Effective weak mixing angle at $O(\alpha^2)$

Małgorzata Awramik

DESY, Zeuthen
&
INP PAS, Cracow

Snowmass, 18.08.2005

Outline:

- Motivation

- Effective weak mixing angle prediction
  - the fermionic part

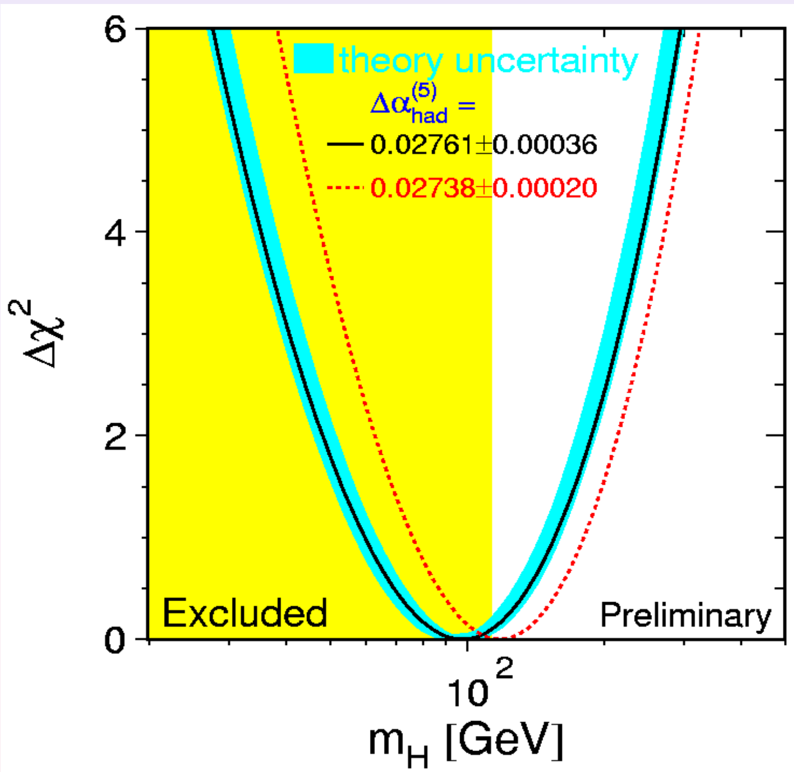
    in collaboration with : Michał Czakon (Wurzburg Uni.)
    Ayres Freitas (Fermilab)
    Georg Weiglein (Durham Uni.)

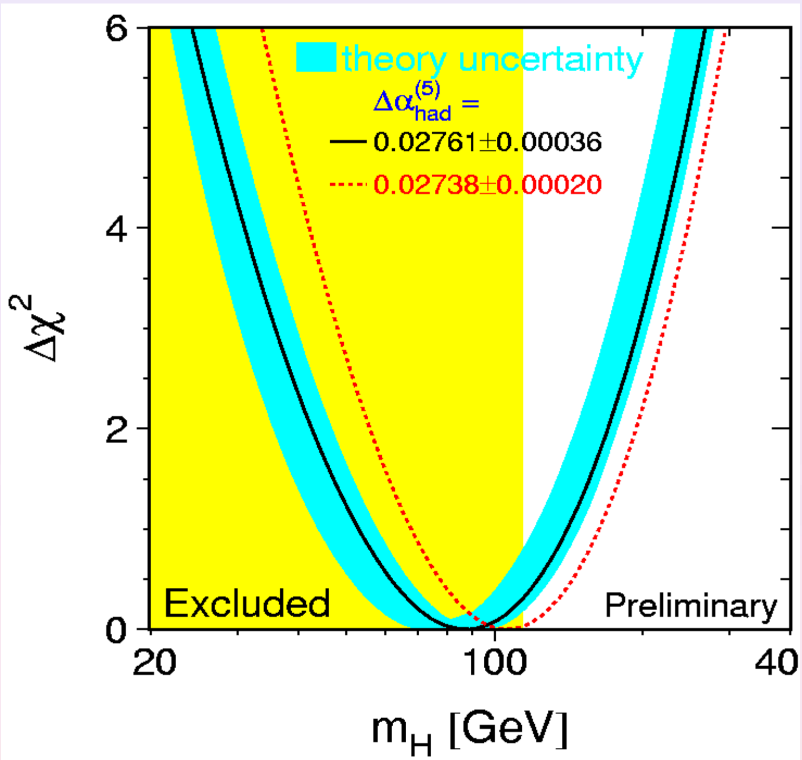
  - the bosonic part

    in collaboration with: Michał Czakon & Ayres Freitas

Motivation: Standard Model precision tests

Winter 2001 | Summer 2001





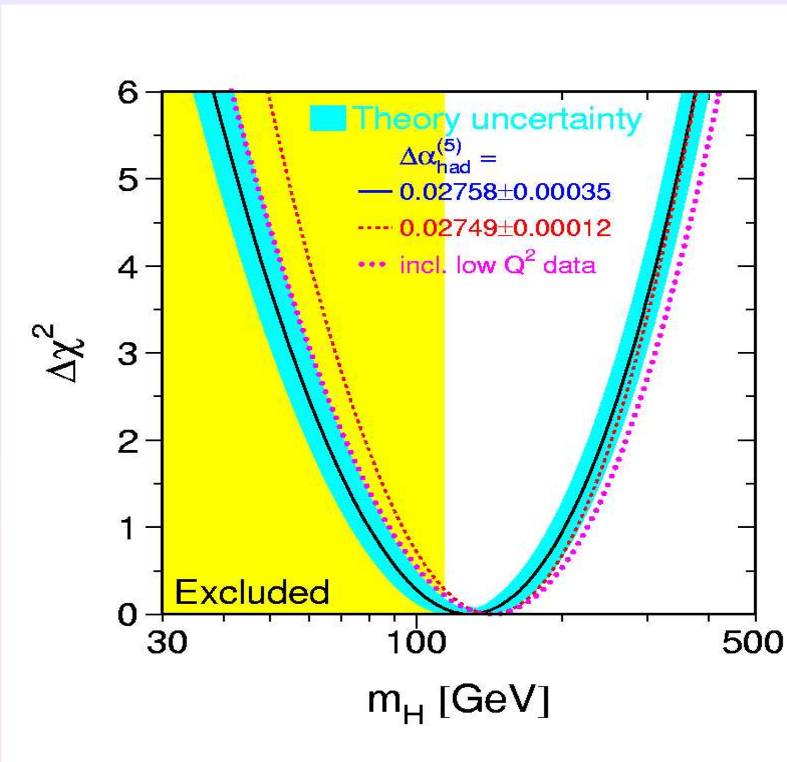
theory prediction of $\sin^2\theta^{lep}_{eff}$, M_W:

expansions up to m_t^2
(Degrassi et al)

