$K^+ \to \pi^+ \nu \bar{\nu}$ at NNLO

Ulrich Haisch

in collaboration with A.J. Buras, M. Gorbahn & U. Nierste,
hep-ph/0508165

LoopFest IV, August 19, 2005, Snowmass, CO

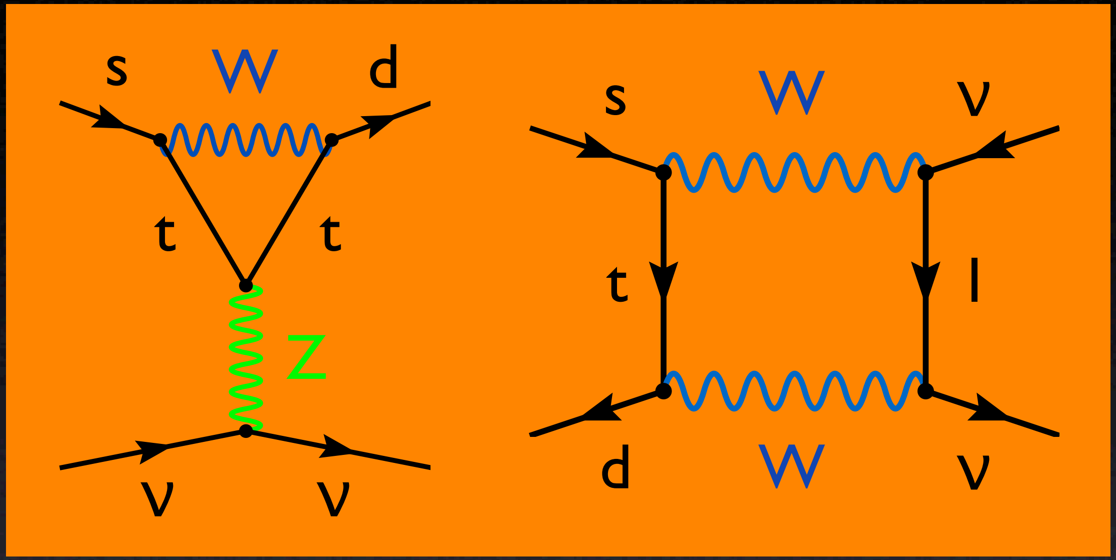
Next 29 minutes before

- Introduction
- Theoretical status of $K \to \pi\nu\bar{\nu}$
- NNLO calculation of $K^+ \to \pi^+\nu\bar{\nu}$
- Determination of unitarity triangle
- Conclusions

Introduction

- FCNC processes strongly suppressed in SM by loop and CKM factors
- SD effects are significant and calculable with high precision
- LD hadronic effects are small and under good theoretical control

precise determination of flavor structure of SM



Introduction

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precise determination of flavor structure of SM

$$\sigma(\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})) = \pm(1-2)\%$$
$$\sigma(\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})) = \pm(3-4)\%$$
$$\sigma(\mathcal{A}_{\rm FB}(B \to X_s l^+ l^-)) = \pm(6-9)\%$$
$$\sigma(\mathcal{B}(B \to X_s \gamma)) = \pm(8-14)\%$$
$$\sigma(\mathcal{B}(B \to X_s l^+ l^-)) = \pm(10-17)\%$$
$$\sigma(\mathcal{B}(B_s \to \mu^+ \mu^-)) = \pm15\%$$
$$\sigma(\mathcal{A}_{\rm FB}(B \to K^{(*)} l^+ l^-)) = \pm15\%$$
$$\sigma(\mathcal{B}(B \to (K^*, \rho, \omega)\gamma)) = \pm(15-30)\%$$
$$\sigma(\mathcal{B}(B \to K^* l^+ l^-) = \pm(30-35)\%$$
$$\ldots$$

Introduction

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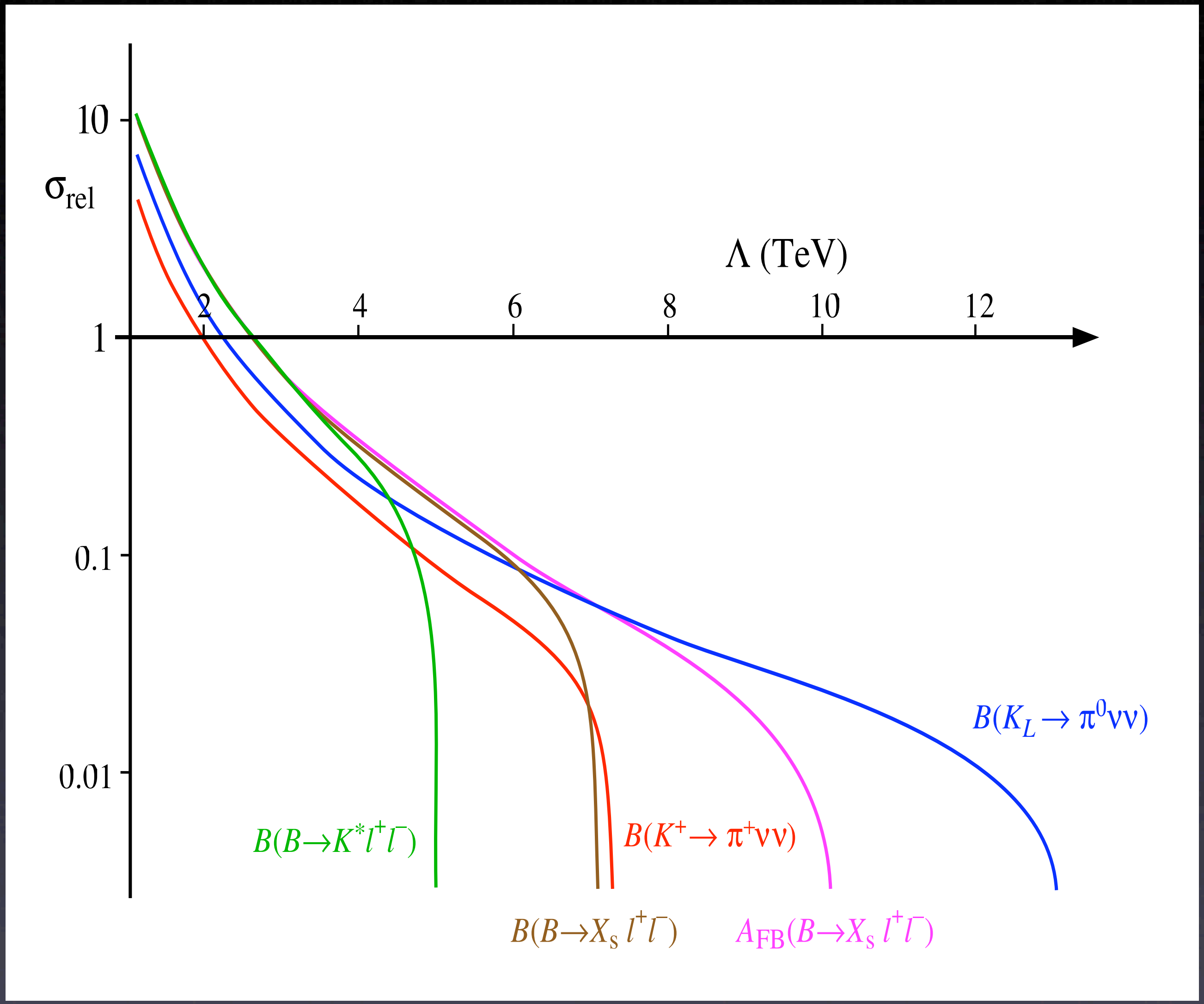
enhanced sensitivity to flavor dynamics of NP



Bryman, Buras, Isidori & Littenberg '05

General properties of $K \longrightarrow \pi\nu\bar{\nu}$

$$\mathcal{L}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\sin^2\theta_W} \sum_{i=u,c,t} C^i(\mu) Q_\nu$$

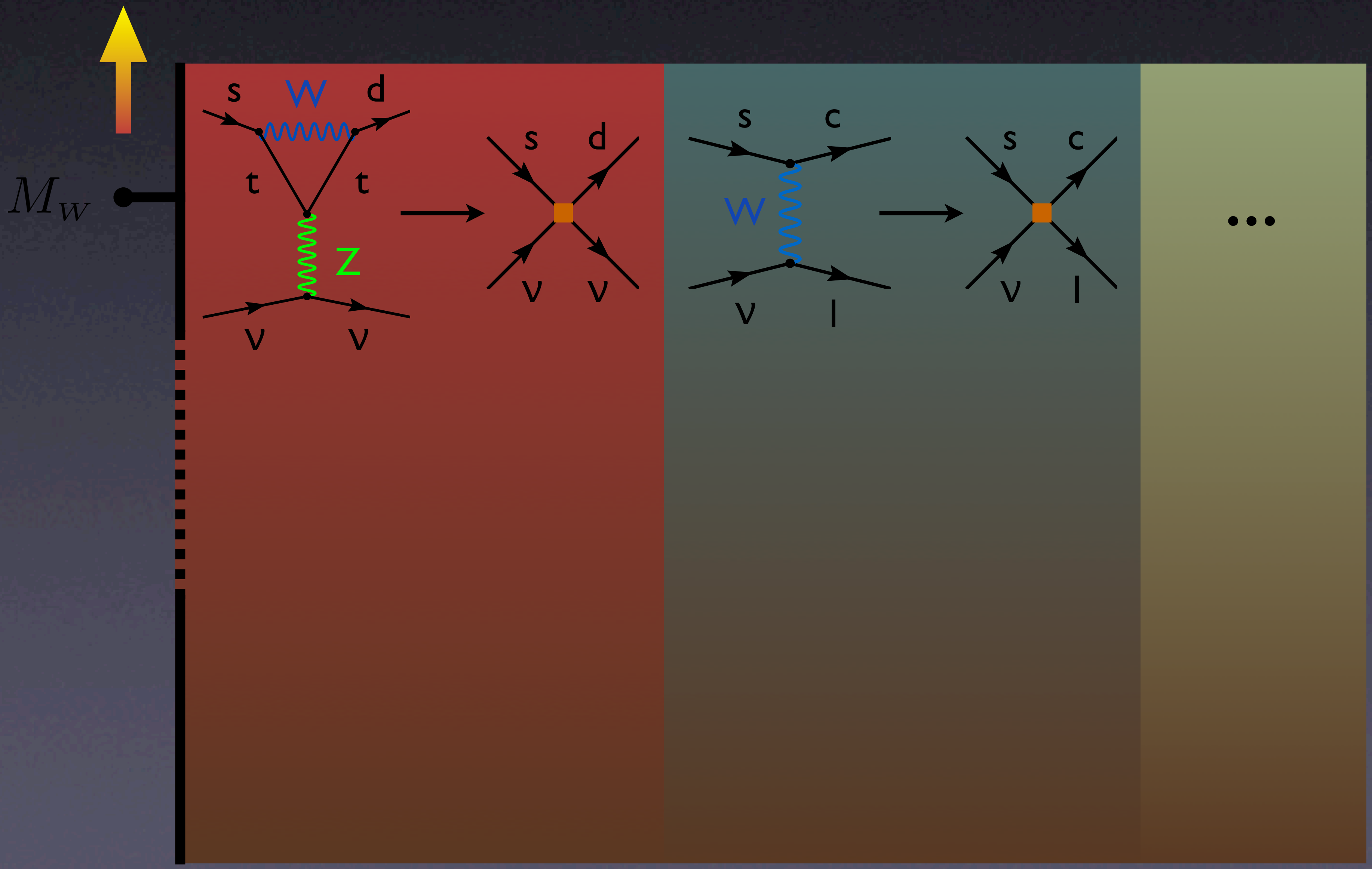
$$Q_\nu = \sum_{l=e,\mu,\tau} (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_{lL} \gamma^\mu \nu_{lL})$$

$$C^i(M_W) \propto m_i^2 V_{is}^* V_{id} \propto \begin{cases} \Lambda_{\text{QCD}}^2 \lambda \\ m_c^2(\lambda + i\lambda^5) \\ m_t^2(\lambda^5 + i\lambda^5) \end{cases}$$

u

c

t



- power-like GIM mechanism
- top quark contributions dominates
- QCD corrections are small
- large $\not{C}\not{P}$ phase

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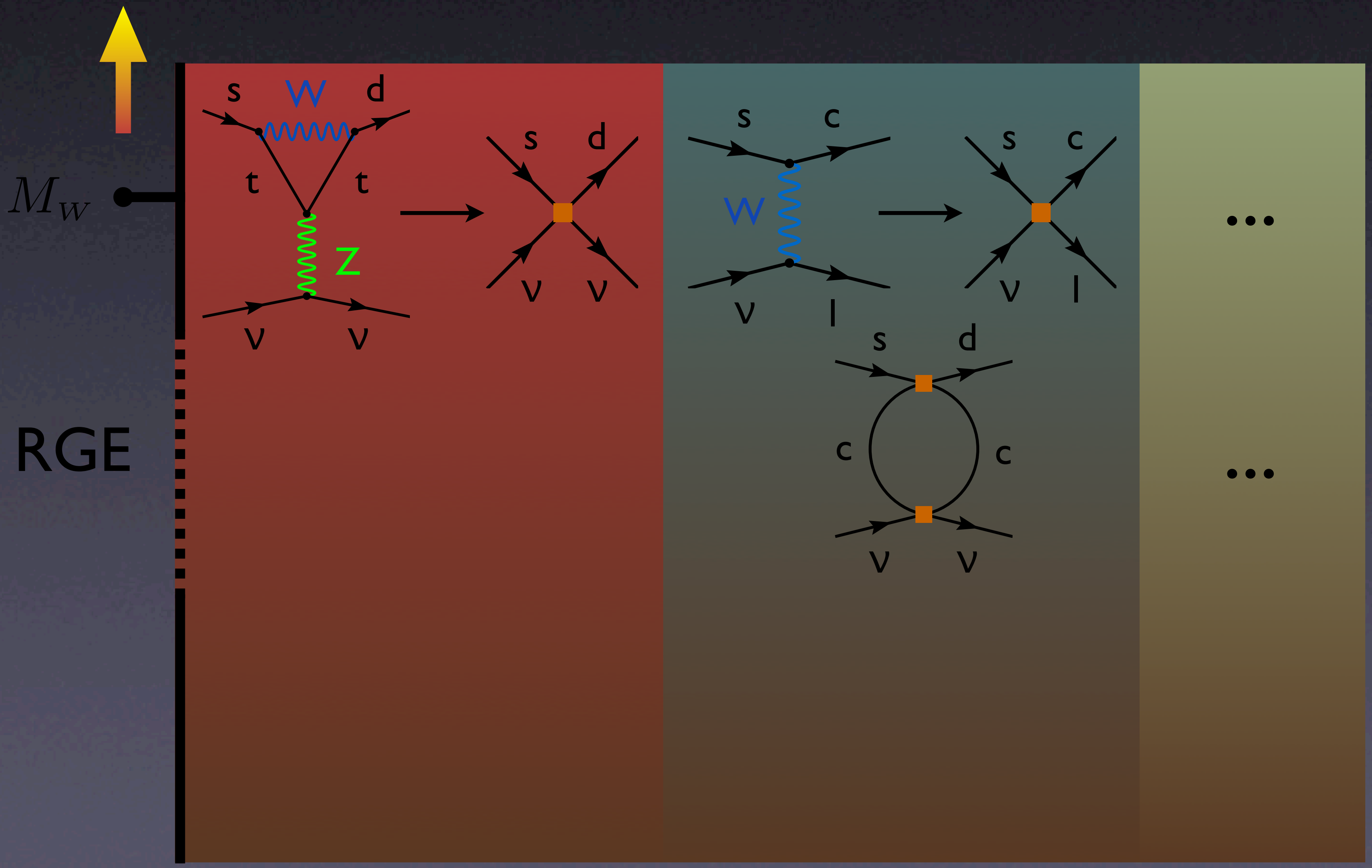
$$Q_\nu = \sum_{l=e,\mu,\tau} (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_{lL} \gamma^\mu \nu_{lL})$$

$$\mu \frac{d}{d\mu} C_i(\mu) = \sum_j \underbrace{\gamma_{ji}(\mu) C_j(\mu)}_{\text{ADM}}$$

u

c



- top quark contribution does not evolve
- charm effects moderate for K$^+$ while negligible for K$_L$

General properties of $K \to \pi\nu\bar{\nu}$

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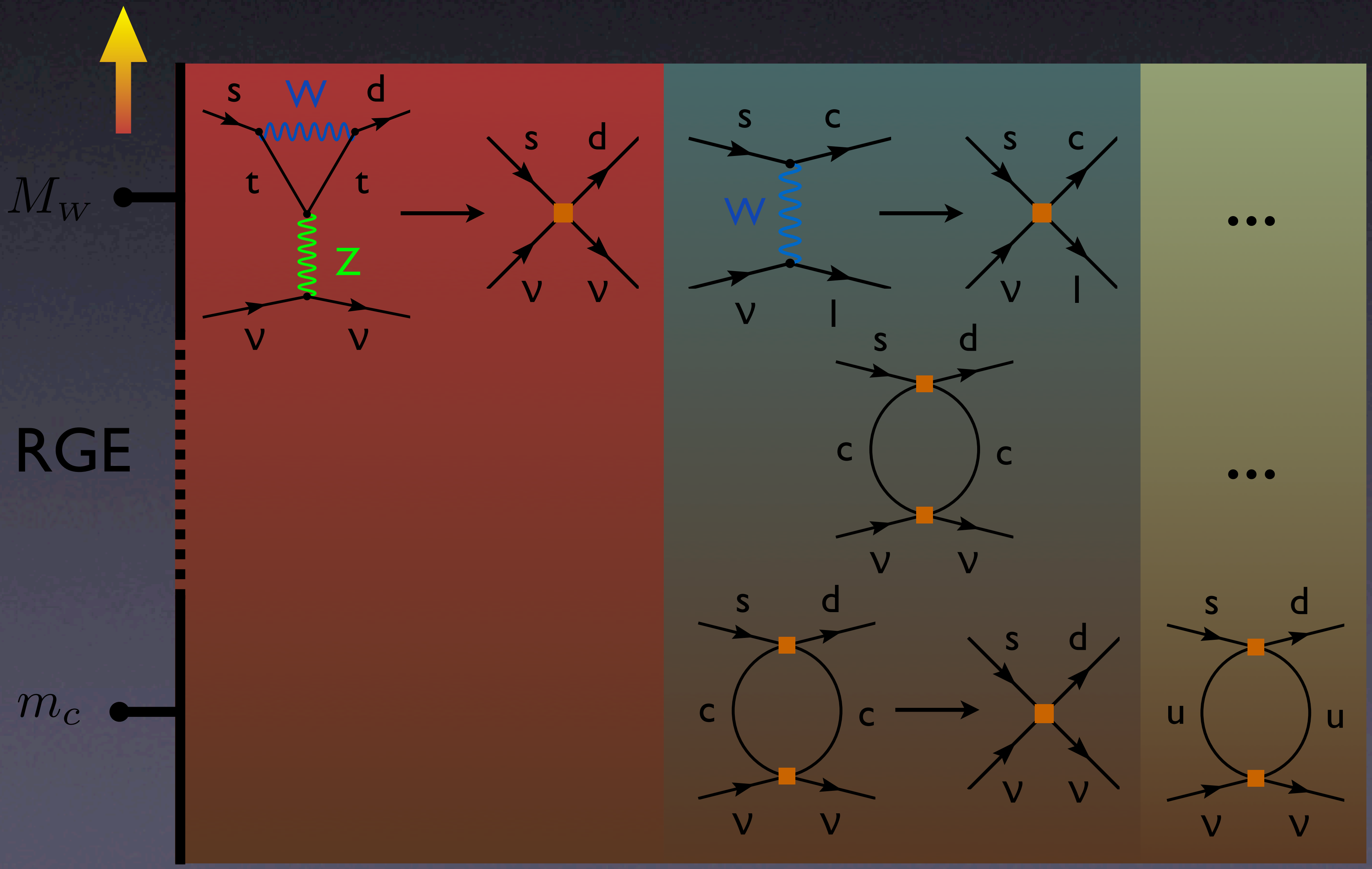
u

$$\langle\pi|\bar{s}_L\gamma_\mu d_L|K\rangle \propto \langle\pi^0|\bar{s}_L\gamma_\mu u_L|K^+\rangle$$

$$Q_\nu = \sum_{l=e,\mu,\tau} (\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_{lL}\gamma^\mu\nu_{lL})$$



