Measuring the W Mass: Electroweak Radiative Corrections

1. Why are Electroweak Radiative Corrections important?
2. M_W, $\sin^2\theta_{eff}$ and M_H
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1 – Why are Electroweak Radiative Corrections important?

- Precise measurements have to be matched by precise theoretical predictions

  ☞ present and future collider experiments aim at measuring observables (cross section, mass, width,...) at the % level or better

  ☞ need to take into account higher order corrections

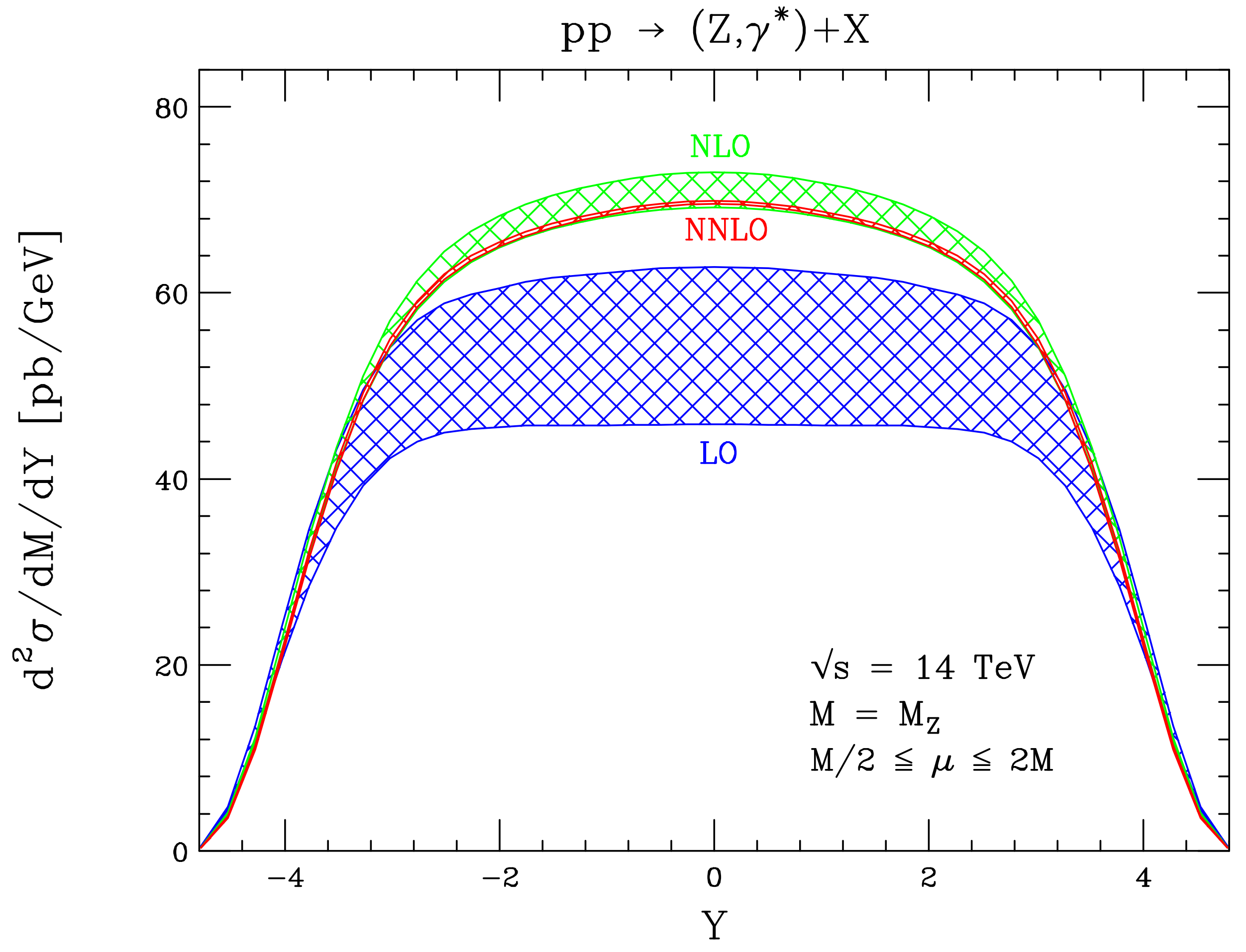
- QCD corrections:

  ☞ NLO: typically $20 - 30\%$

  ☞ NNLO: typically a few %

  ☞ taking into account QCD corrections reduces (sometimes dramatically) the renormalization and factorization scale uncertainty

☞ example: Z boson rapidity distribution (Anastasiou, Dixon, Melnikov, Petriello)

