Higher Loops: Summary and Prospects

Impression and observations

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My perspective in the light of work with
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- **Why so many loops?**
- **How we get away with perturbation theory in QCD**
- **Themes of this Loopfest**
- **What resummation says about singularities**
- **Concluding comments**

No figures: please imagine loops as necessary

- Why so many loops?
  - Coupling to the decoupled

    All New Physics is embedded in Standard Model observables but only through values of observable parameters: M_W, α_s, etc. Effect of massive ($M_{\rm new} \gg E_{\rm ext}$) states is local
  - Discovering the quantum mechanical stories

    But final states are generally indistinguishable from standard model processes event by event.

    At high enough energies ($E_{\rm ext} \geq M_{\rm new}$) these effects become nonlocal; producing deviations from Standard Model predictions

    But only by precision in rates & distributions of Standard Model and New Physics signals can the nonlocality be quantified and New Physics discovered.

- **How we get away with perturbative QCD**

- **The problem for perturbation theory**

1. **Confinement**

$$\int \mathrm{e}^{-iq\cdot x}\langle 0|\, T[\phi_a(x)\ldots]\,|0\rangle$$

   has no $q^2 = m^2$ pole for ϕ_a that transforms nontrivially under color (confinement)

2. **The pole at $p^2 = m_\pi^2$**

$$\int \mathrm{e}^{-iq\cdot x}\langle 0|\, T[\pi(x)\ldots]\,|0\rangle$$

   is not accessible to perturbation theory (χSB etc., etc.)

- **And yet we use infrared safety & asymptotic freedom:**

$$Q^2\ \hat{\sigma}_{\rm SD}(Q^2,\mu^2,\alpha_s(\mu)) \;=\; \sum_n c_n(Q^2/\mu^2)\ {\alpha_s}^n(\mu) + \mathcal{O}\left(1/Q^p\right)$$
$$= \sum_n c_n(1)\ {\alpha_s}^n(Q) + \mathcal{O}\left(1/Q^p\right)$$

- **What are we really calculating? PT for color singlet operators**

  - $\int {\rm e}^{-iq\cdot x}\langle 0|\, T[J(x)J(0)\dots]\,|0\rangle$ **for color singlet currents**

    ${\rm e}^+{\rm e}^-$ **total, sum rules etc. “no scale”** **(Dixon)**

– **Another class of color singlet matrix elements:**

$$\lim_{R\to\infty} \int dx_0 \int d\hat{n}\, f(\hat{n})\, \mathrm{e}^{-iq\cdot y} \langle 0|\, J(0) T[\hat{n}_i \Theta_{0i}(x_0, R\hat{n}) J(y)]\, |0\rangle$$

With Θ_{0i} the energy momentum tensor

– **These are what we really calculate**

"Weight" $f(\hat{n})$ introduces no new dimensional scale

Short-distance dominated if all $d^k f/d\hat{n}^k$ bounded

Individual final states have IR divergences, but these cancel in sum over collinear splitting/merging and soft parton emission because they respect energy flow

We regularize these divergences dimensionally (typically) and "pretend" to calculate the long-distance enhancements only to cancel them in infrared safe quantities

It is this intermediate step that makes the calcualtions tough, and is part [not all] of why higher-order calculations are hard!

The goals of experiment are remarkably similar – to control late stage interactions in calorimeters. J. Repond

– Jet, event shape, energy flow observables
(Tkachov 95, Korchemsky, Oderda, GS 96)

– Light quarks ($m \ll \Lambda_{\rm QCD}$): hadronization respects energy flow

– Parton-hadron duality

– Were it not for light quarks all of QCD would be NRQCD

– Analogies to calculations:

  * Energy flow expectations $\Leftrightarrow$ calorimetric measurements

  * Event generators $\Leftrightarrow$ multi-particle cross sections

- **But sometimes want to introduce new scales say $(1-T)Q$, mass of narrow jets in $\mathrm{e^+e^-}$ annihilation**

