

# New Results for Higgs Boson Masses and Mixings in the r/cMSSM Higgs Sector

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Snowmass, 08/2005

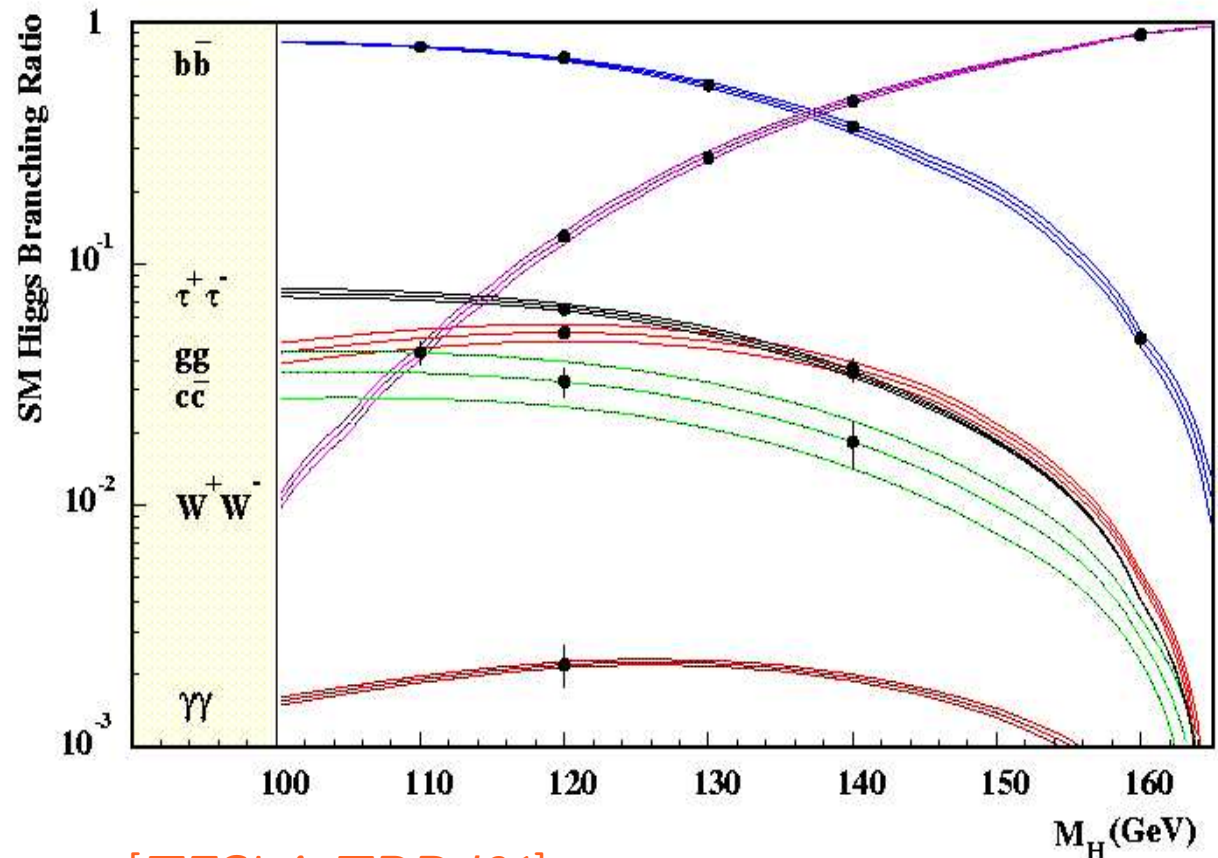
1. Motivation
2. Corrections of  $\mathcal{O}(\alpha_b\alpha_s)$  in the rMSSM
3. Corrections of  $\mathcal{O}(\alpha_t\alpha_s)$  in the cMSSM
4. Conclusions

# 1. Motivation

SM Higgs @ ILC:

Precise measurement of:

1. Higgs boson mass,  
 $\delta M_H \approx 50 \text{ MeV}$
2. Higgs boson width  
(direct/indirect)
3. Higgs boson couplings,  
 $\mathcal{O}(\text{few}\%) \Rightarrow$
4. Higgs boson quantum  
numbers: *spin*, ...



