt{2}\, m_\mathrm{b}}{v\ \cos\beta}$$

$\Rightarrow$ leading Yukawa corrections can be obtained in gauge-less limit:

$$g_{1,2} \to 0, \qquad M_\mathrm{W}^2 = \frac{1}{2} g_2^2 v^2 \to 0, \qquad M_\mathrm{Z}^2 = \frac{1}{2}(g_1^2 + g_2^2) v^2 \to 0,$$

$$c_\mathrm{w} \equiv \frac{M_\mathrm{W}}{M_\mathrm{Z}} : \ \text{fixed}, \qquad v : \ \text{fixed}$$

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⇒ in MSSM, 1-loop: only fermion/sfermion contributions

Higgs sector of general 2HDM contributes
contribution in MSSM vanishes due to symmetry relations

Two-loop Yukawa corrections: $\mathcal{O}(\alpha_f^2)$

Gauge-less limit at 2-loop yields 2-loop Yukawa corr. $\mathcal{O}(\alpha_f^2)$

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$$\Delta\rho^{\mathrm{SM},\alpha_t^2}_{2-\mathrm{loop}|M_\mathrm{H}=0} = 3\,\frac{G_\mu^2}{128\pi^4}\,m_\mathrm{t}^4\,\left(19 - 2\pi^2\right)$$

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SM $\mathcal{O}(\alpha_t^2)$ result for arbitrary M_{H} [*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '92*], [*J. Fleischer, O. Tarasov, F. Jegerlehner '93*]

$\Rightarrow$ much bigger correction for realistic values of $M_{\mathrm{H}}^{\mathrm{SM}}$

Consequences of the gauge-less limit

- Higgs sector:

$$M^2_{\mathrm{H}^\pm} = M^2_{\mathrm{H}} = M^2_{\mathrm{A}}, \quad \sin\alpha = -\cos\beta, \quad \cos\alpha = \sin\beta$$

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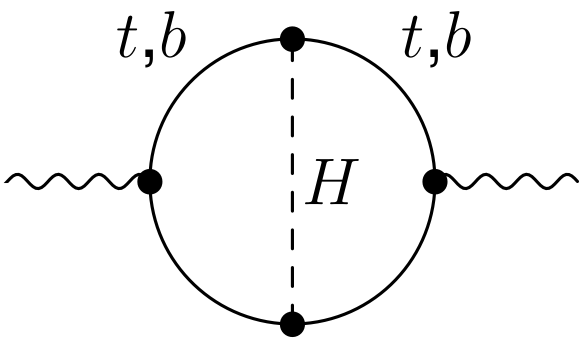
- Higgsino sector:

Diagonal mass matrices of charginos, neutralinos simplify to

$$m_{\tilde{\chi}_i^\pm} = (0, +\mu), \quad m_{\tilde{\chi}_i^0} = (0, 0, +\mu, -\mu)$$

M_{h} *dependence: pure fermion contributions*

Full M_{h} dependence can be kept (i.e. can use true MSSM value for M_{h}) in pure fermion contributions of class (q):



(q)

t/b loops with Higgs and Goldstone boson exchange

[*S. Heinemeyer, G. W. '02*]

Reason: diagrams $+$ counterterm contribution of class (q) correspond to a special case of a general 2HDM (where M_{h} is a free parameter)

[*J. Haestier, S. Heinemeyer, D. Stöckinger, G. W. '05*]

Renormalisation in the $\tilde{\mathrm{t}}/\tilde{\mathrm{b}}$ *sector*

Mass matrices in the stop/sbottom sector:

$$\mathcal{M}^2_{\tilde{\mathrm{t}}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m^2_{\mathrm{t}} + \cos 2\beta\,(\frac{1}{2} - \frac{2}{3}s^2_{\mathrm{W}})M^2_{\mathrm{Z}} & m_{\mathrm{t}} X_{\mathrm{t}} \\ m_{\mathrm{t}} X_{\mathrm{t}} & M^2_{\tilde{t}_R} + m^2_{\mathrm{t}} + \frac{2}{3}\cos 2\beta\; s^2_{\mathrm{W}} M^2_{\mathrm{Z}} \end{pmatrix},$$

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$\Rightarrow$ relation between $m_{\tilde{\mathrm{t}}_1}$, $m_{\tilde{\mathrm{t}}_2}$, $\theta_{\tilde{\mathrm{t}}}$, $m_{\tilde{\mathrm{b}}_1}$, $m_{\tilde{\mathrm{b}}_2}$, $\theta_{\tilde{\mathrm{b}}}$

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$\Rightarrow$ not all parameters can be renormalised independently

$\Rightarrow$ choose $\delta m^2_{\tilde{\mathrm{b}}_1}$ as dependent counterterm