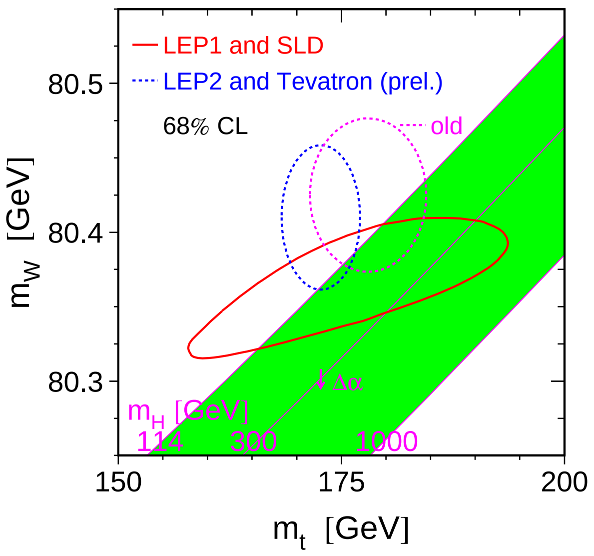


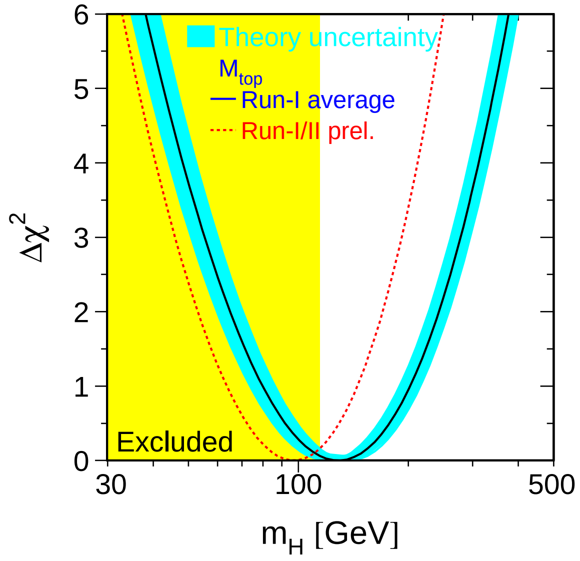
- electroweak radiative corrections:
  - ☞ 1-loop: naively of $\mathcal{O}(\alpha) \leq 1\%$
  - ☞ why bother?
- possible exceptions:
  - ☞ logarithmic enhancement factors
  - → collinear: $\log(\hat{s}/m_f^2)$,
  - → Sudakov: $\log(\hat{s}/M_{W/Z}^2)$
  - ☞ QCD corrections are small (example: W/Z cross section ratio)
  - ☞ and/or very precise measurements (M_W, $\sin^2\theta_{eff}$)
- in some cases need $\geq$ 2-loop EWK corrections
  - ☞ should be able to make use of techniques developed for NNLO QCD corrections

2 – M_W, $\sin^2\theta_{eff}$ and M_H

- 1-loop corrections to M_W and $\sin^2\theta_{eff}$ depend quadratically on the top quark mass, m_t, and logarithmically on M_H

☞ measuring M_W ($\sin^2\theta_{eff}$) and m_t one can extract information on M_H





- fit results depend on

  ☞ experimental uncertainties

  ☞ and theoretical uncertainties

  → primordial theoretical uncertainties: associated with the extraction of (pseudo)observable from measured quantities

  example: M_W from transverse mass distribution

  → intrinsic theoretical uncertainties: from unknown higher order corrections

  example: “blueband”

Experimental Uncertainties: Looking into the Crystal Ball

	present	Tev. run2	LHC	LC	GigaZ
$\delta \sin^2\theta_{eff}$ $(\times 10^{-5})$	14	63	14 – 20	6	1.3
δM_W [MeV]	34	27	10 – 15	10	7
δm_t [GeV]	5.1	2.7	1.0	0.2	0.13
$\delta M_H/M_H$ (indirect)	60%	35%	20%	15%	8%

- need intrinsic theoretical uncertainties which are considerably smaller than experimental uncertainties
- estimate size of missing higher order corrections to M_W and $\sin^2\theta_{eff}$ (Erler)
  - ☞ collect all relevant enhancement and suppression factors
  - ☞ set remaining coefficient (from loop integrals) to unity
  - ☞ choose largest group theory factor

- estimate largest theoretical uncertainties come from
  - ☞ $\mathcal{O}(\alpha^2\alpha_s)$ corrections for M_W (Awramik et al.)
  - ☞ $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha^2\alpha_s^2)$ and $\mathcal{O}(\alpha^2\alpha_s)$ beyond m_t^4 corrections for $\sin^2\theta_{eff}$
- estimated intrinsic theoretical uncertainty

$$\delta M_W^{th} \approx 4\ \mathrm{MeV} \qquad \delta\sin^2\theta_{eff} \approx 5\times 10^{-5}$$