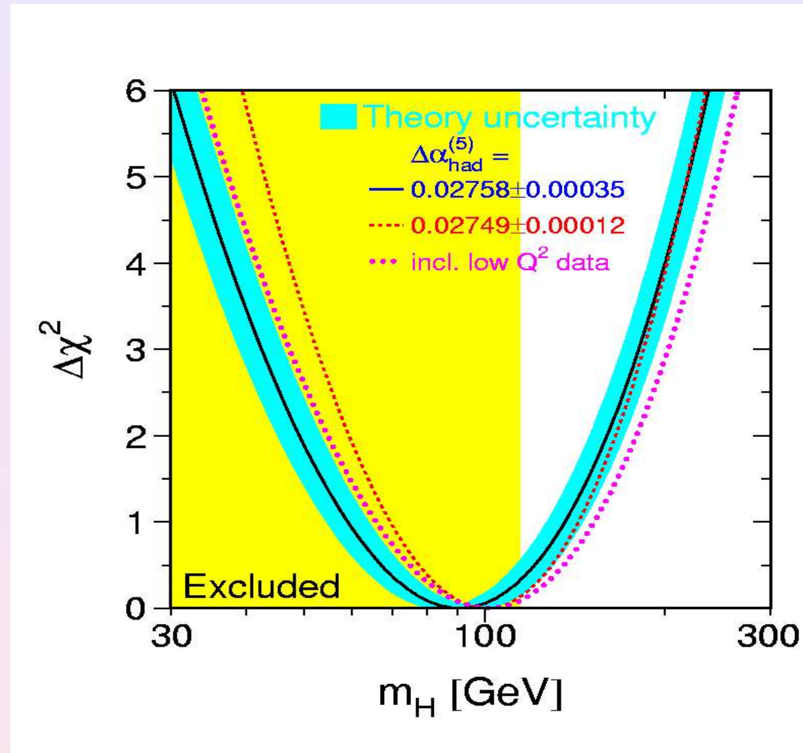
exact calculation of M_W (Freitas et al)
also included in error estimation
of $\sin^2\theta^{lep}_{eff}$

Motivation: Standard Model precision tests

Early Summer 2005



Summer 2005



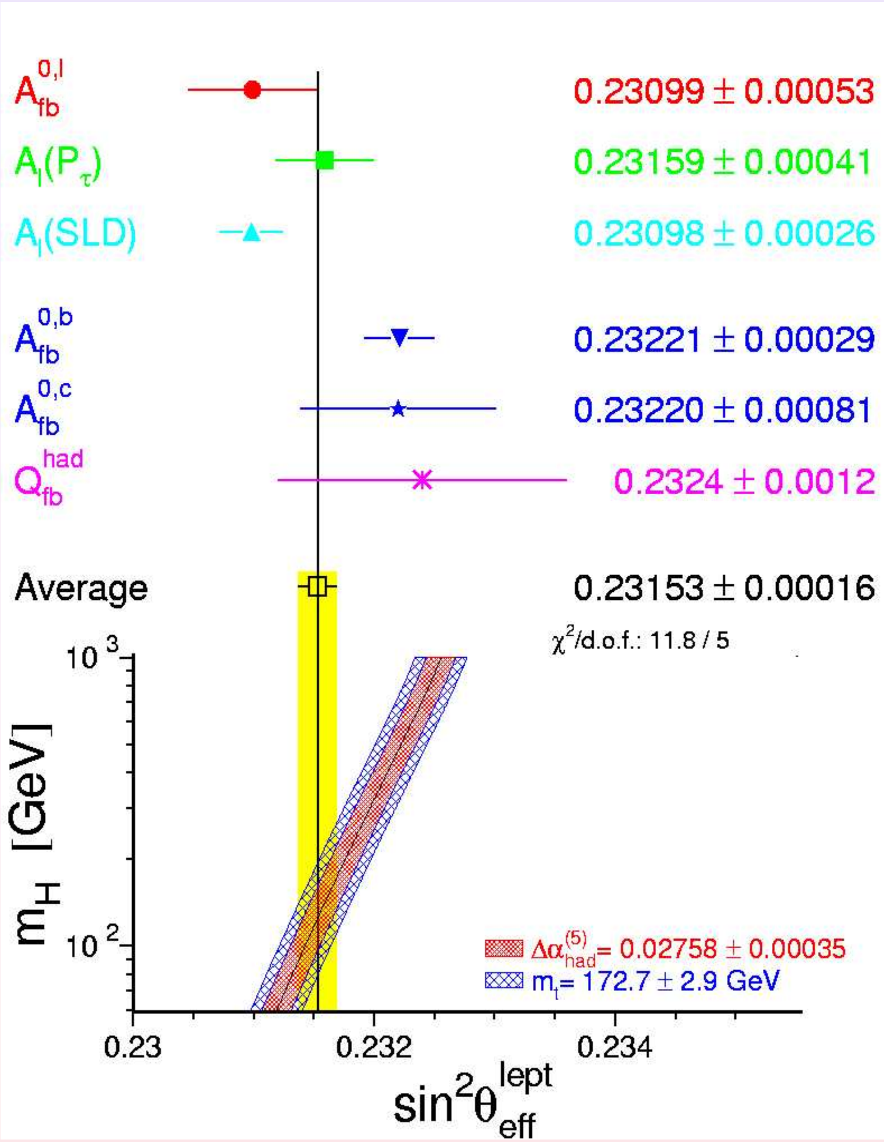
M_H^0=129 GeV, M_H<285 GeV

M_H^0=90.7 GeV, M_H<186 GeV

ZFITTER:

- written by: Dima Bardin, Penka Christova, Mark Jack, Lida Kalinovskaya, Sacha Olshevski, Sabine Riemann, Tord Riemann
- for e+e- annihilation into a fermion pair calculates several observables:
  - M_W
  - Γ_Z and Γ_W
  - cross sections (differential and total)
  - asymmetries
- since May 2005: http://www.ifh.de/~riemann/Zfitter/zf.html
  - supported by: Andrej Arbuzov, Malgorzata Awramik, Michal Czakon, Ayres Freitas, Martin Gruenewald, Klaus Moenig, Sabine Riemann, Tord Riemann
  - recent changes:
    - Higher order QED corrections to fermion-pair production, of importance at energies off the Z boson peak
    - Electroweak corrections to the weak charge Q_W, describing the parity violation effects
    - in atoms, of importance for so-called global Standard Model fits
    - Electroweak corrections to νe νe production, of importance for a precise description of $\nu\nu\gamma$ production;
    - Electroweak two-loop corrections to M_W and the effective weak mixing angle $\sin^2\theta_{eff}$, of importance for global Standard Model fits and for precise predictions of the Higgs mass M_H

Effective weak mixing angle:



- experimental uncertainty
  - 0.00016 - present
  - 0.00014 - at LHC
  - 0.000013 - at LC
- theoretical uncertainty
  - parametric
    - m_t: 0.00014
    - $\Delta\alpha$: 0.00013
  - intrinsic
    - 0.000049

Known contributions to the effective weak mixing angle:

One-loop corrections $O(\alpha)$

W.J. Marciano, A. Sirlin, '80

G. Degrassi, A. Sirlin, '93, P. Gambino, A. Sirlin, '94

QCD corrections $O(\alpha\, \alpha_s)$ and $O(\alpha\, \alpha_s^2)$

A. Djouadi, '88, F. Halzen, B.A. Kniehl, '91

L. Avdeev et al., '94

K. Chetyrkin, J. Kuhn, M. Steinhauser, '95

Expansions in M_H, m_t of the $O(\alpha^2)$ contributions

J. v.d.Bij, M. Veltman, '84

J. v.d.Bij, F. Hoogeveen, '87

R. Barbieri et al., '93

J. Fleischer, O.V. Tarasov, F. Jegerlehner, '95

G. Degrassi, P. Gambino, A. Sirlin '97 $O(\alpha^2\, m_t^2/M_Z^2)$

Exact fermionic of $O(\alpha^2)$

M. Awramik, M. Czakon, A. Freitas, G. Weiglein, '04

W.Hollik, U. Meier, S. Uccirati, '05

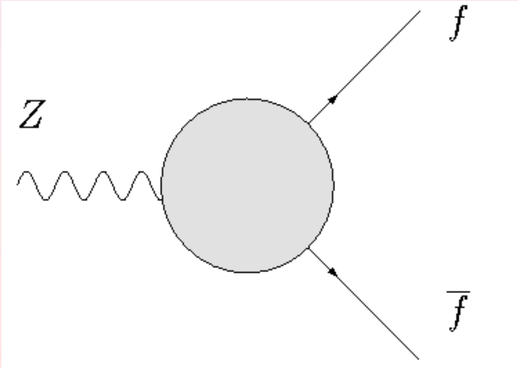
The amplitude for four fermion process at e+e- colliders:

$$\mathcal{A}(e^+e^- \to Z \to f\overline{f}) = \frac{4ie^2 I_e^{(3)} I_f^{(3)}}{s - M_Z^2 + iM_Z\Gamma_Z}\rho_{ef}$$
$$\times \left[\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu(1+\gamma_5)\right.$$
$$-4|Q_e|s_W^2\kappa_e\gamma_\mu\otimes\gamma^\mu(1+\gamma_5) - 4|Q_f|s_W^2\kappa_f\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu$$
$$\left.+16|Q_eQ_f|s_W^4\kappa_{ef}\gamma_\mu\otimes\gamma^\mu\right]$$

defines effective weak mixing angle at Z pole:

$$\Rightarrow \qquad \sin^2\theta_{\text{eff}}^{\text{lept}} = \text{Re}[\kappa_l(s = M_Z^2)]s_W^2$$

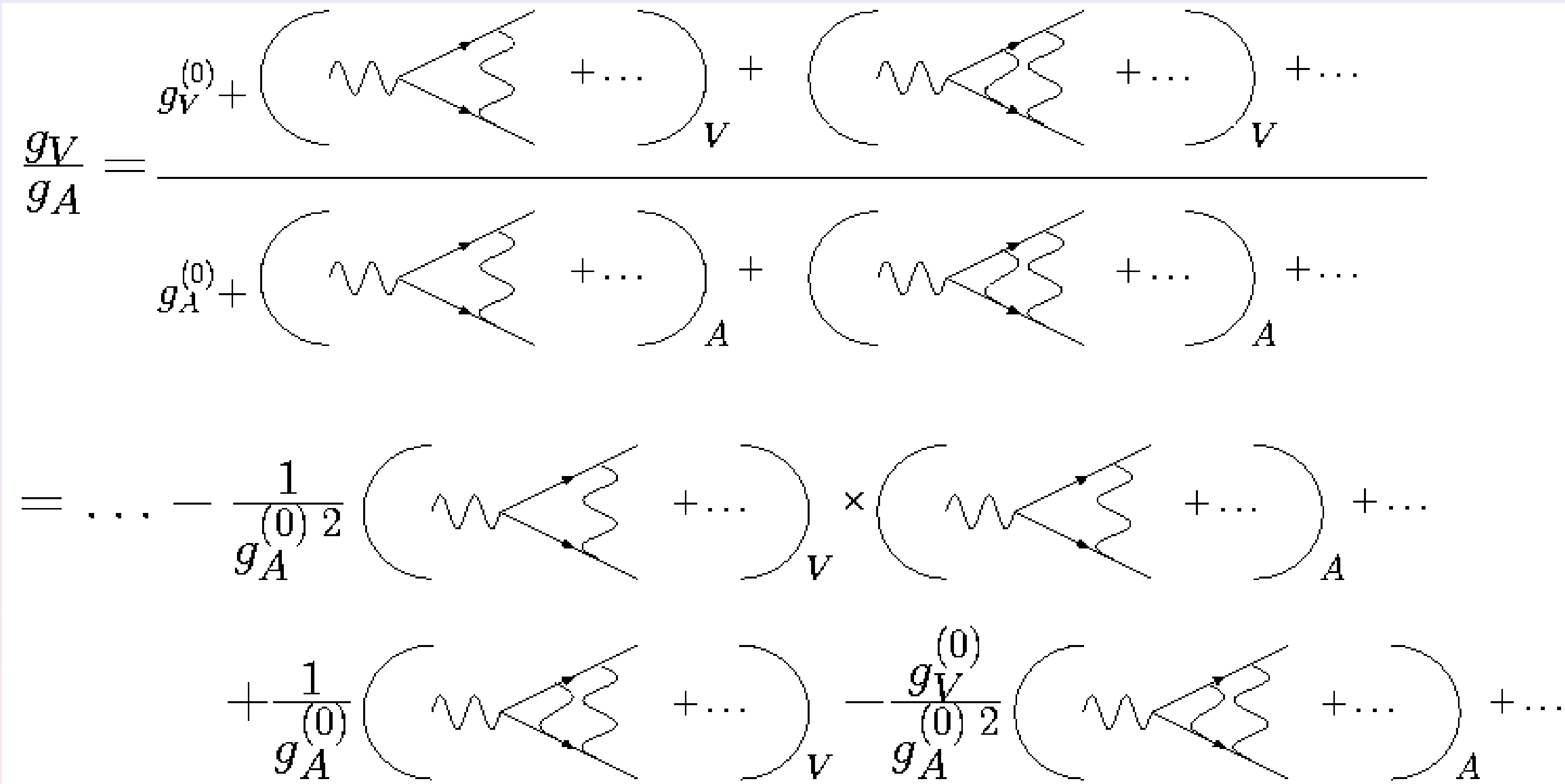
equivalently, the effective Z boson vertex



