# Merging Parton Showers with NLO QCD

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# INTRODUCTION

## Why do we need NLO or higher order?

- We need accuracy.
- The strong coupling is big and the leading order predictions are very poor.
- The leading order results has very strong dependence on the arbitrary non-physical scales (renormalization and factorization)

## Why do we need parton shower?

- We want more realistic final state.
- In the fix order calculations we are able to calculate processes only with few partons in the final state.
- But in the detector we see lots of hadrons.
- If we want more realistic picture we have to deal with the hadronization. The hadronization is long distance physics. We cannot calculate but we can "measure" it. It is universal.
- We need "bridge" between the short distance and long distance part. $\Longrightarrow$ Parton shower

# Born Level Calculation

The Born level cross section is an $m$-parton phase space integral:

$$\sigma^{LO} = \int d\Gamma(\{p\}_m, Q) |M(\{p, f\}_m)|^2 F^{(m)}(p_1, .., p_m) + \mathcal{O}(\alpha_s^{m-1})$$

- Trivially no UV singularities. (No integral over infinite phase space.)
- No IR singularities from the phase space integral ensured by the $F^{(m)}$ measurement function.
- At this level the main task (challenge) is to calculate the matrix element squares.
  - the matrix element automatically generated up to $2 \to 6$ or even $2 \to 8$ (MADGRAPH, ALPGEN, HELAC, AMEGIC++,...)
  - plus automatic integration over the phase space (PHEGAS, MADEVENT, SHERPA,...)

# Parton Shower

Take the primary hard process (in $e^+e^-$ annihilation it is the $e^+e^- \to q\bar{q}$) and calculate the rest of the event by parton shower algorithm

$$\sigma^{LO,S} = \int d\Gamma(\{p\}_2, Q)|M(\{p,f\}_2)|^2 \big(S(\{p,f\}_2)\big|F\big)\big(1 + \mathcal{O}(\alpha_s L)\big)$$

- Based on that physical picture that every parton produce a jet (jet: A "spray" of collinear hadrons).
- In the collinear limit the QCD matrix element has factorization properties. This factorization property allows us to calculate the $\big(S(\{p,f\}_2)\big|F\big)$ recursively.
- There are several program available: APACIC++, ARIADNE, COJET, HERWIG(++), PYTHIA(++),...
- This is an all order expression but gives good approximation only at LL level. Out of this region the performance is very poor.

# Matrix Element + Parton Shower

(CKKW: Catani-Krauss-Kuhn-Webber Method)

Defining the jet clustering sequence for a tree level $m$-parton process using the $k_\perp$ jet algorithm, that is $d_2 > d_3 > \cdots > d_n > d_{\rm ini}$, the cross section is given by

$$\sigma^{LO+S}[F] = \sum_{m=2}^{m_{\rm max}} \tilde{\sigma}_m^{B+S}[F] \quad ,$$

where

$$\tilde{\sigma}_m^{B+S}[F] = \int_m d\Gamma(\{p\}_m, Q)\, |M(\{p,f\}_m)|^2\, \theta(d_m > d_{\rm ini}) \times W_m(\{p,f\}_m)\, \left(S(\{p,f\}_m; d < d_{\rm ini}) \middle| F\right)$$

- The scale $d_{\rm ini} > 0$ helps to keep away from the singular region in the hard matrix element. It is arbitrary but not zero.
- The $\theta(d_m > d_{\rm ini})$ introduces large logarithms in the variable $d_{\rm ini}$ but they are cancelled at NLL level.
- This cancellation is ensured by the Sudakov reweighting and the vetoed shower.

# NLO Cross Sections

To eliminate the IR singularities from the real part the best way is the dipole method:

$$\sigma^{NLO} = \int_m d\sigma^B + \int_{m+1} [d\sigma^R|_{\epsilon=0} - d\sigma^A|_{\epsilon=0}] + \int_m [d\sigma^V + \int_1 d\sigma^A]_{\epsilon=0}$$

massless case : S. Catani and M.H. Seymour

massive case : S. Catani S. Dittmaier, M.H. Seymour, Z. Trócsányi

The $d\sigma^A$ is a local counterterm for $d\sigma^R$ with same pointwise behaviour as $d\sigma^R$. Furthermore it is integrable in $d = 4 - 2\epsilon$ dimension over the single parton subspaces.

