Virtual Corrections to Bremsstrahlung with Applications to Luminosity Processes and Radiative Return

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Outline

We will describe radiative corrections to Bremsstrahlung and related processes – focusing on applications to luminosity, fermion pair production, and radiative return.

- BHLUMI and the Bhabha Luminosity Process
- The KK MC and fermion pair production
- Radiative Return Applications
- Comparisons to other related results

Radiative Corrections to Bhabha Scattering

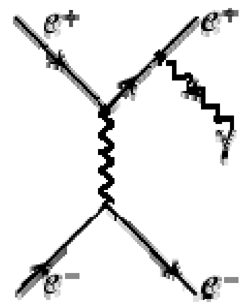
In the 1990s, S. Jadach, M. Melles, B.F.L. Ward and S.A. Yost calculated all two-photon real and virtual corrections to the small angle Bhabha scattering process used in the luminosity monitor at LEP and SLC.

These corrections were used to bring the theoretical uncertainty in the luminosity measurement, as calculated by the BHLUMI Monte-Carlo program, to within a **0.06%** precision level.

The e^+e^- Luminosity Process

- Calculating normalized cross sections requires knowing the beam luminosity $\mathcal{L}$. If N events are observed, the normalized cross section is $\sigma = N/\mathcal{L}$.
- In e^+e^- scattering (SLC, LEP), the luminosity is calibrated using small angle Bhabha scattering

$$e^+e^- \longrightarrow e^+e^- + n\gamma$$



- This process has both experimental and theoretical advantages:

  •A large, clean signal

  •Almost pure QED (3% Z exchange)

- The angle cuts were 1-3 degrees at LEP1, 3-6 degrees at LEP2.

The BHLUMI Monte Carlo Program

BHLUMI was developed into an extremely precise tools for computing the Bhabha luminosity process in $e^+ e^-$ colliders. The project was begun by S. Jadach, B.F.L. Ward, E. Richter-Was, and Z. Was and continued with contributions by S. Yost, M. Melles, M. Skrzypek, W. Placzek and others.

The cross section is calculated by computing electroweak amplitudes to the order needed to meet the desired theoretical uncertainty.

Improvements are added by systematically including higher order radiative corrections as needed. strong interactions.

Historical Progress

	Year	Experiment	Theory
	1982	2%	2%
	1990	0.8%	1%
	1992	0.6%	0.25%
BHLUMI	1996-7	0.1 - 0.15%	<0.11%
	1999	0.05%	**<0.061%**

Progress in the Luminosity Calculation

To obtain a precision level comparable to experiment at LEP1 and LEP2 we found it necessary to eliminate the leading source of error, which was an uncalculated component of the photon radiative corrections to the scattering process.

- Exact calculation of two-photon Bremsstrahlung: all graphs radiating two photons from any fermion lines have been calculated.
- Order α correction to one-photon Bremsstrahlung: include all graphs where one real plus one virtual photon are emitted from a fermion line.
- Order α^2 correction to Bhabha scattering: two virtual photons can be included by adapting known results.

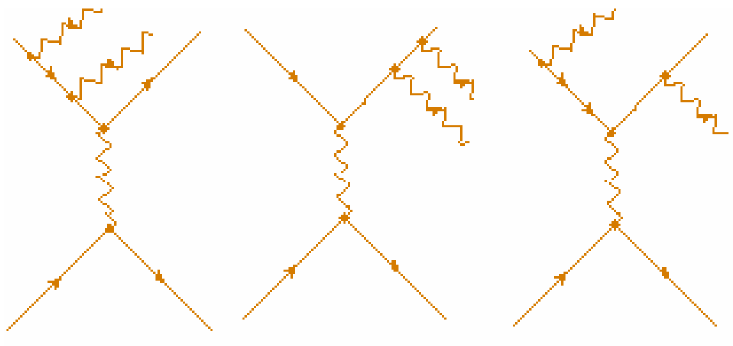
Completing these calculations reduced the uncertainty from order α^2 photon effects from 0.1% for LEP1 down to 0.027%, and from 0.2% for LEP2 down to 0.04%.

Exact 2 Photon Bremsstrahlung

Historically, we started with the two-photon Bremsstrahlung process. The exact two photon Bremsstrahlung cross section was calculated by Jadach, Ward, and Yost in 1991 - 1992. This process is

$$e^+(p_1) + e^-(p_2) \rightarrow e^+(q_1) + e^-(q_2) + \gamma(k_1) + \gamma(k_2)$$

This corresponds to Feynman diagrams of the form



These diagrams were calculated exactly using helicity spinor techniques (which treat the electron as massless), adding electron masses perturbatively.

YFS Soft Photon Techniques

Precision calculations require considerable care, because the cross sections are always dominated by soft (low-energy) photons. But the contribution of soft photons is well known due to results of Yennie, Frautschi and Suura, who exponentiated them to all orders. The soft photon contribution generally factorizes as a soft factor S times a lower order cross section.

Obtaining precision results for multi-photon Bremsstrahlung requires subtracting these dominant, but known contributions and carefully calculating the "YFS residuals" left over. In the case of 2 real photons, the new result we calculated was actually β_2 in the differential cross section

$$d\sigma(k_1, k_2) = \sigma_0\, S(k_1)\, S(k_2) + \beta_1(k_1)\, S(k_2) + \beta_1(k_2)\, S(k_1) + \beta_2(k_1, k_2).$$

2 soft photons

1 soft, 1 hard photon

(β_1 was already known.)

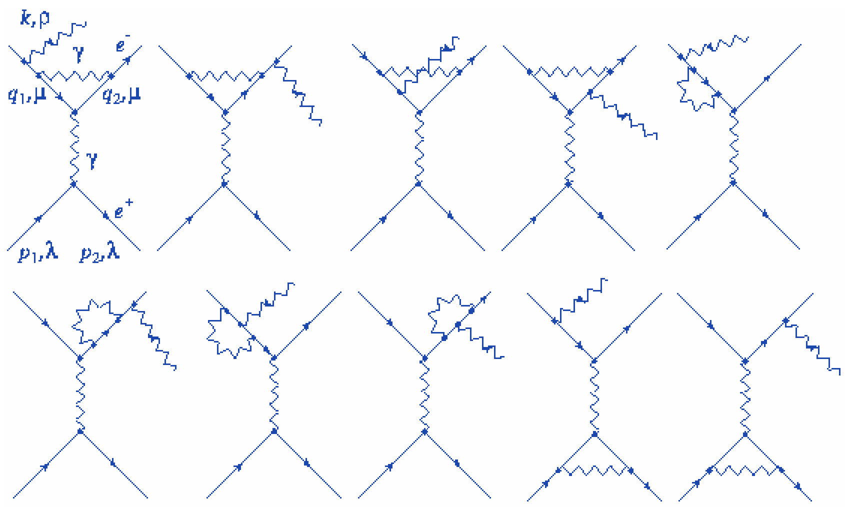
2 hard photons

Real + Virtual Photon Emission

The second, and considerably more difficult contribution completed was the case where one real photon is emitted, but another is virtual, present only in an internal "loop" in the graph. The complexity of the calculations increases rapidly with the addition of loops. (In many calculations, 2 loops is state of the art.) In fact, refinements of this process are still in progress, since some approximations were made.

$$e^+(p_1) + e^-(p_2) \rightarrow e^+(q_1) + e^-(q_2) + \gamma(k)$$

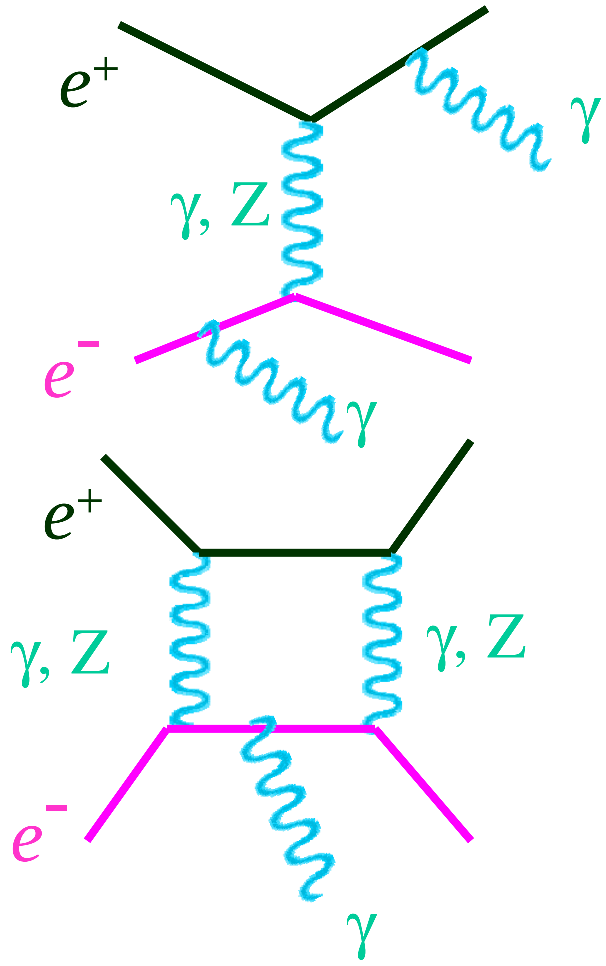
We need this process to order α^2. This requires including an internal photon in a loop.