- **And anyway the formation of initial-state hadrons is never short-distance . . .**

- **Generalization: factorization**

$$Q^2\sigma_{\rm phys}(Q,m) = \omega_{\rm SD}(Q/\mu,\alpha_s(\mu)) \otimes f_{\rm LD}(\mu,m) + \mathcal{O}\left(1/Q^p\right)$$

  - μ **= factorization scale;** m**= IR scale (**m may **be perturbative)**
  - **New physics in** $\omega_{\rm SD}$**;** $f_{\rm LD}$ **"universal"**
  - **Deep-inelastic (**$p=2$**),** ${\rm p\bar{p}} \to Q\bar{Q}$ **. . .**
  - **Exclusive decays:** $B \to \pi\pi$
  - **Exclusive limits:** ${\rm e^+e^-} \to {\rm JJ}$ **as** $m_J \to 0$

- **Whenever there is factorization, there is evolution**

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q, m)$$

$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

- **Wherever there is evolution there is resummation**

$$\ln \sigma_{\rm phys}(Q, m) = \exp \left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

- **Infrared safety & factorization proofs**:
  - **(1) ω_{SD} incoherent with long-distance dynamics**
  - **(2) Mutual incoherence when $v_{\mathrm{rel}} = c$:**
    Jet-jet factorization Ward identities.
  - **(3) Wide-angle soft radiation sees only total color flow:**
    jet-soft factorization Ward identities.
  - **(4) Dimensionless coupling and renormalizability**
    $\Leftrightarrow$ no worse that logarithmic divergence in the IR:
    fractional power suppression $\Rightarrow$ finiteness

– **Summary for e^+e^-: factorization into universal jets + soft**

$$\sigma = \prod_{\text{jets j}} J_j(p_j)\ S_{\{j\}}$$

we'll come back to this

- **Themes of this Loopfest**

  A. Bringing new physics to the foreground in precision measurements matched with precision theory

  - **Instrinsic theoretical uncertainties in the Standard Model can be smaller than those of extensions like SUSY. Why wait for experiment?**

  - **Venturing to higher loops in extensions of the Standard Model requires consistent treatment of renormalization in addition to calculational power.**

- **Two loop Yukawa corrections in MSSM: M_W and weak mixing.** **G. Weiglein, S. Heinemeyer**

- **The high price of giving up custodial $SU(2)$ in extensions of the Standard Model.** **T. Krupovnickas**

- **Fermionic and bosonic corrections to weak mixing in the Standard Model.** **M. Awramik**

- **$\mathcal{O}(\alpha^2)$ corrections to $d\Gamma/dx$ for μ decay.** **K. Melnikov**

- **Exploration of EW corrections and uncertainties in M_W.** **U. Baur**

- In QCD pecial requirements of $\mathrm{t\bar{t}}$ near threshold resummations in α_s/v and $\alpha_s \ln v$: advances to NNLL/νNRQCD A. Hoang

- NNLO and NNLL in $K^+ \to \pi^+ \nu \bar{\nu}$. $\mathcal{O}({\alpha_s}^2)$ in coefficients and $\mathcal{O}({\alpha_s}^3)$ in anomalous dimensions. U. Haisch

- Qualitative advances of a few years ago are today's commonplace tools (requiring uncommon skill to use)
  (approximate) Quote of the workshop: "Only a few diagrams, about 300."

- The exploitation of advances in computing power

B. The background to New Physics: QCD corrections

analytic and numerical tracks

– **Taming NNLO cross sections: how to use infrared safety?**

$\ast$ **Subtractions and antennae:**
Implementing soft-jet factorization

Organized into the number of "unresolved" partons

$$\sigma^{\mathrm{NNLO}} = \int_{n+2} \Big(d\sigma^{(0)}_{n+2} - d\alpha^{(0)}_{n+2} - d\gamma^{(0)}_{n+2} + d\beta^{(0)}_{n+2} \Big) + \ldots$$