$$\sigma^{NLO} = \int_m d\sigma^B + \int_{m+1} \Big[d\sigma^R\Big|_{\epsilon=0} - \sum_{\text{dipoles}} d\sigma^B \otimes dV\Big|_{\epsilon=0}\Big]$$

$$+ \int_m \Big[d\sigma^V + d\sigma^B \otimes \boldsymbol{I}^R(\epsilon)\Big]_{\epsilon=0}$$

where the $\boldsymbol{I}(\epsilon)$ singular factor in the massless case is

$$\boldsymbol{I}^R(\epsilon) = -\frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\sum_{j\neq i}\frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{\boldsymbol{T}_i^2}\left(\frac{\mu^2}{s_{ij}}\right)^\epsilon\left[\boldsymbol{T}_i^2\left(\frac{1}{\epsilon^2}-\frac{\pi^2}{3}\right)+\gamma_i\frac{1}{\epsilon}+\gamma_i+K_i+\mathcal{O}(\epsilon)\right]$$

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- several program: EKS, JETRAD, EVENT(2), EERAD, DISENT, DISASTER++,MEPJET, AYLEN/EMILIA, PHOX, ...
- in some case we can calculate up to $2 \rightarrow 3$ (NLOJET++, MCFM )
- some interesting process is computed but many of them still missing
- There is no automated program.

# NLO + SHOWER

There are two other approches on the market:

- MC@NLO approach by Frixione, Webber and Nason
  - the method is not general, not Lorentz covariant
  - the matching is not exact
  - it is worked out for the processes with no final state colored particle or only with massive quarks.
  - The method is specific to a particular Monte Carlo implementation (HERWIG).
- By M. Krämer and D.E. Soper
  - It is worked out only for $e^+e^- \to 3\,\text{jets}$.
  - It is based on a fully numeric NLO method which is also specific to the $e^+e^- \to 3\,\text{jets}$ process.
  - It is not Lorentz covariant.
  - But the basic idea is very general and it gives exact matching.
  - It can work together with any shower program.
- There is no algorithm defined to achive the "NLO matrix element + Parton Shower" project (CKKW@NLO).

# HIGHLIGHTS OF OUR METHOD

In our approach we want the most of all, to have an algorithm that can be used in a reasonably straightforward manner:

- It is based on the Catani-Seymour subtraction method $\Rightarrow$ Lorentz covariance, *easy* to implement
- The first hardest step of the shower is included in the NLO program and the subsequent shower is calculated by the user's favorite shower algorithm $\Rightarrow$ not specific to a particular MC implementation
- We have full control on the first step in every singular regions $\Rightarrow$ exact matching
- The algorithm can deal with any number of the colored particles in the final and initial states.
- We implement the CKKW matching scheme at NLO level.

# NLO + Parton Shower

In the CKKW matching scheme the cross section is sum of the partial cross sections:

$$\sigma^{S}[F] = \sum_{m=2}^{m_{\rm NLO}} \sigma_m^{NLO+S}[F] + \sum_{m=m_{\rm NLO}+1}^{m_{\max}} \tilde{\sigma}_m^{B+S}[F]$$

At NLO level one term in the CKKW cross section should be

$$\sigma_m^{NLO+S}[F] = \sigma_m^{B+S}[F] + \sigma_m^{R+S}[F] + \sigma_m^{V+S}[F]$$

The simplified Born term should be

$$\tilde{\sigma}_m^{B+S}[F] = \sum_{\{f\}_m} \frac{1}{m!} \int d\Gamma(\{p\}_m; Q)\, \theta(d_m > d_{\rm ini})\, W_m(\{p,f\}_m) \times \left|\mathcal{M}(\{p,f\}_m)\right|^2 \left(S(\{\hat{p},\hat{f}\}_m; d < d_{\rm ini})\middle|F\right)$$

