Electron energy spectrum in muon decay

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Outline

- Introduction
- QED corrections with the logarithmic accuracy
- QED corrections beyond the logarithmic accuracy
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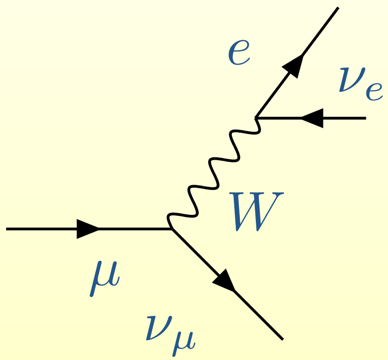
Introduction

- Muon decay to electrons and neutrinos occupies a special place in high energy physics
  - $V - A$ current; the Fermi theory;
  - Kinoshita-Lee-Nauenberg theorem was conceived as the result of explicit computation of QED radiative corrections for $\mu \to e\nu_e\nu_\mu$ in particular;
  - One of the first complete one-loop calculations in the Standard Model;
  - The Fermi coupling constant, G_F, is extracted from the muon lifetime measurements; it is an input parameter for precision electroweak fits.
- Robust experimental program with long history and established tradition.
- Recent highlights
  - Calculation of $\mathcal{O}(\alpha^2)$ QED corrections to muon life-time within the Fermi Model Stuart, van Ritbergen
  - Ongoing measurements of the muon lifetime at μLAN and FAST experiments at PSI; the expected precision is 10^{-6} which is a factor of seventeen better than the current precision.
  - Ongoing measurement of the electron energy spectrum by the TWIST collaboration. Expected to reach the precision of 10^{-4}.

Introduction

- The electron energy spectrum in muon decay offers an opportunity to test the $V - A$ structure of the charged weak current Michel, Bouchiat



- The electron energy spectrum in the decay of polarized muon is given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x\mathrm{d}\cos\theta_e} = \frac{G_F^2 m}{4\pi^3} E_{\max}^5 \beta_e x^2 \left(F(x) - P_\mu \cos\theta_e G(x)\right)$$

$$F(x) = x(1-x) + \frac{2}{9}\rho\left(4x^2 - 3x - x_0^2\right) + \eta x_0(1-x)$$

$$G(x) = \frac{1}{3}\xi\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2\delta}{3}\left(4x - 3 + \sqrt{1 - x_0^2} - 1\right)\right].$$

- $x = E_e/E_{\max}$, $E_{\max} = (m^2 + m_e^2)/(2m)$ and ρ, η, ξ, δ are Michel parameters.

Introduction

- In the Standard Model , Michel parameters

$$\rho = \frac{3}{4}, \quad \eta = 0, \quad \xi = 1, \quad \delta = \frac{3}{4}.$$

  The direct consequence of the $V - A$ structure of weak interactions.

- Current measurements of the electron energy spectra imply

$$\underline{\rho = 0.751(1)} \quad \eta = -0.007(13), \quad \xi = 1.003(84), \quad \underline{\delta = 0.749(1)};;$$

  any deviation from the Standard Model values implies New Physics.

  TWIST collaboration, PDG 2004

- The standard example of New Physics is the left-right symmetric models with heavy W_R.

  Herzceg, Langacker

- If TWIST reaches the expected precision, the bound on the mass of W_R becomes competitive with CDF and D0 bounds $M \geq 500 - 600\ \mathrm{GeV}$ and the bound on the mixing angle becomes very tight $\zeta \sim 10^{-2}$.

QED corrections

- The description of the electron energy spectrum in muon decay in terms of Michel parameters is, of course, oversimplified.
- With the precision $\mathcal{O}(10^{-4})$, the QED radiative corrections have to be calculated; QED corrections change the functional form of $\mathrm{d}\Gamma/\mathrm{d}x$ and, hence, have to be subtracted before the fit to extract the Michel parameters is attempted.
- The total decay rate can be computed assuming that electron is massless; this approximation is invalid for the electron energy spectrum

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \sum_{i=0}^{\infty} \sum_{j=0}^{i} \left(\frac{\alpha}{\pi}\right)^i \ln^j \frac{m}{m_e} f_{ij}(x).$$

- Since $\ln(m/m_e) \approx 5$, radiative corrections are important

$$\frac{\alpha}{\pi} \ln \frac{m}{m_e} \approx 1.2 \times 10^{-2}.$$

Computation of $\mathcal{O}(\alpha^2)$ corrections is required for the interpretation of TWIST results.