$\alpha^{(0)}$, $\gamma^{(0)}$ **single and double-particle subtractions**
$\gamma^{(0)}$ **eliminates double counting** W. Kilgore, T. Gehrmann

Explicit NNLO subtractions for 3-jet cross sections in $\mathrm{e}^+\mathrm{e}^-$ organized around color connections (antennae)

T. Gehrmann, A. Gehrmann-De Ridder

* **Sector decomposition**

  F. Petriello

  Utilize logarithmic bounds on singularities

$$PS = \prod_i \int d\lambda_i \lambda_i^{a_i \varepsilon} (1-\lambda)^{b_i \varepsilon}$$

Chosen such that $|M|^2 \sim 1/\lambda_i$, to develop Laurent series:

$$\frac{1}{\lambda^{1+\varepsilon}} = \frac{1}{-\varepsilon}\delta(\lambda) + \left[\frac{1}{\lambda}\right]_+ + \ldots$$

Transparent implementation of experimental cuts consistent with infrared safety Petriello, Melnikov

Another exploitation of computing capability

* Similar themes in GRACE evaluation of phase space integrals toward NLO QCD generator. Y. Kurihara

* Semi-numerical calculations for virtual corrections to Higgs plus jets in heavy-top effective theory Laurent expansion (again). G. Zanderighi

C. Advances at tree and NLO

What it looks like to one outsider: Degree of difficulty.

$$\mathrm{Difficulty} = C \times E \exp\left[L/(1+\mathcal{N})\right]$$

with E = number of external lines, L = number of loops
$\mathcal{N}$ = number of supersymmetries

- **Progress in QED scattering generators.** S. Yost, A. Lorca

- **Multipurpose automated computation**
  D. Rainwater, K. Yoshimasa, A. Lorca

- **Matching parton showers to NLO** P. Skands, Z. Nagy

– **Recursive trees and the new analytic continuation: spinors, tree and loops.** L. Dixon

$$(k^{\mu}\sigma_{\mu})_{\alpha\dot{\alpha}} = \lambda_{\alpha}\tilde{\lambda}_{\dot{\alpha}}$$

* **Continuation of a story from the previous Loopfest**
* **The newest features come from "on-shell analytic continuation"**

$$\lambda_1 \to \hat{\lambda}_1 = \lambda_1 - z\lambda_n$$

* **Recursion in tree diagrams**

* **Progress toward recursion at NLO**

* **Ultimate role of twistor space not settled**

- **What resummation says about virtual corrections**
  - **Context: Breakthroughs in multiscale NNLO matrix elements, anomalous dimensions and amplitudes**
    (Tausk, Smirnov, Anastasiou, Glover, Oleari Tejeda-Yeomans, Bern, De Freitas, Dixon, Gehrmann, Remiddi . . .)
  - **Progress in the resummation of logarithmic corrections to all orders in perturbation theory**
  - **Challenge of cross sections: especially with realistic cuts**
  - **Synergy between the two in this context?**
  - **Resummation is based on jet-soft-jet factorization with simplified color structure.**

- **The structure of elastic amplitudes in dimensional regularization**
  - **Partonic processes**

$$\begin{aligned} \mathrm{f} &: f_A(\ell_A, r_A) + f_B(\ell_B, r_B) \to f_1(p_1, r_1) + f_2(p_2, r_2) + \ldots \\ \mathrm{f}' &: V(Q) \to f_1(p_1, r_1) + f_2(p_2, r_2) + \ldots \end{aligned}$$

  - **Color tensor**

$$\begin{aligned} \mathcal{M}^{[\mathrm{f}]}_{\{r_i\}}\left(\{\wp_j\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) &= \mathcal{M}^{[\mathrm{f}]}_L\left(\{\wp_j\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)(c_L)_{\{r_i\}} \\ &\equiv \left|\mathcal{M}^{[\mathrm{f}]}\right\rangle \end{aligned}$$

  - **Recursion relations in infrared structure**

    (Catani 98, Tejeda-Yeomans GS (03), Bern Dixon Kosower (05))