$\Rightarrow$ $\delta m^2_{\tilde{\mathrm{b}}_1}\big|_{\mathrm{symm}} = f(\delta m_{\tilde{\mathrm{t}}_1}, \delta m_{\tilde{\mathrm{t}}_2}, \delta\theta_{\tilde{\mathrm{t}}}, \delta m_{\tilde{\mathrm{b}}_2}, \delta\theta_{\tilde{\mathrm{b}}}, \delta m_{\mathrm{t}}, \delta m_{\mathrm{t}})$

Total result for $\Delta\rho$

On-shell renormalisation of the other parameters:

$$\delta m^2_{\tilde{f}_i} = \mathrm{Re}\,\Sigma_{\tilde{f}_i}(m^2_{\tilde{f}_i})$$

for $\tilde{f}_i = \tilde{t}_{1,2}, \tilde{b}_2$

$$\delta\theta_{\tilde{f}} = \frac{\mathrm{Re}\,\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}) + \mathrm{Re}\,\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_2})}{2(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2})}$$

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$\Rightarrow$ total result for $\Delta\rho$ can be written as

$$\Delta\rho^{(q,\tilde{q},\tilde{H})} = \Delta\rho^{(q)}_{\mathrm{MSSM}} + \Delta\rho^{(\tilde{q},\tilde{H})}_{\mathrm{MSSM,\ full\ OS}} + \Delta m^2_{\tilde{\mathrm{b}}_1}\,\partial_{m^2_{\tilde{\mathrm{b}}_1}}\,\Delta\rho^{\mathrm{SUSY}}_{1-\mathrm{loop}},$$

$$\text{where }\ \Delta m^2_{\tilde{\mathrm{b}}_1} = \delta m^2_{\tilde{\mathrm{b}}_1}\big|_{\mathrm{symm}} - \delta m^2_{\tilde{\mathrm{b}}_1}\big|_{\mathrm{OS}}, \qquad \delta m^2_{\tilde{\mathrm{b}}_1}\big|_{\mathrm{OS}} = \mathrm{Re}\,\Sigma_{\tilde{b}_1}(m^2_{\tilde{\mathrm{b}}_1})$$

Alternative renormalisation scheme: $\overline{\mathrm{DR}}$ ren. for soft SUSY-breaking parameters in stop/sbottom sector

M_h *dependence: sfermion and higgsino contributions,* $\Delta\rho^{(\tilde{q},\tilde{H})}$

$\Delta\rho^{(\tilde{q},\tilde{H})}_{\text{MSSM, full OS}}$: full M_h dependence can be kept

$\Delta m^2_{\tilde{\mathrm{b}}_1}\,\partial_{m^2_{\tilde{\mathrm{b}}_1}}\,\Delta\rho^{\mathrm{SUSY}}_{1-\mathrm{loop}}$: needs to be evaluated in full gauge-less limit, i.e. for $M_\mathrm{h} = 0$

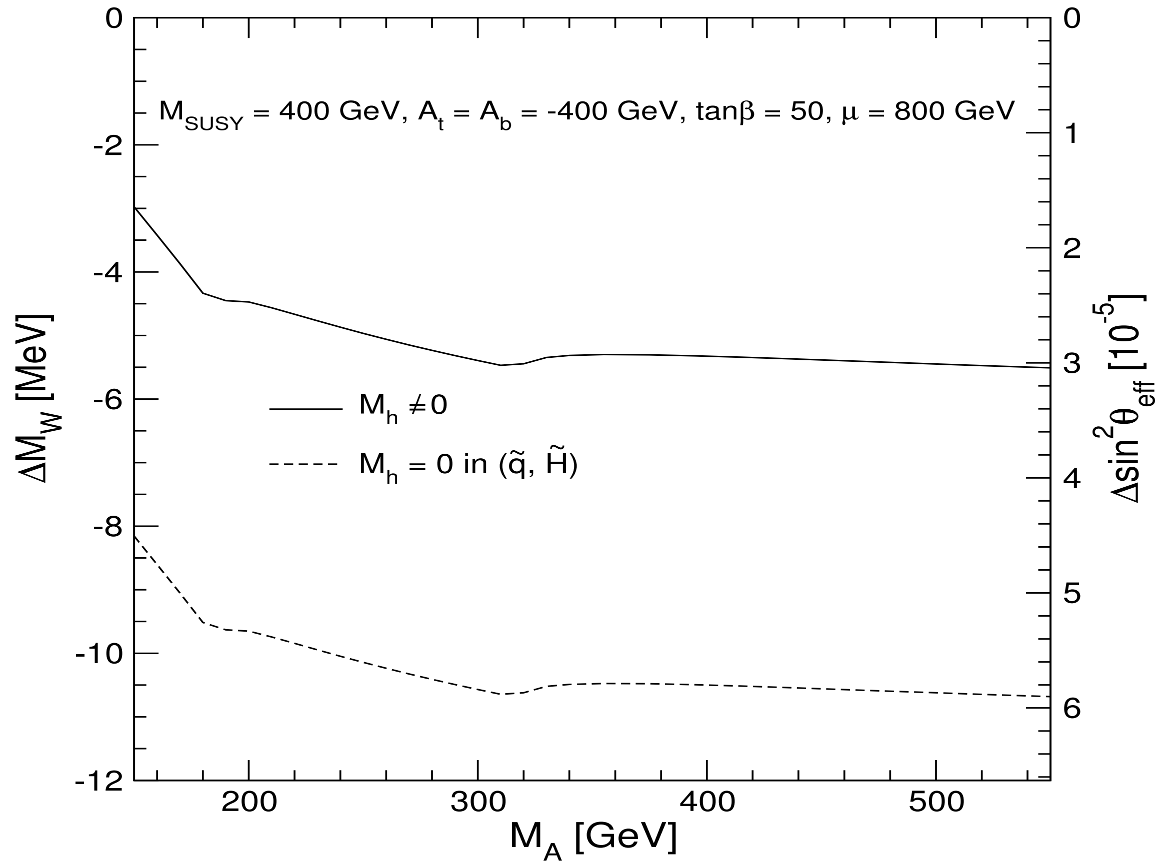
$\Rightarrow$ treat M_h-dependence of $\Delta\rho^{(\tilde{q},\tilde{H})}$ as theoretical uncertainty in the following