QED corrections in the logarithmic approximation

- If we are only interested in $\mathcal{O}(\alpha^2 \ln(m/m_e))$, computations can be simplified using the so-called perturbative fragmentation function Mele and Nason
- Similar to the familiar concept of fragmentation in QCD, we may write ($z = E_j/E_{\max}$) Melnikov and Arbuzov

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \sum_j \int_x^1 \frac{\mathrm{d}z}{z} \frac{\mathrm{d}\Gamma_i^{\overline{\mathrm{MS}}}(\mu_f, z)}{\mathrm{d}z} D_i^e\left(\frac{x}{z}, \mu_f, m\right).$$

- The interpretation: “hard process” at distance scales $1/m$, followed by soft fragmentation at distances $1/m_e$. In the hard process, the dependence on the electron mass can be neglected.
- In contrast to QCD, the fragmentation function D_i^e is fully computable from first principles. Similar to QCD, the dependence on the factorization scale is governed by the QED analog of the DGLAP evolution equation

$$\frac{\mathrm{d}D_i^e(\mu_f, x)}{\mathrm{d}\ln\mu_f} = \frac{\alpha}{2\pi} \int_x^1 \frac{\mathrm{d}z}{z} P_{ij}\left(\frac{x}{z}\right) D_j^e(\mu_f, z).$$

QED corrections with the logarithmic accuracy

- In the above formalism, the logarithms of the muon to electron mass ratio reside in the fragmentation function. Choosing $\mu_f = m$,

$$D_e^e(m,x) = \delta(1-x) + \frac{\alpha}{2\pi}\left(P_{ee}(x)\ln\frac{m}{m_e} + D_e^{e,1}(x)\right) + \ldots$$

- The $\mathcal{O}(\alpha^2 \ln(m/m_e))$ corrections to the electron energy spectrum can easily be computed provided that
  1. $\frac{\mathrm{d}\Gamma_i^{\overline{\mathrm{MS}}}}{\mathrm{d}x}$ is known through $\mathcal{O}(\alpha)$ in the massless approximation ;
  2. $D_i^e(x)$ is known exactly through $\mathcal{O}(\alpha)$.
- The results in midpoint of the spectrum are
  1. $\mathcal{O}(\alpha^2 \ln^2 m/m_e)$ corrections are $\sim 7 \times 10^{-4}$

     Arbuzov, Czarnecki, Gaponenko ;
  2. $\mathcal{O}(\alpha^2 \ln m/m_e)$ corrections are $\sim -3 \times 10^{-4}$

     Melnikov, Arbuzov .
- Hence, to make full use of the TWIST precision, we require $\mathcal{O}(\alpha^2)$ corrections without the $\ln m/m_e$ enhancement.

QED corrections

- There are two ways to approach the calculation of $\mathcal{O}(\alpha^2)$ corrections;
- It is possible to generalize the method based on the fragmentation function. Then we require
  1. $\frac{d\Gamma_i^{\overline{MS}}}{dx}$ through $\mathcal{O}(\alpha^2)$ in the massless approximation
  2. $D^e(x)$ through $\mathcal{O}(\alpha^2)$ Melnikov and Mitov .
- The other option is to approach the calculation numerically, using techniques developed in the context of QCD computations for differential observables Anastasiou, Melnikov and Petriello .
- We choose to pursue the second option since
  1. It is more flexible; computing $d\Gamma/dz$ is equivalent to computing any differential distribution for $\mu \to e\nu_\mu\nu_e$;
  2. Extension of the method to massive particles, with potential applications to heavy quark decay spectra;
  3. Interesting to check if $m_e/m \approx 1/200$ is large enough, to permit a stable numerical evaluation.

