Constraining New Models with Precision Electroweak Data

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LoopFest IV, Snowmass, 2005

EW Sector of the Standard Model

3 parameters in the gauge boson+Higgs sector: g, g' and v. Plus masses of Higgs boson and fermions. α, G_μ and M_Z usually chosen as 3 inputs:

$$g^2 = \frac{4\pi\alpha}{s_W^2}, \qquad g'^2 = \frac{4\pi\alpha}{c_W^2}, \qquad v^2 = \frac{1}{(\sqrt{2}G_\mu)^{\frac{1}{2}}}.$$

Custodial symmetry $SU(2)_c$ is present in the SM. It guarantees that at tree level

$$\rho \equiv \frac{M_W^2}{M_Z^2 c_W^2} = 1.$$

Then s_W is an output of the calculation:

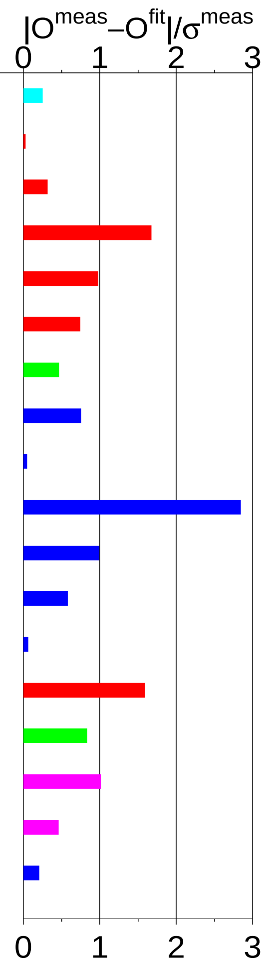
$$s_W^2 c_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}.$$

Tree-level relation $\rho = 1$ remains correct for:

- SM extensions with extra generations of fermions
- MSSM
- *etc.*

LEP EW WG (Summer 2005)

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.410 ± 0.032	80.377
Γ_W [GeV]	2.123 ± 0.067	2.092
m_t [GeV]	172.7 ± 2.9	173.3



If a new model predicts some deviation from the SM in the EW observables, it has to be small.

The Usual

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{NEW},$$

where

$$\mathcal{L}_{NEW} = \Sigma_i c_i \frac{O_i}{\Lambda^2}.$$

Common assumptions:

1. Use the SM renormalization procedure with new particles in the loops.
2. As $\Lambda \to \infty$, New Physics effects become small.

If $\rho = 1$ at tree level, this is correct.

SM Renormalization

At tree level:

$$G_\mu = \frac{\pi\alpha}{\sqrt{2} s_W^2 c_W^2 M_Z^2 \rho}, \quad \rho \equiv \frac{M_W^2}{M_Z^2 c_W^2} = 1.$$

One loop result (on-shell definition):

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2} s_W^2 G_\mu (1 - \Delta r_{SM})}, \quad s_W^2 = 1 - \frac{M_W^2}{M_Z^2}.$$

Here Δr_{SM} collects all radiative corrections:

$$\Delta r_{SM} = -\frac{\delta G_\mu}{G_\mu} + \frac{\delta\alpha}{\alpha} - \frac{\delta s_W^2}{s_W^2} - \frac{\delta M_W^2}{M_W^2}.$$

The SM is non-trivially constrained because of custodial symmetry ($\rho = 1$):

$$\frac{\delta s_W^2}{s_W^2} = \frac{c_W^2}{s_W^2}\left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right] \sim m_t^2.$$

M_W and s_W^2 are outputs. G_μ, α and M_Z + all fermion and Higgs boson masses are inputs.