Assumptions in Calculation

An *exact* calculation of interference effects involving simultaneous emission from both lines has not yet been included.

These contributions are expected to be small below 6^{o} . Calculations are in progress with S. Majhi to verify this, and to extend the range of BHLUMI to larger angles if this should be required in a future ILC design.



Computational Method

The graphs shown were calculated by S. Jadach, M. Melles, B.F.L. Ward and S. Yost in *Phys. Rev.* **D65**, 073030 (2002), based on earlier results for the corresponding *t*-channel graphs by the same authors, *Phys. Lett.* **B377**, 168 (1996).

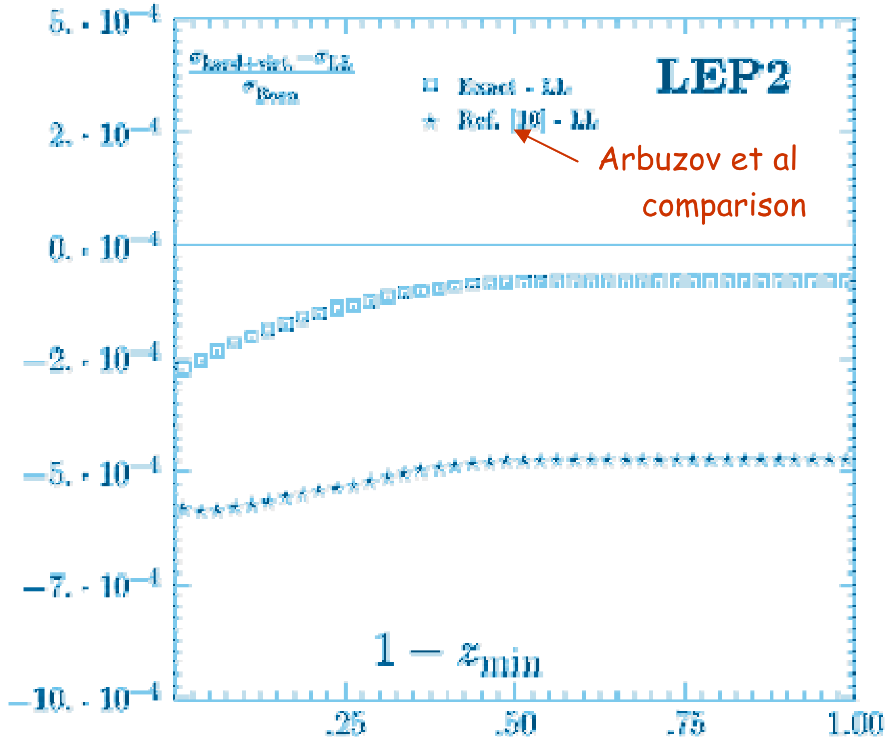
The results were obtained using

- Helicity spinor methods
- Vermaseren's algebraic manipulation program FORM
- Oldenborgh's FF package of scalar one-loop Feynman integrals (later replaced by analytic expressions)
- Mass corrections added via methods of Berends *et al* (CALCUL Collaboration), which were checked to show that all significant collinear mass corrections were included.

Real + Virtual Photon MC Results

The hard photon residual part of the cross section was inserted into MC program which simulates a realistic detector (ALEPH Sical),

allowing us to calculate the size of the part of the real + virtual photon contribution which was previously unavailable.



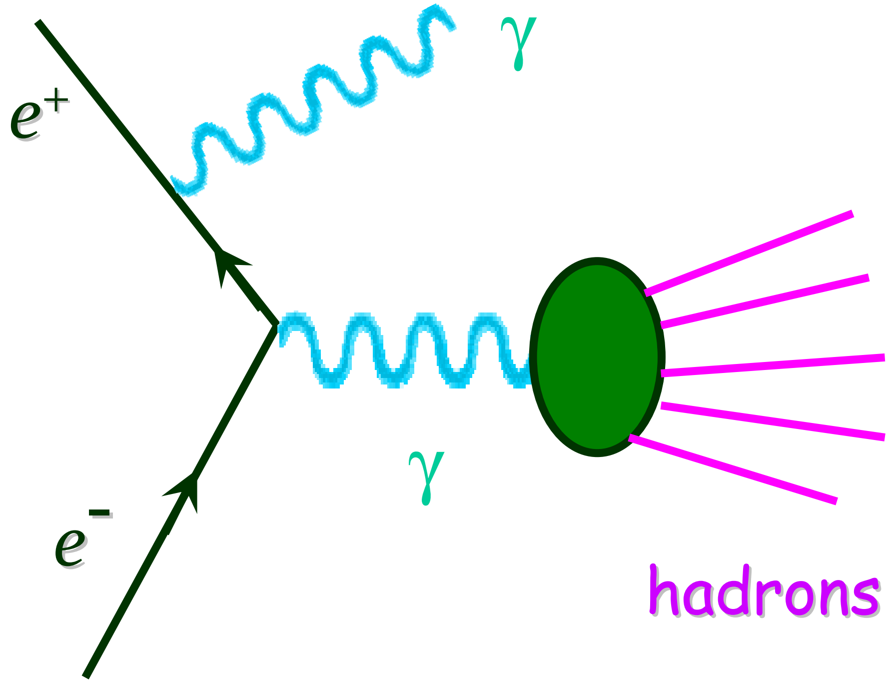
(z is the fraction of energy going into photons.)

Radiative Return

Similar methods can be applied to calculate Radiative return processes – in studies of both hadronic and electroweak physics.

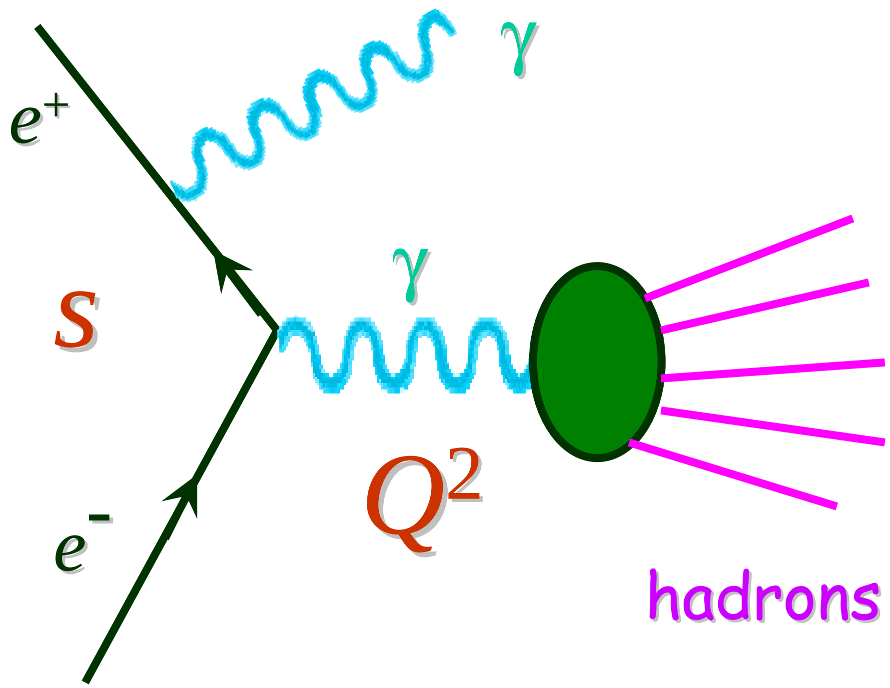
Radiating a photon from the initial state makes a convenient way to study low energy final states in a high-energy collider.