- hadronic matrix elements precisely known from K^+_{e3}
- neutrino pair in CP eigenstate
- K_L decay purely CP

NLO SM prediction of $K^+ \to \pi^+ \nu \bar{\nu}$

$$\mathcal{B}(K^+) = \kappa_+ \left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X \right)^2 + \left(\frac{\mathrm{Re}\lambda_t}{\lambda^5} X + \frac{\mathrm{Re}\lambda_c}{\lambda} (P_c + \delta P_c) \right)^2 \right]$$

t

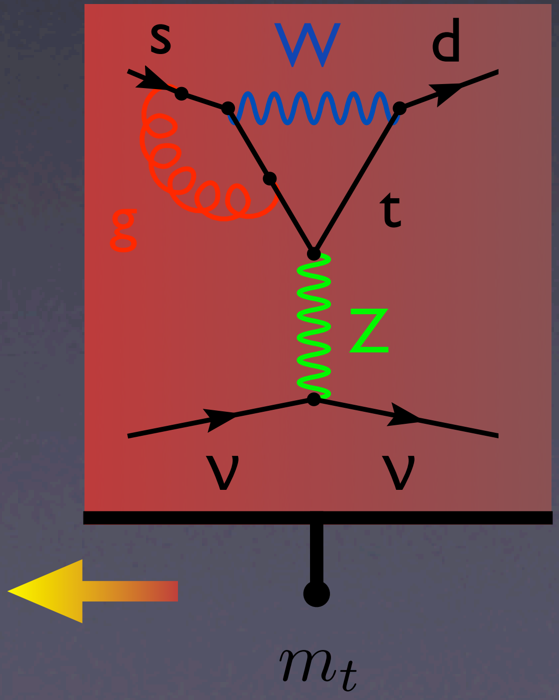
$$\kappa_+ = r_{K^+} \frac{3\alpha^2 \mathcal{B}(K^+_{e3})}{2\pi^2 \sin^4 \theta_W}$$

$$X = 1.46 \pm 0.04 \quad (\mathrm{NLO})$$

Buchalla & Buras '93, '99;
Misiak & Urban '99

$$\lambda_i = V^*_{is} V_{id}$$

$$\lambda = |V_{us}| \approx 0.22$$



NLO SM prediction of $K^+ \to \pi^+ \nu\bar{\nu}$

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c

t

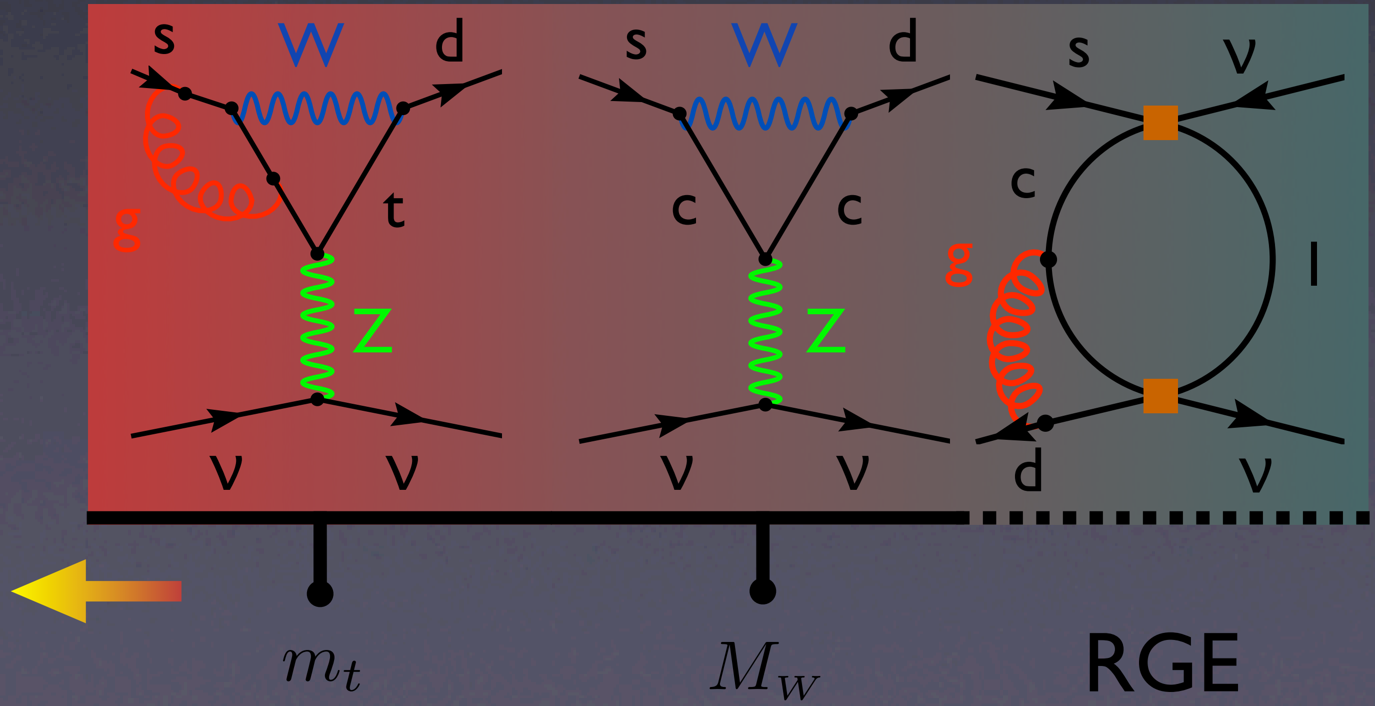
$$\kappa_+ = r_{K^+} \frac{3\alpha^2 \mathcal{B}(K^+_{e3})}{2\pi^2 \sin^4 \theta_W}$$

$$P_c = 0.37 \pm 0.06 \quad (\mathrm{NLO})$$

Buchalla & Buras ‘94, ‘99;
Buras, Gorbahn, Nierste & UH ‘05

$$\lambda_i = V_{is}^* V_{id}$$

$$\lambda = |V_{us}| \approx 0.22$$



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u

c

t

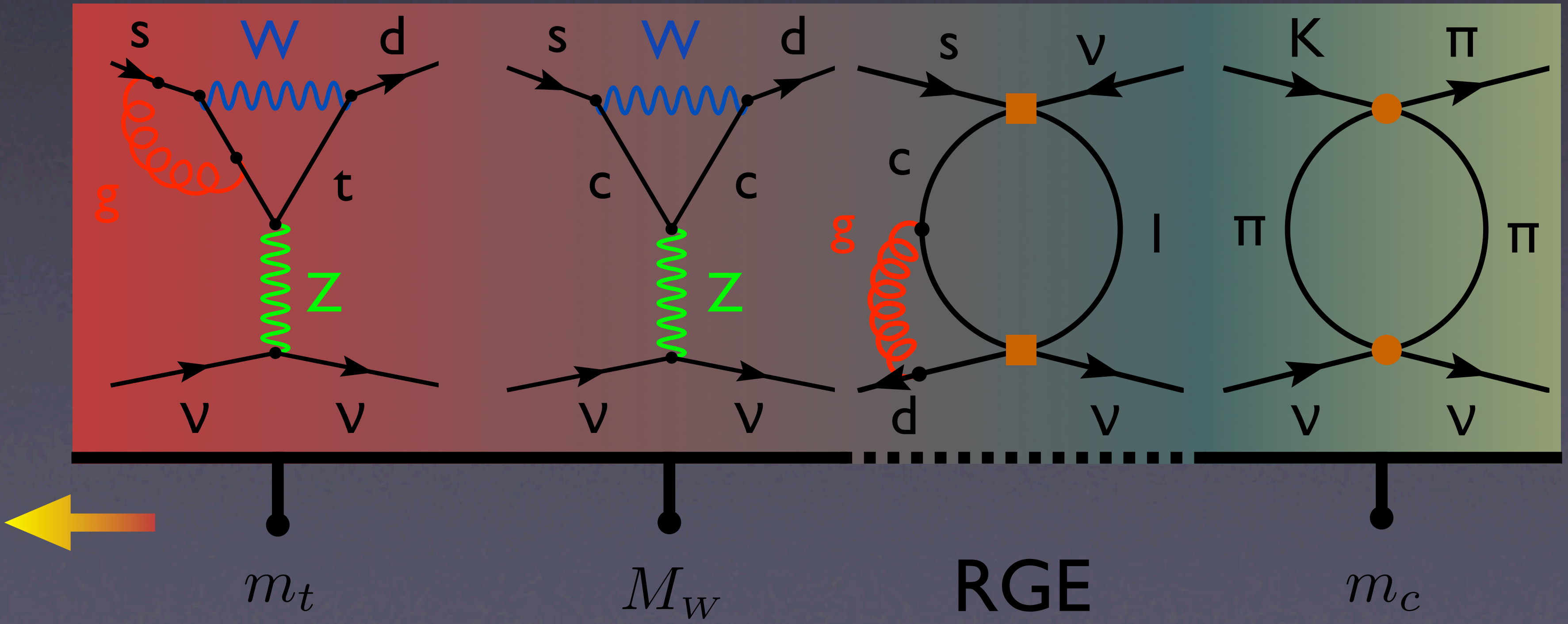
$$\kappa_+ = r_{K^+} \frac{3\alpha^2 \mathcal{B}(K^+_{e3})}{2\pi^2 \sin^4 \theta_W}$$

$$\delta P_c = 0.04 \pm 0.02 \quad (\mathrm{CHPT})$$

Isidori, Mescia & Smith ‘05

$$\lambda_i = V_{is}^* V_{id}$$

$$\lambda = |V_{us}| \approx 0.22$$



NLO SM prediction of $K^+ \to \pi^+\nu\bar{\nu}$

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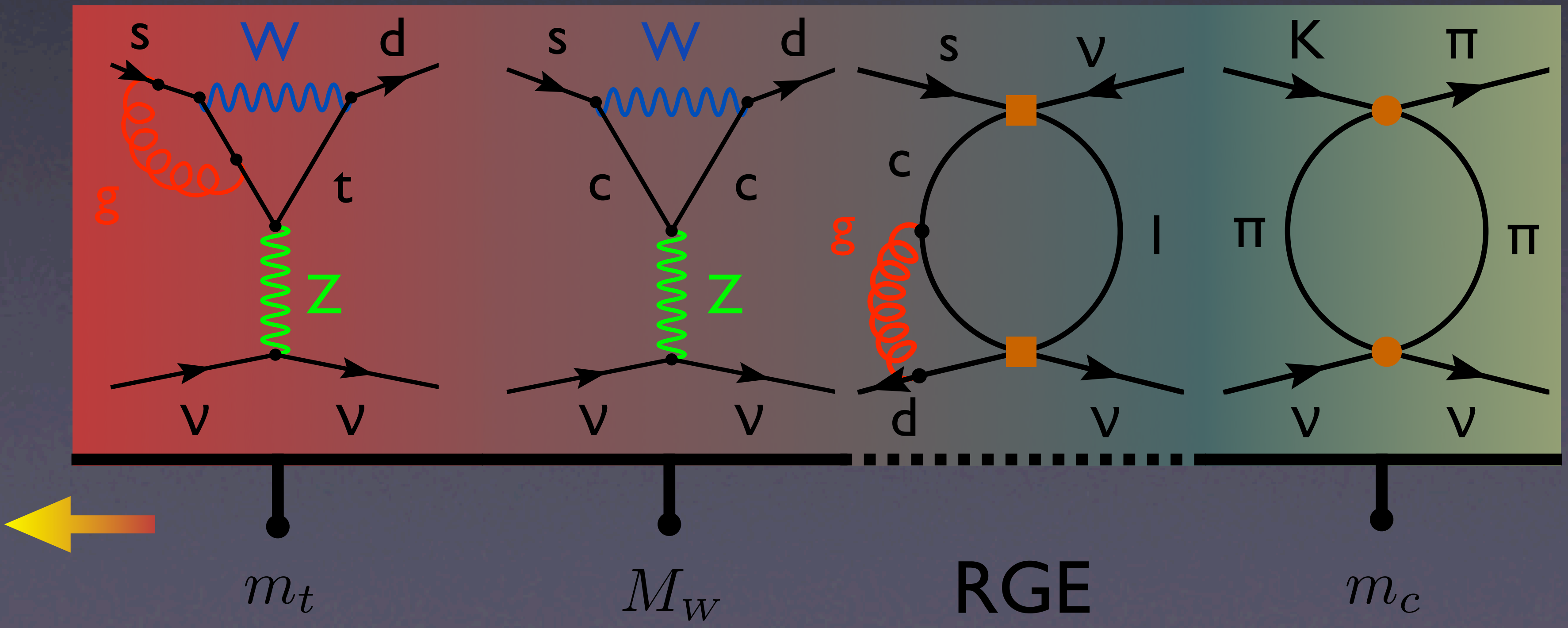
$$\kappa_+ = \boxed{r_{K^+}} \frac{3\alpha^2 \mathcal{B}(K^+_{e3})}{2\pi^2 \sin^4\theta_W}$$

$$r_{K^+} = 0.90 \pm 0.03 \quad (\cancel{\mathrm{SU}(2)})$$

Marciano & Parsa '96



NLO SM prediction of $K^+ \to \pi^+ \nu\bar{\nu}$

$$\mathcal{B}(K^+) = \kappa_+ \left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}X\right)^2 + \left(\frac{\mathrm{Re}\lambda_t}{\lambda^5}X + \frac{\mathrm{Re}\lambda_c}{\lambda}\left(P_c + \delta P_c\right)\right)^2\right] = (7.9 \pm 1.3) \times 10^{-11}$$

u

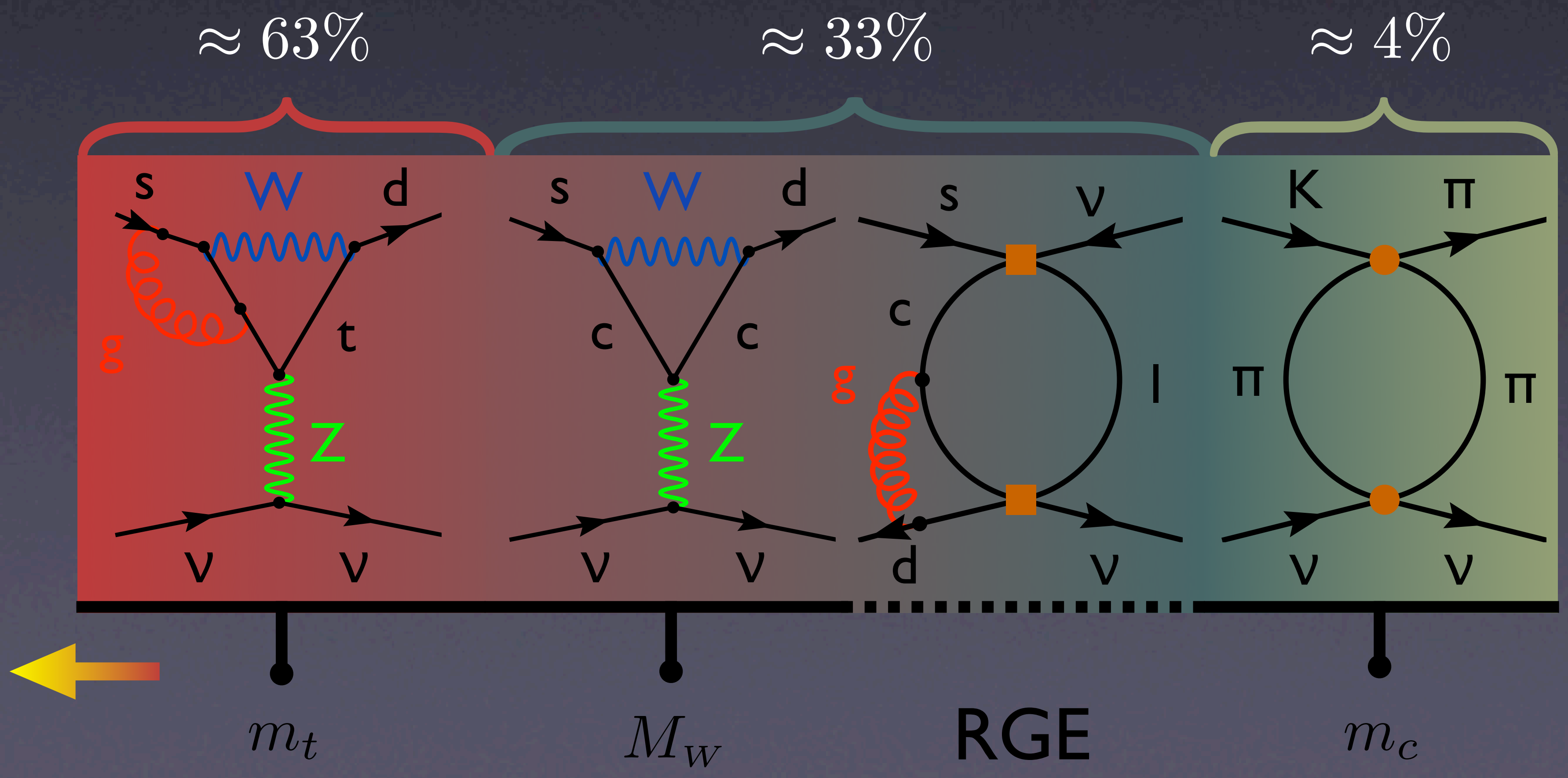
c

t

$$\kappa_+ = r_{K^+}\frac{3\alpha^2\,\mathcal{B}(K^+_{e3})}{2\pi^2 \sin^4\theta_W}$$