SM Prediction for $M_W(m_t)$

In terms of 2-point functions

$$\begin{aligned}\Delta r_{SM} &= \frac{\Pi_{WW}(0)-\Pi_{WW}(M_W^2)}{M_W^2}+\Pi'_{\gamma\gamma}(0)+2\frac{s_W}{c_W}\frac{\Pi_{\gamma Z}(0)}{M_Z^2}\\ &- \frac{c_W^2}{s_W^2}\left[\frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2}-\frac{\Pi_{WW}(M_W^2)}{M_W^2}\right].\end{aligned}$$

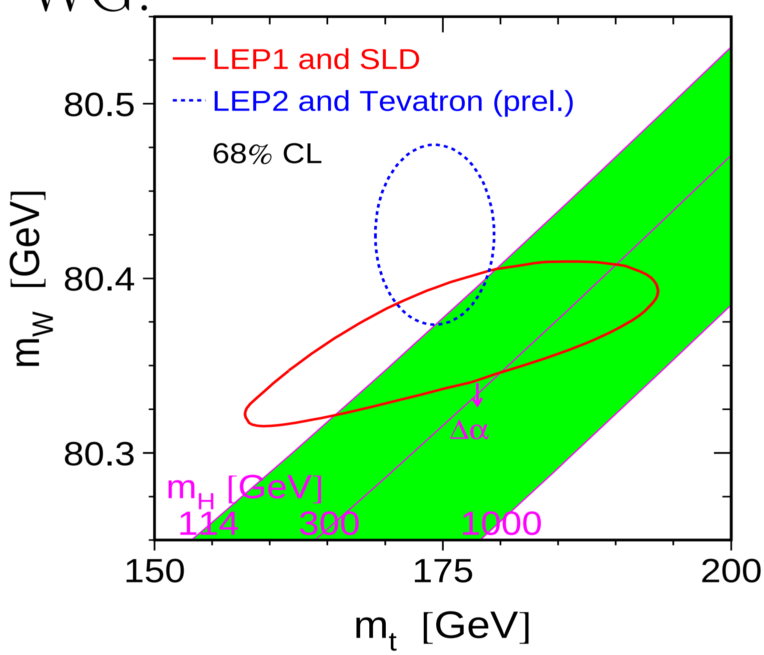
For large m_t top quark-dependent part:

$$\Delta r^t_{SM} \approx -\frac{3G_\mu}{\sqrt{2}8\pi^2}\frac{c_W^2}{s_W^2}m_t^2.$$

A good approximation for Δr is:

$$\Delta r_{SM} \approx 0.067+\Delta r^t_{SM}+\frac{\alpha}{\pi s_W^2}\frac{11}{48}\left(\ln\frac{M_H^2}{M_Z^2}-\frac{5}{6}\right)+\ldots$$

LEP EW WG:



The Simplest Example: SM + Higgs Triplet

B. W. Lynn and E. Nardi, Nucl.Phys.B381:467-500,1992,
T. Blank and W. Hollik, Nucl.Phys.B514:113-134,1998,
M.-C. Chen, S. Dawson and T. K., hep-ph/0504286.

Add a real Higgs triplet to the SM:

$$H = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \phi^0_R + \mathrm{i}\phi^0_I) \end{pmatrix}, \quad \Phi = \begin{pmatrix} \eta^+ \\ \frac{1}{2}v' + \eta^0 \\ -\eta^- \end{pmatrix}.$$

Assume that triplet does not couple to fermions.
The set of model parameters becomes:
g, g', v and v'
+ fermion masses and the masses of the Higgses:
m_{H^0}, m_{K^0} and $m_{H^\pm}$.
In this model

$$c_\theta^2 = \frac{g^2}{g^2 + g'^2},$$

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2)\ v^2, \qquad M_W^2 = \frac{1}{4}g^2(v^2 + v'^2).$$

And

$$\rho = 1 + \frac{v'^2}{v^2} \neq 1.$$

Higgs bosons mix:

$$\begin{pmatrix} H^0 \\ K^0 \end{pmatrix} = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} \phi_R^0 \\ \eta^0 \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} c_\delta & s_\delta \\ -s_\delta & c_\delta \end{pmatrix} \begin{pmatrix} \phi^\pm \\ \eta^\pm \end{pmatrix}.$$

For simplicity set $\gamma = 0$.