3. Numerical results

M_h-dependence of $\Delta\rho^{(\tilde{q},\tilde{H})}$:

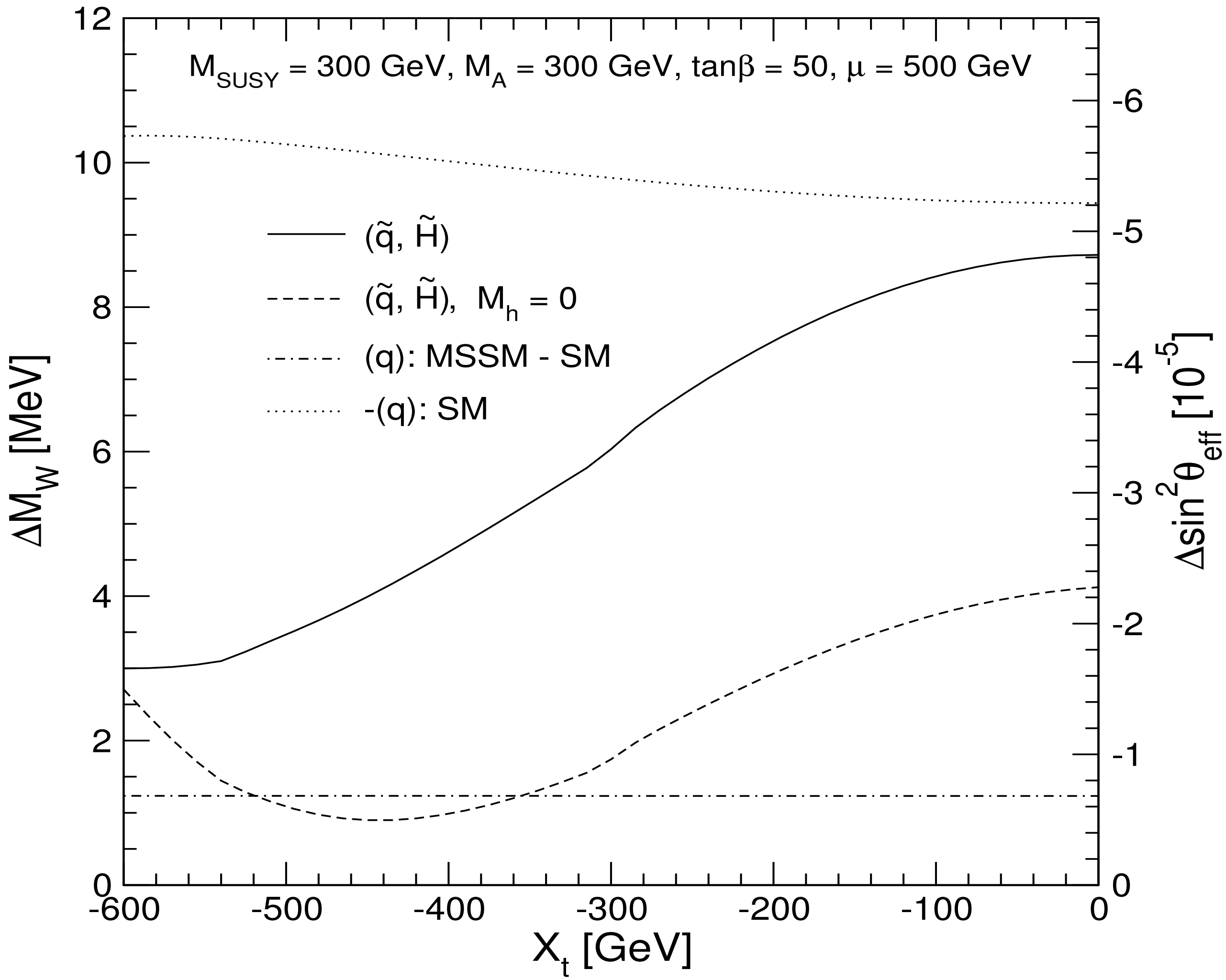
Shift induced by $\Delta\rho^{(\tilde{q},\tilde{H})}$ in M_W, $\sin^2\theta_\mathrm{eff}$:

“extreme scenario”, where M_h dependence of $\Delta\rho^{(\tilde{q},\tilde{H})}$ is particularly large



$\Rightarrow$ M_h-dependence of squark and higgsino contributions induces shift of up to $+5$ MeV in M_W and -3×10^{-5} to $\sin^2\theta_\mathrm{eff}$

Shifts induced in M_{W} and $\sin^2\theta_{\mathrm{eff}}$ by two-loop Yukawa corrections as function of mixing in scalar top sector

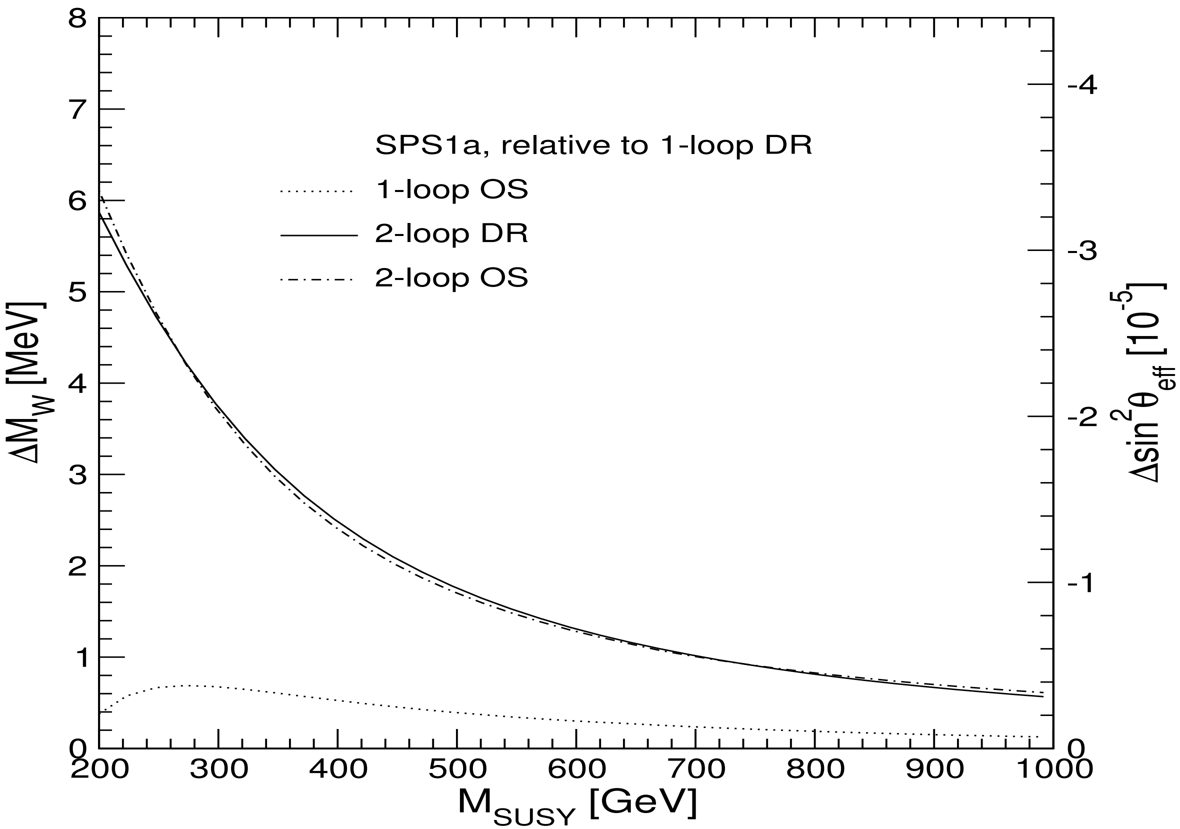


$\Rightarrow$ Corrections up to $\Delta M_{\mathrm{W}} \approx +8\ \mathrm{MeV}$, $\Delta\sin^2\theta_{\mathrm{eff}} \approx -4\times10^{-5}$ from SUSY loops, can be as large as SM quark loops