- **Color tensor factorization**

$$\mathcal{M}_L^{[\mathrm{f}]}\left(\wp_i,\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) = J^{[\mathrm{f}]}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) \times S_{LI}^{[\mathrm{f}]}\left(\wp_i,\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right)\ h_I^{[\mathrm{f}]}\left(\wp_i,\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)$$

- **The factors . . .**
  - **An infrared safe coefficient h_I for each color tensor I**
  - **Coherent virtual soft gluon exchange function S_{LI}: interpolates short to long distance color tensors**
  - **Product of "jets" collinear to external lines: color diagonal**

- **The jet functions:**

$$J^{[\mathrm{f}]}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) \quad\equiv\quad \prod_i J^{[f_i]}_{(\mathrm{virt})}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right)$$

  – **Definitions form** $1_{\mathrm{singlet}} \leftrightarrow 2$**:** $J^{[i]} = J^{[\bar{i}]} = \sqrt{M^{[i\bar{i}\to 1]}}$

$$\mathcal{M}^{[i\bar{i}\to 1]}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) \qquad = \exp\left\{ \frac{1}{2}\int_0^{-Q^2}\frac{d\xi^2}{\xi^2}\left[\mathcal{K}^{[i]}(\alpha_s(\mu^2),\epsilon)\right.\right.$$

$$+\mathcal{G}^{[i]}\left(-1,\bar{\alpha_s}\left(\frac{\mu^2}{\xi^2},\alpha_s(\mu^2),\epsilon,\right)\epsilon\right)$$

$$\left.\left.+\frac{1}{2}\int_{\xi^2}^{\mu^2}\frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\gamma_K^{[i]}\left(\bar{\alpha_s}\left(\frac{\mu^2}{\tilde{\mu}^2},\alpha_s(\mu^2),\epsilon\right)\right)\right]\right\}$$

– **Derived from factorization**
(Mueller 79, Collins-Soper, Sen 80)

– **Compare to fixed-order by re-expansion of α_s in D dimensions**
(Magnea-GS 91)

– **Anomalous dimensions $\mathcal{K}$, $\mathcal{G}$, $\gamma_K \leftrightarrow A$ available to 2, 2, 3 loops**
(Moch,Vermaseren,Vogt,Gehrmann, 2005)

- **The soft functions**

$$\mathbf{S}^{[\mathrm{f}]}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) = \mathrm{P}\ \exp\left[-\frac{1}{2}\int_0^{-Q^2}\frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\mathbf{\Gamma}^{[\mathrm{f}]}\left(\bar{\alpha}_s\left(\frac{\mu^2}{\tilde{\mu}^2},\alpha_s(\mu^2),\epsilon\right)\right)\right]$$

  – **From evolution equation:** $\frac{d}{d\ln\ Q} S_{LI} = -\Gamma^{[\mathrm{f}]}_{LJ}\ S_{JI}$
  (Botts-GS 85, Kidonakis, Oderda-GS 98)

  – **LL in soft $\rightarrow$ NNLL overall:**

  – **"The fifth form factor" (Dokshitzer and Marchesini 08/05)**
  Relation of $t \rightarrow u$ and $N \rightarrow \infty$?

- **What we know; what we need to know**
  - Γ_S **known at 1 loop, "available" at 2**
  - **For** γ_K **to** ${\alpha_s}^{n+1}$**,** $\mathcal{K}$**,** $\mathcal{G}$**,** Γ_S **to** ${\alpha_s}^n$**:** $1/\epsilon^P$**,** $P > 1$**.** $m \to m'$
  - **For** $1/\epsilon$ **need only Sudakov form factor and** Γ_S **to** ${\alpha_s}^{n+1}$
  - **Color evolution is entirely in the soft function. Could indicate simplifications in subtraction color structure.**
  - **Reproduces** ϵ **structure of QCD** $2 \to 2$ **amplitudes**
  - **A recent surprise, motivated by study of SYM and heroic calculation of 3 loop planar diagrams . . .**