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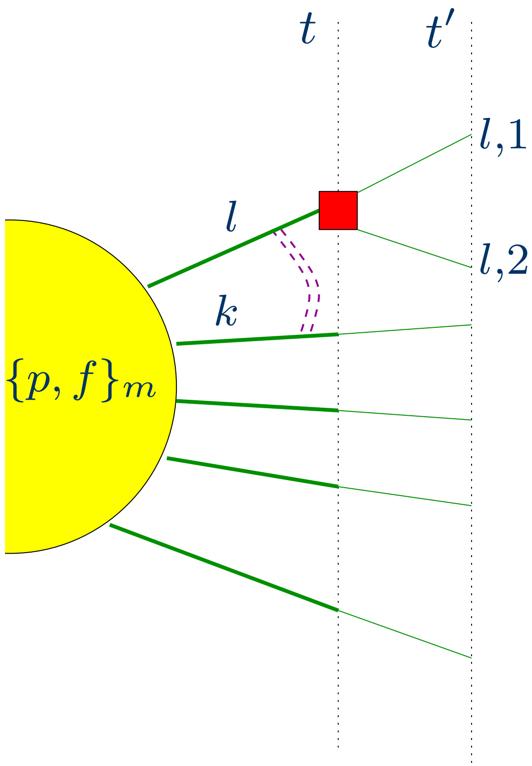
But the Born term in the NLO part must be matched to the NLO calculation

$$\begin{aligned}\sigma_m^{B+S}[F] = & \sum_{\{f\}_m} \frac{1}{m!} \int d\Gamma(\{p\}_m; Q)\, \theta(d_m > d_{\rm ini})\, W_m(\{p,f\}_m) \\ & \times \sum_{l=1}^{m} \sum_{k\neq l} \langle \mathcal{M}(\{p,f\}_m) \big| \int dY_l\, \boldsymbol{E}_{l,k}(Y_l) \big| \mathcal{M}(\{p,f\}_m)\rangle \\ & \times \big(S(\{\hat{p},\hat{f}\}_{m+1}; d < d_{\rm ini}) \big| F\big)\end{aligned}$$

# Emission Operator




$$\boldsymbol{E}_{l,k}(Y_l) = \frac{\boldsymbol{T}_l \cdot \boldsymbol{T}_k}{-\boldsymbol{T}_l^2} \int_0^\infty dt\, \delta(t - T_l(z_l, y_l))$$

$$\times \frac{\alpha_s(t)}{2\pi} \boldsymbol{S}_l(Y_l)\, \theta(d_l(z_l, y_l) < d_{\rm ini})$$

$$\times \prod_{l'} \exp\left( - \int_0^1 \frac{dy}{y} \int_0^1 dz\, \theta(T_{l'}(z, y) > t) \right.$$

$$\left. \times\, \theta(d_{l'}(z, y) < d_{\rm ini}) \frac{\alpha_s}{2\pi} S(y, z, f_{l'}) \right)$$

The $Y_l = \{z, y, \phi, \hat{f}_{l,1}, \hat{f}_{l,2}\}$ represent the splitting variables

$$\int dY_l \equiv \int_0^1 \frac{dy}{y} \int_0^1 dz \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{1}{2} \sum_{\hat{f}_{l,1}, \hat{f}_{l,2}} \delta^{f_l}_{\hat{f}_{l,1} + \hat{f}_{l,2}}$$

It is not a simple object but its integral is very simple

$$\sum_{k \neq l} \frac{\boldsymbol{T}_l \cdot \boldsymbol{T}_k}{-\boldsymbol{T}_l^2} = 1 \qquad \Longrightarrow \qquad \sum_l \sum_{k \neq l} \int dY_l\, \boldsymbol{E}_{l,k}(Y_l) = 1 \ .$$

# SPLITTING KINEMATICS

The daugther partons those are emitted from the line $l$ are labeled by $l, 1$ and $l, 2$. Their flavors must correspond to a QCD vertex that $f_l \to \hat{f}_{l,1} + \hat{f}_{l,2}$.

The momenta are defined according to the Sudakov parametrization

$$\hat{p}_{l,1} = z p_l + y(1-z) p_k + k_\perp \quad , \qquad \hat{p}_{l,2} = (1-z) p_l + yz p_k - k_\perp \quad ,$$

and the spectator (recoiled) momentum is $\hat{p}_k = (1-y)p_k$.

$$p_l + p_k = \hat{p}_{l,1} + \hat{p}_{l,2} + \hat{p}_k \quad , \qquad \hat{p}_{l,1}^2 = \hat{p}_{l,2}^2 = 0 \quad .$$

The transverse momentum is perpendicular both to the emitter and spectator

$$k_\perp \cdot p_l = k_\perp \cdot p_k = 0 \quad , \qquad k_\perp^2 = -2 p_l \cdot p_k yz(1-z) \quad .$$

The phase space can be written in factorized form

$$d\Gamma^{(m+1)}(\{\hat{p}\}_{m+1}; Q) \frac{1}{2\hat{p}_{l,1} \cdot \hat{p}_{l,2}} = d\Gamma^{(m)}(\{p\}_m; Q) \, \frac{dy}{y} \, dz \, \frac{d\phi}{2\pi} \, \frac{1-y}{16\pi^2}$$

$\Longrightarrow$ There is no approximation in the phase space.