Buras, Gorbahn, Nierste & UH '05

- dominant theoretical error of ≈6% due to perturbative charm contribution P_c
- ≈50% of total error of ≈16% due to m_c and CKM elements



NLO SM prediction of $K_L \to \pi^0 \nu\bar{\nu}$

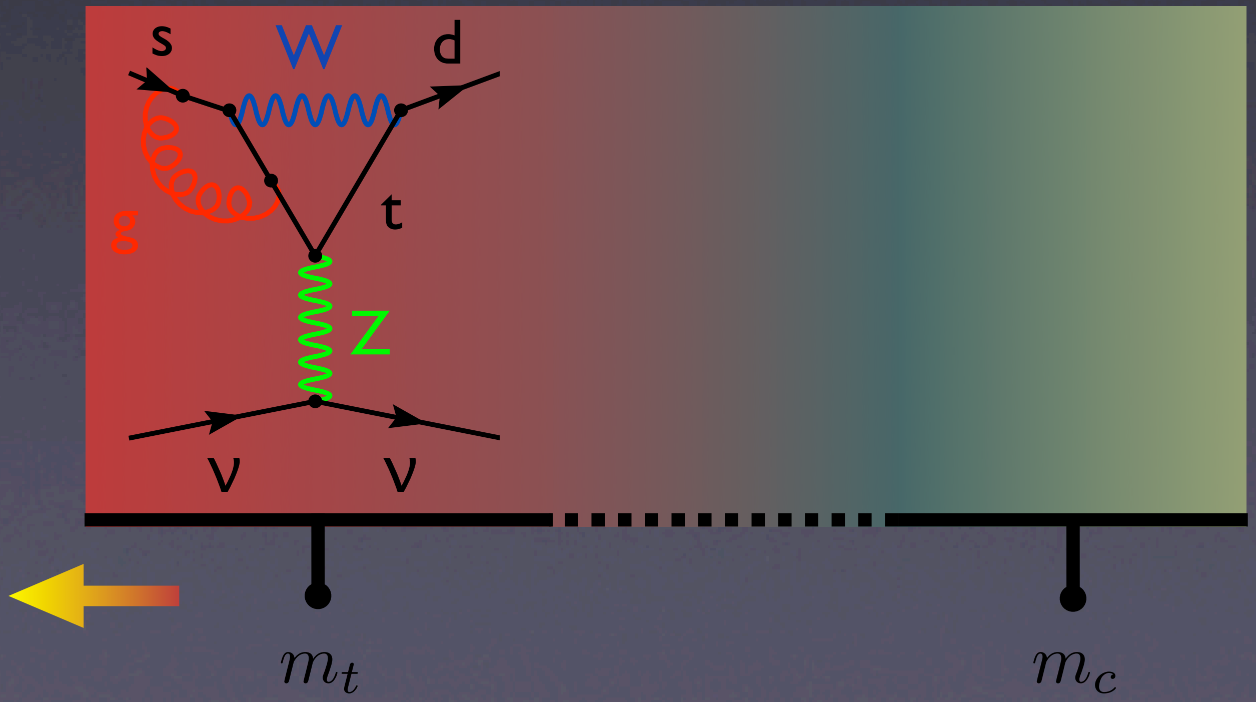
$$\mathcal{B}(K_L) = \kappa_L \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X \right)^2$$

t

$$\kappa_L = \kappa_+ \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)}$$

$$r_{K_L} = 0.94 \pm 0.03 \quad (\mathrm{S}\cancel{\mathrm{U}(2)})$$

Marciano & Parsa '96



NLO SM prediction of $K_L \to \pi^0 \nu \bar{\nu}$

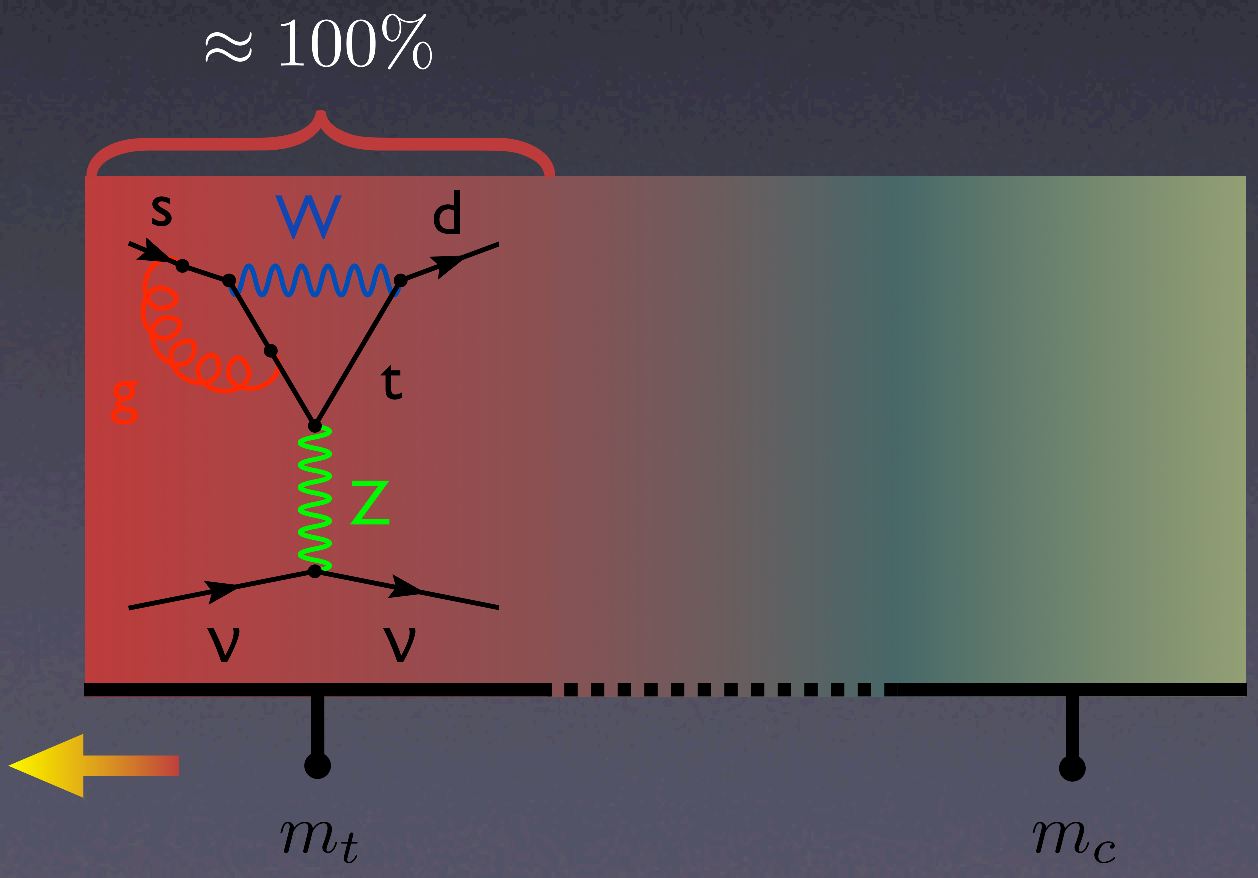
$$\mathcal{B}(K_L) = \kappa_L \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X \right)^2 = (2.9 \pm 0.4) \times 10^{-11}$$

t

$$\kappa_L = \kappa_+ \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)}$$

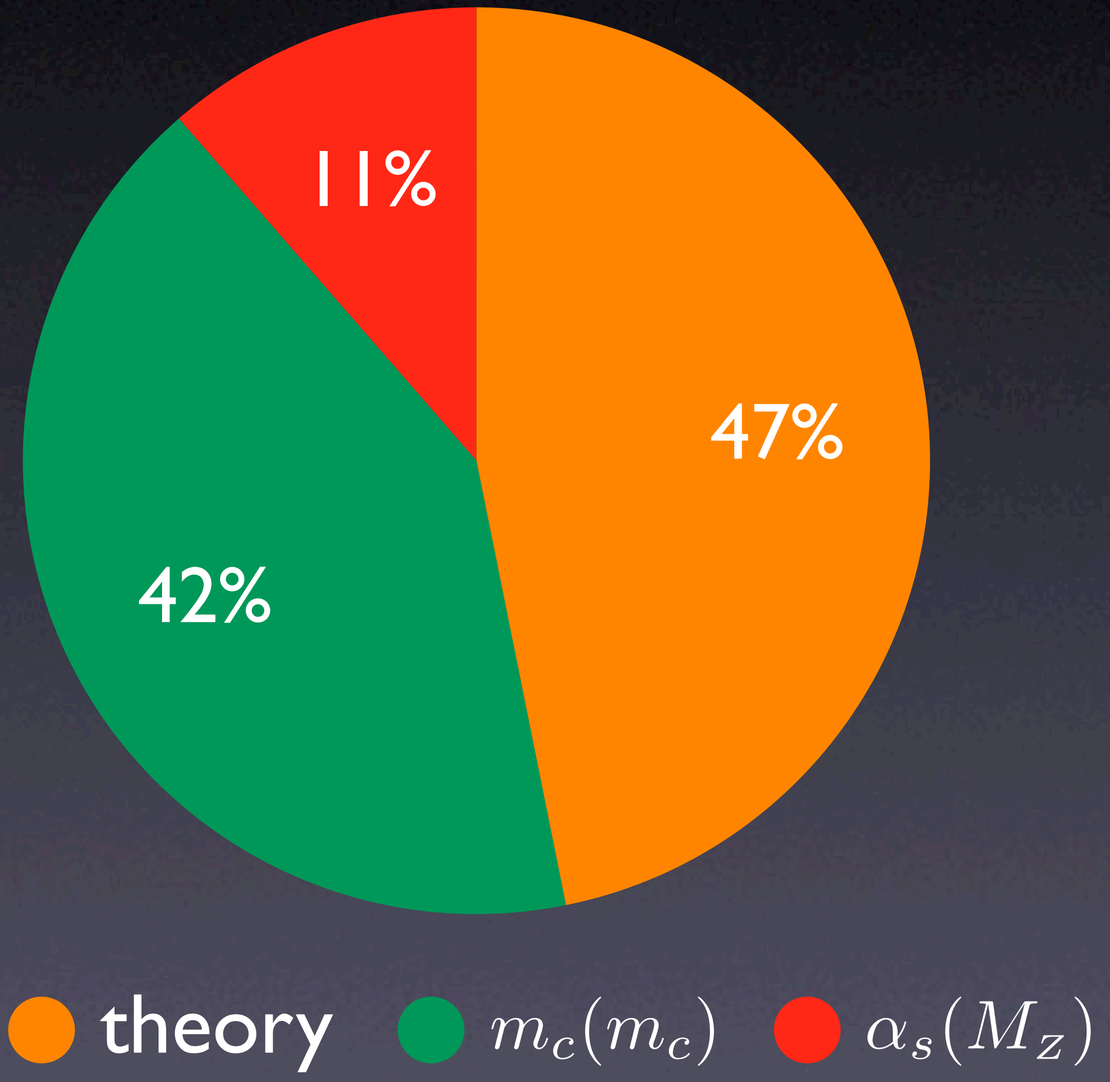
Littenberg ‘89;
Buchalla & Buras ‘96;
Buchalla & Isidori ‘98;
Buras, Gorbahn, Nierste & UH ‘05

- very small theoretical error of only ≈1%
- ≈90% of total error of ≈15% due to CKM parameters
- within SM amount of $\not{C}\!\not{P}$ can in principle be determined with unmatched precision



Error budget of P_c at NLO

$$P_c = 0.367 \pm 0.037 \pm 0.033 \pm 0.009$$



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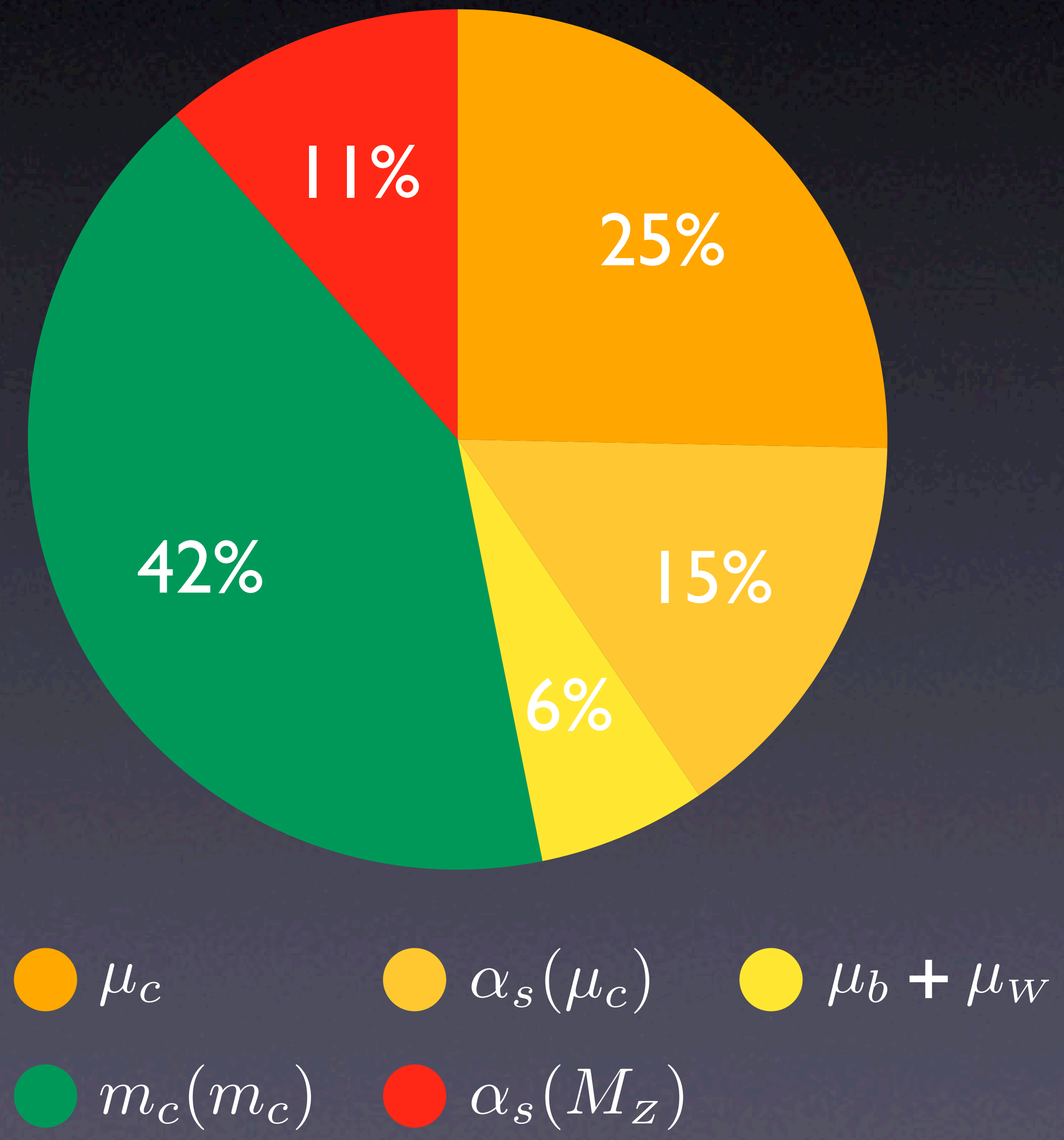