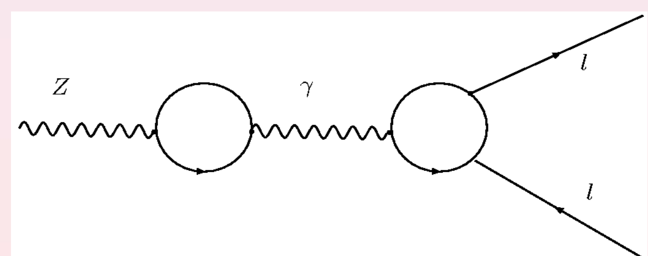
defines:

$$\sin^2\theta_{\text{eff}}^{\text{lept}} = \frac{1}{4}\left(1 - \text{Re}\left(\frac{g_V}{g_A}\right)\right)$$

Contributions to the effective weak mixing angle:

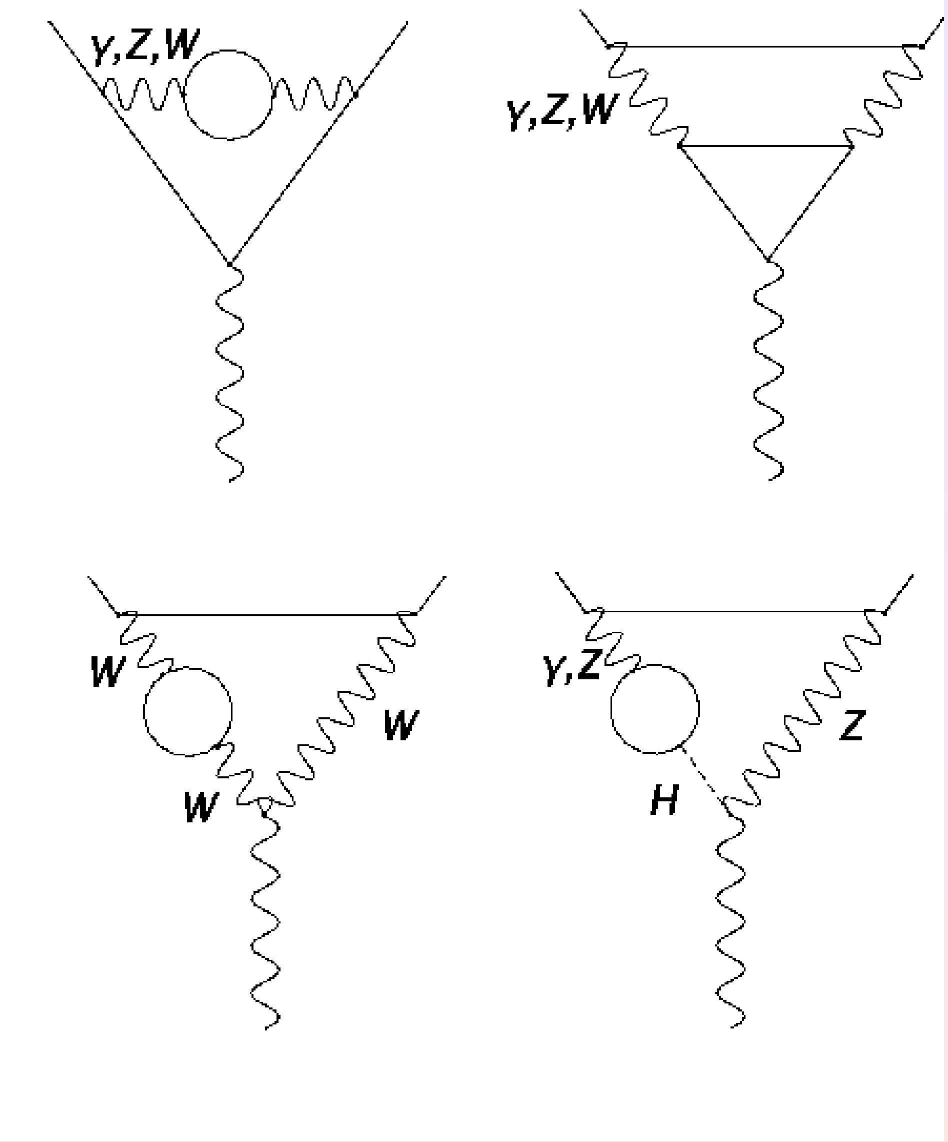


plus some 1PR contributions, as:



Two loop vertex contributions: the fermionic part

M.A.,M.Czakon, A.Freitas, G.Weiglein Phys.Rev.Lett.93:201805,2004



- diagrams with Higgs cancel
- diagrams with top quark:

  semi-analytic result
- diagrams with light fermions:

  analytic result

Light fermion contributions:

- one scale only: M_W/M_Z
- reduction by Integration By Parts Identities (K.Chetyrkin, Tkachov '81)

  and Lorentz Invariance Identities (Gehrman, Remiddi '00)
- use IdSolver: C++ program (M. Czakon)
- for new master integrals differential equations work
- exact result in terms of polylogarithms

Top Quark Contributions:

- depend on two scales:

  m_t/M_Z, M_W/M_Z

- use asymptotic expansions

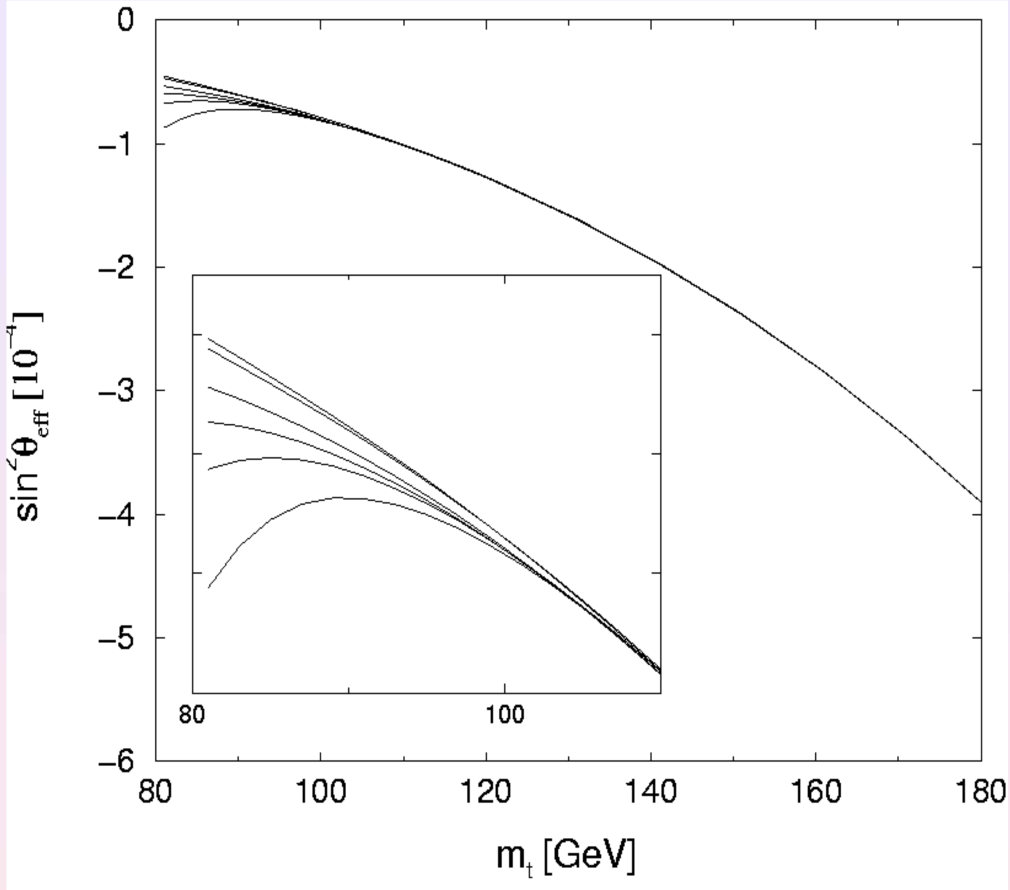
  in heavy top quark mass

  - justified by size of

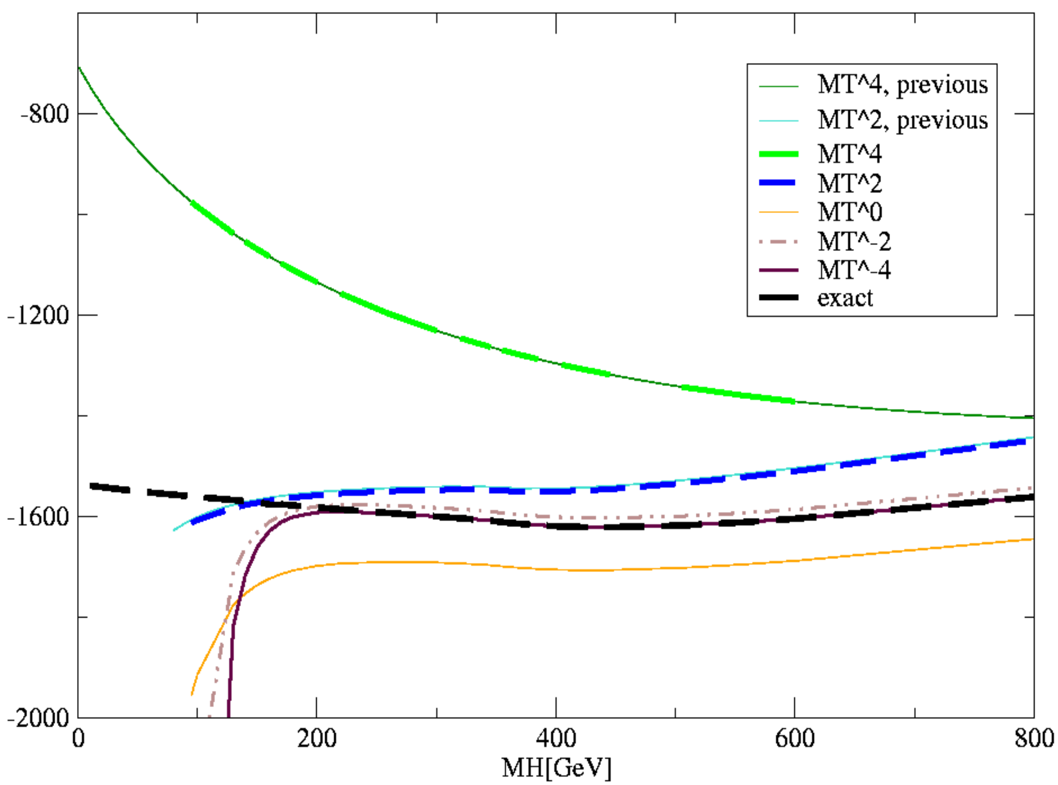
    $M_Z^2/m_t^2 \sim 1/4$

  - up to two loop vacuum diagrams required -> easy

  - 10^{th} term gives relative error around $\sim 10^{-5}$



Tests:



1) expansion of the whole result
- agreement of leading and next to leading term with results of Gambino and Degrassi

2) some integrals tested with low momentum expansion

3) some integrals tested by means of Pade re-summed Mellin-Barnes representation

4) independent internal calculation

Size of different contributions to $\sin^2\theta^{lep}_{eff}$:

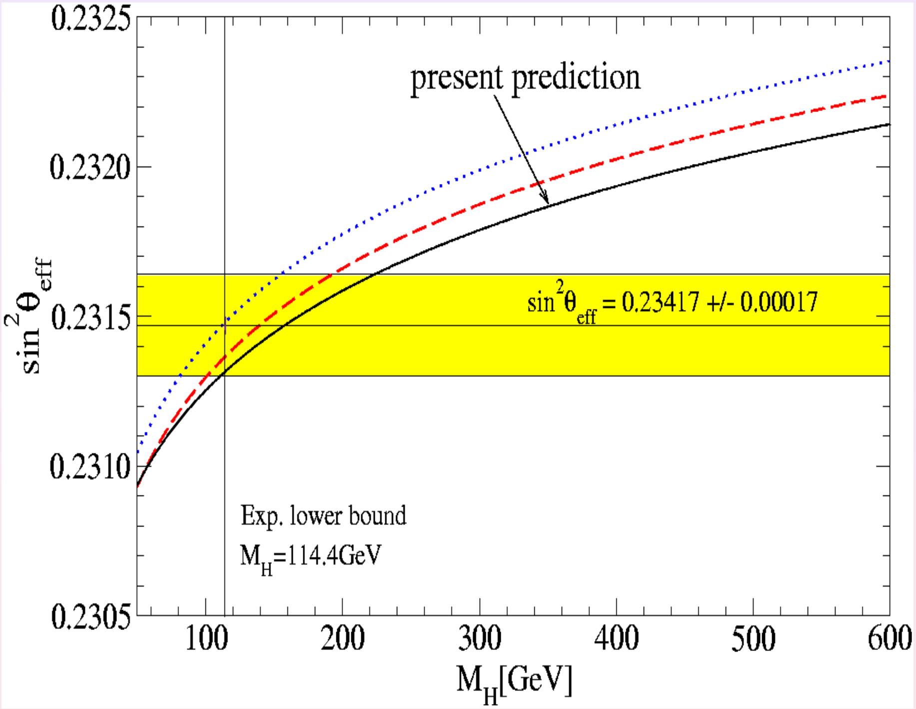
M_H [GeV]	$\mathcal{O}(\alpha)$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{ferm}}$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{tb}}$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{lf}}$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{tr}\gamma_5}$ $\times 10^{-4}$
100	97.47	-0.14	-3.77	-0.63	0.06
200	93.17	-0.48	-3.80	-0.68	0.06
600	84.28	-1.11	-3.75	-0.84	0.06
1000	79.63	-1.05	-3.31	-0.94	0.06

agreement in recent calculation of Hollik, Meier, Uccirati

M_H [GeV]	$\mathcal{O}(\alpha\alpha_s)$ $\times 10^{-4}$	$\mathcal{O}(\alpha\alpha_s^2)$ $\times 10^{-4}$	$c_W^2 \Delta\rho^{\alpha^2\alpha_s}$ $\times 10^{-4}$	$c_W^2 \Delta\rho^{\alpha^3}$ $\times 10^{-4}$	reducible $\times 10^{-4}$
100	-8.18	-1.60	0.28	0.04	0.20
200	-8.18	-1.60	0.46	0.02	0.21
600	-8.18	-1.60	0.90	0.02	0.21
1000	-8.18	-1.60	1.11	0.22	0.22

Faisst, Kuhn, Seidensticker, Veretin, '03

Theory versus experiment:



The newest theory prediction for $\sin^2\theta^{lep}_{eff}$:

- M.A.,M.Czakon,A.Freitas,G.Weiglein

  Phys.Rev.Lett.93:201805,2004

- contains all known corrections, but two loop bosonic part is still missing

- error from unknown higher order terms: $4.9 \cdot 10^{-5}$ (previous: $14 \cdot 10^{-5}$)

M^0_H = 125 GeV (for m_t=172.7GeV)

shift due to new formula: +15GeV

shift due to measurements:

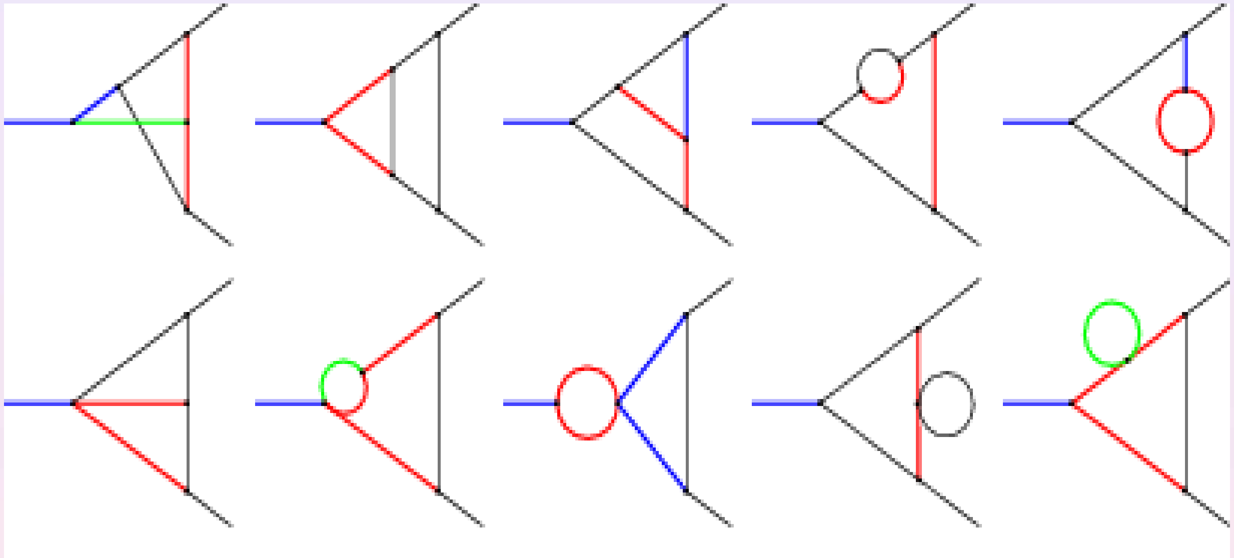
of m_t: ±30GeV

of $\sin^2\theta^{lep}_{eff}$: -10GeV

Two loop vertex contributions: the bosonic part

(under way, in collaboration with Michal Czakon and Ayres Freitas)

Possible topologies:



Possible problems:

- number of scales: M_Z (blue line), M_W (red line) and M_H (green line)