In terms of mixing angle $c_\delta^2 = \frac{v^2}{v^2+v'^2}$:

$$M_W^2 = \frac{g^2 v^2}{4c_\delta^2}, \qquad \rho \equiv \frac{M_W^2}{M_Z^2 c_\theta^2} = \frac{1}{c_\delta^2} \geq 1.$$

$\rho \approx 1$ experimentally $\Rightarrow v' \ll v$.

Renormalization

s_θ^2 is no longer a calculable quantity. It has to be fixed by experiment. Choose effective leptonic mixing angle as input:

$$L = -\mathrm{i}\bar{e}(v_e + \gamma_5 a_e)\gamma_\mu e Z^\mu, \qquad v_e = \frac{1}{2} - 2s_\theta^2, \quad a_e = \frac{1}{2},$$

$$1 - 4s_\theta^2 \equiv \frac{\mathrm{Re}(v_e)}{\mathrm{Re}(a_e)}.$$

LEP result: $s_\theta^2 = 0.23150 \pm 0.00016$.

Return to muon decay:

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}s_\theta^2 c_\theta^2 M_Z^2 \rho(1-\Delta r_{TM})},$$

where

$$\Delta r_{TM} = -\frac{\delta G_\mu}{G_\mu} + \frac{\delta\alpha}{\alpha} - \frac{\delta M_Z^2}{M_Z^2} - \frac{c_\theta^2 - s_\theta^2}{c_\theta^2}\frac{\delta s_\theta^2}{s_\theta^2} - \frac{\delta\rho}{\rho}.$$

In SM the following relation was used to derive s_θ^2 counterterm:

$$\frac{\delta\rho}{\rho} = \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} + \frac{s_W^2}{c_W^2}\frac{\delta s_W^2}{s_W^2} = 0.$$

Now this relation can only be used as an expression for $\frac{\delta\rho}{\rho} \neq 0$.

Do $Ze\bar{e}$ vertex renormalization to calculate:

$$\frac{\delta s_\theta^2}{s_\theta^2} = \frac{c_\theta}{s_\theta}\mathrm{Re}\left[\frac{\Pi_{\gamma z}(M_Z^2)}{M_Z^2} + \delta_{self-energy+vertex}\right].$$

$$\Pi_{\gamma z}(M_Z^2) \sim \ln\frac{m_t^2}{Q^2} \Rightarrow \frac{\delta s_\theta^2}{s_\theta^2} \sim \ln\frac{m_t^2}{Q^2}.$$

Top quark dependence in TM

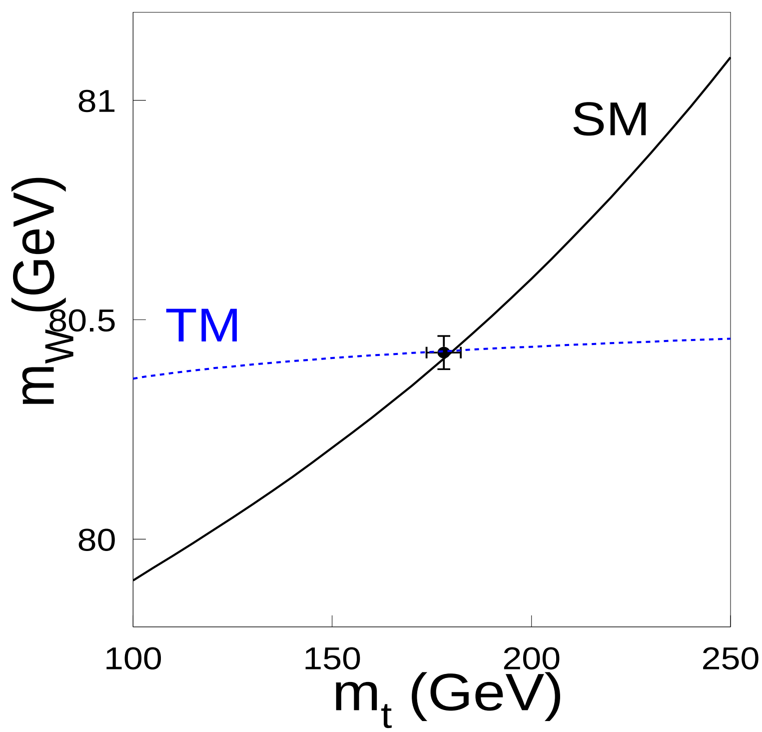
Put it all together:

$$\Delta r_{TM} = \frac{\Pi_{WW}(0) - \Pi_{WW}(M_W^2)}{M_W^2} + \Pi'_{\gamma\gamma}(0) + 2\frac{s_\theta}{c_\theta}\frac{\Pi_{\gamma Z}(0)}{M_Z^2} - \frac{c_\theta}{s_\theta}\frac{\Pi_{\gamma Z}(M_Z^2)}{M_Z^2} + \delta_{self-energy+vertex+box}.$$

W mass is calculated from

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu s_\theta^2(1-\Delta r_{TM})}.$$

For large m_t: $M_W^2 \sim \ln\frac{m_t^2}{Q^2}$!



$M_W(Higgs)$

Higgs mass dependence is quadratic vs. logarithmic dependence in the SM!

If $M_{H^0} \approx M_{K^0} \approx M_{H^\pm}$, then

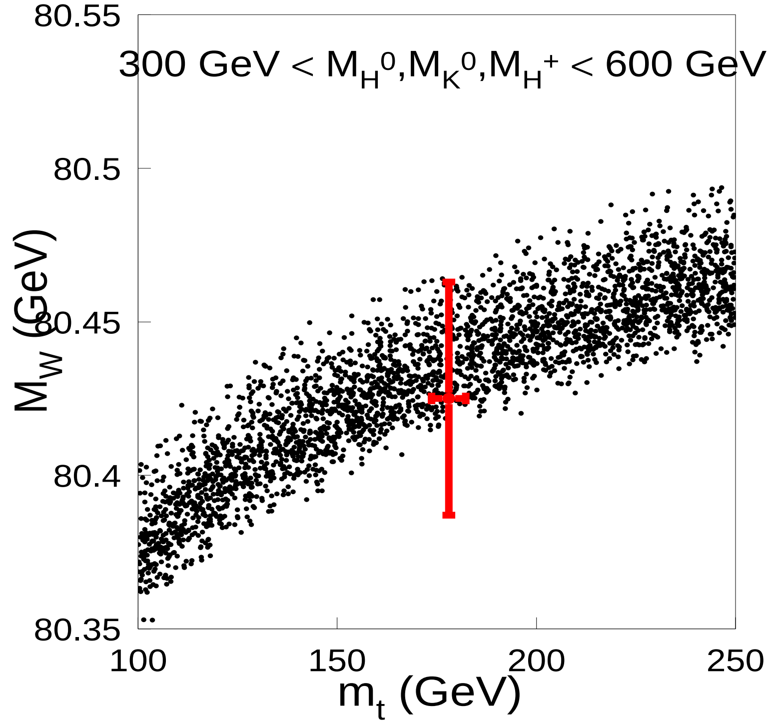
$$\begin{aligned}
\Delta r_{TM}^{scalars} &\to \frac{\alpha}{4\pi s_\theta^2}[-\frac{1}{2}[c_\delta^2 \frac{M_{H^0}^2}{M_W^2}\ln\frac{M_{H^0}^2}{M_W^2} + 4s_\delta^2\frac{M_{K^0}^2}{M_W^2}\ln\frac{M_{K^0}^2}{M_W^2} \\
&+ s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^4}\ln\frac{M_{H^\pm}^2}{M_W^2}] \\
&+ \frac{5}{72}[s_\delta^2\frac{M_{H^\pm}^2 - M_{H^0}^2}{M_{H^0}^2} + 4c_\delta^2\frac{M_{H^\pm}^2 - M_{K^0}^2}{M_{K^0}^2}]].
\end{aligned}$$