- ultimate goal: bring intrinsic theoretical uncertainties down to
  - ☞ $\mathcal{O}(1\ \mathrm{MeV})$ for M_W
  - ☞ and $\mathcal{O}(\mathrm{few}\times 10^{-6})$ for $\sin^2\theta_{eff}$
  - → if we want GigaZ option
- probably need full 3-loop corrections to $\sin^2\theta_{eff}$ and $\mathcal{O}(\alpha^2\alpha_s)$ corrections to M_W

3 – Electroweak Radiative Corrections to W and Z Boson Production

- example for primordial theoretical uncertainties
- for W mass measurement, need radiative corrections for W and Z boson production:

  ☞ $Z \to \ell^+\ell^-$ data constrain lepton scale and resolution

  ☞ calibrate using using LEP data

  ☞ need to use the same theoretical input that has been used to extract Z parameters at LEP:

  → include QED corrections (change the Z mass extracted from data)

  → include purely weak corrections

  → include $\mathcal{O}(G_F^2 m_t^2 M_W^2)$ corrections to $\sin^2\theta_{eff}$

- Treatment of collinear singularities:

  ☞ Final state collinear singularities are regulated by finite lepton masses

  ☞ Initial state collinear singularities are universal to all orders and are absorbed into the parton distribution functions (PDF's), in complete analogy to QCD

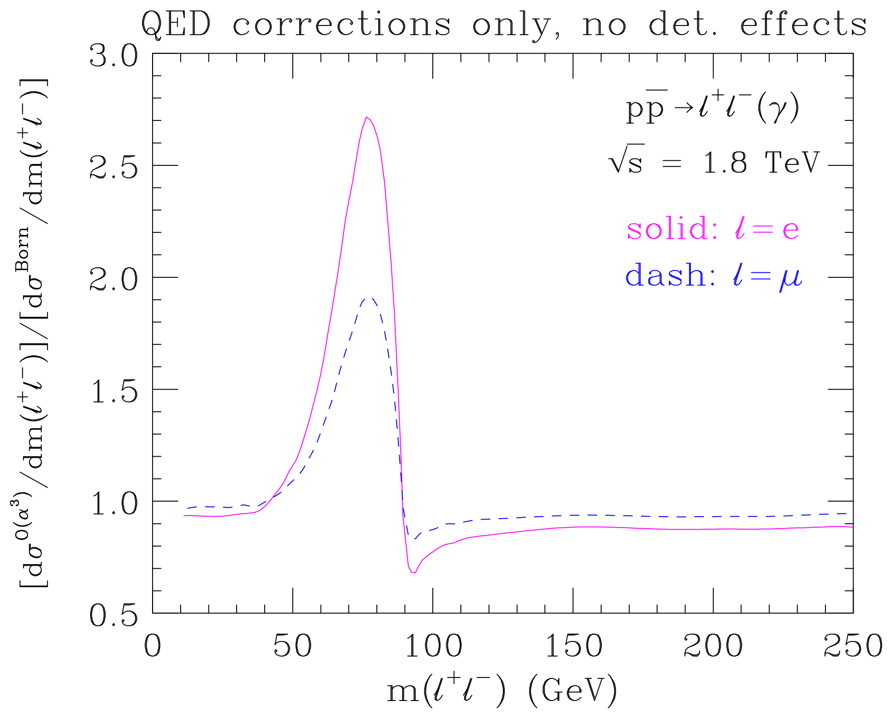
  → for a consistent treatment of the $\mathcal{O}(\alpha)$ initial state corrections, QED corrections have to be incorporated into the global fitting of PDF's

  → PDF's with QED corrections exist now: MRSTQED2004

- 1-loop EWK corrections shift W and Z masses by $\mathcal{O}(100\text{ MeV})$
  - ☞ most of the effect comes from final state photon radiation
  - ☞ proportional to

$$\frac{\alpha}{\pi}\log\left(\frac{\hat{s}}{m_\ell^2}\right)$$

  → these terms significantly influence the $\ell^+\ell^-$ inv. mass distribution
  - ☞ taking only QED corrections into account



- integrating over $m(\ell\ell)$, the large positive and negative corrections cancel (KLN theorem)
- Detector effects may significantly influence the QED corrections:

☞ It is difficult to discriminate electrons and photons which hit the same calorimeter cell

→ recombine e and γ momenta to an effective electron momentum in that case

→ an inclusive quantity is formed

→ the mass singular terms ($(\alpha/\pi)\log(\hat{s}/m_\ell^2)$) disappear (KLN again...)