Method: the basics

- Automated, numerical method for extracting and canceling the infra-red singularities.
  - The NNLO decay rate:

$$d\Gamma_{\rm NNLO} = d\Gamma_{VV} + d\Gamma_{RV} + d\Gamma_{RR}.$$

  - For each component, obtain an expansion:

$$d\Gamma_{AB} = \sum_{j=j_{\rm min}}^{j=0} \frac{M_j^{\rm AB}}{\epsilon^j},$$

    where $M_j^{\rm AB}$ are ϵ-independent and integrable throughout the phase-space.
  - $M_j^{\rm AB}$ can be computed numerically. Poles in ϵ cancel, when all $d\Gamma_{AB}$ are combined.
- In principle, the method deals with the differential cross-sections $\Rightarrow$ arbitrary cuts on the final states are allowed.
- For the purposes of this talk, I focus on $d\Gamma/dx$.

Method: the sketch of the algorithm

- The method applies to VV, RV and RR, with minimal modifications. I focus on RR since it is where the bottleneck usually is.
- The algorithm:
  - map the differential phase-space onto the unit hypercube:

$$\int \prod_i \frac{\mathrm{d}p_i^{-1}}{2p_i^0} \delta^d \left(P_{\rm in} - \sum p_i \right) .... \Rightarrow \int_0^1 \prod_j \mathrm{d}x_j x_j^{-a_j \epsilon} (1-x_j)^{-b_j \epsilon} ...$$

  - use the “sector decomposition” to disentangle overlapping singularities;
    Binoth, Heinrich , Denner, Roth.
  - use “plus”-distribution expansion for book-keeping:

$$\frac{1}{x^{1+a\epsilon}} = -\frac{1}{a\epsilon}\delta(x) + \left[\frac{1}{x}\right]_+ - a\epsilon \left[\frac{\ln x}{x}\right]_+ + ...$$

- The outcome: all the singularities from RR diagrams are extracted without a single integration.

Method: what changes with massive particles

- For massless particles, the double real emission correction is, usually, the most difficult to deal with. This changes, if massive particles are involved.
- Convenient parametrization of the phase-space is the key for the efficiency of the method; trivially achieved if fermions are massive since only soft, $\omega_\gamma \to 0$, photons contribute to the divergencies.
- With massless particles, many two-loop integrals are known analytically. With massive particles, this is not the case.
- Loops are amenable to sector decomposition.

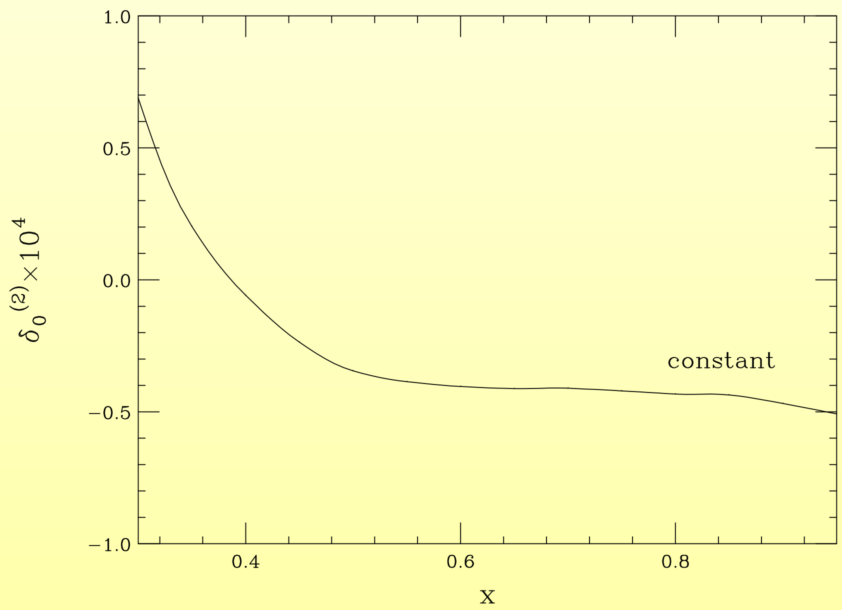
  Binoth, Heinrich , Denner, Roth.
- Dealing with loop integrals for $\mu \to e \nu_e \nu_\mu$ is, in principle, straightforward. It is possible to deal numerically with the complete diagram at once; neither the Passarino-Veltman reduction nor the reduction to master integrals is required. Not quite trivial, but can be worked out.
- We do not use the fact that $m_e \ll m$; keeping m_e arbitrary is good for checks, for example $m_e \to m$ limit can be deduced from the known $\mathcal{O}(\alpha_s^2)$ corrections to $b \to c$ transitions at zero recoil. We achieve good numerical precision for $m_e/m \approx 200$.