A wide range of energies can be studied by varying the energy of the radiated photon.



Radiative Return

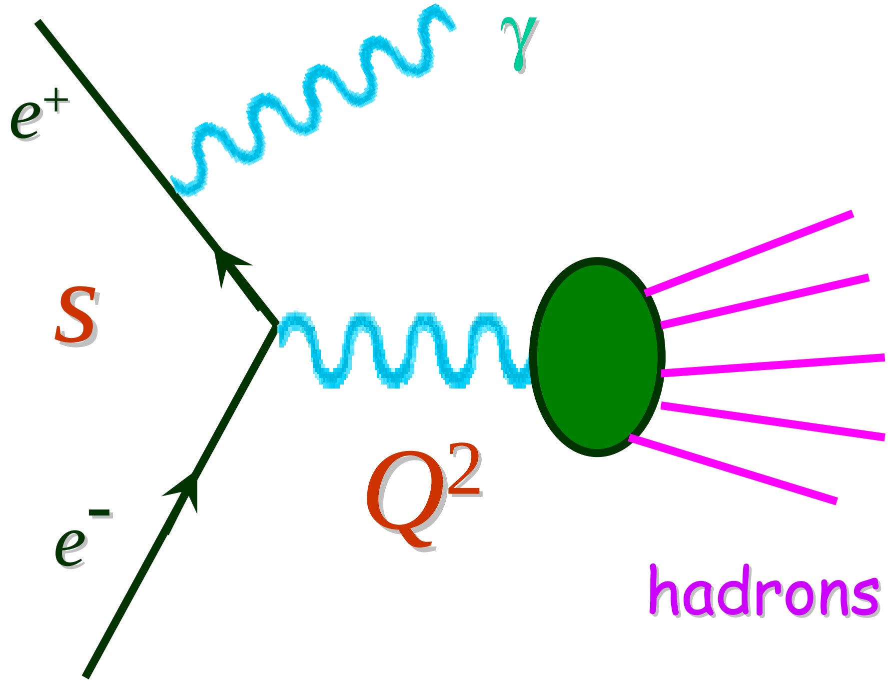
At high energies (10 GeV, for example) this is a two-step process with a clear separation between a photonic process and a hadronic process at a lower effective energy scale.



Radiative Return

This technique can be used for studying low energy hadronic physics when most of the energy is carried off by the photon, leaving

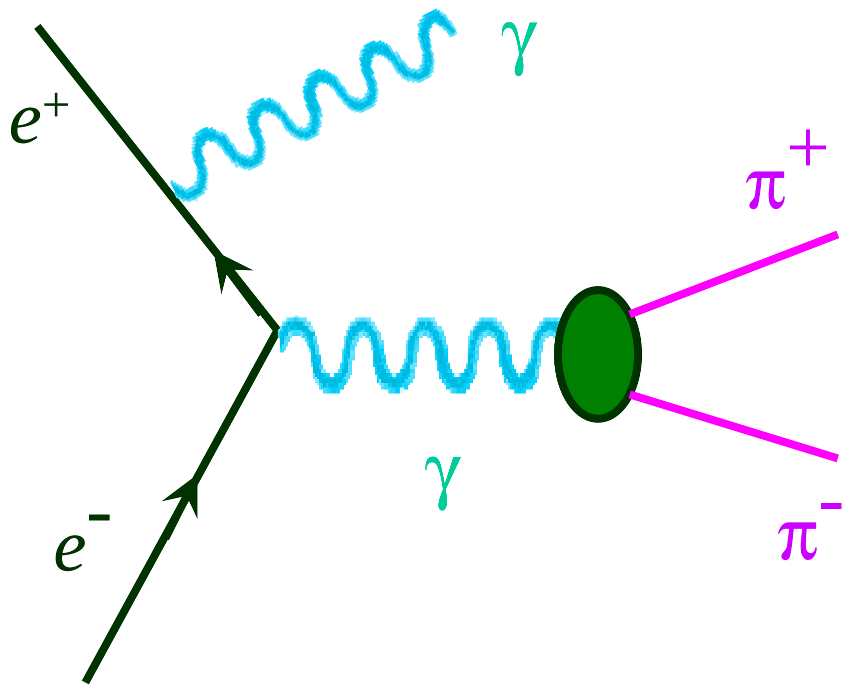
$Q^2 << s$



Radiative Return

One application of this method is to measure the pion form factor.

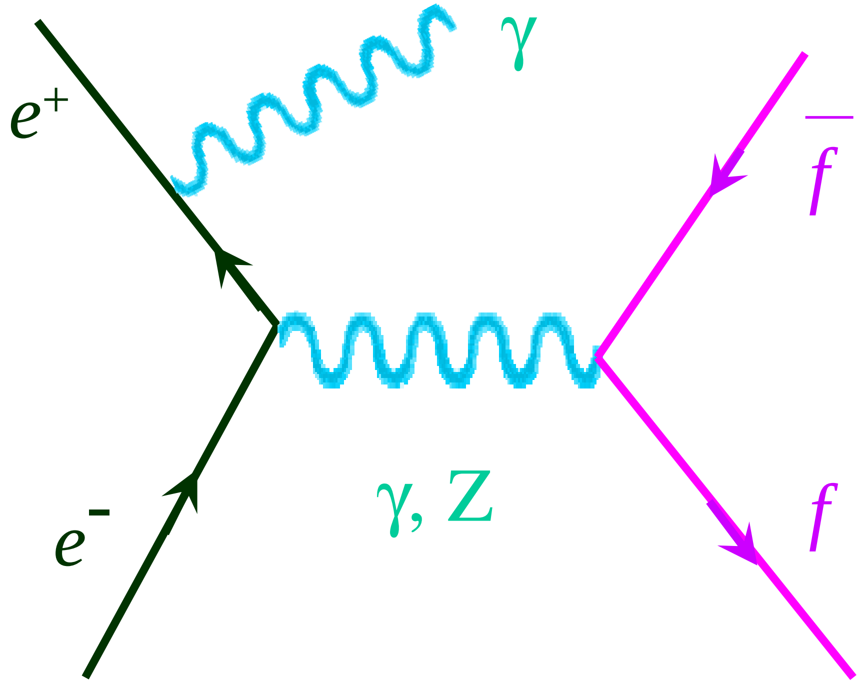
J. Kuhn *et al* have developed a MC program to calculate this process for Φ and B factories.



KK MC

Radiative return can also be used to study electroweak physics at a variety of energies.

Radiating an initial state photon in a high-energy collider can be used to study the Z resonance region, for example.



PHOKHARA MC

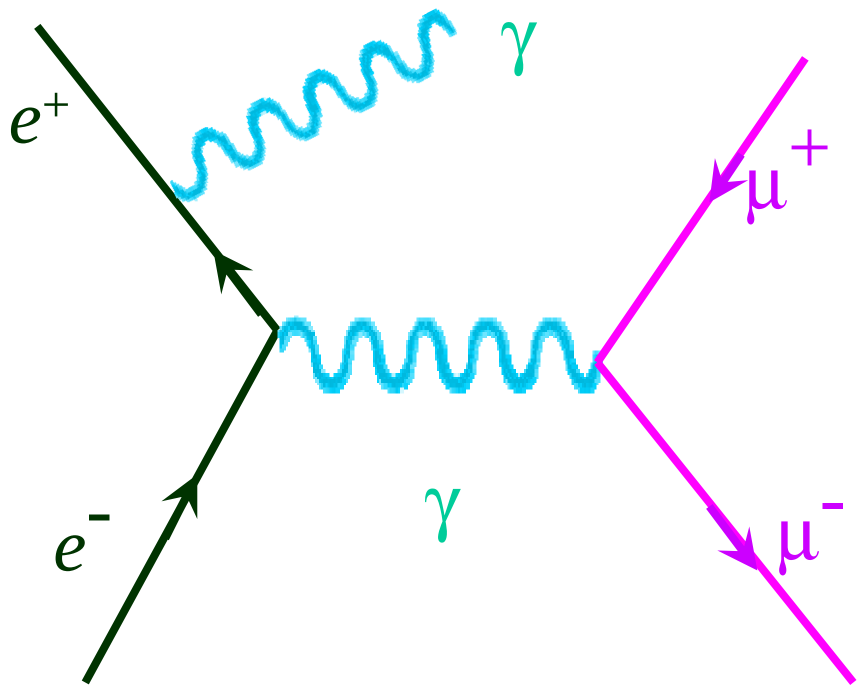
J. Kuhn *et al.*

This program is called
PHOKHARA

PHOtons from
Karlsruhe
HAdronically
RAdiated

PHOKHARA MC

PHOKHARA also calculates leptonic final states ($\mu^+ \mu^-$) and includes $O(\alpha)$ real and virtual radiative corrections to this basic process, both in the initial and final states.