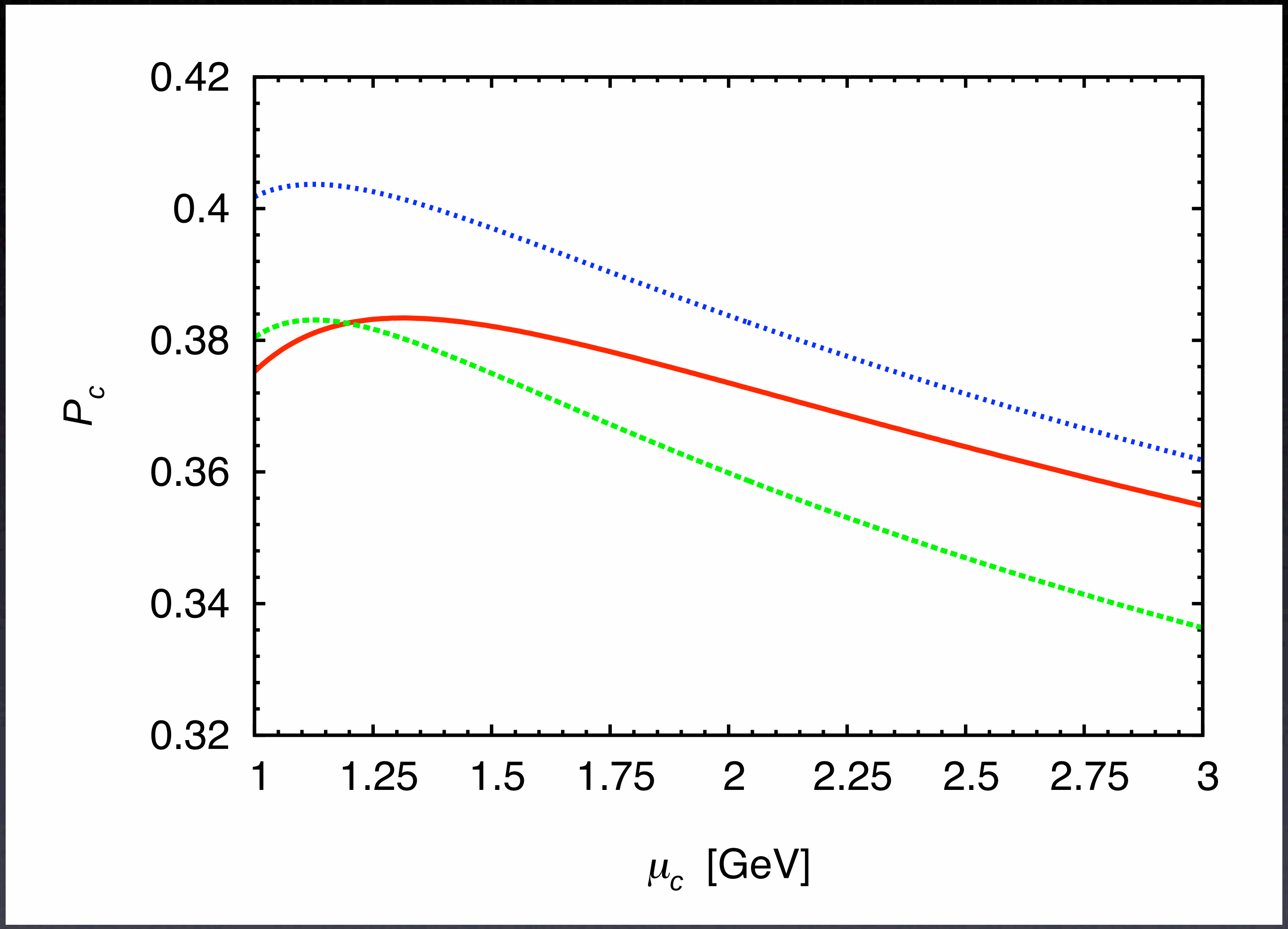

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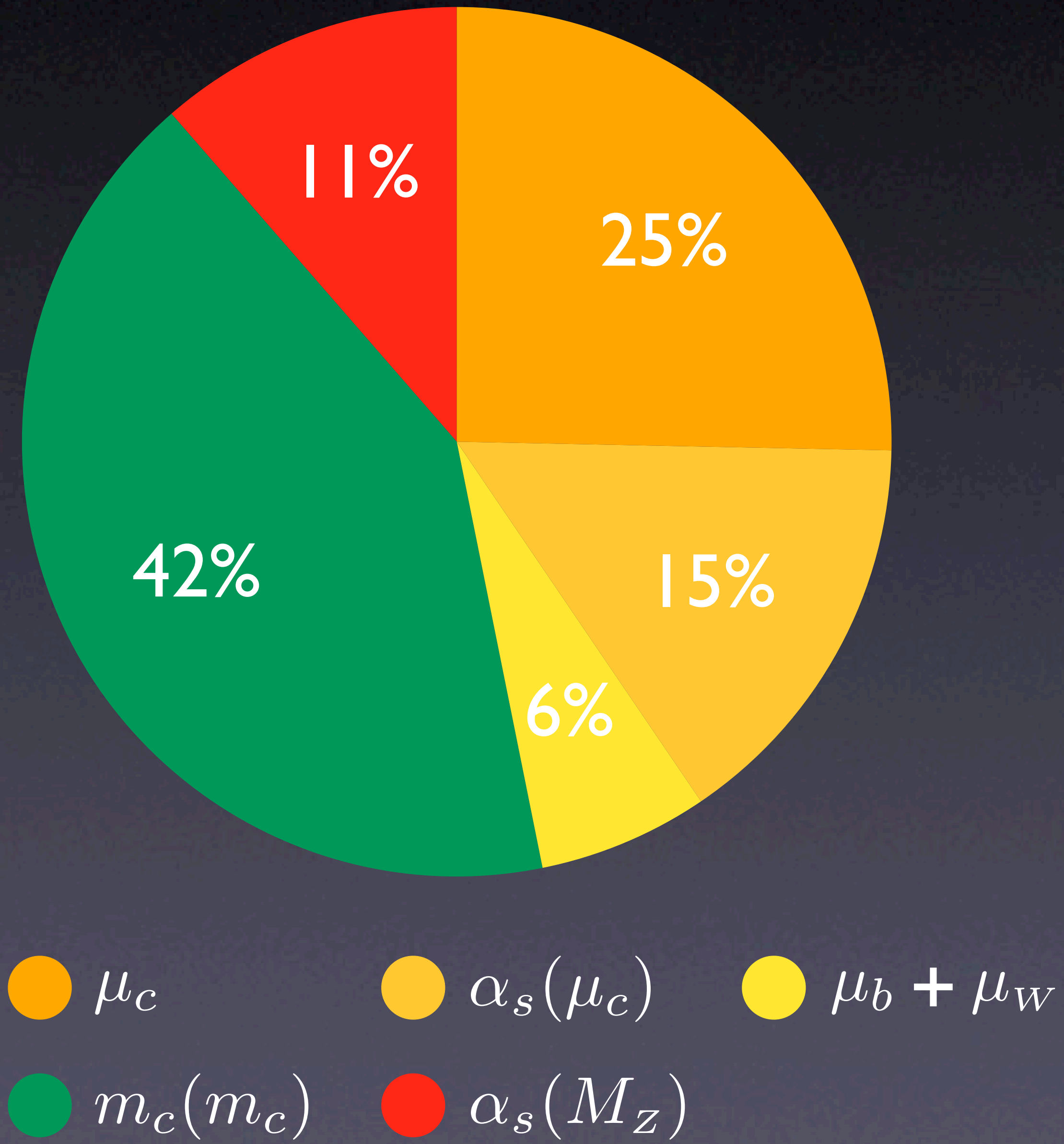
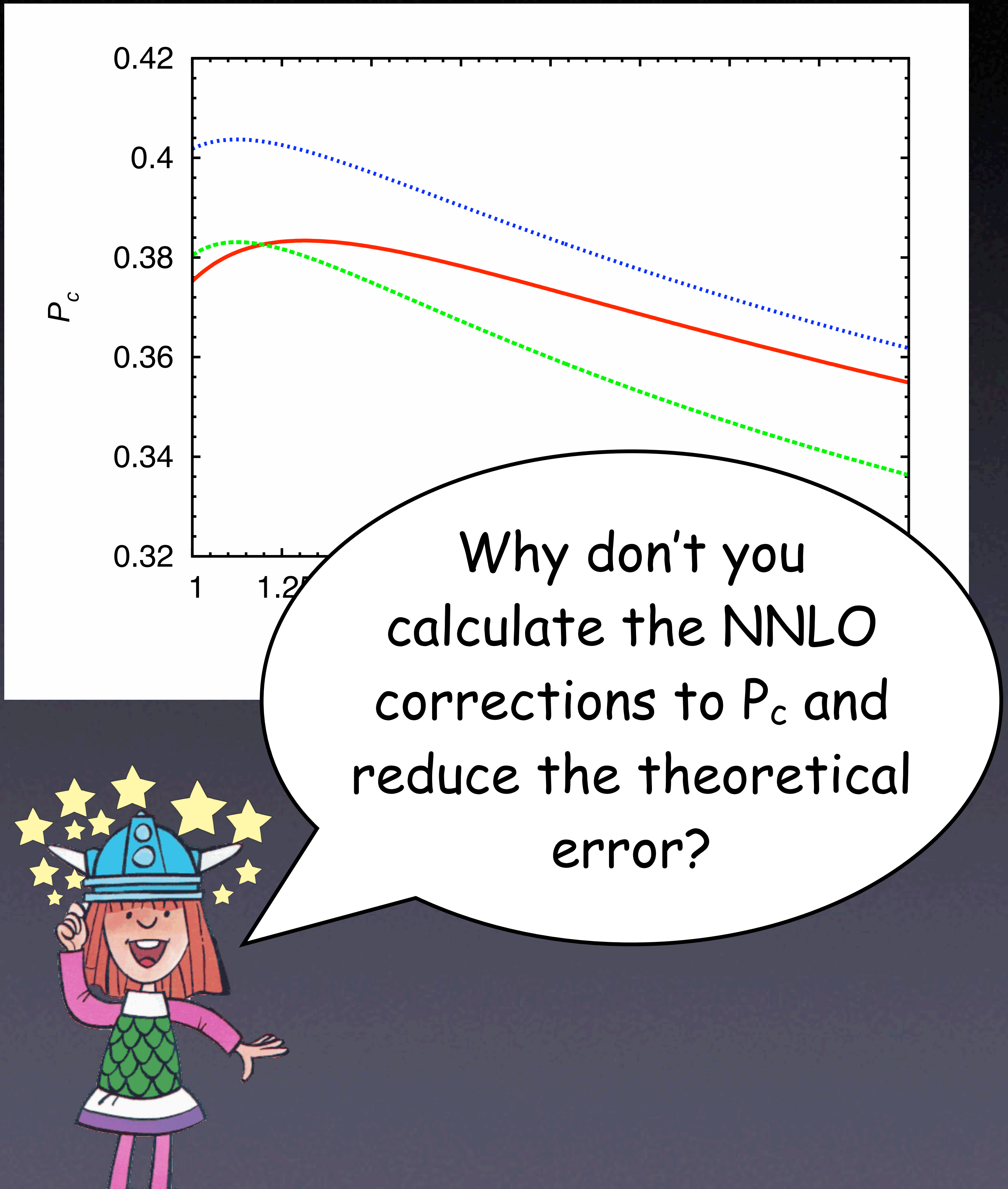
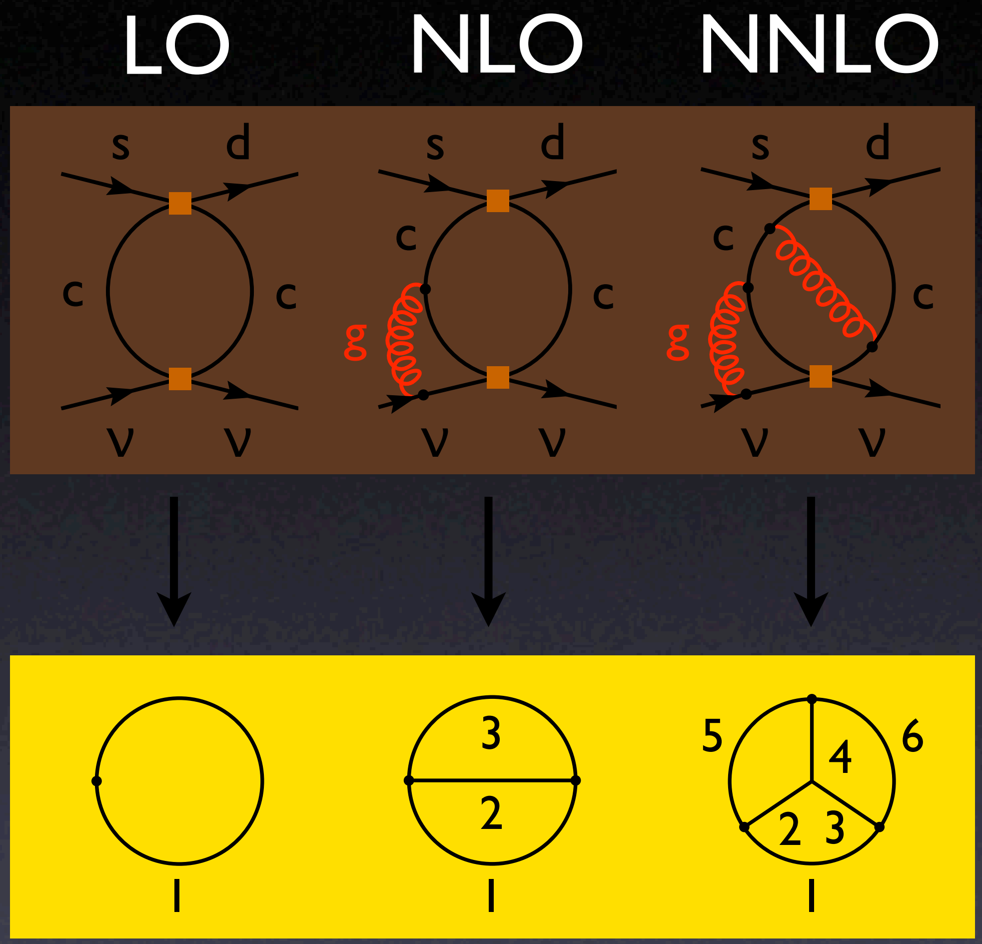




NNLO ADM calculation of P_c

- ADM is determined from 1/ϵ_{UV} of 1-, 2- and 3-loop diagrams
- integrals have 1/ϵ_{IR} and 1/ϵ_{UV} that are indistinguishable in DR
- in $\overline{\text{MS}}$ scheme 1/ϵ_{UV} are polynomial in masses and momenta after subtraction of subdivergences
- calculation of counterterms reduces to computation of massive tadpoles



$$\frac{1}{(k+p)^2-m^2} = \frac{1}{k^2-M^2} - \frac{p^2+2k\cdot p-m^2+M^2}{k^2-M^2}\,\frac{1}{(k+p)^2-m^2}$$

Chetyrkin, Misiak & Münz '98

New features in NNLO calculation of P_c

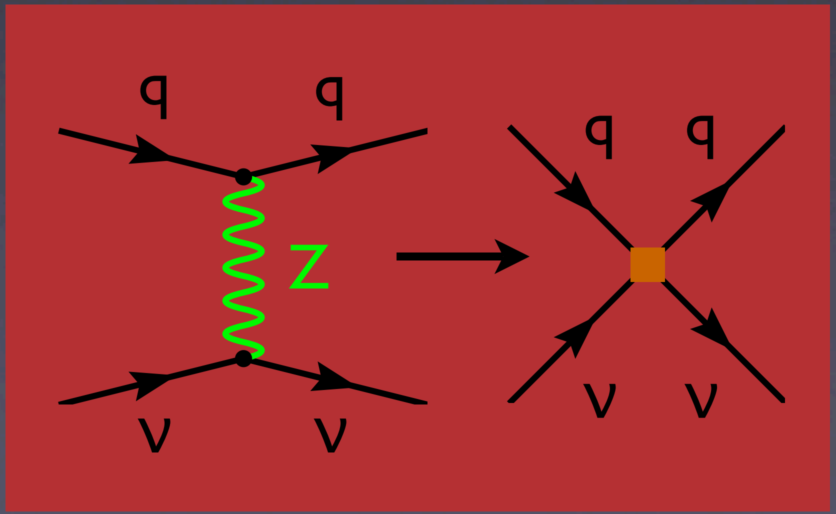
$$Q_Z = \sum_q ((I_q^3 - 2e_q \sin^2\theta_W) Q_V^q - I_q^3 (Q_A^q + Q_{CS}))$$

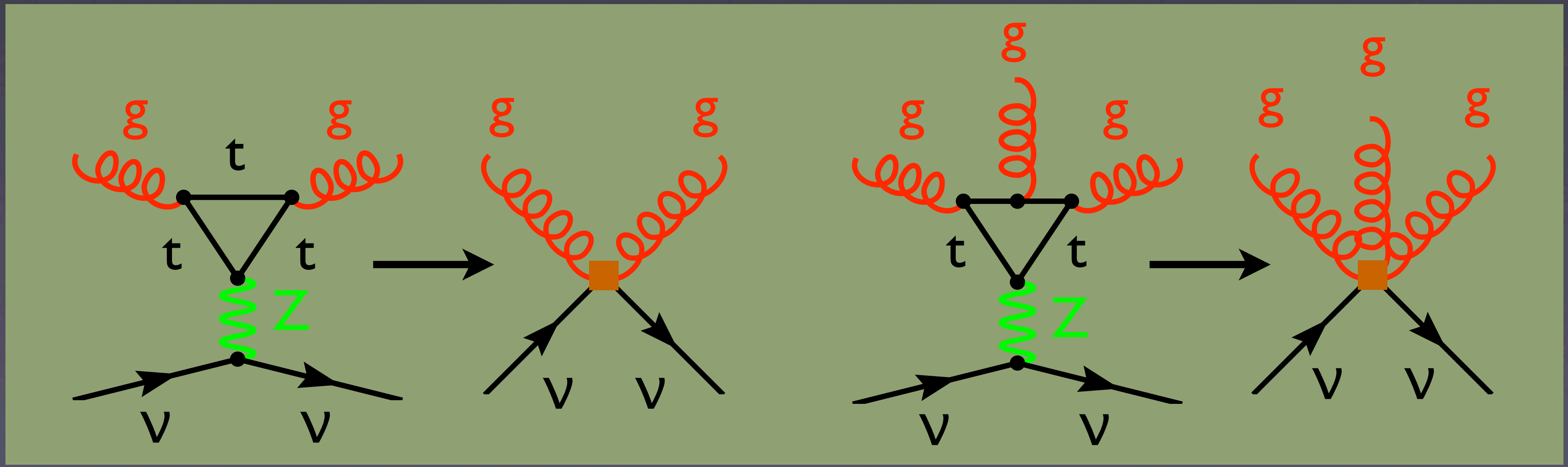
$$a_{CS} = \begin{cases} 2 & \text{HV} \\ \frac{2}{3} & \text{DRED} \end{cases}$$

$$Q_V^q = \sum_{l=e,\mu,\tau} (\bar{q}\gamma_\mu q)(\bar{\nu}_{lL}\gamma^\mu \nu_{lL})$$

$$Q_A^q = \sum_{l=e,\mu,\tau} (\bar{q}\gamma_\mu \gamma_5 q)(\bar{\nu}_{lL}\gamma^\mu \nu_{lL})$$

$$Q_{CS} = \frac{g^2}{16\pi^2} a_{CS} \epsilon^{\mu\nu\lambda\kappa} \left(G_\mu^a \partial_\nu G_\lambda^a + \frac{1}{3} g f^{abc} G_\mu^a G_\nu^b G_\lambda^c \right) \sum_{l=e,\mu,\tau} (\bar{\nu}_{lL}\gamma_\kappa \nu_{lL})$$





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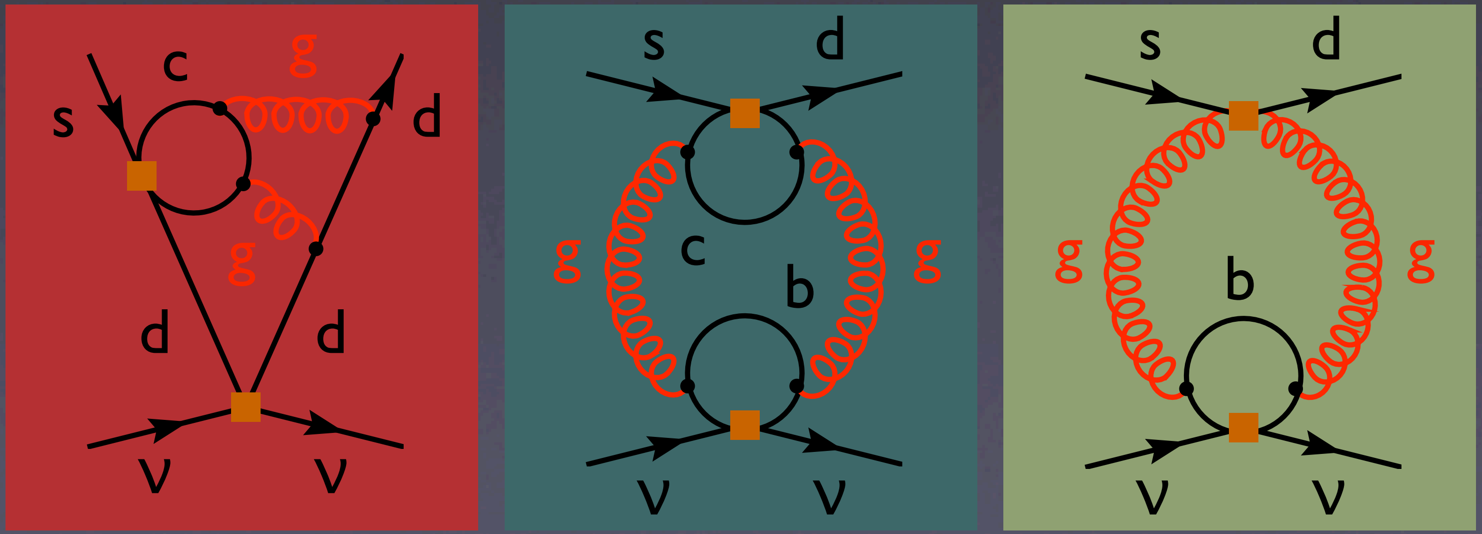