Result in SPS 1a scenario: 2-loop on-shell and 2-loop $\overline{\mathrm{DR}}$ result relative to 1-loop result with $\overline{\mathrm{DR}}$ parameters

Shifts induced by squark and higgsino corrections

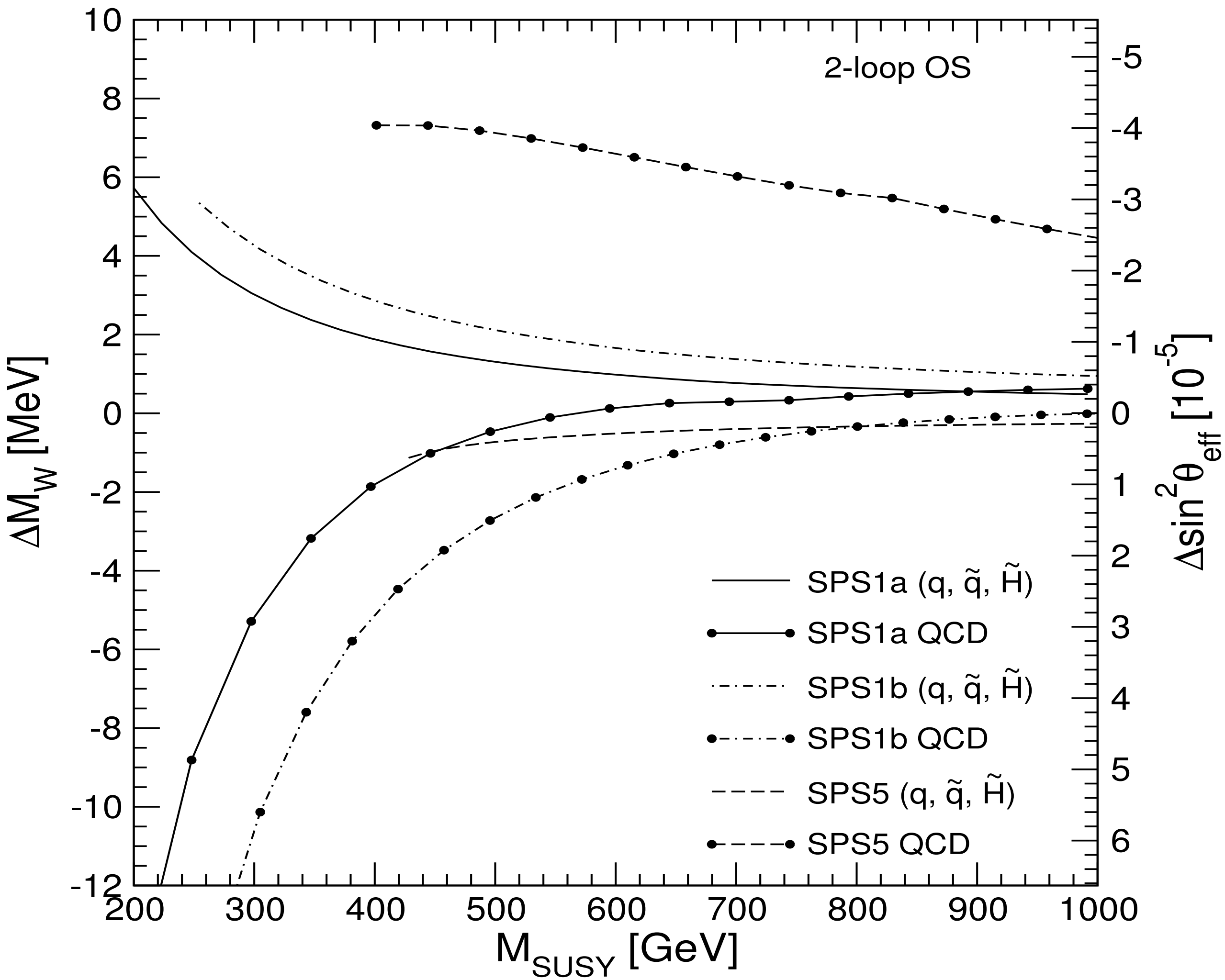
M_{SUSY}, A_{t}, A_{b}, μ, $\mu^{\overline{\mathrm{DR}}}$ varied using common scale factor