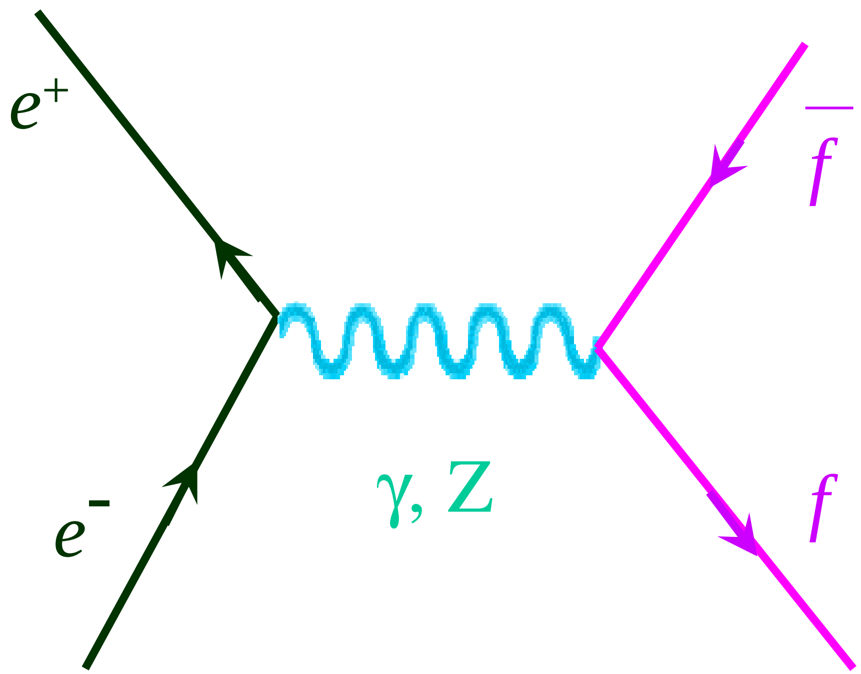


KK MC

These radiative corrections were also calculated earlier for the KK Monte Carlo of S. Jadach *et al* which was designed to calculate

$$e^+ e^- \longrightarrow f\bar{f}$$

including radiative corrections at the same order.



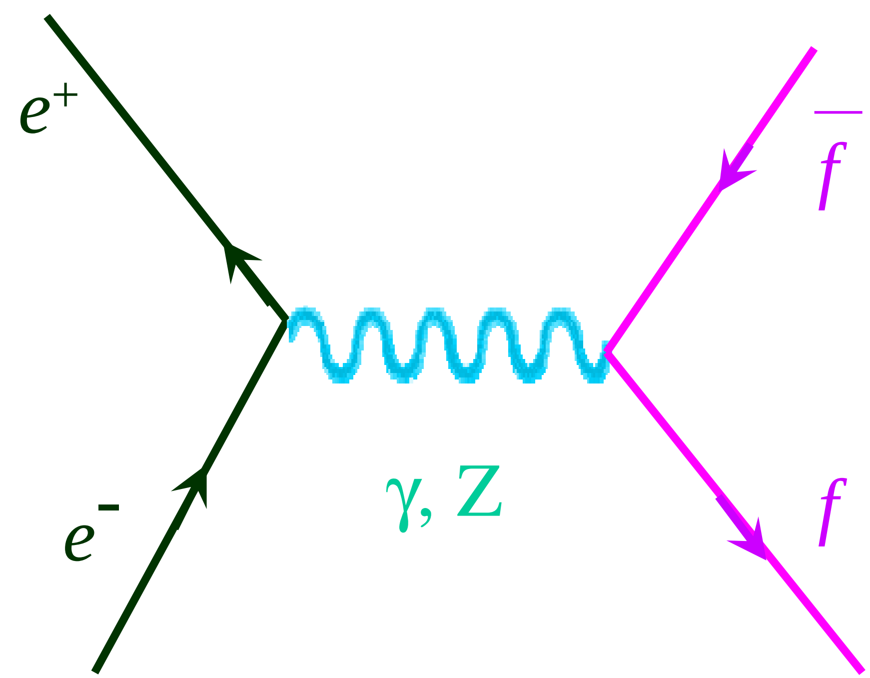
KK MC

The KK MC includes initial state photon radiation, and all other two-photon (real or virtual) to the basic process.

It also includes YFS exponentiation, and the effects of Z boson exchange

KK MC

The KKMC was designed for LEP and LEP2 physics, so the energies are higher (up to 200 GeV), but the relevant QED radiative corrections are the same, so we can compare the radiative corrections calculated for each.



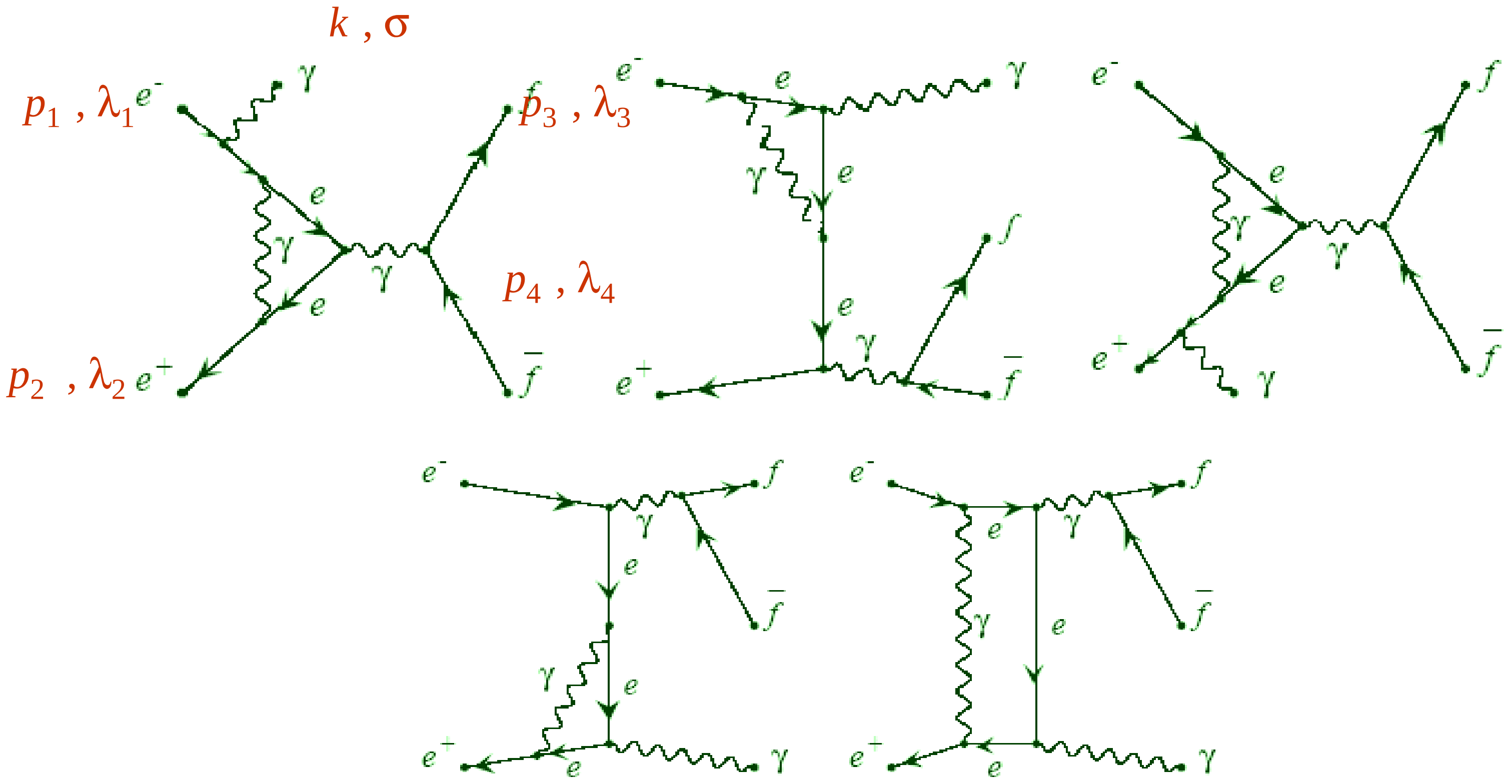
Comparison of Radiative Corrections

Since the initial state radiation is calculated exactly at order α^2 in both PHOKHARA and KK, and since both expressions are available analytically, it is useful to know how the expressions compare.