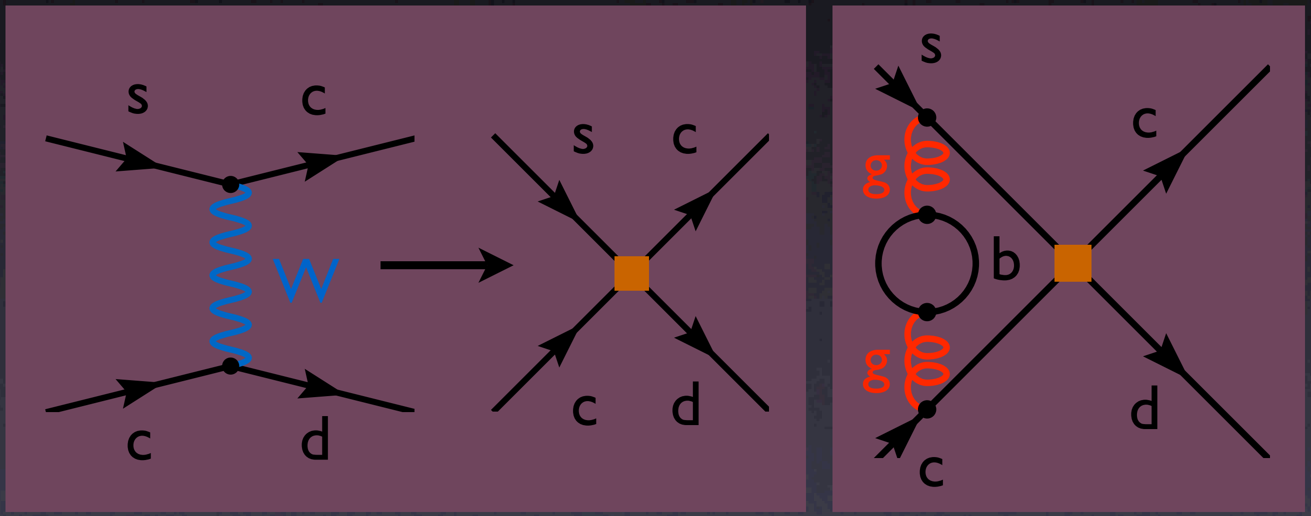
- vector part of neutral-current coupling starts to contribute
- contributions from axial-vector and Chern-Simons operator cancel exactly

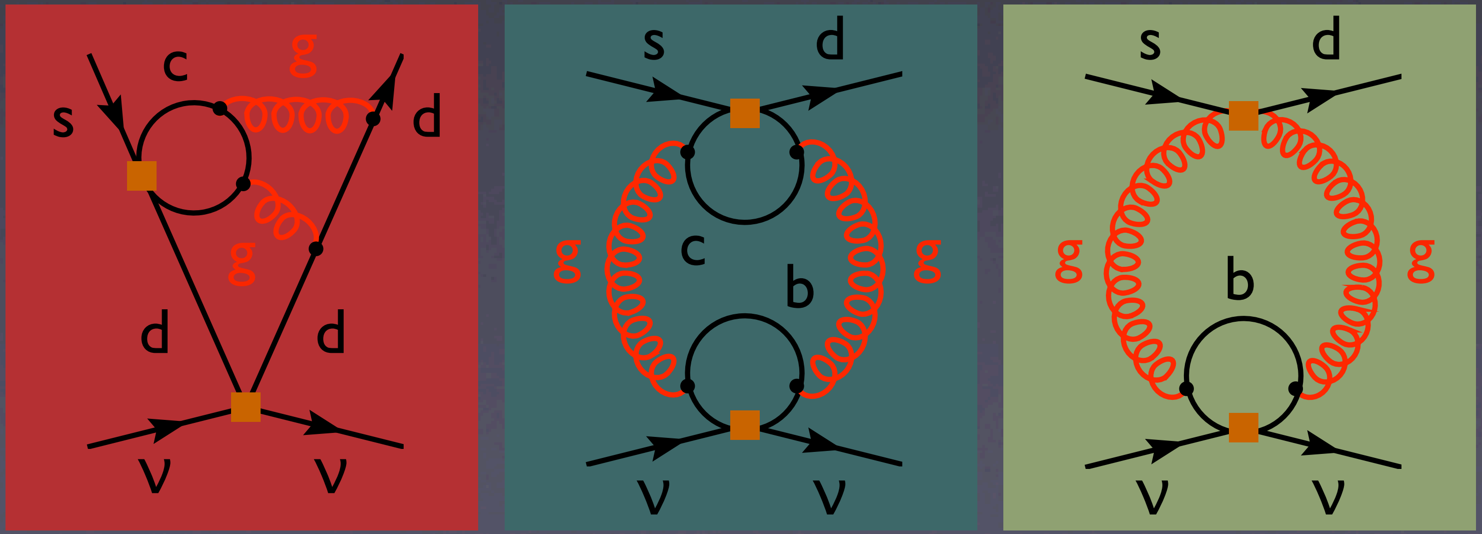
New features in NNLO calculation of *P_c*

$$Q_Z = \sum_q ((I_q^3 - 2e_q \sin^2\theta_W) Q_V^q - I_q^3 (Q_A^q + Q_{CS}))$$

$$Q_1 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu d_L)$$

$$Q_2 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a d_L)$$

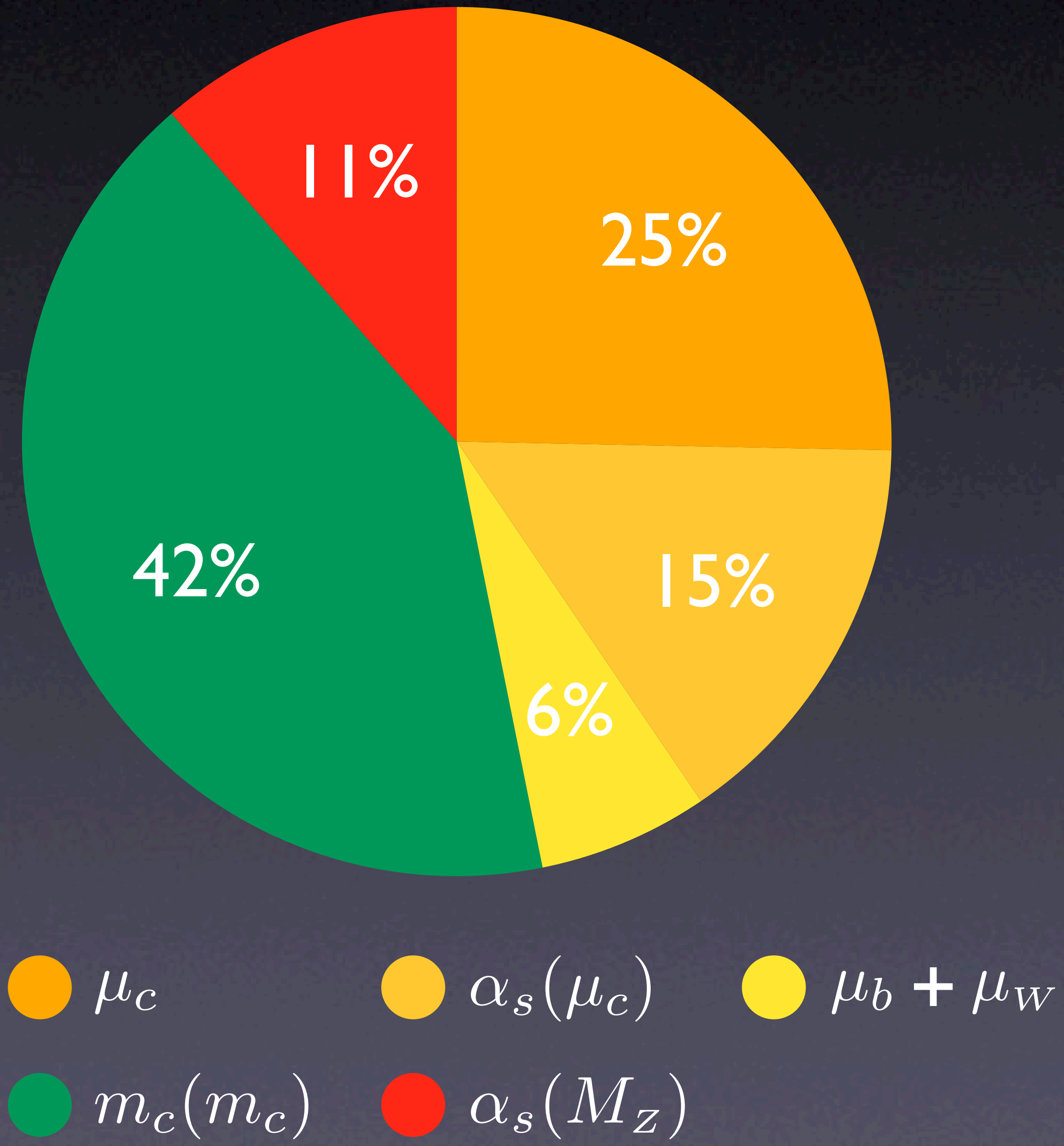


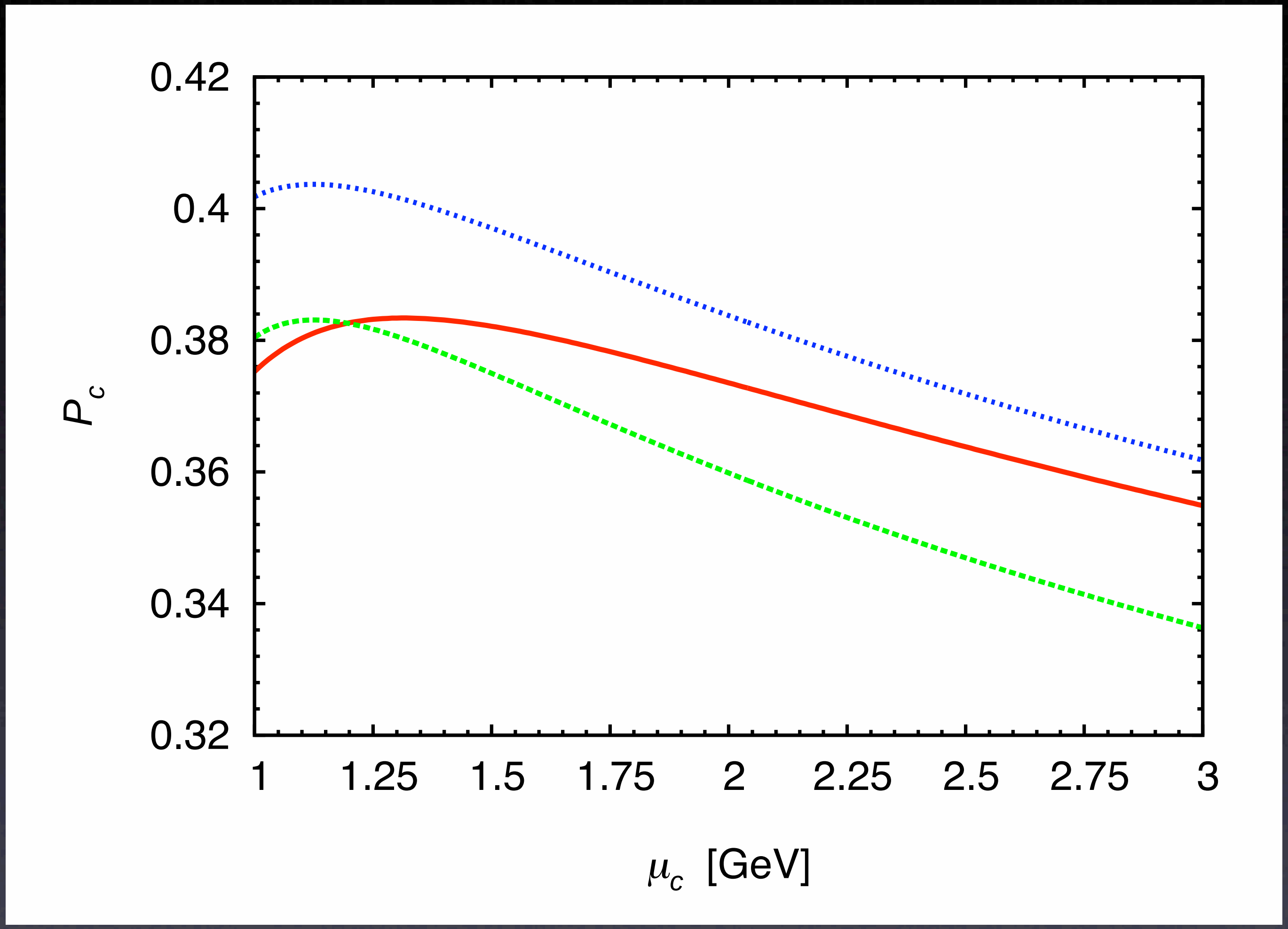


- non-trivial matching corrections at bottom quark threshold arise for current-current operators

Error budget of P_c and $B(\mathrm{K}^+)$ at NLO

$$P_c = 0.367 \pm 0.037 \pm 0.033 \pm 0.009$$



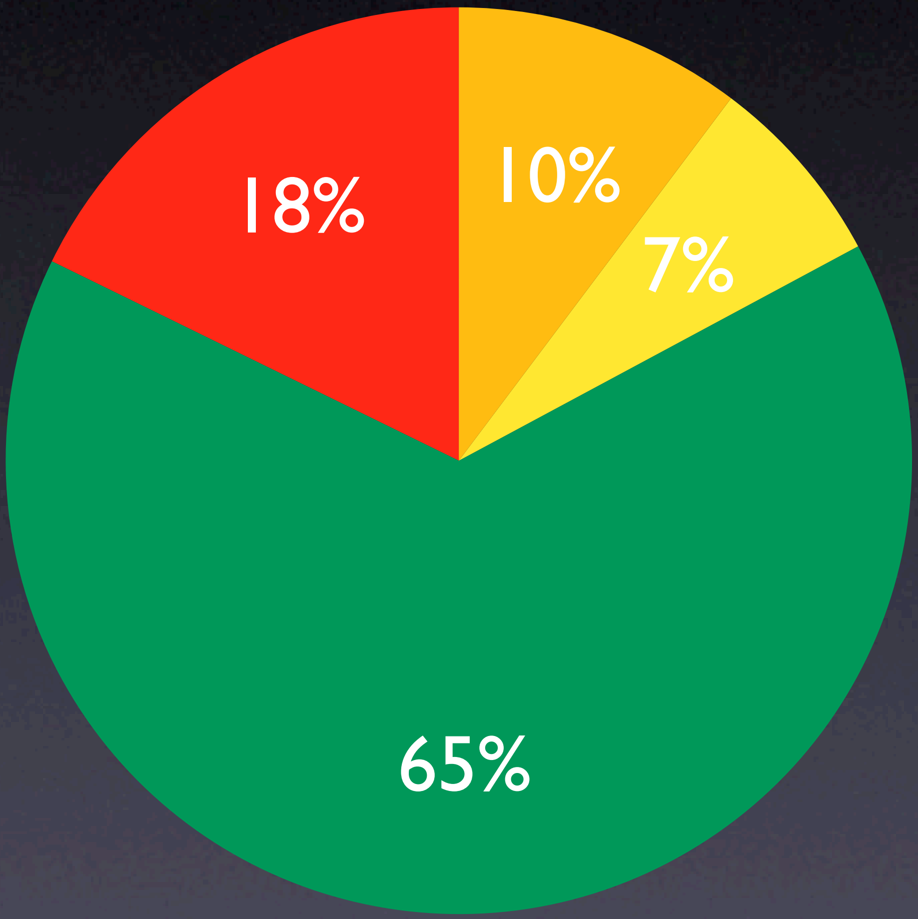


$$\mathcal{B}(K^+) = (7.93 \pm 0.77_{P_c} \pm 0.84_{\text{other}}) \times 10^{-11}$$

Buras, Gorbahn, Nierste & UH '05

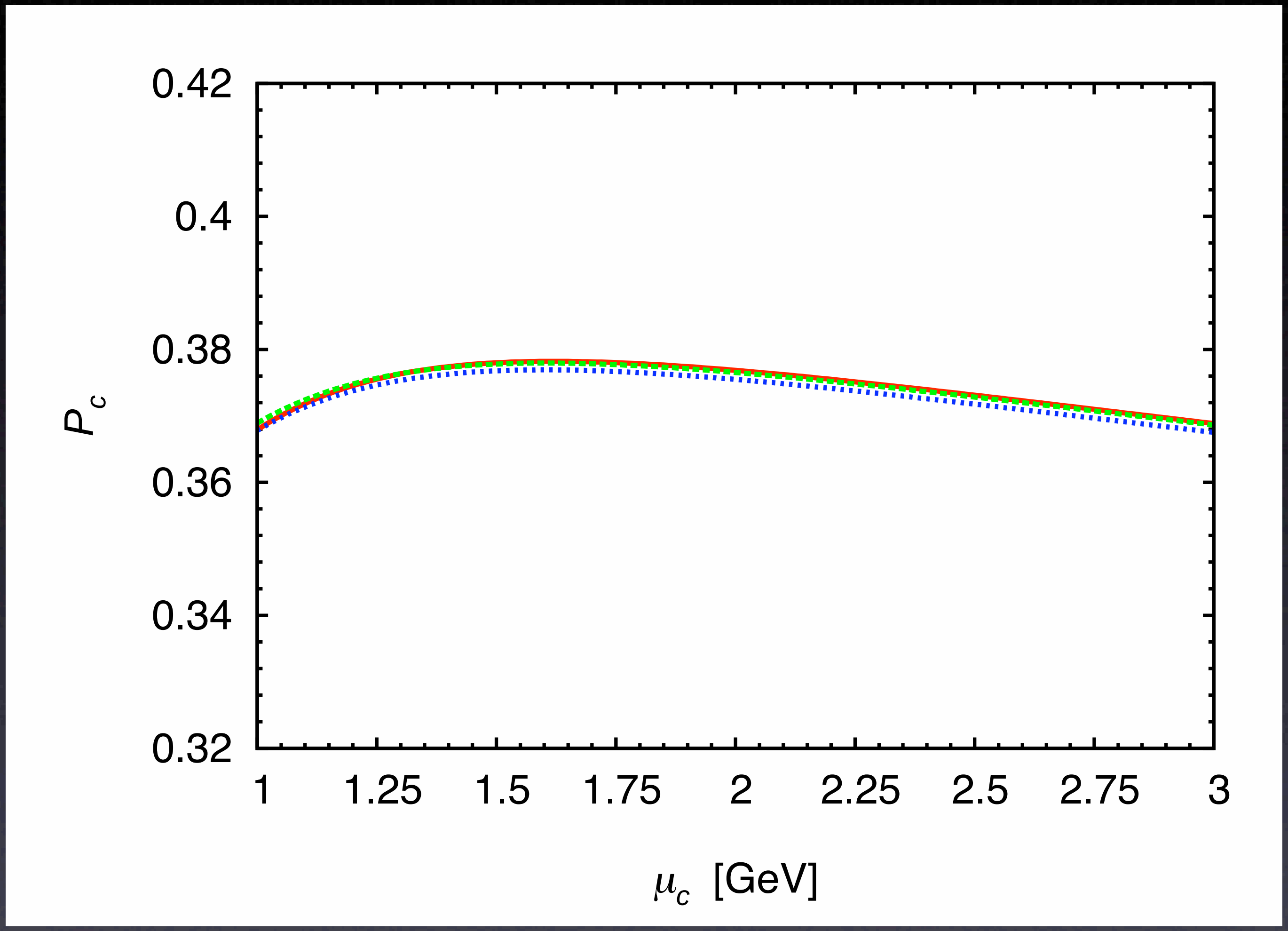
Error budget of P_c and $B(K^+)$ at NNLO

$$P_c = 0.371 \pm 0.009 \pm 0.031 \pm 0.009$$



● $\mu_c + \alpha_s(\mu_c)$ ● $\mu_b + \mu_W$

● $m_c(m_c)$ ● $\alpha_s(M_Z)$



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Buras, Gorbahn, Nierste & UH ‘05