→ the effect of the QED corrections is reduced

☞ Muons must be consistent with a minimum ionizing particle

→ require $E_\gamma < 2$ GeV in cell traversed by muon

→ this reduces the hard photon part

→ the mass singular terms survive

- calculations of the complete $\mathcal{O}(\alpha)$ EWK corrections to

☞ $p\overset{(-)}{p} \to W^\pm \to \ell^\pm\nu$ (Dittmaier+Krämer, UB+Wackeroth)

☞ and $p\overset{(-)}{p} \to \gamma,\, Z \to \ell^+\ell^-$, including $\mathcal{O}(G_F^2 m_t^2 M_W^2)$ corrections to $\sin^2\theta_{eff}$ (UB et al.) exist now

- if final state photon radiation shifts W mass by $\mathcal{O}(100)$ MeV:

☞ need to worry about multiple (final) state photon radiation in W *and* Z production

☞ effect should be more pronounced in Z case since both final state leptons radiate

☞ two photon radiation is known to significantly change the shape of the $m(\ell\ell)$ and M_T distributions (UB, Stelzer)

- recent progress in taking multi-photon radiation into account: two approaches

  ☞ YFS exclusive exponentiation (Jadach, Placzek)

  → currently only at parton level and for W decay

  → procedure used is gauge invariant

  ☞ QED structure function approach (Montagna et al.)

  → only final state corrections are presently incorporated

  → procedure used is not gauge invariant

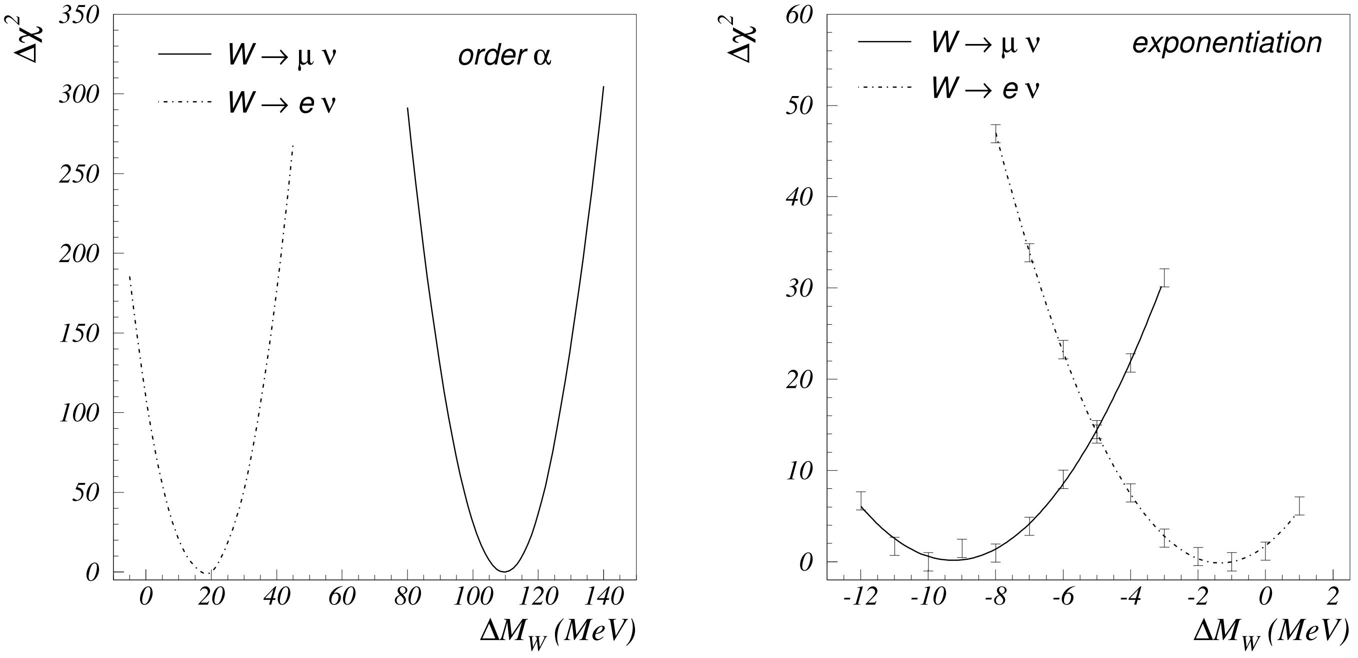
  → however, terms violating gauge invariance are numerically small ($< 0.1\%$)

- Montagna et al. calculate shift in M_W using simplified detector model:

  → combine e and γ momenta for $\Delta R(e, \gamma) < 0.2$

  → reject μ events if $E_\gamma > 2$ GeV and $\Delta R(\mu, \gamma) < 0.2$

☞ result:



☞ shift of M_W caused by multi-photon radiation is about 10% of that caused by one photon radiation

☞ Note: absolute value of shift caused by $\mathcal{O}(\alpha)$ corrections smaller than value observed by CDF/DØ, due to simplified detector model