If $M_{H^0} \ll M_{K^0} \approx M_{H^\pm}$, then

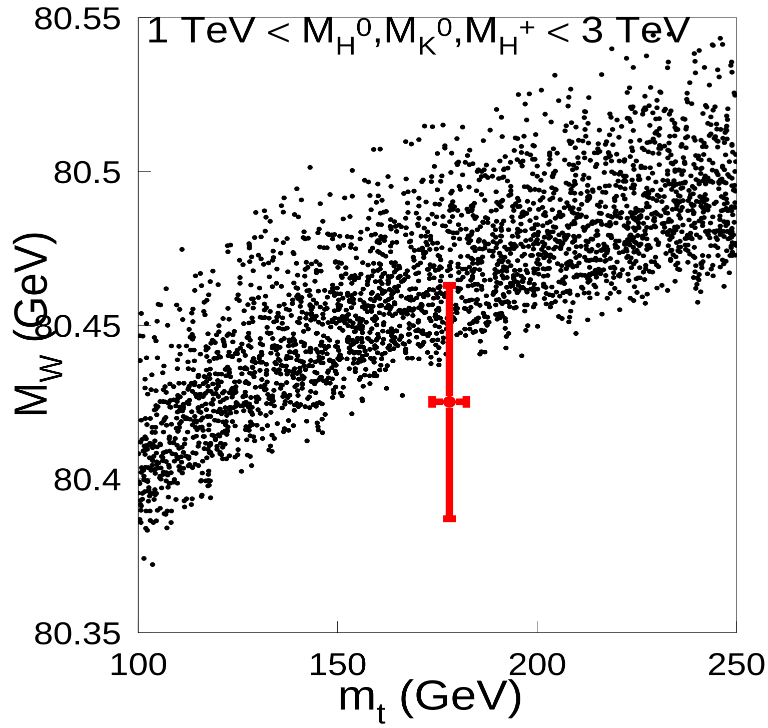
$$\begin{aligned}
\Delta r_{TM}^{scalars} &\to \frac{\alpha}{4\pi s_\theta^2}[-\frac{1}{2}[c_\delta^2 \frac{M_{H^0}^2}{M_W^2}\ln\frac{M_{H^0}^2}{M_W^2} + 4s_\delta^2\frac{M_{K^0}^2}{M_W^2}\ln\frac{M_{K^0}^2}{M_W^2} \\
&+ s_\delta^2 \frac{M_{H^\pm}^2 M_Z^2}{M_W^4}\ln\frac{M_{H^\pm}^2}{M_W^2}] \\
&- s_\delta^2\frac{M_{H^\pm}^2 M_{H^0}^2}{2M_W^4}\ln\frac{M_{H^\pm}^2}{M_{H^0}^2} + \frac{5}{18}c_\delta^2\frac{M_{H^\pm}^2 - M_{K^0}^2}{M_{K^0}^2}.
\end{aligned}$$

Take $M_{K^0} = M_{H^\pm} >> M_{H^0}$ and assume $M_{H^0} \approx M_Z$:

$$\Delta r_{TM}^{scalars} \to -\frac{\alpha}{8\pi s_\theta^2}[c_\delta^2 \frac{M_{H^0}^2}{M_W^2}\ln\frac{M_{H^0}^2}{M_W^2} + 4s_\delta^2\frac{M_{K^0}^2}{M_W^2}\ln\frac{M_{K^0}^2}{M_W^2}].$$