# SPLITTING KERNELS

The $\boldsymbol{S}$ splitting kernels are based on the Catani-Seymour dipole factorization formulas. For example for the $q \to q + g$ splitting

$$\langle s|\boldsymbol{S}_{qg}(z,y)|s'\rangle = C_{\mathrm{F}}\,(1-y)\left[\frac{2}{1-z(1-y)} - (1+z)\right]\delta_{ss'} \quad ,$$

and similarly for the $g \to g + g$ splitting

$$\langle s|\boldsymbol{S}_{gg}(p_l,z,y,k_\perp)|s'\rangle =$$

$$2C_{\mathrm{A}}\,(1-y)\,\epsilon^*_\mu(p,s)\Big[-g^{\mu\nu}\left(\frac{1}{1-z(1-y)} + \frac{1}{1-(1-z)(1-y)} - 2\right) + 2\,z(1-z)\,\frac{k_\perp^\mu k_\perp^\nu}{-k_\perp^2}\Big]\epsilon_\nu(p,s') \quad .$$

The evolution variable is chosen to be proportional to the transverse momentum

$$T_l(z,y) = s_l\, yz(1-z) \quad ,$$

where $s_l$ is a hard scale but it is independent off the spectator label $k$.

# NLO Correction

At NLO level one term in the CKKW cross section should be

$$\sigma_m^{NLO+S}[F] = \sigma_m^{B+S}[F] + \sigma_m^{R+S}[F] + \sigma_m^{V+S}[F]$$

The contributions of the $m+1$ parton matrix element should be

$$\sigma_m^{R+S}[F] = \sum_{\{f\}_{m+1}} \frac{1}{(m+1)!} \int_{m+1} d\Gamma^{(m+1)}(\{p\}_{m+1};Q)$$
$$\times \Big\{ \big|\mathcal{M}(\{p\}_{m+1})\big|^2 \, \theta(d_m > d_{\mathrm{ini}})\, \theta(d_{m+1} < d_{\mathrm{ini}})$$
$$- \sum_{\substack{i,j \\ \text{pairs}}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(\{p\}_{m+1})\, \theta(d_m > d_{\mathrm{ini}})\, \theta(d_{ij} < d_{\mathrm{ini}}) \Big\}$$
$$\times \big(S(\{p,f\}_{m+1})\big|F\big)$$

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The contributions of the $m$ parton one-loop matrix element is

$$\begin{aligned}\sigma_m^{V+S}[F] = & \sum_{\{f\}_m} \frac{1}{m!} \int_m d\Gamma^{(m)}(\{p\}_m; Q) \\ & \times \Big\{ Re \left\langle \mathcal{M}\left(\{p,f\}_m\right) \middle| \Big[ \boldsymbol{I}(\epsilon) \middle| \mathcal{M}\left(\{p,f\}_m\right) \right\rangle + 2 \left| \mathcal{M}^{(1)}\left(\{p,f\}_m; \epsilon\right) \right\rangle \Big]_{\epsilon=0} \\ & \quad - \frac{\alpha_s}{2\pi} W_m^{(1)}(\{p,f\}_m) \left\langle \mathcal{M}\left(\{p,f\}_m\right) \middle| \mathcal{M}\left(\{p,f\}_m\right) \right\rangle \\ & \quad - \sum_l \sum_{k \neq l} C_{l,k}(\{p\}_m, d_{\mathrm{ini}}) \Big\} \\ & \times \left( S(\{p,f\}_m) \middle| F \right)\end{aligned}$$

# NLO Expansion

The secondary shower has the property that

$$\big(S(\{p,f\}_n)\big|F\big) = F(\{p\}_n) + \mathcal{O}(\alpha_s) + \mathcal{O}(1\,\mathrm{GeV}/\sqrt{s})$$