  => around 100 of different mass patterns with up to 3 scales

- finiteness of the result and treatment of divergences

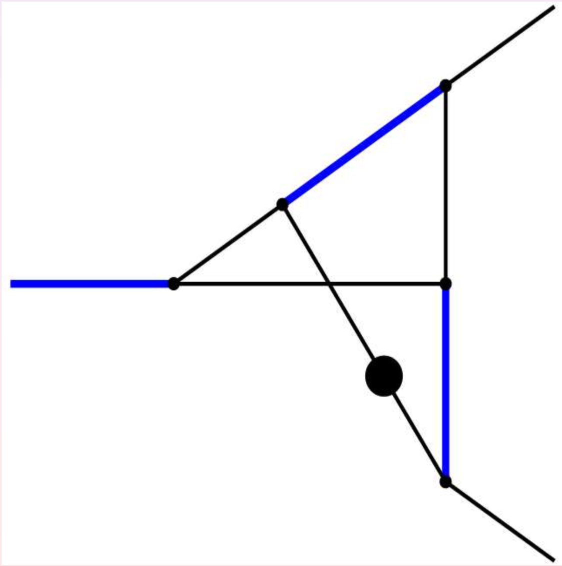
General approach:

- to reduce number of scales first perform
  mass difference expansion, in $\sin^2\theta_W$ (except for threshold lines)
- for the desired precision of 3-5 digits
  number of integrals rises: ~350 -> ~20 000
- only up to two scale integrals left
- **either** perform reduction to masters on two scale integrals
  -> would require a few weeks on a three 3.GHz 64 bit computer
- **or** perform further (re-)expansion before

Reduction of integrals to a set of master integrals

- feasible only with automatic program (IdSolver)
- arbitrary set of masters, often required up to ε^2 terms
- sometimes can be used to identify unknown and complicated integrals, e.g.:

here divergences are given "for free"



Reduction of two scale masters/integrals:

- re-expansions in $\sin^2\theta^{lep}_{eff}$

- threshold expansions

  (for diagrams with Z boson and W or Higgs boson)

- large mass expansion

  (for diagrams with Higgs boson)

Threshold expansions:

- hard-hard contribution from the region:

$k1, k2 \sim 1$

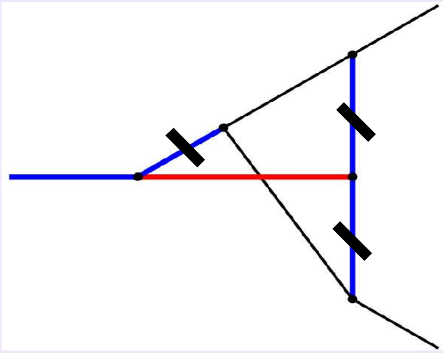
=> one scale integrals

- ultrasoft-ultrasoft contribution from the region:

$k1, k2 \sim \sin^2\theta_W$

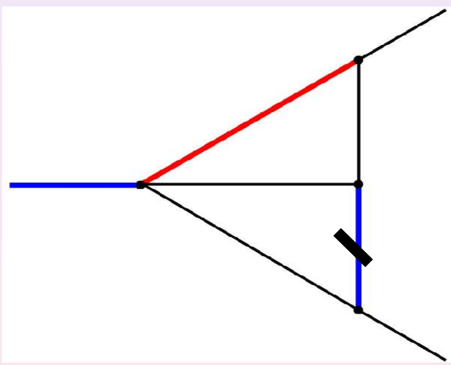
=> new non-trivial integrals

three types of ultrasoft integrals:



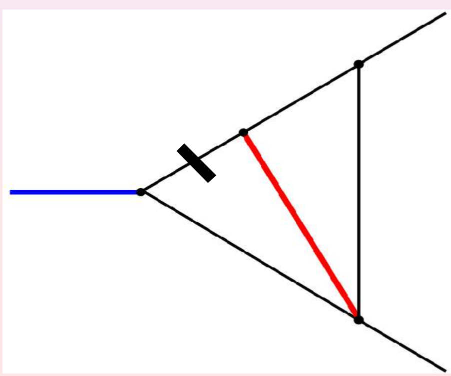
=> propagator integral (Smirnov)

$$\frac{1}{(k2.k2(k1+k2)^2(-2k1.p+sw^2))}$$



=> two loop vertex integral

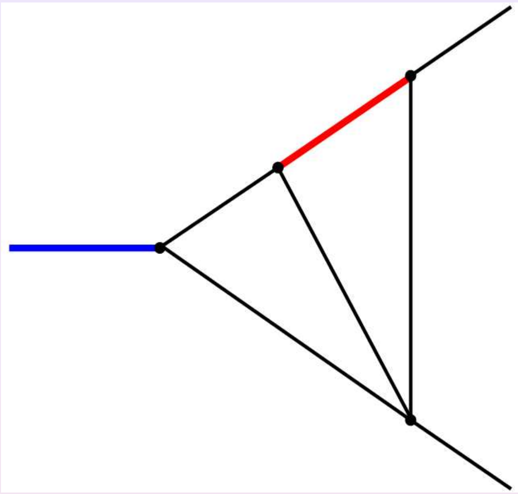
$$\frac{1}{(k2.k2(k1+k2)^2(-2k1.p+sw2)(-2k1.p2))}$$



=> two loop vertex integral

$$\frac{1}{(k2.k2(k1+k2)^2(-2k1.p+sw2)(2k2.p2))}$$

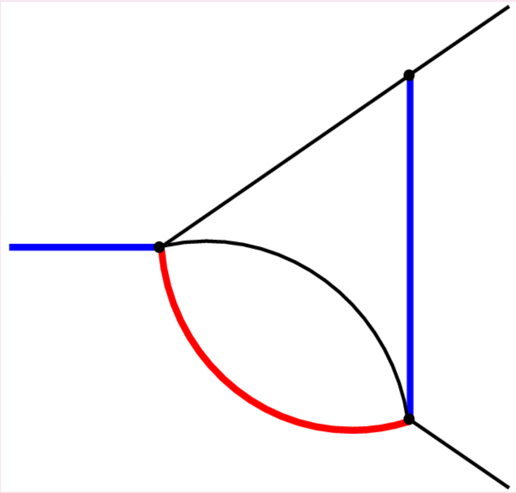
Precision of threshold expansions:



exact result: 0.9097786

threshold expansion (to $\sin^{18}\theta_W$): 0.9097779

relative error: $0.7 \cdot 10^{-6}$



relative error: $1.6 \cdot 10^{-8}$

Evaluation of one scale master integrals:

- numerical methods:

➢ Sector Decomposition method (T.Binoth and G. Heinrich, '02)

with Cuba library (T. Hahn)

-> poor precision

-> applicable only in some cases

-> only for tests

➢ via dispersion relations and reduction with Feynman parameters

-> applicable to most of the required integrals

-> possible problems with evaluation of ε terms

-> further complications for integrals with collinear divergences

Numerical integrations:

- topologies with self-energy sub-loop
  using dispersion relations for B_0 function (S. Bauberger, et. al. '95)

$$B_0(p^2, m_1^2, m_2^2) = -\int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s \, \frac{\Delta B_0(s, m_1^2, m_2^2)}{s - p^2}$$

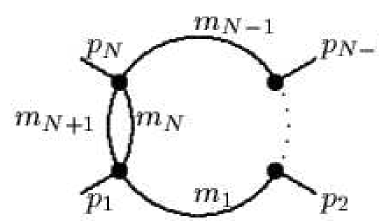
with

$$\Delta B_0(s, m_1^2, m_2^2) = (4\pi\mu^2)^{4-D} \frac{\Gamma(D/2-1)}{\Gamma(D-2)} \frac{\lambda^{(D-3)/2}(s, m_1^2, m_2^2)}{s^{D/2-1}}$$

$$\lambda(a,b,c) = (a-b-c)^2 - 4bc$$

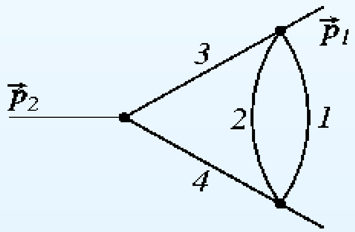
in general:

$$T_{N+1}(p_i; m_i^2) = -\int_{s_0}^{\infty} \mathrm{d}s \, \Delta B_0(s, m_N^2, m_{N+1}^2) \times \int \mathrm{d}^4 q \, \frac{1}{q^2 - s} \frac{1}{(q+p_1)^2 - m_1^2} \cdots \frac{1}{(q+p_1+\cdots+p_{N-1})^2 - m_{N-1}^2}$$



Numerical integrations:

example of a topology with a sub-loop



$$= -\int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s\ \Delta B_0(s, m_1^2, m_2^2) \times C_0(p_2^2, (p_1+p_2)^2, p_1^2; m_3^2, m_4^2, s)$$

- reduce to one dimensional integration – fast evaluation
- divergent parts need to be subtracted in integrand
  e.g. terms that evaluate to products of one-loop functions

Numerical integrations:

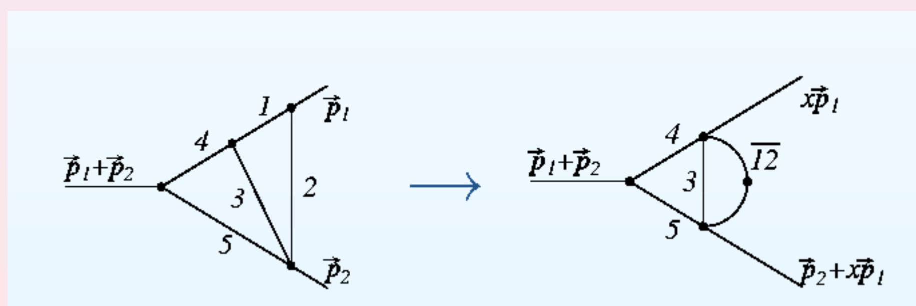
topologies with a triangle sub-loop:

- dispersion relations possible but difficult
- can be reduced to self-energy sub-loop case:
  method of van der Bij and Ghinculov by introducing Feynman parameters

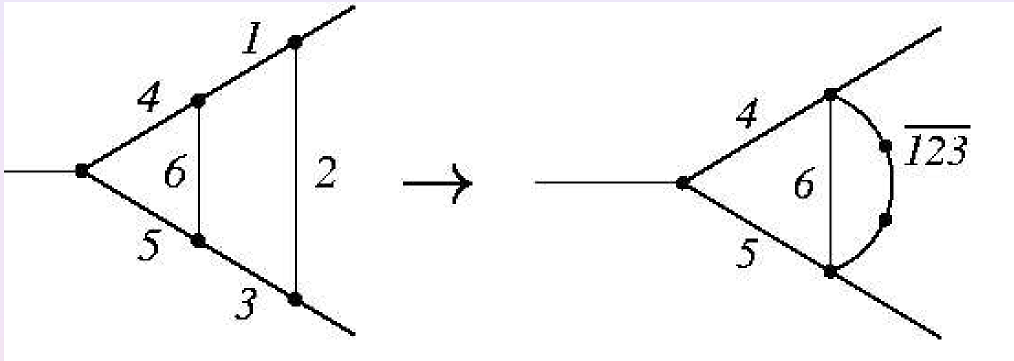
$$\frac{1}{(q+p_1)^2-m_1^2}\,\frac{1}{(q+p_2)^2-m_2^2} = \int_0^1 \mathrm{d}x\, \frac{1}{[(q+\bar{p})^2-\overline{m}^2]^2}$$

$$\bar{p} = x\,p_1 + (1-x)p_2, \qquad \overline{m} = x\,m_1 + (1-x)m_2 - x(1-x)(p_1-p_2)^2$$

reduces triangle sub-loop to self-energy subloop,
e.g:



or in more complicated case,
with two Feynman parameters



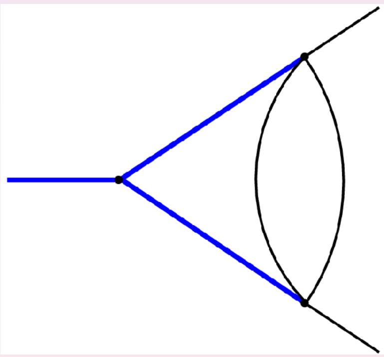
- integration over Feynman parameters is performed numerically
- at worst: three dimensional integrations
- numerical integration performed with discrete Gauss-Kronrod algorithm

Evaluation of one scale master integrals:

- Differential equations (for collinear cases)

- Mellin-Barnes integration

- Taylor expansions (v. high precision)



subsequent terms of the expansion:

3.0954706667539833641838236
3.0954706667539833641838253

numerical result:

3.0954706678

Summary:

- the theory error on Higgs boson mass prediction is dominated by unknown corrections to the effective weak mixing angle

- the fermionic corrections give large numerical contributions, with the shift ~ 15GeV on the Higgs boson mass prediction

- the bosonic contributions pose new and interesting problems; will be available soon

- to match the precision at LC/GigaZ higher order terms: $O(\alpha^2\alpha_s)$ beyond leading term, $O(\alpha^3)$, $O(\alpha^2\alpha_s^2)$ will be required