- **Recursive infrared structure of $2 \to 2$ at 3 loops**

(Tejeda Yeomans GS (03), Bern, Dixon, Smirnov (05) [Maximal SYM])

$$
\begin{aligned}
|\mathcal{M}^{[\mathrm{f}(3)]}\rangle &= \boldsymbol{F}^{[\mathrm{f}(1)]}(\epsilon)|\mathcal{M}^{[\mathrm{f}(2)]}\rangle + \boldsymbol{F}^{[\mathrm{f}(2)]}(\epsilon)|\mathcal{M}^{[\mathrm{f}(1)]}\rangle \\
&\quad + \boldsymbol{F}^{[\mathrm{f}(3)]}(\epsilon)|\mathcal{M}^{[\mathrm{f}(0)]}\rangle + |\mathcal{M}_{UV}^{[\mathrm{f}(3)]}\rangle
\end{aligned}
$$

– **where for example . . .**

– **The coefficient of** $|\mathcal{M}^{[\mathrm{f}(0)]}\rangle$

$$\begin{aligned}
\boldsymbol{F}^{[\mathrm{f}(3)]}(\epsilon) \;=\; & -\frac{1}{3}\left[\boldsymbol{F}^{[\mathrm{f}(1)]}(\epsilon)\right]^3 - \frac{1}{3}\boldsymbol{F}^{[\mathrm{f}(1)]}(\epsilon)\boldsymbol{F}^{[\mathrm{f}(2)]}(\epsilon) - \frac{2}{3}\boldsymbol{F}^{[\mathrm{f}(2)]}(\epsilon)\boldsymbol{F}^{[\mathrm{f}(1)]}(\epsilon) \\
& - \left(\frac{\beta_0}{4\epsilon}\right)^2 \boldsymbol{F}^{[\mathrm{f}(1)]}(3\epsilon) + \left(\frac{\beta_0}{4\epsilon}\right)\left\{-\frac{1}{2}\left[\boldsymbol{F}^{[\mathrm{f}(1)]}(\epsilon)\right]^2 - \boldsymbol{F}^{[\mathrm{f}(2)]}(\epsilon)\right. \\
& \quad + \frac{1}{2}\left(\mathrm{K} + \frac{\beta_0}{2\epsilon}\right)\left[2\boldsymbol{F}^{[\mathrm{f}(1)]}(3\epsilon) - \boldsymbol{F}^{[\mathrm{f}(1)]}(2\epsilon)\right] \\
& \quad \left. + \boldsymbol{L}^{[\mathrm{f}(2)]}(3\epsilon) - \frac{1}{2}\boldsymbol{L}^{[\mathrm{f}(2)]}(2\epsilon)\right\} + \frac{1}{2}\boldsymbol{L}^{[\mathrm{f}(3)]}(3\epsilon),
\end{aligned}$$

– **All F's, L's on the right are combinations of** γ_K **to** ${\alpha_s}^3$, $\mathcal{K}$, $\mathcal{G}$, Γ_S **to** ${\alpha_s}^2$, **and** $\frac{1}{\epsilon}$

- **Concluding Comments**
  - **Perturbative quantum field theory is vibrant, opportunistic and inspires total dedication. There seems no other way to get things right.**
  - **The capabilities of experiment and theory are well matched and mutually inspiring.**
  - **The field was advanced qualitatively by 2-loop computations and three-loop anomalous dimensions, and applications are still being found.**

- **Amazing (to me at least) advance in analytic results within the previous year,**

- **As well as in the power of numerical approaches.**

- **There is further potential for applications of resummation whose power is greatly enhanced by exact 2-loop results.**

- **Is it possible to combine the nominal flexibility of sector decomposition with physically-motivated subtraction formalism that makes use of the universality in final-state evolution?**

- **The somewhat coarser resolutions and backgrounds at the LHC may paradoxically provide the time to fully realize the potential of techniques that are now being developed and reach fruition at a future (but not too far future) ILC**