$\Rightarrow$ Corrections up to $\Delta M_{\mathrm{W}} \approx +6\ \mathrm{MeV}$, $\Delta \sin^2\theta_{\mathrm{eff}} \approx -3 \times 10^{-5}$

large reduction of scheme dependence

Yukawa corrections vs. $\mathcal{O}(\alpha\alpha_s)$ *corrections to* M_W*,* $\sin^2\theta_{\text{eff}}$ *as function of* M_{SUSY} *for three SPS scenarios*



$\Rightarrow$ Corrections have similar size, large compensations for small M_{SUSY}

Electroweak precision tests: SM vs. MSSM

Prediction for $M_{\rm W}$ in the SM and the MSSM ($m_{\rm t} = 172.7 \pm 2.8$ GeV):

[A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97]

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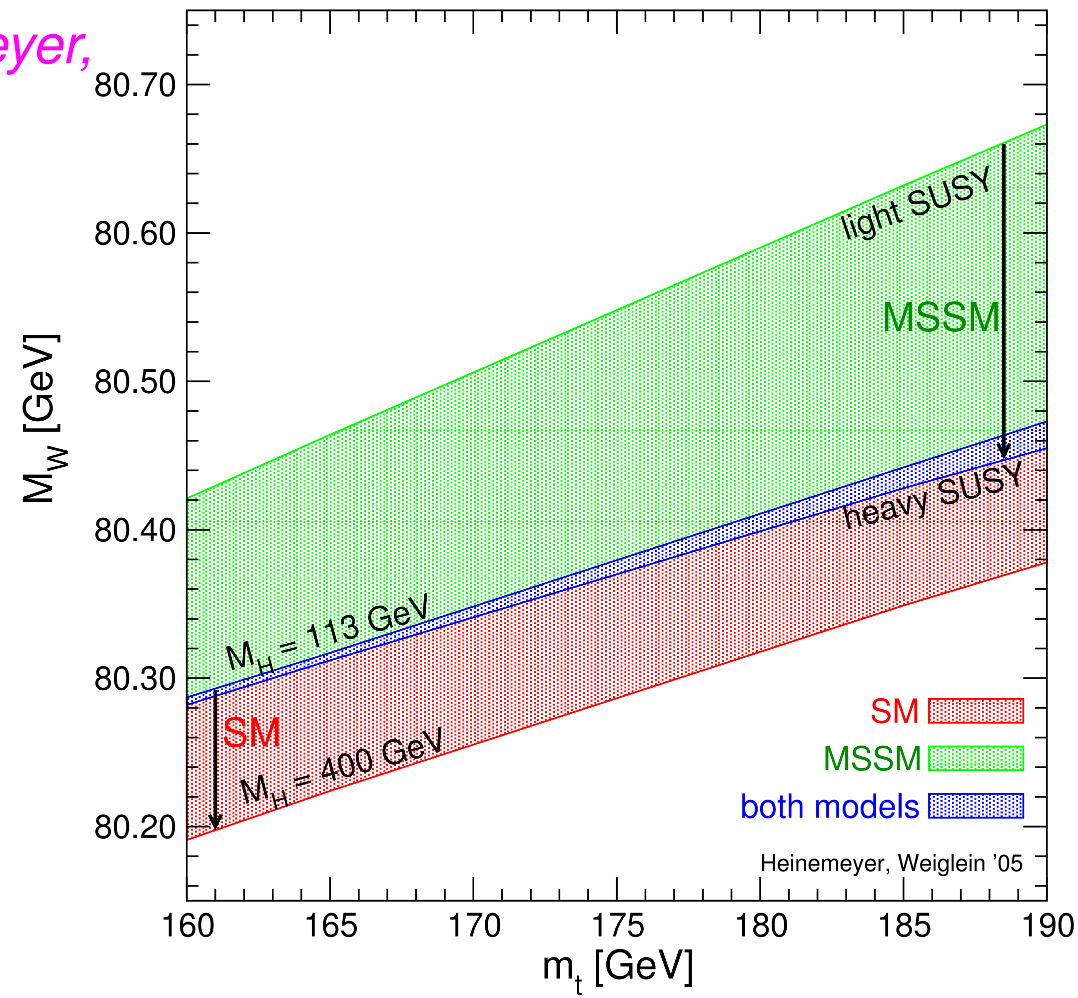

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MSSM: SUSY parameters varied