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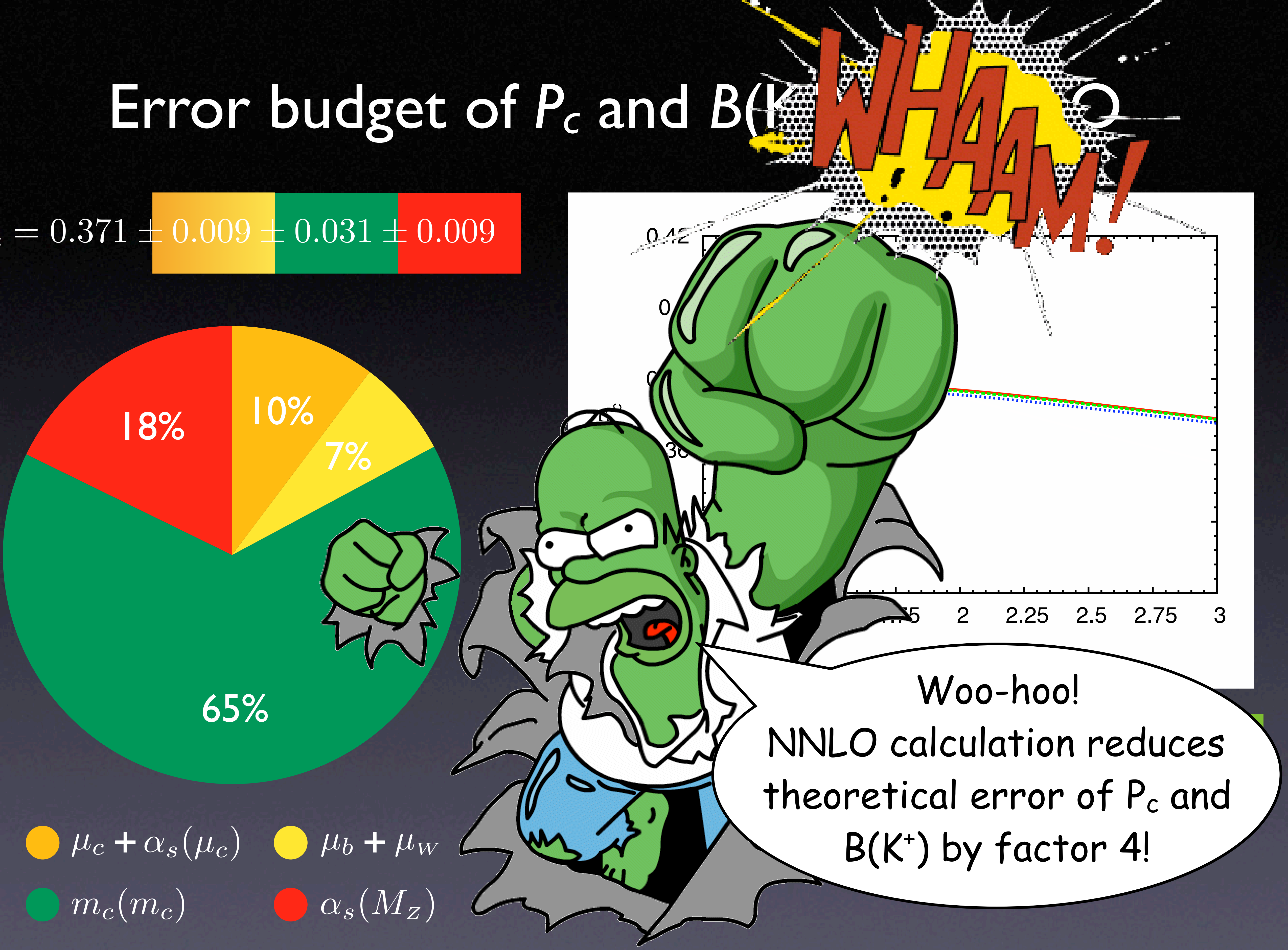
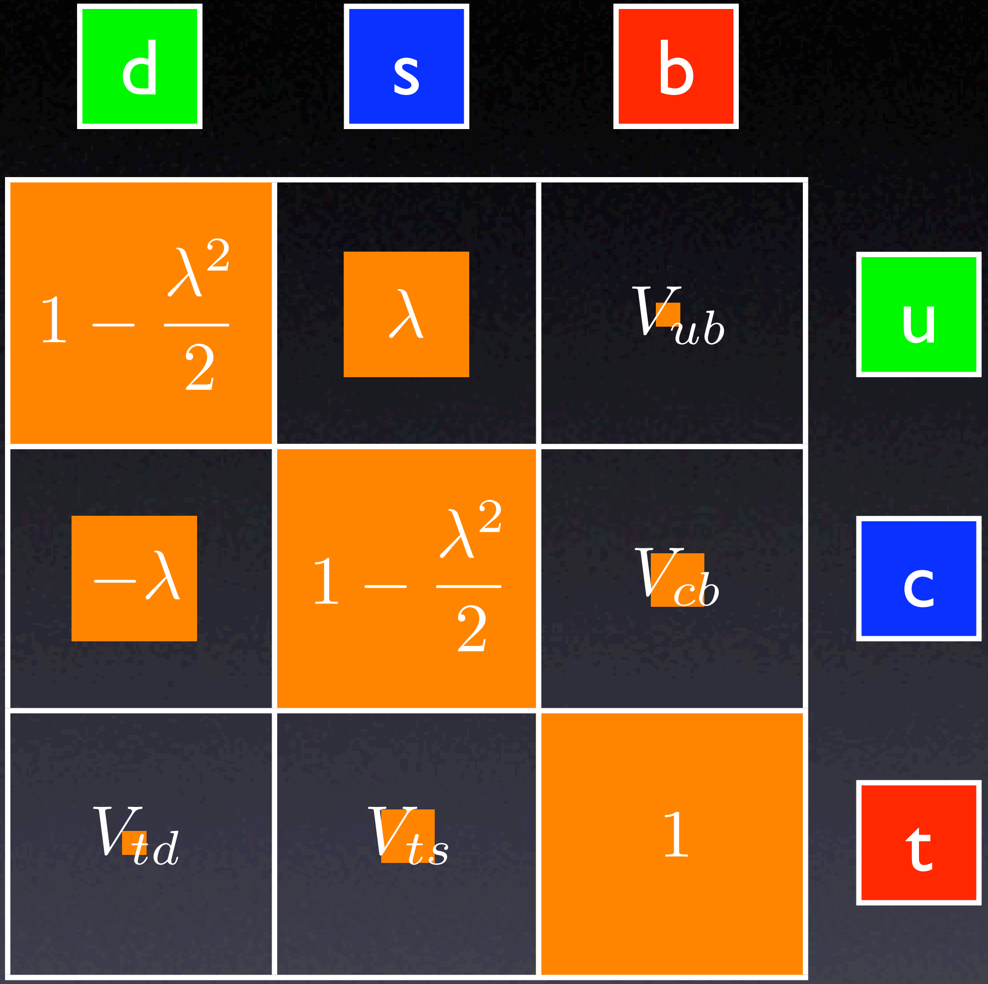


Unitarity triangle from $K \to \pi\nu\bar{\nu}$



$$\begin{array}{cccc} d & s & b & \\ 1-\dfrac{\lambda^2}{2} & \lambda & V_{ub} & u \\ -\lambda & 1-\dfrac{\lambda^2}{2} & V_{cb} & c \\ V_{td} & V_{ts} & 1 & t \end{array}$$

$$A \approx 0.82$$

$$\bar{\rho} = \rho\left(1-\frac{\lambda^2}{2}\right)$$

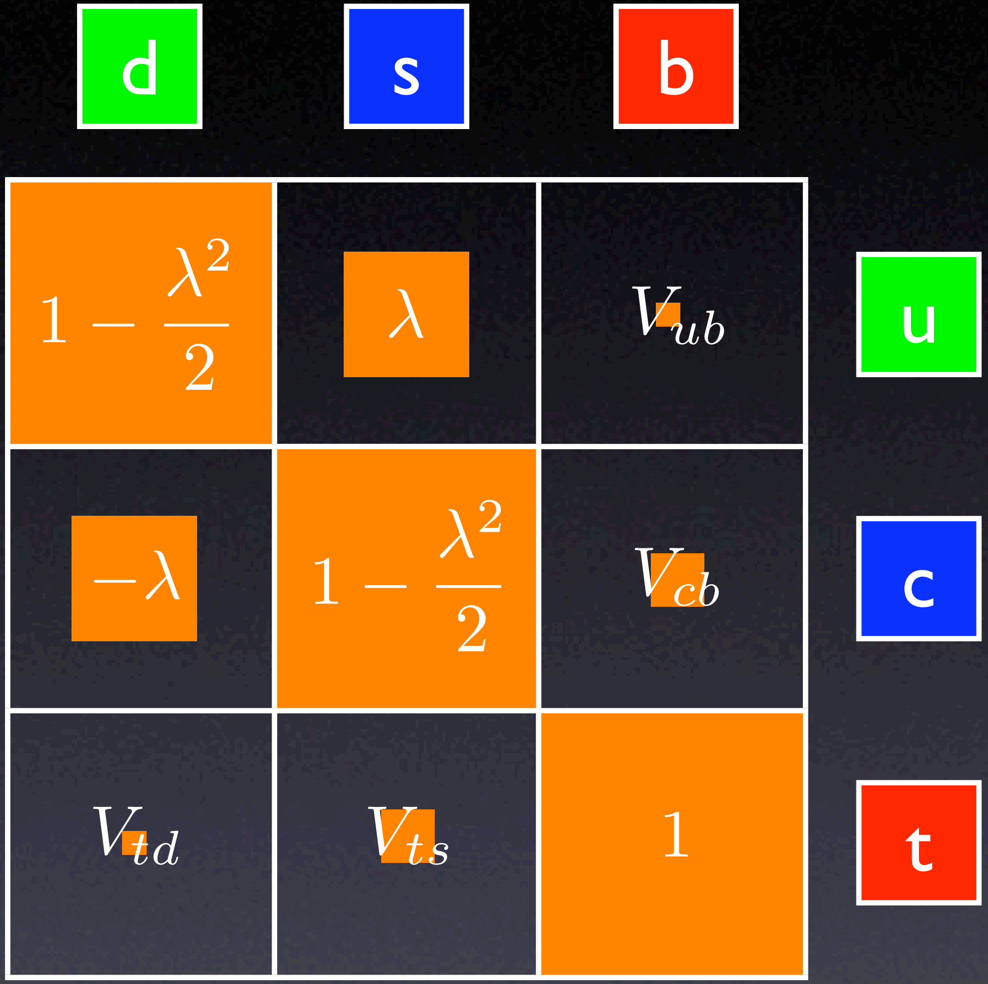
$$\bar{\eta} = \eta\left(1-\frac{\lambda^2}{2}\right)$$

$$V_{cb} = -V_{ts} = A\lambda^2$$

$$V_{ub} = A\lambda^3(\rho - i\eta)$$

$$V_{td} = A\lambda^3(1-\bar{\rho}-i\bar{\eta})$$

Unitarity triangle from $K \to \pi\nu\bar{\nu}$



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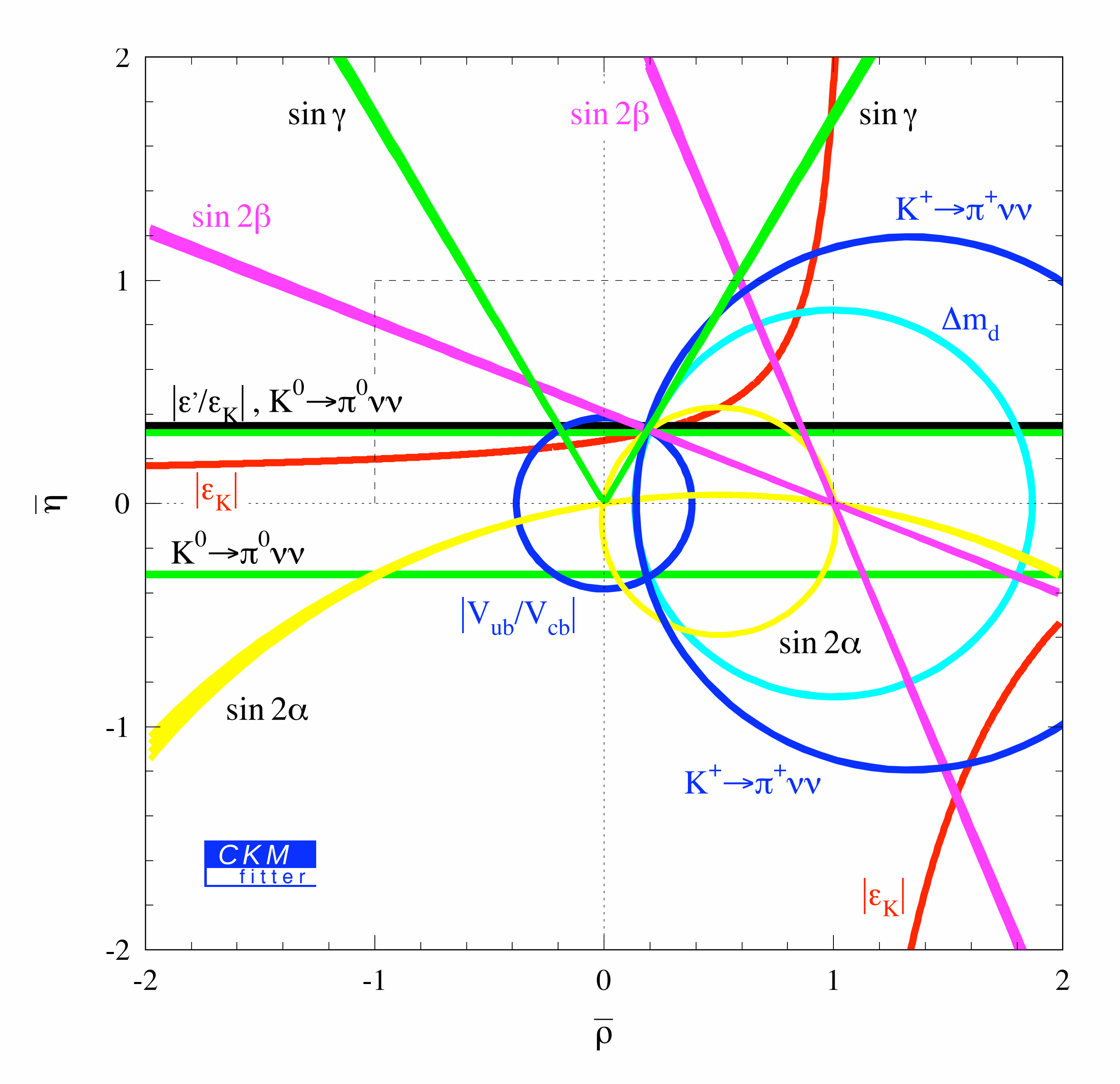