☞ a similar calculation in the Z case also exists now

- for W mass analysis need calculation of W and Z production including electroweak and resummed QCD corrections

  ☞ need accurate knowledge of W p_T distribution to determine $\not{p}_T$ resolution

  ☞ $\not{p}_T$ resolution determines how “sharp” the edge in the M_T distribution at $M_T \approx M_W$ is

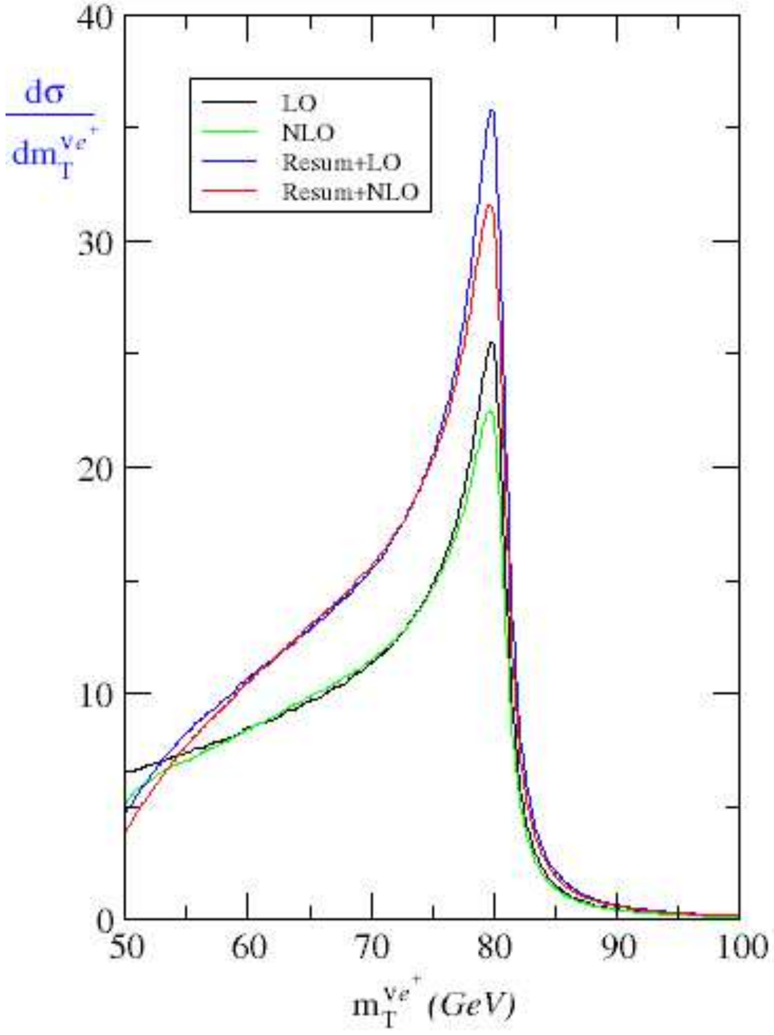
  ☞ which in turn determines how well M_W can be measured

- first step towards this lofty goal:

  ☞ incorporate final state photon radiation effects into RESBOS calculation (Cao, Yuan)