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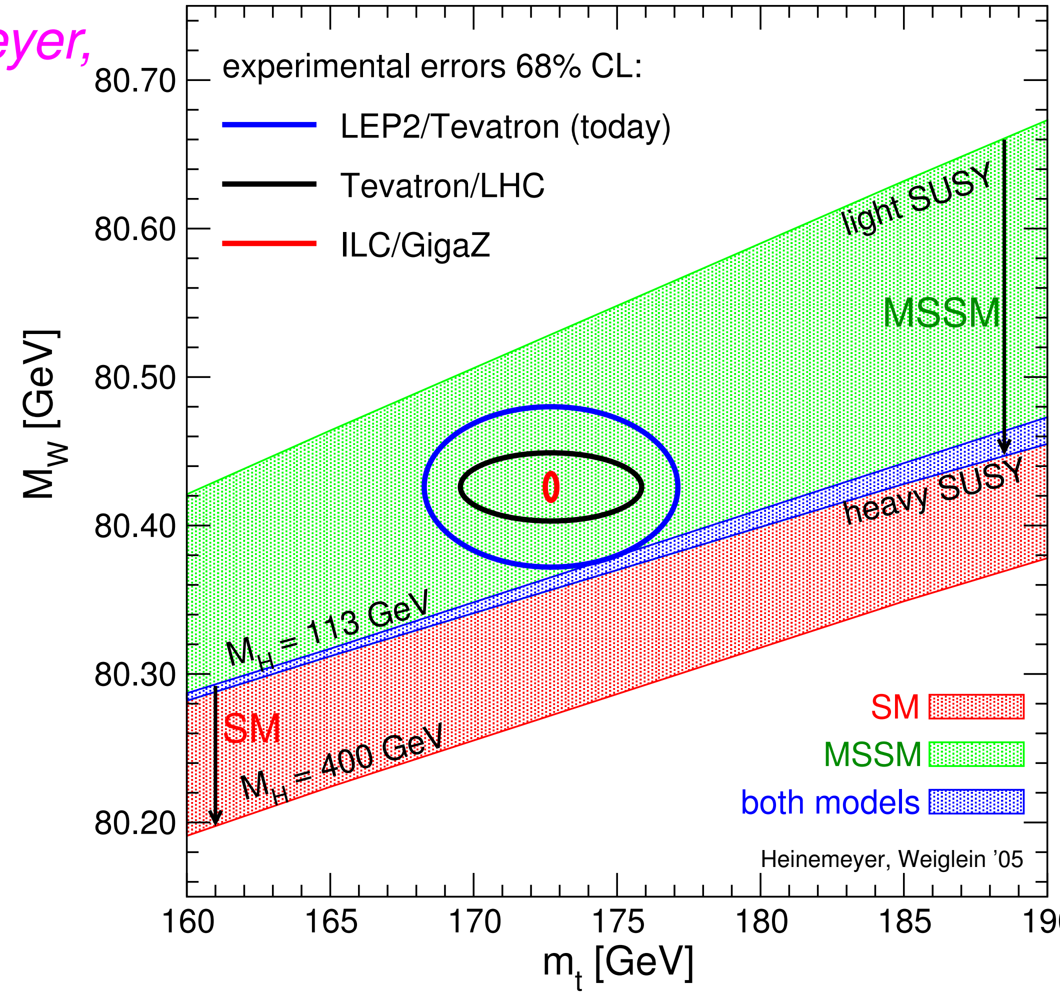

SM: $M_{\rm H}$ varied

MSSM: SUSY parameters varied

4. Estimate of remaining uncertainties from unknown higher-orders

Theoretical evaluation of precision observables is more advanced in the SM than in the MSSM

⇒ Write MSSM prediction for observable O ($O = M_\mathrm{W}, \sin^2\theta_\mathrm{eff}, \ldots$) as

$$O_\mathrm{MSSM} = O_\mathrm{SM} + O_\mathrm{MSSM-SM}$$

⇒ takes all known SM corrections into account

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$\Rightarrow$ Write MSSM prediction for observable O ($O = M_\mathrm{W}, \sin^2\theta_\mathrm{eff}, \ldots$) as

$$O_\mathrm{MSSM} = O_\mathrm{SM} + O_\mathrm{MSSM-SM}$$

$\Rightarrow$ takes all known SM corrections into account

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⇒ Need estimate of further uncertainties from SUSY loop corrections

Estimate of uncertainties from SUSY loop corrections depending on M_{SUSY}

- Electroweak 2-loop corrections beyond the leading Yukawa corrections:
  - assume ratio of subleading ew 2-loop corr. to 2-loop Yukawa corr. is the same in the MSSM as in the SM
  - M_{h} dependence of squark and higgsino contributions

- $\mathcal{O}(\alpha\alpha_s)$ corrections beyond the $\Delta\rho$ approximation:

  assume ratio of contribution entering via $\Delta\rho$ to full result is the same as in the SM