Agreement is not obvious from examining the expressions – some work is required to compare them.

Radiative Corrections to Fermion Pair Production

For ISR, the relevant radiative corrections include these -



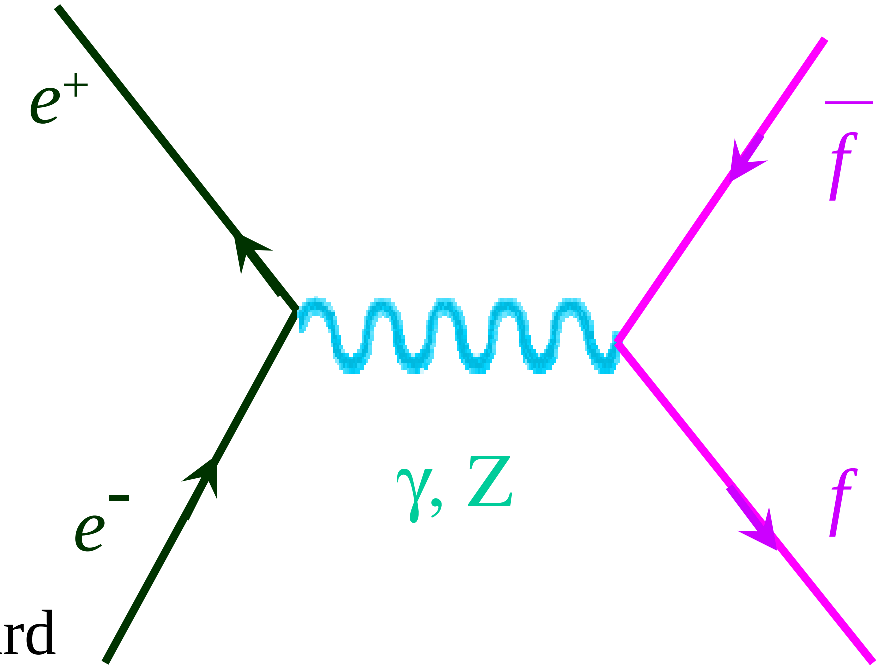
Radiative Corrections for Fermion Production

Virtual photon corrections to the process

$$e^+ e^- \longrightarrow f\bar{f} + \gamma$$

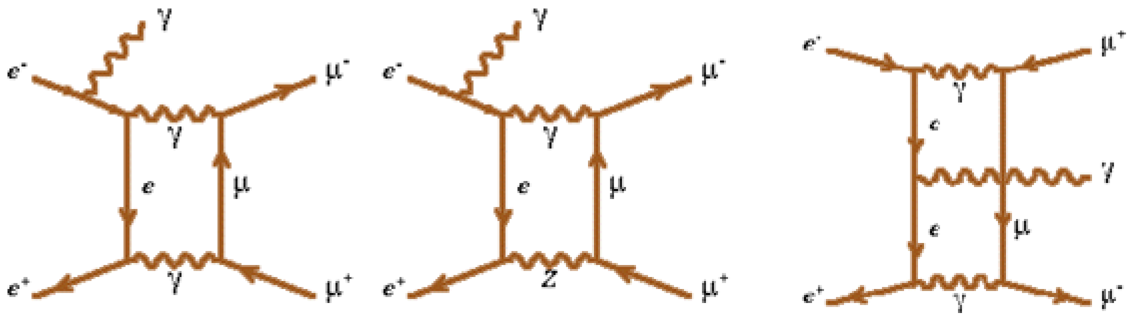
were presented in

S. Jadach, M. Melles, B.F.L. Ward and S. Yost, *Phys. Rev.* **D65**, 073030 (2002)



Assumptions Made In The Calculation

A small angle approximation was made, which suppresses initial-final state interference terms, including the box diagrams of the type shown below. The addition of such diagrams is in progress with S. Majhi, since they will be useful if the proposed NLC is built.



The lightness of the fermions permits masses to be neglected except when the multiply denominators which vanish in collinear limits. This was used to eliminate many very small mass terms from the calculation.

The Complete Result

The differential cross section for initial state radiation (ISR) can be expressed as

$$\frac{d\sigma_1^{\mathrm{ISR}(1)}}{d^2\Omega dr_1 dr_2} = \frac{1}{(4\pi)^4 s'} \sum_{\lambda_i,\sigma} \mathrm{Re}[(\mathcal{M}_1^{\mathrm{ISR}(0)})^* \mathcal{M}_1^{\mathrm{ISR}(1)}].$$

$r_i = 2\, p_i{\cdot}k$

and the differential cross section with virtual corrections can be expressed as

$$\frac{d\sigma_1^{\mathrm{ISR}(0)}}{d^2\Omega dr_1 dr_2} = \frac{1}{2(4\pi)^4 s'} \sum_{\mathrm{helicities}} \left|\mathcal{M}_1^{\mathrm{ISR}(0)}\right|^2$$

$$\left|\mathcal{M}_1^{\mathrm{ISR}(0)}\right|^2 = \frac{e^6}{s^2 r_1 r_2}(t_1^2 + t_2^2 + u_1^2 + u_2^2) + \text{mass corrections}$$

The Complete Result

The complete amplitude with virtual corrections can be expressed in terms of the amplitude without virtual corrections (pure hard Bremsstrahlung) as

$$\mathcal{M}_1^{\mathrm{ISR}(1)} = \frac{\alpha}{4\pi}(f_0 + f_1 I_1 + f_2 I_2)\mathcal{M}_1^{\mathrm{ISR}(0)}$$

where I_1 and I_2 are spinor factors which vanish in the collinear limits $p_i{\cdot}k = 0$:

Spinor product: $|s_\lambda(p,k)|^2 = 2p\cdot k$

$$\begin{aligned} I_1 &= \sigma\lambda_3 s_{\lambda_1}(p_1,k)s_{-\lambda_1}(p_2,k) \\ &\quad\times\frac{s_{\lambda_3}(p_4,p_2)s_{-\lambda_3}(p_2,p_3) - s_{\lambda_3}(p_4,p_1)s_{-\lambda_3}(p_1,p_3)}{s_{-\sigma}(p_1,p_2)s_{-\sigma}(p_3,p_4)s_\sigma^2(p_{21},p_{34})}, \\ I_2 &= \lambda_1\lambda_3\frac{s_{\lambda_1}(p_1,k)s_{-\lambda_1}(p_2,k)s_{\lambda_3}(p_4,k)s_{-\lambda_3}(p_3,k)}{s_{-\sigma}(p_1,p_2)s_{-\sigma}(p_3,p_4)s_\sigma^2(p_{21},p_{34})}, \end{aligned}$$

with $p_{ij} = p_i$ or p_j when $\sigma = \lambda_i$ or λ_j.

Form Factors

For helicities $\sigma = \lambda_1$. Otherwise interchange r_1 and r_2: (mass terms added later)

$$
\begin{aligned}
f_0 &= 2\left\{\ln\left(\frac{s}{m_e^2}\right) - 1 - i\pi\right\} + \frac{r_2}{1-r_2} + \frac{r_2(2+r_1)}{(1-r_1)(1-r_2)}\left\{\ln\left(\frac{r_2}{z}\right) + i\pi\right\} \\
&- \left\{3v + \frac{2r_2}{1-r_2}\right\}\mathrm{Lf}_1(-v) + \frac{v}{(1-r_2)} R_1(r_1,r_2) + r_2\, R_1(r_2,r_1),
\end{aligned}
$$

$$
\begin{aligned}
f_1 &= \frac{r_1 - r_2}{2(1-r_1)(1-r_2)} + \frac{z(1+z)}{2(1-r_1)^2(1-r_2)}\left\{\ln\left(\frac{r_2}{z}\right) + i\pi\right\} \\
&+ \frac{z}{1-r_2}\left\{\frac{1}{2}R_1(r_1,r_2) + r_2 R_2(r_1,r_2)\right\} + \frac{v}{4}\{R_1(r_1,r_2)\delta_{\sigma,1} + R_1(r_2,r_1)\delta_{\sigma,-1}\}
\end{aligned}
$$

$$
\begin{aligned}
f_2 &= 2 - \frac{1+z}{2(1-r_1)(1-r_2)} + \frac{z(r_2-r_1)}{2(1-r_1)^2(1-r_2)}\left\{\ln\left(\frac{r_2}{z}\right) + i\pi\right\} \\
&+ 2z\,\mathrm{Lf}_2(-v) + \frac{z}{1-r_2}\left\{\frac{1}{2}R_1(r_1,r_2) + (2-r_2)R_2(r_1,r_2)\right\} \\
&+ \frac{r_1 - r_2}{4}\{R_1(r_1,r_2)\delta_{\sigma,1} + R_1(r_2,r_1)\delta_{\sigma,-1}\}
\end{aligned}
$$