QED corrections: results

- We write the second order correction to the electron energy spectrum as

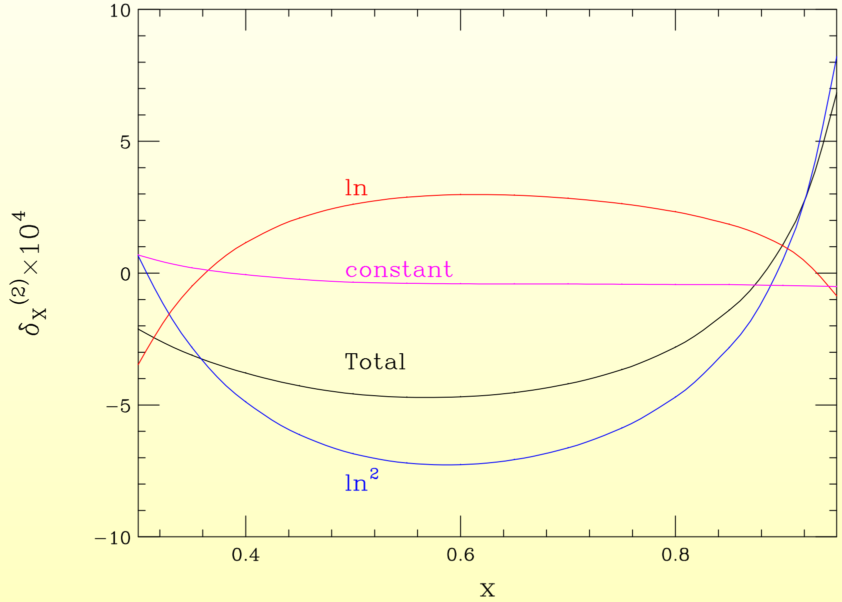
$$f^{(2)}(x) = f_2^{(2)}(x)\ln^2(m_\mu/m_e) + f_2^{(2)}(x)\ln(m_\mu/m_e) + f_0^{(2)}(x).$$

- The interesting piece is $f_0^{(2)}(x)$. Define $\delta_0^{(2)} = f_0^{(2)}(x)/f_0^{(0)}(x)$.



- QED corrections not enhanced by the logarithm of the muon to electron mass ratio are $\sim 0.5\times10^{-4}$, i.e. one half of the expected experimental precision.

QED corrections: results



- A hierarchy of $\ln^j(m/m_e),\ j = 2..0$ corrections exists, but it does not quite follow the naive expectation.
- The leading $\ln^2$-enhanced correction is good for estimates within a factor $1.5 - 2$.
- The theory uncertainty in the prediction for the electron energy spectrum is conservatively estimated to be 5×10^{-6} using $\alpha^3 \ln^3(m/m_e) \sim \text{few} \times 10^{-6}$, hadronic vacuum polarization correction and finite W mass effects.

Arbuzov, Davydichev, Schilcher, Spiesberger

Conclusions

- First calculation of the energy spectrum of any charged particle through $\mathcal{O}(\alpha^2)$; for $\mu \to e\nu_e\nu_\mu$, almost half a century after the $\mathcal{O}(\alpha)$ corrections to the electron energy spectrum were obtained.
- $\mathcal{O}(\alpha^2)$ contribution to the electron energy spectra are in the range -5 to 8×10^{-4} depending on the value of x. The largest contribution comes from the $\ln m/m_e$-enhanced corrections.
- The remaining theoretical uncertainty on the electron energy spectrum is estimated to be 5×10^{-6}, well below the requirements of the TWIST experiment.
- The calculation I just discussed applies to unpolarized muon decay only; to make full use of the TWIST measurement, the calculation has to be extended to include the muon polarization.
- The computational methods developed in the context of this calculation are applicable to decays of heavy particles (top, Higgs, bottom).