If the measurement function $F$ sensitive for the $N$-jet region then the main contribution comes from the partial cross section $\sigma_{m=N}^{NLO+S}$.
The expansion of the Born term is

$$\begin{aligned}
\sigma_N^{B+S}[F] = \sum_{\{f\}_N} \frac{1}{N!} \int d\Gamma(\{p\}_N; Q)\, \big\langle \mathcal{M}(\{p,f\}_N)\big|\mathcal{M}(\{p,f\}_N)\big\rangle \\
\times F(\{p\}_N)\, \theta(d_N > d_{\mathrm{ini}}) \left(1 + \frac{\alpha_s}{2\pi} W_N^{(1)}\right) \\
+ \sum_{\{f\}_{m+1}} \frac{1}{(N+1)!} \int_{N+1} d\Gamma^{(N+1)}(\{p\}_{N+1}; Q) \\
\times \sum_{\substack{i,j\\ \text{pairs}}} \sum_{k\neq i,j} \mathcal{D}_{ij,k}(\{p\}_{N+1})\, \theta(d_{ij} < d_{\mathrm{ini}} < d_N^{ij,k}) \\
\times \Big\{ F(\{p\}_{N+1}) - F\big(\{\tilde{p}\}_N^{ij,k}\big) \Big\} \\
+ \mathcal{O}(\alpha_s^2) + \mathcal{O}(1\,\mathrm{GeV}/\sqrt{s})
\end{aligned}$$

# NLO Expansion

Finally, the perturbative expansion of the N-jet cross section is

$$
\begin{aligned}
\sigma^{S}[F] = \sigma_m^{NLO}[F] + \\
& + \sum_{\{f\}_N} \frac{1}{N!} \int d\Gamma(\{p\}_N; Q)\, \theta(d_N > d_{\rm ini}) F(\{p\}_N) \\
& \quad \times \Big[ \big\langle \mathcal{M}(\{p,f\}_N) \big| \mathcal{M}(\{p,f\}_N) \big\rangle \big( W_N^{(1)} - W_N^{(1)} \big) \\
& \qquad - \sum_{l} \sum_{k \neq l} C_{l,k}(\{p\}_N, d_{\rm ini}) \Big] \\
& + \sum_{\{f\}_{N+1}} \frac{1}{(N+1)!} \int_{m+1} d\Gamma^{(N+1)}(\{p\}_{N+1}; Q) \\
& \quad \times \sum_{\substack{i,j \\ \text{pairs}}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(\{p\}_{N+1})\, \theta(d_N^{ij,k} > d_{\rm ini}) \\
& \quad \times \Big\{ F(\{p\}_{N+1}) - F\big(\{p\}_{N+1}\big) + F\big(\{p\}_N^{ij,k}\big) \theta(d_{ij} \geq d_{\rm ini}) \Big\} \\
& + \mathcal{O}(\alpha_s^2) + \mathcal{O}(1\, \text{GeV}/\sqrt{s})
\end{aligned}
$$

# CONCLUSIONS AND OUTLOOKS

- With some modifications in the first step of the shower we are able to merge the parton shower with the NLO cross section avoiding the double counting.
  - The method is Lorentz covariant.
  - It is based on the Catani-Seymour dipole subtraction method.
  - The method is not specific to a particular Monte Carlo implementation.
  - The algorithm is fully accurate in the soft region (in every singular regions). There is no left over singularities that needs special treatment.
- This method works for the processes with incoming hadrons,
- and with massive particles
- With this method we can also add "NLO matrix element" corrections to the parton shower (CKKW@NLO).
- The coding is a big challenge. We need a general NLO program that can work together with the automated matrix element generators.

# CONCLUSIONS AND OUTLOOKS

- Based on the C-S dipole factorization one can define a new transverse ordered shower.
  - The shower is Lorentz invariant/covariant.
  - The phase space is the exact $m$-body phase space with the exact phase space weight.
  - The angular ordering is provided by the kinematics. $\Longrightarrow$ There is no awkward cut parameters in the algorithm.
  - Actually there is only one external parameter, the infrared cutoff parameter.

- The $k_\perp$ ordered shower helps to have better understanding on the "Matrix element + Shower" macthing
  - Two ways: Slicing (CKKW) and Subtraction ($\Rightarrow$ Peter Skands talk)
  - It is not obvious but they are completely equivalent.
  - The slicing method is *artificially* complicated.
  - The subtraction method is more suitable for NLO+Shower matching.

- The shower with exact phase space is the best way to include higher order effects. $\Rightarrow$ complete NNLO subtraction method with exact phase space factorization at least at leading color level