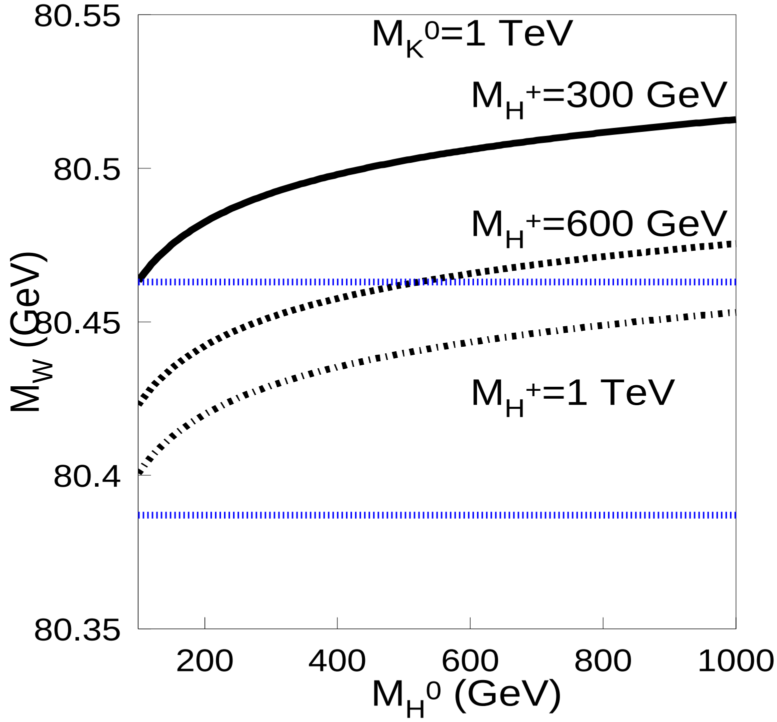


Random scan over 300 GeV $< M_{H^0} <$ 600 GeV, 300 GeV $< M_{K^0} <$ 600 GeV, 300 GeV $< M_{H^\pm} <$ 600 GeV.

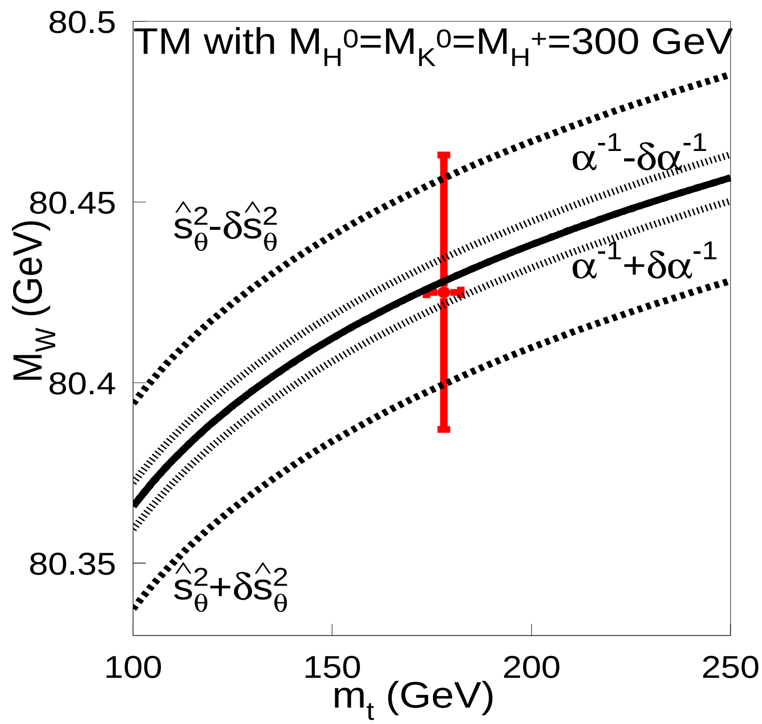


Random scan over 1 TeV $< M_{H^0} <$ 3 TeV, 1 TeV $< M_{K^0} <$ 3 TeV, 1 TeV $< M_{H^\pm} <$ 3 TeV.

M.-C. Chen, S. Dawson and T. K., hep-ph/0504286.



$M_W(M_{H^0})$ for fixed M_{K^0} and $M_{H^\pm}$.



Experimental error impact on M_W prediction.
M.-C. Chen, S. Dawson and T. K., hep-ph/0504286.

(Non)-Decoupling

Shouldn't we recover the SM results when either
1. $m_{K^0} \to \infty$ and $m_{H^\pm} \to \infty$,
or 2. $s_\delta \to 0$ (no mixing between $\eta^\pm$ and $\phi^\pm$)?