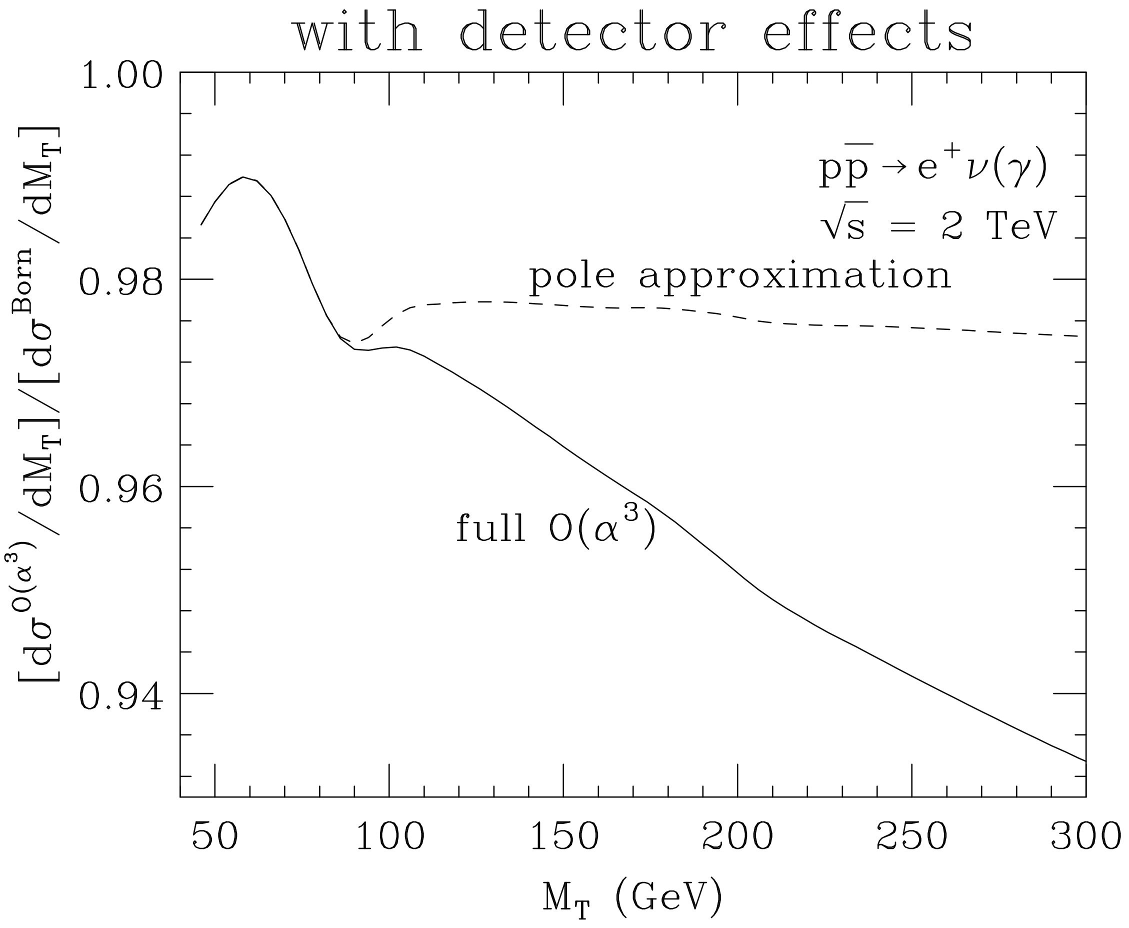
NLO: $\mathcal{O}(\alpha)$ QED final state radiation

Resum: resummed QCD corrections (RESBOS)