$r_i = 2\, p_i{\cdot}k$

$v = r_1 + r_2$

$z = 1 - v$

Special Functions

The form factors depend on the following special functions, designed to be stable in the collinear limits (small r_i) which are most important in the cross section.

$$\begin{aligned}
R_1(x,y) &= \mathrm{Lf}_1(-x)\left\{\ln\left(\frac{1-x}{y^2}\right) - 2\pi i\right\} \\
&+ \frac{2(1-x-y)}{1-x}\,\mathrm{Sf}_1\left(\frac{y}{1-x}, \frac{x(1-x-y)}{1-x}\right), \\
R_2(x,y) &= z + \frac{1}{1-x}\left\{\ln\left(\frac{y}{1-x-y}\right) + i\pi\right\} \\
&+ \mathrm{Lf}_2(-x)(\ln y + i\pi) - (1-x-y)\,\mathrm{Lf}_1(-x) - \frac{1}{2}\,\mathrm{Lf}_1^2(-x) \\
&+ \frac{1-x-y}{(x+y)(1-x)}\left\{x\,\mathrm{Lf}_1\left(\frac{-y}{1-x}\right) - y\,\mathrm{Lf}_2\left(\frac{-y}{1-x}\right)\right\} \\
&+ \left(\frac{1-x-y}{1-x}\right)^2 \mathrm{Sf}_2\left(\frac{y}{1-x}, \frac{x(1-x-y)}{1-x}\right),
\end{aligned}$$

with

$$\mathrm{Lf}_0(x) = \ln(1-x), \qquad \mathrm{Lf}_{n+1}(x) = \frac{1}{x}\left(\mathrm{Lf}_n(x) - \mathrm{Lf}_n(0)\right),$$

$$\mathrm{Sf}_0(x,y) = \mathrm{Sp}(x+y), \qquad \mathrm{Sf}_{n+1}(x,y) = \frac{1}{y}\left(\mathrm{Sf}_n(x,y) - \mathrm{Sf}_n(x,0)\right)$$

Dilogarithm(Spence function)

Next to Leading Log Approximation

For Monte Carlo use, it is desirable to have the shortest possible equation with sufficient accuracy. For most of the range of hard photon energies, the leading log (LL) and next to leading log (NLL) contributions suffice. These include all terms important in the collinear limits (r_i small).

- To NLL order, the spinor terms I_1 and I_2 can be dropped, since f_1 and f_2 are at most logarithmically divergent for small r_i.
- The spin-averaged collinear limit of f_0 is ...

$$\begin{aligned}\langle f_0\rangle^{\rm NLL} &= 2\{L-1\} + \frac{r_1(1-r_1)}{1+(1-r_1)^2} + \frac{r_2(1-r_2)}{1+(1-r_2)^2} + 2\ln r_1 \ln(1-r_2) \\ &+ 2\ln r_2 \ln(1-r_1) - \ln^2(1-r_1) - \ln^2(1-r_2) + 3\ln(1-r_1) \\ &+ 3\ln(1-r_2) + 2\,{\rm Sp}(r_1) + 2\,{\rm Sp}(r_2) + \langle f_0\rangle_m^{\rm NLL}\end{aligned}$$

big log $L = \ln\left(\frac{s}{m_e^2}\right)$

mass corrections

Mass Corrections

Mass corrections were added following Berends, *et al* (CALCUL collaboration). We checked that all significant mass corrections are obtained in this manner.

The most important corrections for a photon with momentum k radiated collinearly with each incoming fermion line p_1 and p_2 are added via the prescription

$$\left|\mathcal{M}_{1\gamma}^{(m)}\right|^2 = -\sum_i \frac{e^2 m_e^2}{p \cdot k} \left|\mathcal{M}_{\mathrm{Born}}(p_i - k)\right|^2$$

At the cross-section level, the net effect is that the spin-averaged form factor f_0 receives an additional mass term

$$\begin{aligned} \langle f_0 \rangle^m &= \frac{2m_e^2}{s} \left(\frac{r_1}{r_2} + \frac{r_2}{r_1} \right) \frac{z}{(1-r_1)^2 + (1-r_2)^2} \\ &\times \left\{ \langle f_0 \rangle + \ln\left(\frac{s}{m_e^2}\right)(\ln z - 1) - \frac{3}{2}\ln z + \frac{1}{2}\ln^2 z + 1 \right\} \end{aligned}$$

YFS Residuals

The Monte Carlo program will calculate YFS residuals, which are obtained by subtracting the YFS factors containing the infrared singularities. This amounts to subtracting a term

$$4\pi B_{\rm YFS}(s,m) = \left(4\ln\frac{m_0}{m}+1\right)\left(\ln\frac{s}{m^2}-1-i\pi\right)-\ln^2\left(\frac{s}{m^2}\right)-1+\frac{4\pi^2}{3}+i\pi\left(2\ln\frac{s}{m^2}-1\right)$$

from the form factor f_0 . At NLL order, we would have

$$\overline{\beta}_1^{(2)} = \overline{\beta}_1^{(1)}\left(1+\frac{\alpha}{2\pi}\langle f_0\rangle^{\rm NLL}\right)$$

In our comparisons we will actually subtract this NLL term and look at the NNLL behavior of each expression.

Comparisons

IN Igarashi and Nakazawa, *Nucl. Phys.* **B288** (1987) 301

- spin-averaged cross section, fully differential in r_1 and r_2, no mass corrections

BVNB Berends, Van Neerven and Burgers, *Nucl. Phys.* **B297** (1988) 429

- spin-averaged cross section, differential only in $v = r_1 + r_2$, includes mass corrections

KR Kuhn and Rodrigo, *Eur. Phys. J.* **C25** (2002) 215

- spin-averaged Leptonic tensor, fully differential in r_1 and r_2, includes mass corrections

The **KR** comparison is new, and closest to our calculation in its assumptions.

The New Comparison

The new comparison is to the leptonic tensor of Kuhn and Rodrigo, which was constructed for radiative return in hadron production, but can be adapted to fermion pairs by changing the final state tensor. The ISR result is expressed as a leptonic tensor

$$
\begin{aligned}
L_0^{\mu\nu} \quad = \quad & \frac{e^6}{ss'^2}\{a_{00}s\eta^{\mu\nu} + a_{11}p_1^{\mu}p_1^{\nu} + a_{22}p_2^{\mu}p_2^{\nu} + a_{22}(p_1^{\mu}p_2^{\nu} + p_1^{\nu}p_2^{\mu}) \\
& + i\pi a_{-1}(p_1^{\mu}p_2^{\nu} + p_1^{\nu}p_2^{\mu})\}
\end{aligned}
$$

which can be contracted with a final state tensor to get the squared matrix element:

$$
H^{\mu\nu} = e^2(p_3^{\mu}p_4^{\nu} + p_3^{\nu}p_4^{\mu} - p_3 \cdot p_4\eta^{\mu\nu})
$$

$$
|\mathcal{M}^{\rm ISR}|^2 = zL_0^{\mu\nu}H_{\mu\nu}.
$$

The New Comparison

The coefficients in the leptonic tensor can be separated into a tree-level term and a one-loop correction,

$$a_{ij} = a_{ij}^{(0)} + \frac{\alpha}{\pi} a_{ij}^{(1)}$$

An infrared term is subtracted to get a finite result: $c_{ij} = a_{ij}^{(1)} - a_{ij}^{\text{IR}}$

with $$a_{ij}^{\text{IR}} = a_{ij}^{(0)} \left[2(L-1)\ln v_{\min} + \frac{1}{2}L - 1 + \frac{\pi^2}{3} \right]$$

cutoff on $v = 2E_\gamma/\sqrt{s}$

Calculating the coefficients requires considerable attention to numerical stability!