As long as $s_\delta \neq 0$, no matter how small, custodial symmetry is broken and one needs an additional counterterm for ρ: $\frac{1}{\hat{\epsilon}}$ is floating around otherwise! One does not have a smooth transition from 4 to 3 input parameters.

To understand what is going on, examine the tree level potential of TM:

$$\begin{aligned} V(\Phi, H) &= \mu_1^2 H^\dagger H + \frac{1}{2}\mu_2^2 \Phi^\dagger \Phi + \lambda_1 (H^\dagger H)^2 + \frac{1}{4}\lambda_2 (\Phi^\dagger \Phi)^2 \\ &+ \frac{1}{2}\lambda_3 (H^\dagger H)(\Phi^\dagger \Phi) + \lambda_4 v \Phi_U^i H^\dagger \sigma_i H. \end{aligned}$$

J. R. Forshaw, D. A. Ross and B. E. White, JHEP 0110:007,2001.
Here

$$\Phi_U = U^\dagger \Phi, \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\mathrm{i} & 0 \\ 0 & 0 & \sqrt{2} \\ -1 & -\mathrm{i} & 0 \end{pmatrix}.$$

Define $t_\beta \equiv \frac{v'}{v}$. Minimization with respect to ϕ_R^0 and η^0 yields:

$$\begin{aligned} 8\mu_1^2 + 8\lambda_1 v^2 + \lambda_3 t_\beta^2 v^2 - 4\lambda_4 t_\beta v^2 &= 0, \\ 4\mu_2^2 t_\beta + \lambda_2 t_\beta^3 v^2 + 2\lambda_3 t_\beta v^2 - 4\lambda_4 v^2 &= 0. \end{aligned}$$

We have assumed that there is no mixing between neutral Higgs components $\Rightarrow \left.\frac{\partial^2 V(\Phi,H)}{\partial\eta^0 \partial\phi_R^0}\right|_{<>} = 0$. That implies

$$2\lambda_4 = \lambda_3 t_\beta.$$

And then minimization equations become

$$\begin{aligned} 8\mu_1^2 + 8\lambda_1 v^2 - \lambda_3 t_\beta^2 v^2 &= 0, \\ t_\beta \left(4\mu_2^2 + \lambda_2 t_\beta^2 v^2\right) &= 0. \end{aligned}$$

1. Let mixing between $\eta^\pm$ and $\phi^\pm$ vanish. That can be achieved by setting $v' \to 0$. But then custodial symmetry is restored and the 1-loop renormalization should be done as in the SM.

2. Let the mass of $\eta^\pm$ go to ∞. Then $\left.\frac{\partial^2 V(\Phi,H)}{\partial\eta^{\pm 2}}\right|_{<>} \to \infty$:

$$4\mu_2^2 + \lambda_2 t_\beta^2 v^2 + 2\lambda_3 v^2 \to \infty.$$

v and t_β cannot get too large. In the TM
$v^2 + v'^2 = v_{SM}^2 = (246\ \text{GeV})^2$.
λ_3 cannot get too large, otherwise $\mu_1^2 \to \infty$ and also theory becomes non-perturbative.
Therefore $4\mu_2^2 + \lambda_2 t_\beta^2 v^2 \to \infty$ is the only solution.

If $4\mu_2^2 + \lambda_2 t_\beta^2 v^2 \to \infty$, then minimization condition

$$t_\beta \left(4\mu_2^2 + \lambda_2 t_\beta^2 v^2\right) \;=\; 0$$

implies that $t_\beta \to 0$. But this case corresponds to the restoration of custodial symmetry, $\rho = 1$ at tree level.

There is one more option to eliminate the mixing between neutral components of triplet and doublet:

$\left.\frac{\partial^2 V(\Phi,H)}{\partial {\eta^0}^2}\right|_{<>} \to \infty$, while keeping $\left.\frac{\partial^2 V(\Phi,H)}{\partial \eta^0 \partial \phi_R^0}\right|_{<>}$ fixed.