Estimate of uncertainties from SUSY loop corrections depending on M_{SUSY}

- $\mathcal{O}(\alpha\alpha_s^2)$ corrections:
  - assume ratio of supersymmetric $\mathcal{O}(\alpha\alpha_s^2)$ contributions to $\mathcal{O}(\alpha\alpha_s)$ supersymmetric contributions is the same as for corresponding corrections in the SM
  - geometric progression from lower orders: assume ratio is the same as of $\mathcal{O}(\alpha\alpha_s)$ supersymmetric contributions to $\mathcal{O}(\alpha)$ supersymmetric contributions
  - variation of renormalisation scale of $\alpha_s(\mu^{\overline{\mathrm{DR}}})$ entering the $\mathcal{O}(\alpha\alpha_s)$ result according to $m_{\mathrm{t}}/2 \leq \mu^{\overline{\mathrm{DR}}} \leq 2m_{\mathrm{t}}$

Estimate of uncertainties from SUSY loop corrections depending on M_{SUSY}

- $\mathcal{O}(\alpha^2\alpha_s)$ corrections:
  - assume ratio of supersymmetric $\mathcal{O}(\alpha^2\alpha_s)$ contributions to $\mathcal{O}(\alpha^2)$ leading Yukawa supersymmetric contributions is the same as for corresponding corrections in the SM
  - geometric progression from lower orders: assume ratio is the same as of $\mathcal{O}(\alpha\alpha_s)$ supersymmetric contributions to $\mathcal{O}(\alpha)$ supersymmetric contributions
  - change value of m_{t} in result for 2-loop supersymmetric Yukawa corrections from $m_{\mathrm{t}}^{\mathrm{OS}}$ to $m_{\mathrm{t}}(m_{\mathrm{t}}) = m_{\mathrm{t}}^{\mathrm{OS}}/(1 + 4/(3\pi)\,\alpha_s(m_{\mathrm{t}}))$

- Electroweak three-loop corrections:

  renormalisation scheme dependence of result for 2-loop supersymmetric Yukawa corrections

Resulting estimates for uncertainty in M_W (in MeV) for different values of M_SUSY

Values obtained for SPS 1a, SPS 1b, SPS 5 (largest value taken as estimate):

M_SUSY	$<$500 GeV	500 GeV	1000 GeV
$\mathcal{O}(\alpha^2)$ sublead.	6.0	2.0	0.8
$\mathcal{O}(\alpha\alpha_s)$ sublead.	1.8	0.9	0.5
$\mathcal{O}(\alpha\alpha_s^2)$	3.0, 5.3, 1.5	1.4, 1.1, 0.7	0.9, 2.2, 0.5
$\mathcal{O}(\alpha^2\alpha_s)$	1.5, 2.2, 1.4	0.6, 0.8, 0.4	0.2, 0.2, 0.2
$\mathcal{O}(\alpha^3)$	0.3	0.3	0.3

Estimates for uncertainties in M_{W} and $\sin^2\theta_{\mathrm{eff}}$ from unknown higher-order SUSY contrib.

$$
\begin{aligned}
\delta M_{\mathrm{W}} &= 8.5\ \mathrm{MeV}\ \text{for}\ M_{\mathrm{SUSY}} = 200\ \mathrm{GeV}\\
\delta M_{\mathrm{W}} &= 2.7\ \mathrm{MeV}\ \text{for}\ M_{\mathrm{SUSY}} = 500\ \mathrm{GeV}\\
\delta M_{\mathrm{W}} &= 2.4\ \mathrm{MeV}\ \text{for}\ M_{\mathrm{SUSY}} = 1000\ \mathrm{GeV}
\end{aligned}
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\begin{aligned}
\delta\sin^2\theta_{\mathrm{eff}} &= 4.7\times 10^{-5}\ \text{for}\ M_{\mathrm{SUSY}} = 200\ \mathrm{GeV}\\
\delta\sin^2\theta_{\mathrm{eff}} &= 1.5\times 10^{-5}\ \text{for}\ M_{\mathrm{SUSY}} = 500\ \mathrm{GeV}\\
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$\Rightarrow$ Total uncertainty including higher-order SM corrections:

$$\delta M_{\mathrm{W}} = (5-9)\ \mathrm{MeV}, \quad \delta \sin^2\theta_{\mathrm{eff}} = (5-7) \times 10^{-5}$$

5. Conclusions

- New result for 2-loop Yukawa corr. to $\Delta\rho$ in the MSSM:
  $\mathcal{O}(\alpha_{\rm t}^2)$, $\mathcal{O}(\alpha_{\rm t}\alpha_{\rm b})$, $\mathcal{O}(\alpha_{\rm b}^2)$ contributions from SM fermions, sfermions, Higgs bosons and higgsinos

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  $\delta M_W = (5 - 9)$ MeV, $\delta \sin^2\theta_{\text{eff}} = (5 - 7) \times 10^{-5}$
  $\Rightarrow$ Further efforts needed to reduce theor. uncertainties in the MSSM to the level achieved for the SM