Analytical Comparison

The expression for the leptonic tensor is very different from our earlier exact result, but it is possible to show that in the massless NLL limit, they agree analytically.

The virtual correction to the YFS residual can be expressed in the NLL limit as

$$\overline{\beta}_1^{(2)} = \overline{\beta}_1^{(1)} \left(1 + \frac{\alpha}{2\pi} C_1 \right) + C_2$$

with coefficients

$$\begin{aligned} C_1 &= \frac{1}{2a_{00}^{(0)}} \left(\frac{c_{11}}{z} + z c_{22} - 2c_{12} \right), \\ C_2 &= \frac{c_{11}}{4z} + \frac{z c_{22}}{4} - \frac{c_{12}}{2} - c_{00}, \end{aligned}$$

We find that

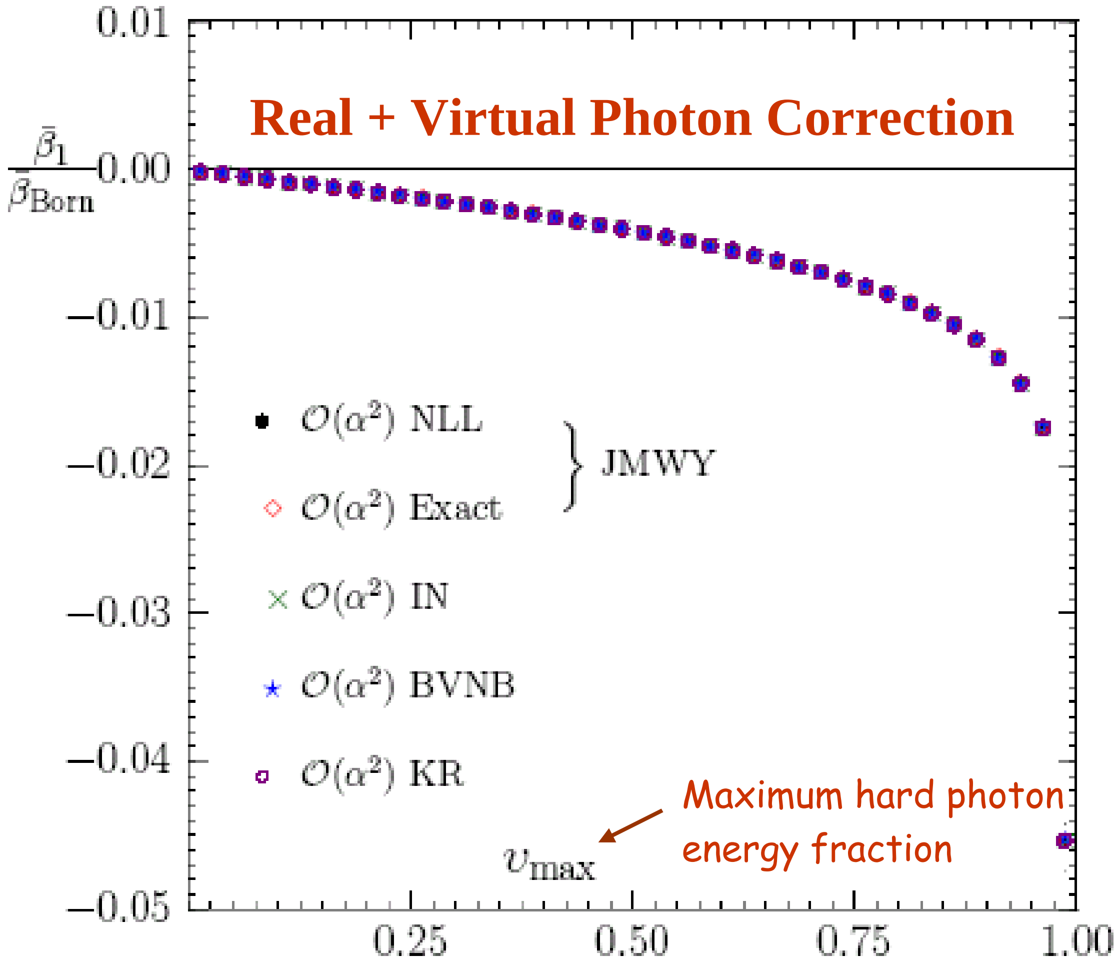
$$C_1 = \langle f_0 \rangle^{\mathrm{NLL}}$$

Monte Carlo Results

Results of a KK Monte Carlo run with 10^8 events at E_{CMS} = 200 GeV.

This figure shows the complete real + virtual photon radiative correctionto muon pair production.

The standard YFS infrared term $4\pi B_{YFS}$ has been subtracted to create a finite result.

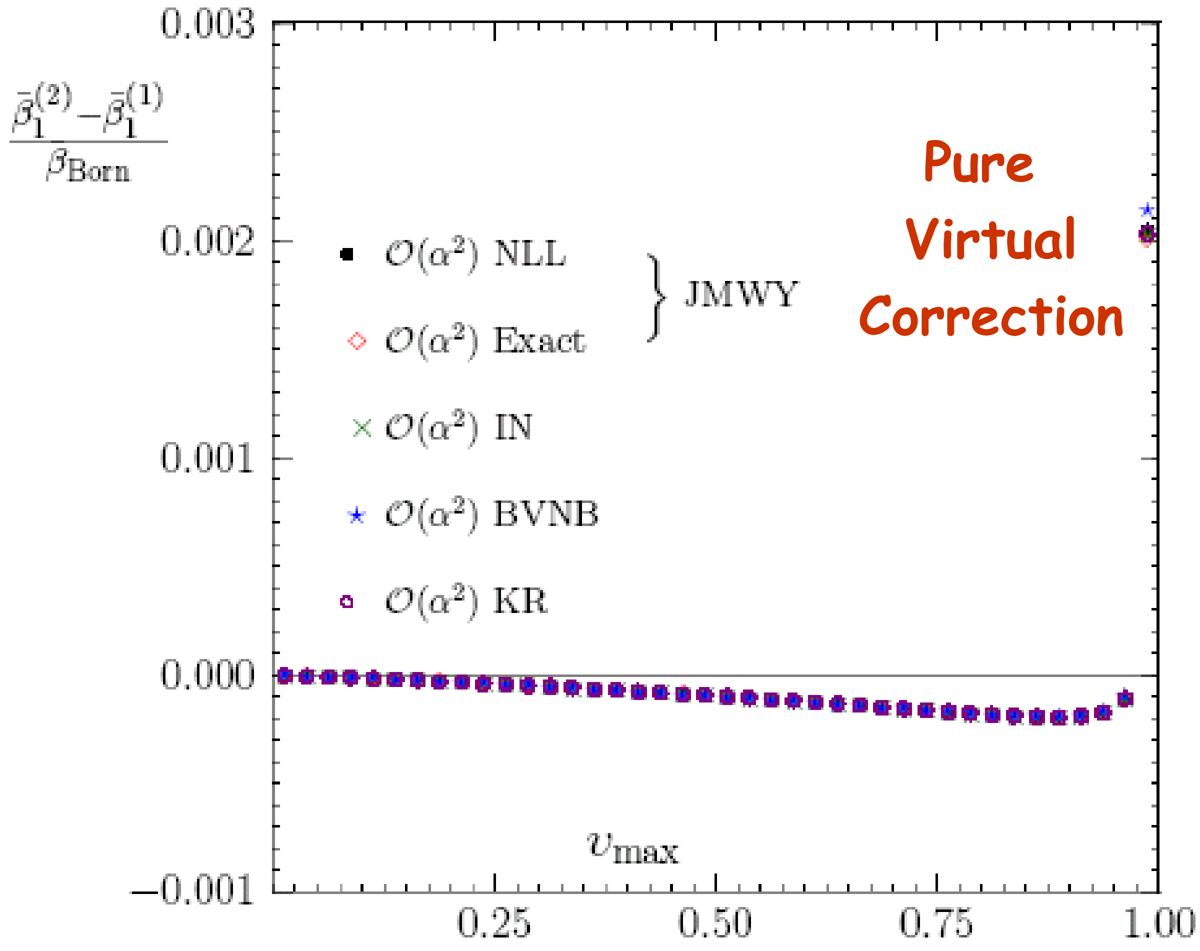


Monte Carlo Results

Results of a KK Monte Carlo run with 10^8 events at E_{CMS} = 200 GeV.