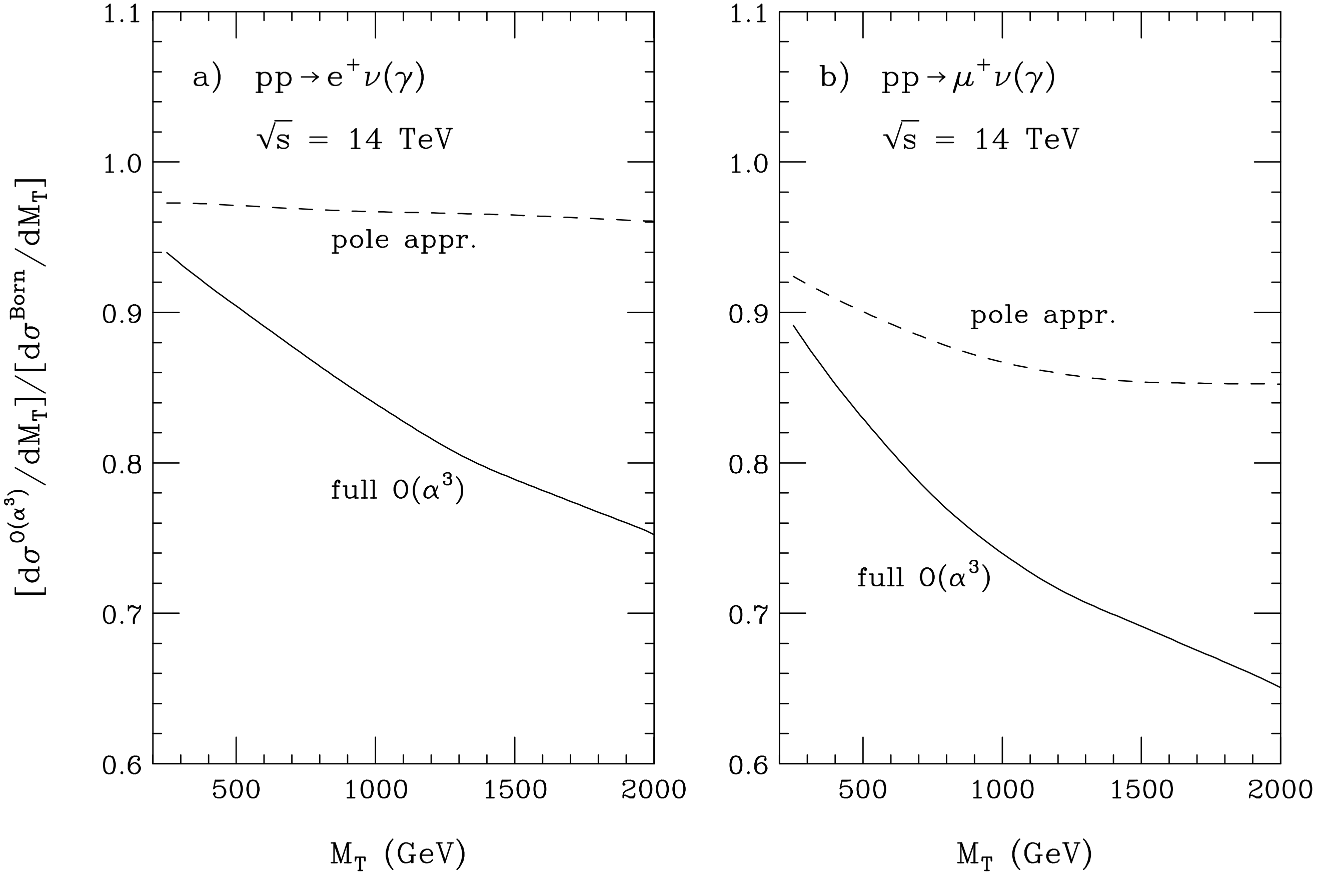
Electroweak Sudakov Logs

- for $\hat{s} \gg M^2_{W/Z}$, the weak corrections become large and negative



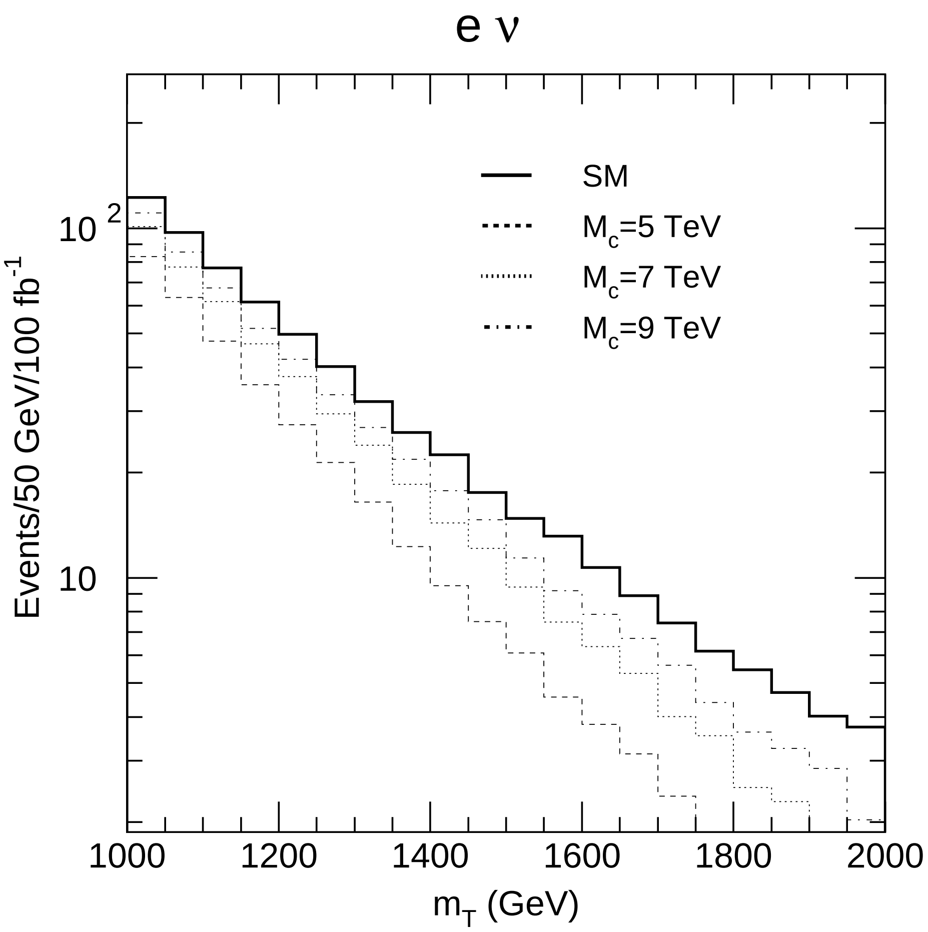
dashed: evaluate weak form factors for $\hat{s} = M^2_W$

- reason: terms $\sim \alpha \log^2(\hat{s}/M_W^2)$ from vertex and box corrections
  - ☞ need to resum?
  - ☞ certainly for the LHC this is necessary (not done yet)



- important for new physics searches:

☞ example: KK excitations of W boson: a slight reduction in cross section could signal a heavy KK excitation beyond reach for direct production (Polesello, Prata)



- effect on W width extracted from high M_T tail

☞ the non-resonant weak corrections which contain the Sudakov logs have not been taken into account in previous exp. analyses

☞ they change the shape of the M_T distribution

☞ performing a χ^2 analysis:

non-resonant weak corrections shift W width by

$$\delta\Gamma_W \approx -7.2 \text{ MeV}$$

☞ expected exp. precision in Tevatron run2 (2 fb^{-1}, $e+\mu$, CDF+DØ combined):

$$\Delta\Gamma_W \approx 25 - 30 \text{ MeV}$$

not negligible!