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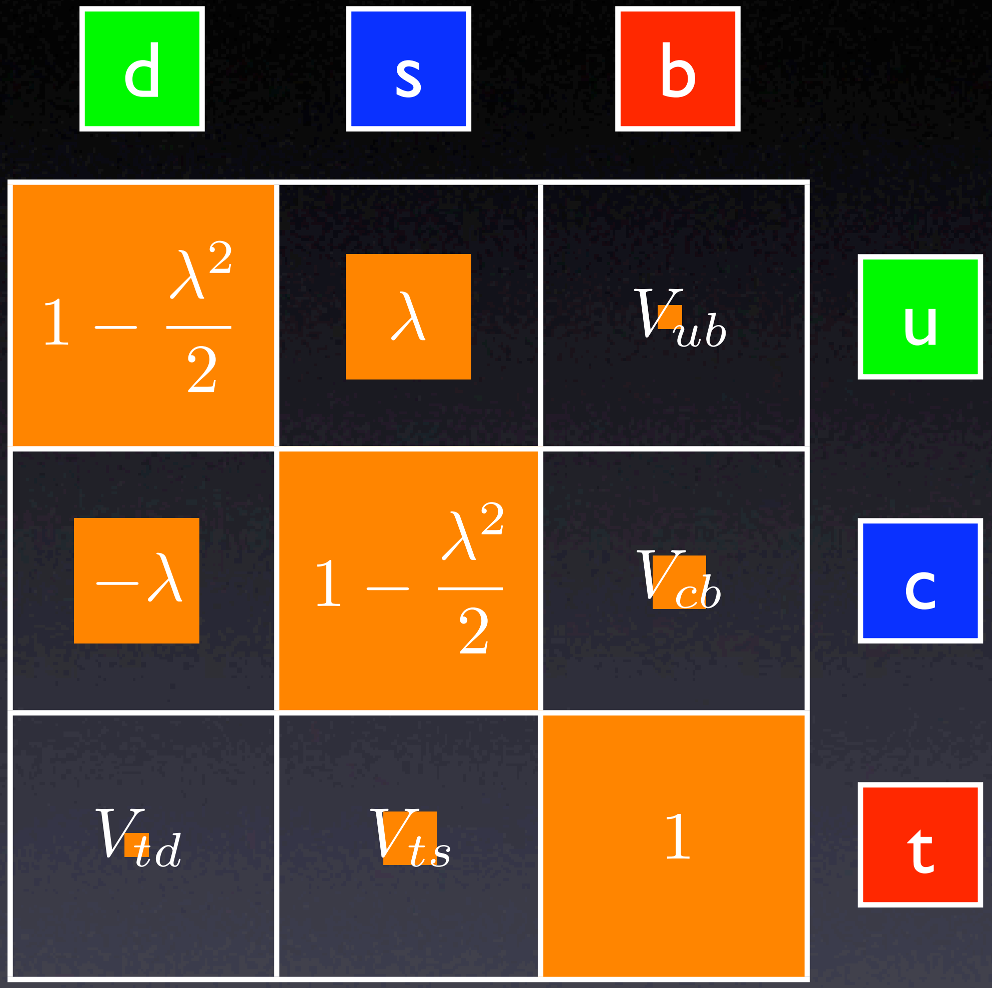
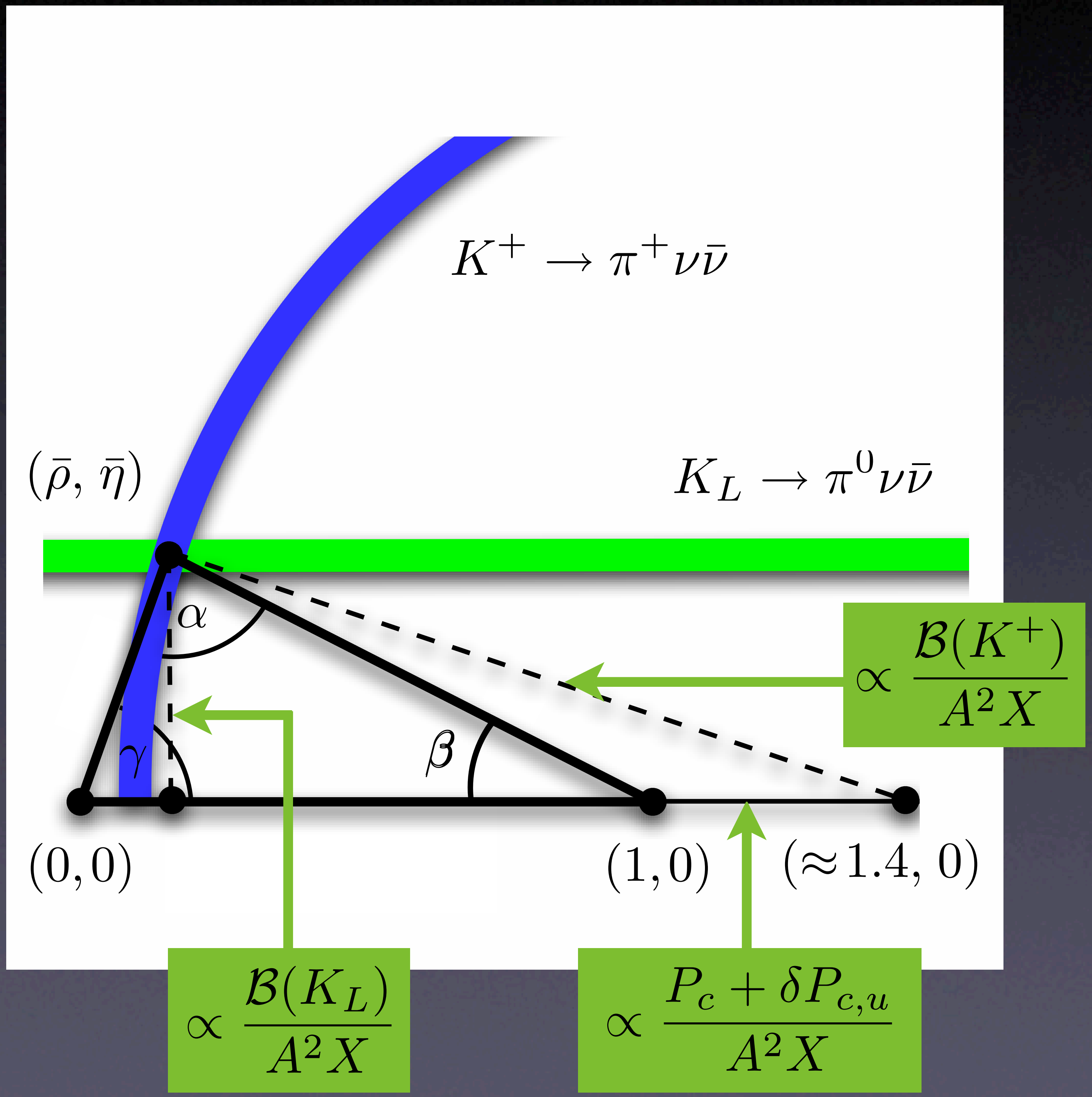


$$\begin{array}{c|ccc} & d & s & b \\ \hline u & 1-\frac{\lambda^2}{2} & \lambda & V_{ub} \\ c & -\lambda & 1-\frac{\lambda^2}{2} & V_{cb} \\ t & V_{td} & V_{ts} & 1 \end{array}$$

$$V_{cb} = -V_{ts} = A\lambda^2$$
$$V_{ub} = A\lambda^3(\rho - i\eta)$$
$$V_{td} = A\lambda^3(1-\bar{\rho}-i\bar{\eta})$$

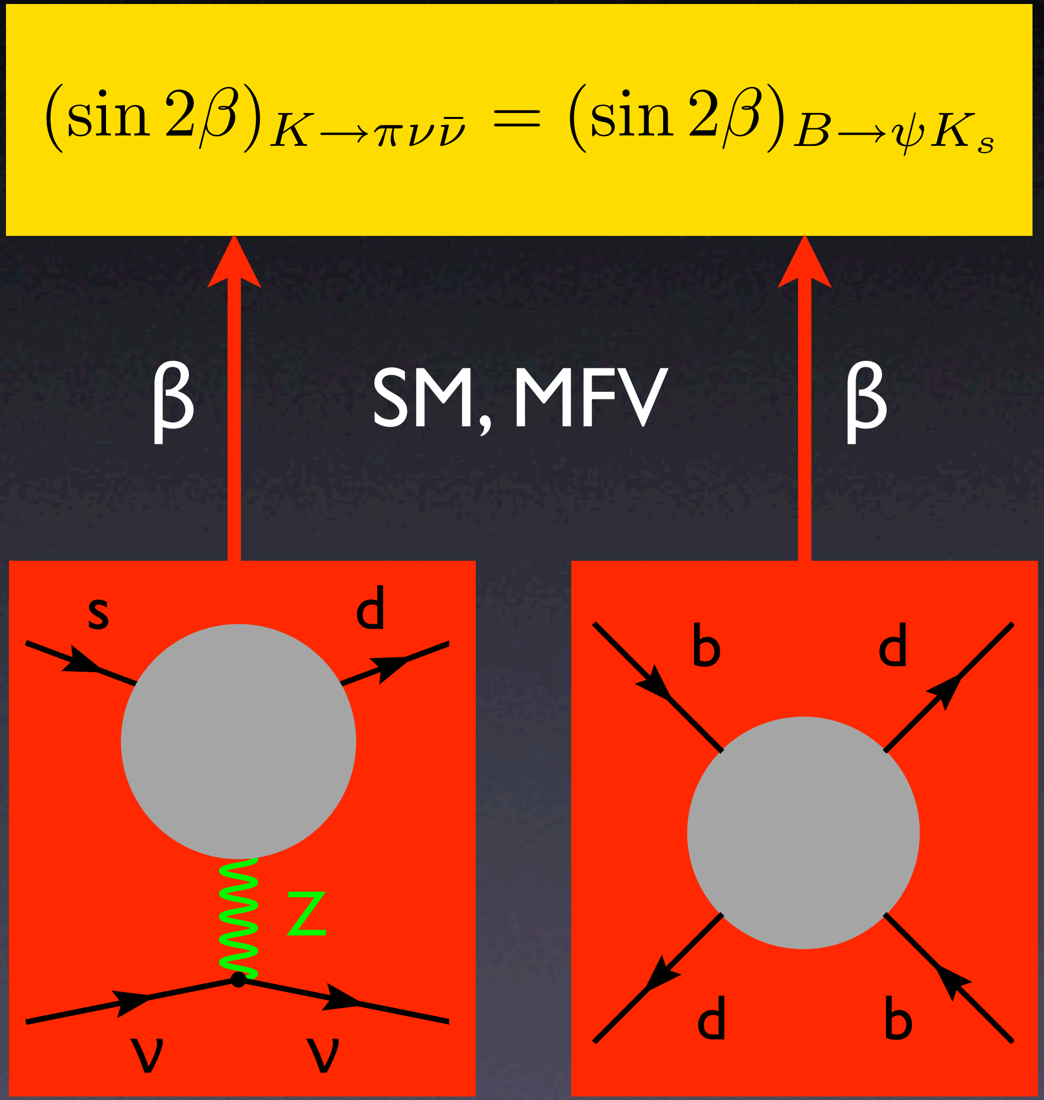


$$\propto \frac{\mathcal{B}(K^+)}{A^2 X}$$

$$\propto \frac{\mathcal{B}(K_L)}{A^2 X}$$

$$\propto \frac{P_c + \delta P_{c,u}}{A^2 X}$$

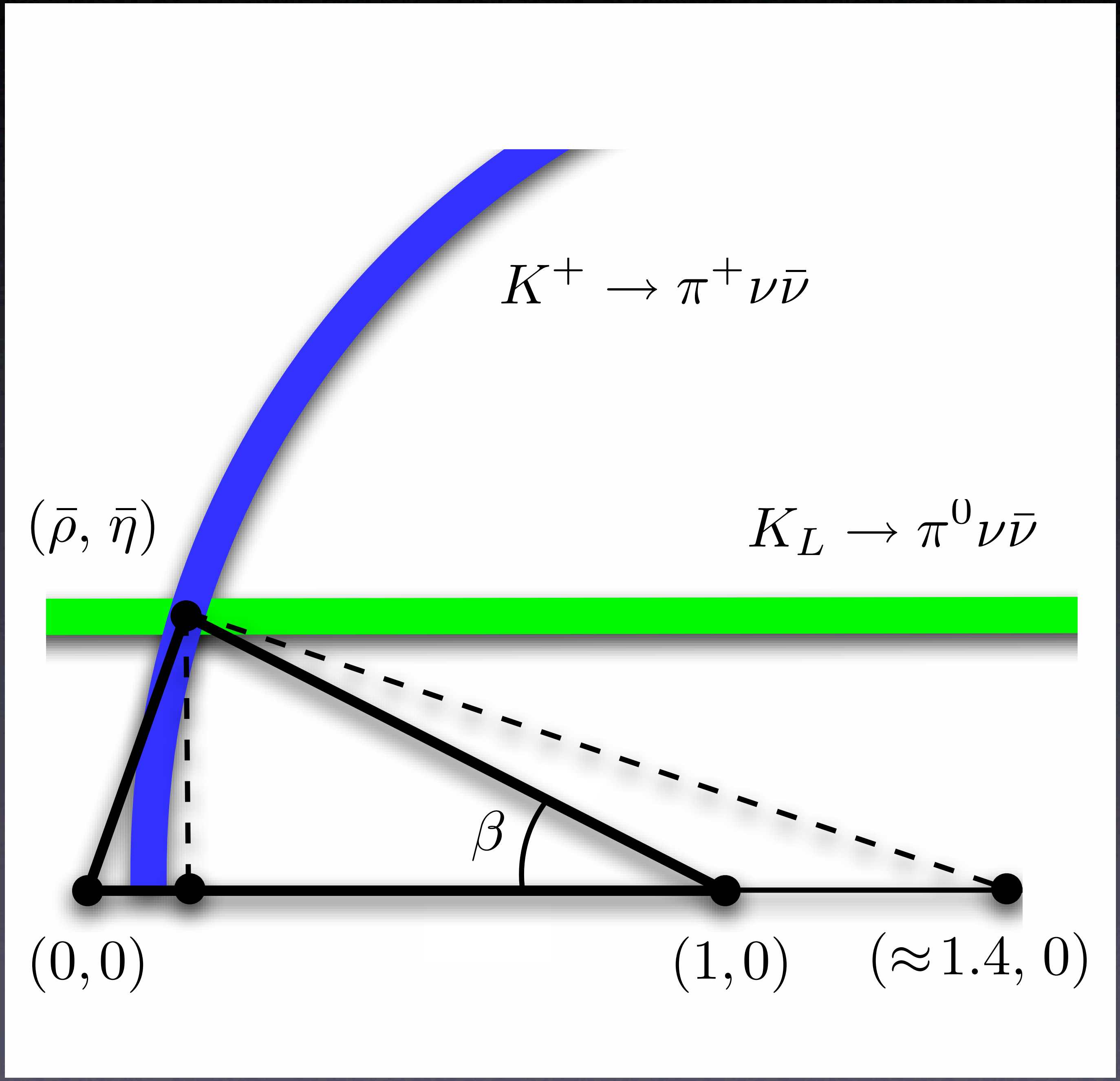
Unitarity triangle from $K \to \pi\nu\bar{\nu}$