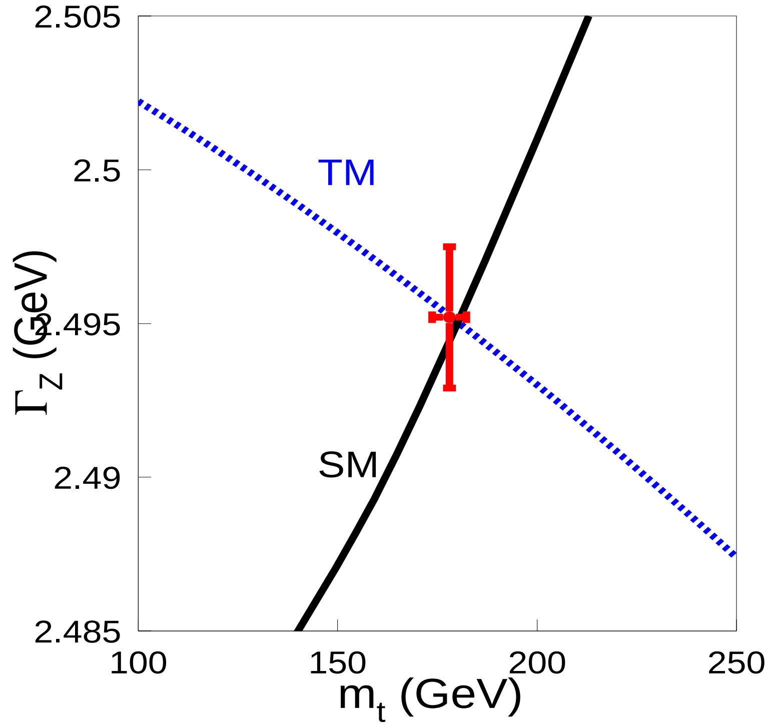
Then $4\mu_2^2 + 3\lambda_2 t_\beta^2 v^2 + 2\lambda_3 v^2 \to \infty$ and $2\lambda_4 - \lambda_3 t_\beta$ is finite. The minimization condition implies:

$$t_\beta(4\mu_2^2 + \lambda_2 t_\beta^2 v^2) = 2v^2(2\lambda_4 - \lambda_3 t_\beta).$$

If one restores the custodial symmetry, $t_\beta \to 0$, then $\mu_2 \to \infty$ can be accomodated. However, if $t_\beta \neq 0$, then λ_3 has to $\to \infty$, and therefore $\mu_1^2 \to \infty$ and theory becomes non-perturbative.

It is impossible to decouple the triplet without either restoring the custodial symmetry, or else blowing up the perturbative framework.

Including all electroweak observables in the analysis provides a better constraint on m_t. Example:



T. Blank and W. Hollik, Nucl.Phys.B514:113-134,1998,

M.-C. Chen, S. Dawson and T. K., in preparation.

Global fit is coming soon.

A (very incomplete) list of models with $\rho \neq 1$ at tree level (with even less complete list of references):

Littlest Higgs

N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207:034,2002,

M.-C. Chen and S. Dawson, Phys.Rev.D70:015003,2004.

$SU(2)_L \times SU(2)_R$ model

J. C. Pati and A. Salam, Phys.Rev.D10:275-289,1974,

M. Czakon, M. Zralek and J. Gluza, Nucl.Phys.B573:57-74,2000.

Warped extra dimensions without $SU(2)_c$

L. Randall and R. Sundrum, Phys.Rev.Lett.83:3370-3373,1999, M. Carena, A. Delgado, E. Ponton, T. M. P. Tait and C. E. M. Wagner, Phys.Rev.D71:015010,2005.

Conclusions

• Models with $\rho \neq 1$ at tree level require careful treatment. SM renormalization is not applicable.

• Top quark mass constraint from electroweak precision measurements may be loosened.

• One can have massive Higgs bosons in the theory.

• Constraints on some models need to be reexamined.