This figure shows only the pure virtual photon correction to single hard bremmstrahlung.

The standard YFS infrared term $4\pi B_{YFS}$ has been subtracted to create a finite result.

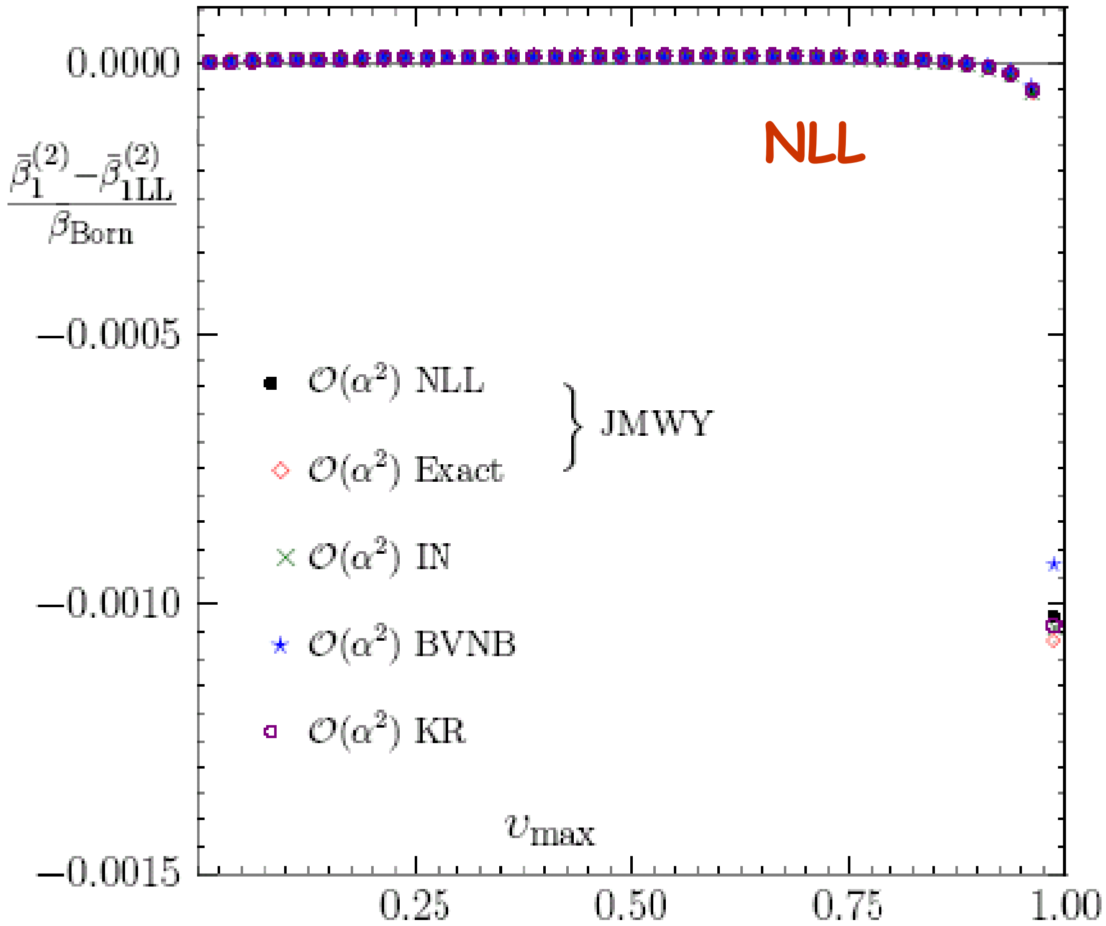


Monte Carlo Results

Results of a KK Monte Carlo run with 10^8 events at E_{CMS} = 200 GeV.

This figure shows the next to leading log (NLL) contribution to the real + virtual photon cross section.

The leading log (LL) contribution has been subtracted from each expression.

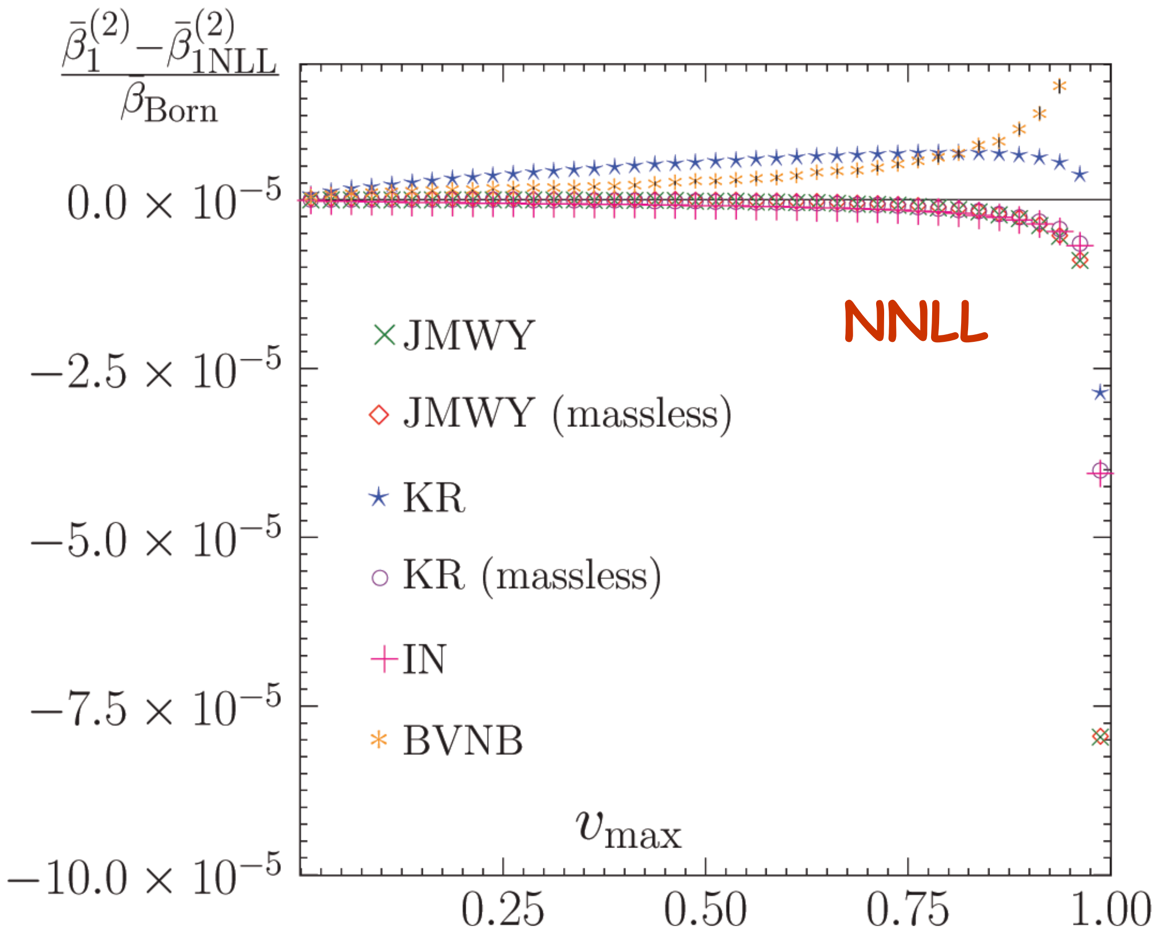


Monte Carlo Results

Results of a KK Monte Carlo run with 10^8 events at E_{CMS} = 200 GeV.

This figure shows the sub-NLL contribution to the real + virtual photon correction to muon pair production

The NLL expression of JMWY has been subtracted in each case to reveal the NNLL contributions.



Summary

- The size of the NNLL corrections for all of the compared “exact” expressions is less than $2x10^{-6}$ in units of the Born cross section for photon energy cut $v_{max} < 0.75$.

- For $v_{max} < 0.95$, all the results except BVNB agree to within $2.5x10^{-6}$ of the Born cross section.

- For the final data point, $v_{max} = 0.975$, the KR and JMWY results differ by $5x10^{-5}$ (with mass terms) or $3.5x10^{-5}$ (without mass terms) of the Born cross section.

- These comparisons show that we have a firm understanding of the precision tag for an important part of the order α^2 corrections to fermion pair production in precision studies of the final LEP2 data analysis, radiative return at Φ and B-factories, and future NLC physics.