$$(\sin 2\beta)_{K\to\pi\nu\bar{\nu}} = (\sin 2\beta)_{B\to\psi K_s}$$

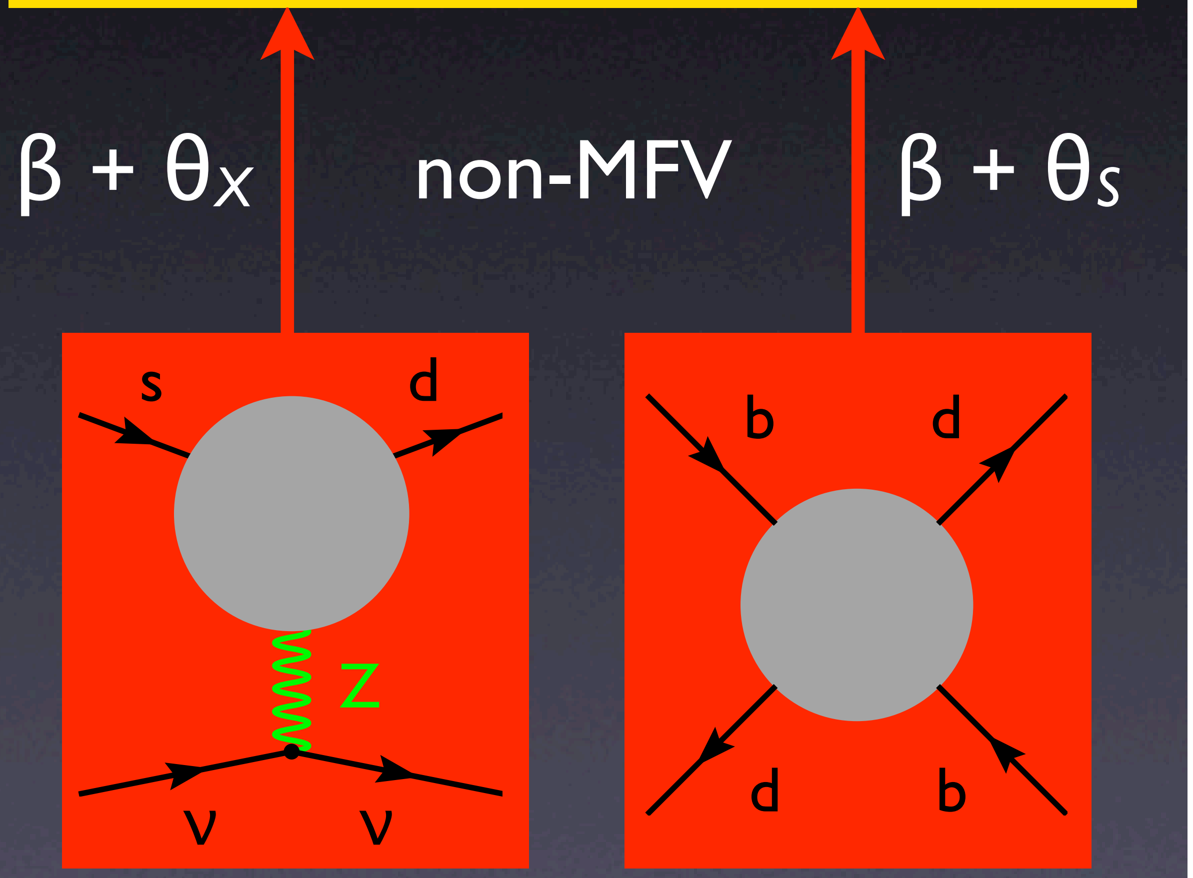
β SM, MFV β

K physics ⟷ B physics



Unitarity triangle from $K \to \pi\nu\bar{\nu}$

$$(\sin 2\beta_X)_{K\to\pi\nu\bar{\nu}} \neq (\sin 2\beta_S)_{B\to\psi K_S}$$

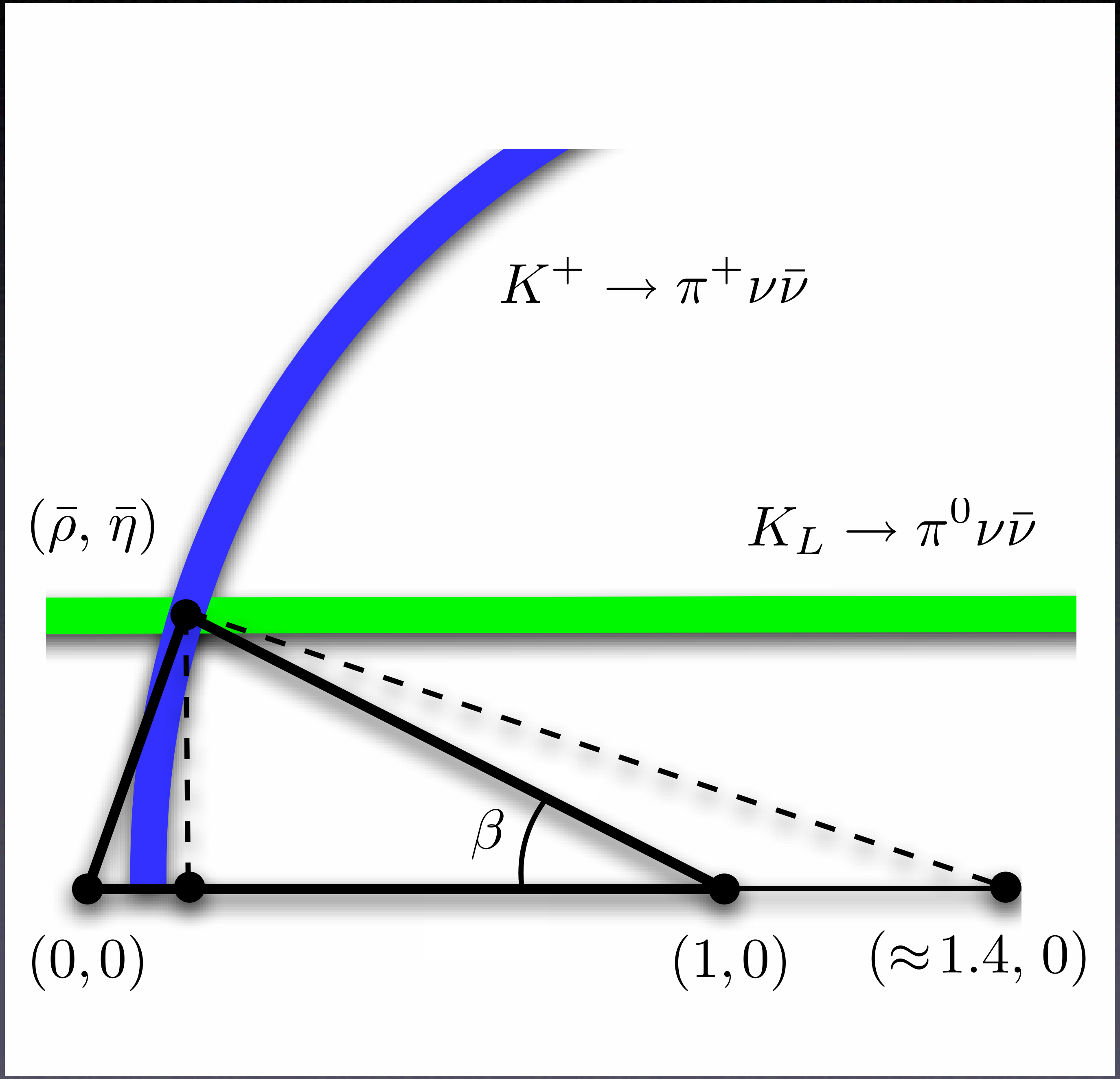


$\beta + \theta_X$

non-MFV

$\beta + \theta_S$

K physics ↔ B physics



Unitarity triangle fit from $K \rightarrow \pi\nu\bar{\nu}$

$$\mathcal{B}(K^+) = (14.7^{+13.0}_{-8.9}) \times 10^{-11}$$

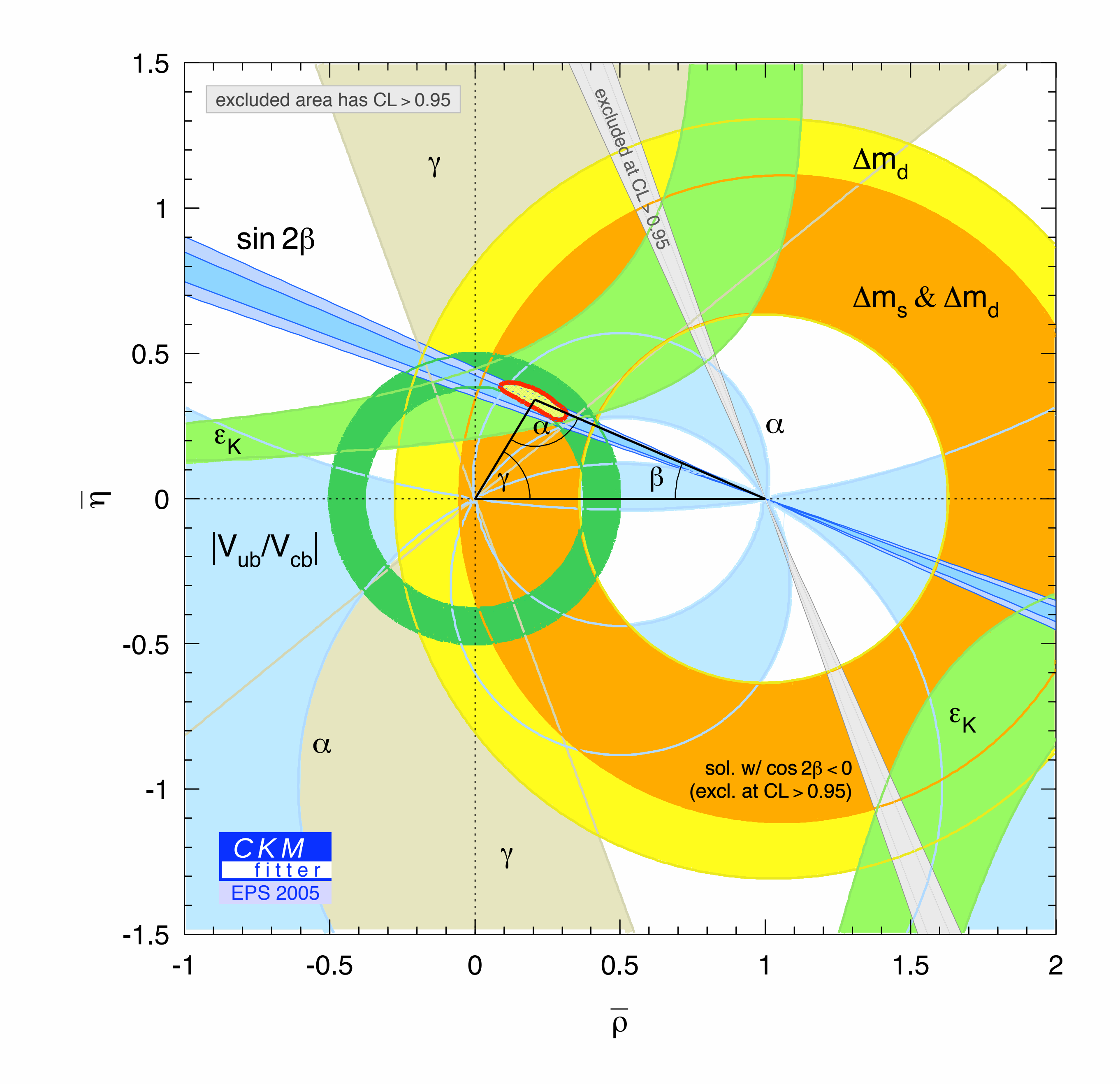
BNL AGS E787 & E949 '04

$$\mathcal{B}(K_L) < 5.9 \times 10^{-7} \quad (90\%\ \mathrm{CL})$$

FNAL KTeV E799-II '00

$$\mathcal{B}(K_L) < 2.86 \times 10^{-7} \quad (90\%\ \mathrm{CL})$$

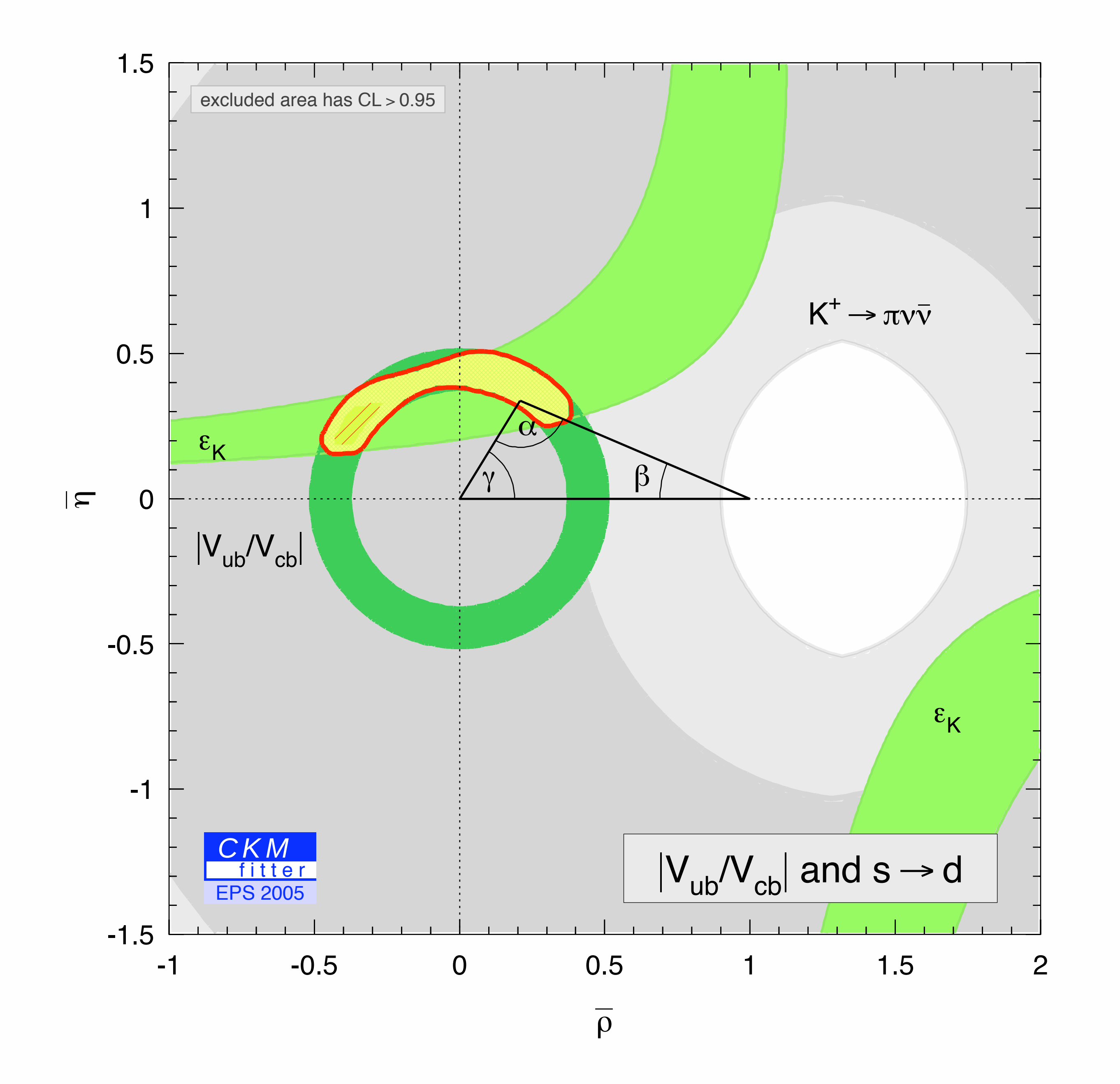
KEK PS E391a '05



Unitarity triangle fit from $K \to \pi\nu\bar{\nu}$

$$\mathcal{B}(K^+) = \left(14.7^{+13.0}_{-8.9}\right) \times 10^{-11}$$

BNL AGS E787 & E949 ‘04



Unitarity triangle fit from $K \to \pi\nu\bar{\nu}$

$$\mathcal{B}(K^+) = (7.50 \pm 0.75) \times 10^{-11}$$

$$\mathcal{B}(K_L) = (2.67 \pm 0.27) \times 10^{-11}$$

Future (?)

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 4.1\%$$

$$\sigma(\sin 2\beta) = \pm 0.025$$

$$\sigma(\gamma) = \pm 4.9^\circ$$

NLO

(theory error only)

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 1.0\%$$

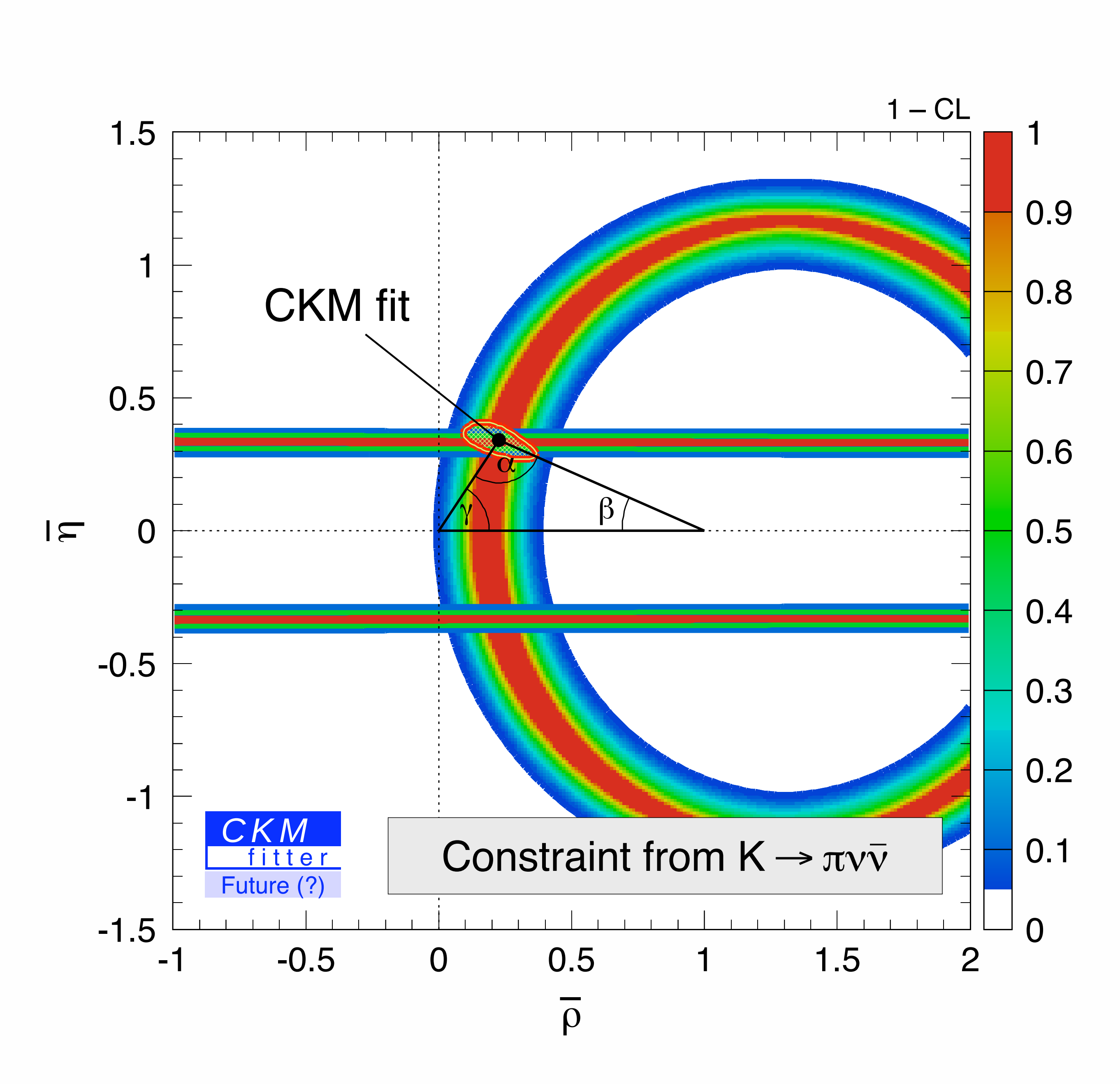
$$\sigma(\sin 2\beta) = \pm 0.006$$

$$\sigma(\gamma) = \pm 1.2^\circ$$

NNLO

(theory error only)

Buras, Gorbahn, Nierste & UH '05



Unitarity triangle fit from $K \to \pi\nu\bar{\nu}$

$$\mathcal{B}(K^+) = (7.50 \pm 0.75)\times 10^{-11}$$

$$\mathcal{B}(K_L) = (2.67 \pm 0.27)\times 10^{-11}$$

Future (?)

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 1.0\%$$

$$\sigma(\sin 2\beta) = \pm 0.006$$

$$\sigma(\gamma) = \pm 1.2^\circ$$

NNLO

(theory error only)

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 8.4\%$$

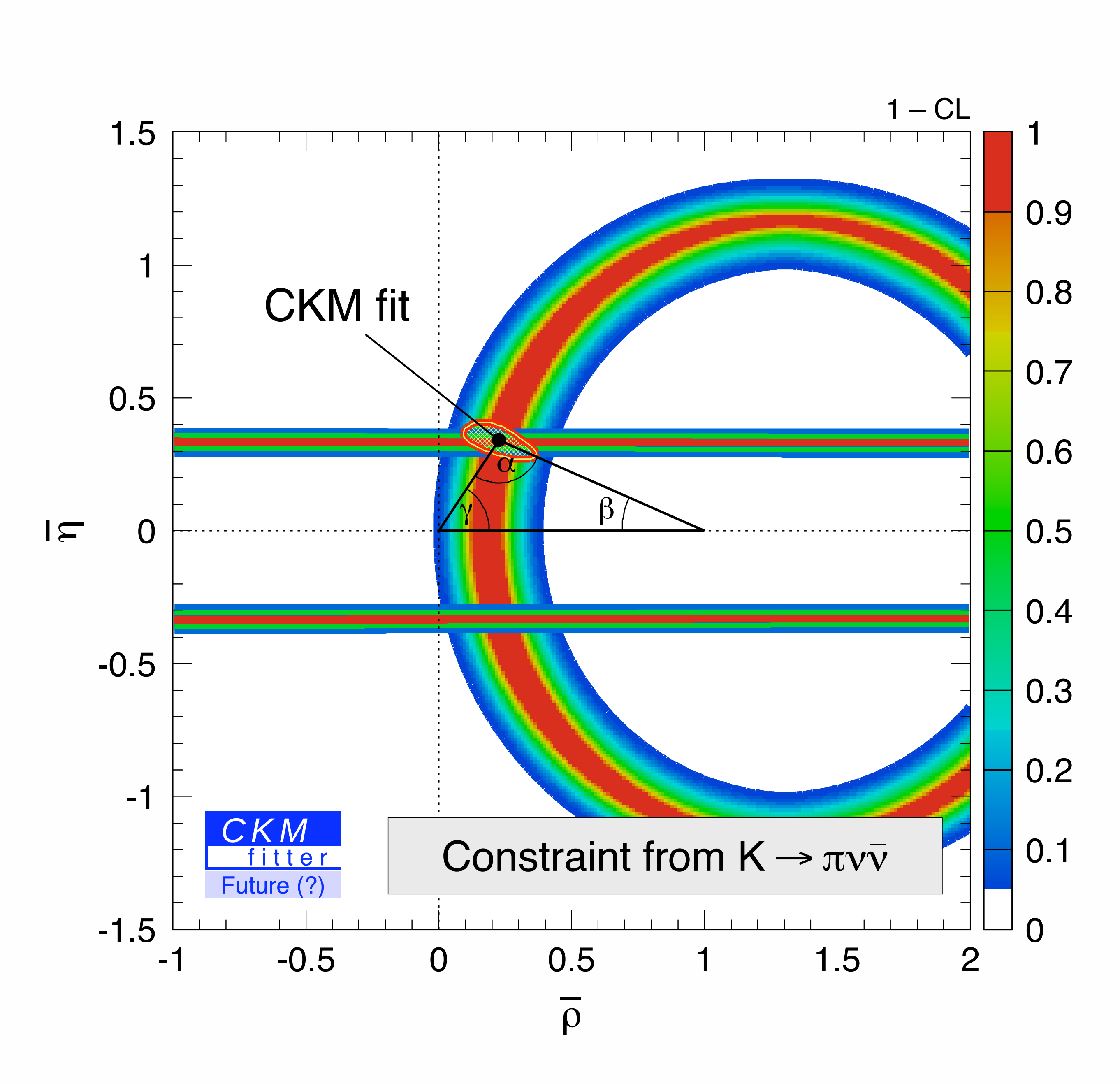
$$\sigma(\sin 2\beta) = \pm 0.056$$

$$\sigma(\gamma) = \pm 11^\circ$$

NNLO

(all uncertainties)

Future (?)



Conclusions

- SD dominated rare K^+ and K_L decays offer powerful and complementary test of flavor sector of SM
- NNLO calculation of charm contribution to K^+ is now available
- there is sizeable room for NP in this golden modes and their clean theoretical character remains valid in essential all extensions of SM
- measurements of branching ratios at ≈10% would substantially improve our understanding of flavor dynamics at TeV scale

Credits

A.J. Buras, A. Höcker, M. Gorbahn & U. Nierste