4 – Radiative Corrections to $e^+e^- \to 4f$

- Measuring M_W at the ILC:

☞ continuum measurement ($\sqrt{s} > 2M_W$):

➞ reconstruct W's from decay products (similar to method employed by LEP II exps.)

➞ expect to achieve $\delta M_W \approx 10$ MeV for $\int \mathcal{L} dt = 500$ fb^{-1} (uncertainty dominated by systematic uncertainty)

☞ threshold scan: $\sqrt{s} \approx 161$ GeV (Wilson, Sitges Workshop)

$e^+e^- \to 4$ fermion cross section is sensitive to M_W in threshold region

- the threshold scan under the magnifying glass:
  - ☞ statistical uncertainty: (Stirling)

$$\delta M_W^{stat} = 90 \text{ MeV} \left[\frac{\epsilon \int \mathcal{L} dt}{100 \text{ pb}^{-1}} \right]^{-1/2}$$

for $\epsilon = 0.67$ (efficiency) and $\int \mathcal{L} dt = 100 \text{ fb}^{-1}$:

$$\delta M_W^{stat} \approx 3.5 \text{ MeV}$$

  - ☞ add systematic errors

For a multiplicative factor C:

$$\delta M_W^{sys} = 17 \text{ MeV} \left[\frac{\Delta C}{C} \times 100\% \right]$$

assume $\Delta \epsilon \approx 0.25\%$, $\Delta \mathcal{L} \approx 0.1\%$:

$$\delta M_W \approx 6 \text{ MeV}$$

  - ☞ detailed simulations yield $\delta M_W \approx 7$ MeV (Mönig)

- theoretical uncertainties:

☞ if one wishes to achieve $\delta M_W \approx 7$ MeV, one needs $\delta M_W^{theor} \sim 1$ MeV

☞ need to know cross section in threshold region with

$$\frac{\Delta\sigma}{\sigma} \approx 0.05\%$$

☞ present situation: only calculation valid in threshold region: GENTLE, includes full (improved) Born $e^+e^- \to 4$ fermion cross section, including non-resonant graphs, finite W width, Coulomb corrections and ISR effects

☞ uncertainty of GENTLE cross section in threshold region (CERN LEP2 Yellow Report):

$$\frac{\Delta\sigma}{\sigma} \approx 1.4\%$$

☞ need full $\mathcal{O}(\alpha)$ corrections in threshold region

☞ finite W width effects are important in threshold region:

→ must go beyond double pole approximation

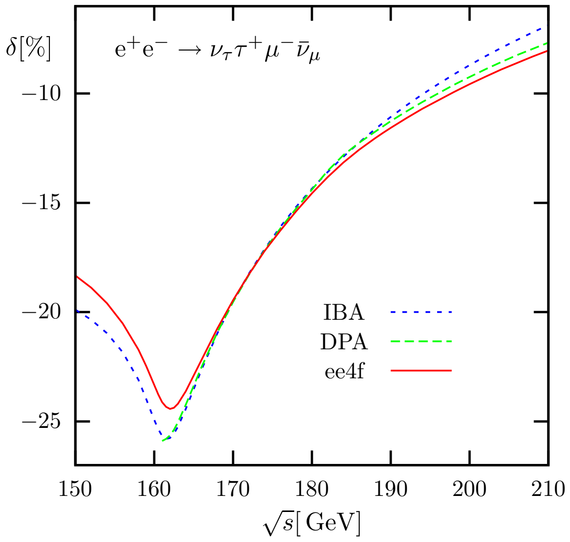
☞ new: full $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to 4$ fermions known (`ee4f`; Denner et al.)

☞ remaining theoretical uncertainties:

→ NLL corrections ($(\alpha/\pi)^2 \log(m_e^2/s)$): $\mathcal{O}(0.1\%)$

→ higher order effects of coulomb singularity: $\sim 0.2\%$ (Fadin et al., Bardin et al.)

☞ still a way to go to reach goal....



5 – Conclusions

- controlling electroweak radiative corrections is essential for future high precision tests of the SM
- significant progress has been made over the last few years
- a long shopping list of things to do remains:

  ☞ higher order corrections to M_W and $\sin^2\theta_{eff}$

  ☞ higher order EWK corrections to W and Z production in the pole region

  ☞ resummation of EWK Sudakov logs

  ☞ ....