[TESLA TDR '01]

MSSM: similar precision expected (possible problems from loop corrections)

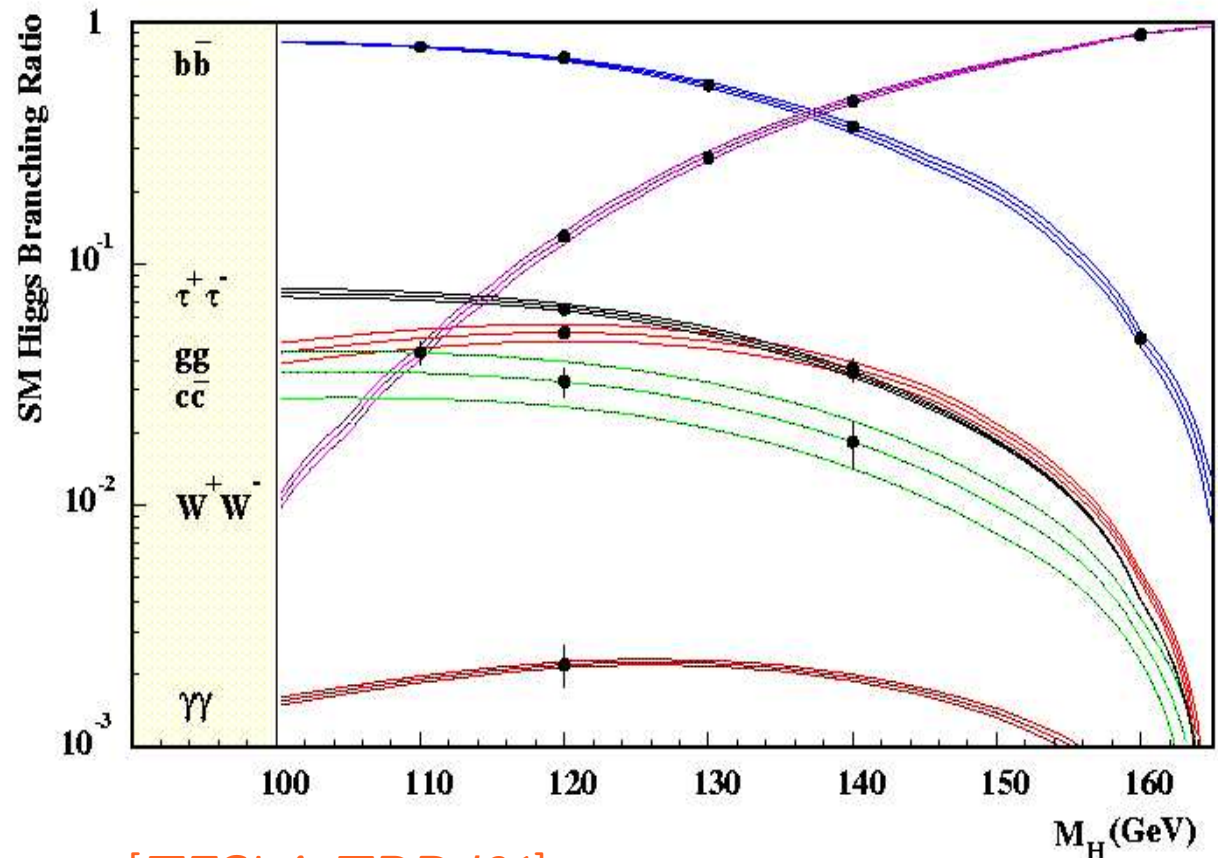
**Q:** Can this precision be utilized in the MSSM Higgs sector?

# 1. Motivation

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[TESLA TDR '01]

MSSM: similar precision expected (possible problems from loop corrections)

**Q:** Can this precision be utilized in the MSSM Higgs sector?

**A:** Yes! ... if the theory predictions are as precise

# The Minimal Supersymmetric Standard Model (MSSM)

## Superpartners for Standard Model particles

$$\begin{array}{llll} [u, d, c, s, t, b]_{L,R} & [e, \mu, \tau]_{L,R} & [\nu_{e,\mu,\tau}]_L & \text{Spin } \frac{1}{2} \\ [\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}]_{L,R} & [\tilde{e}, \tilde{\mu}, \tilde{\tau}]_{L,R} & [\tilde{\nu}_{e,\mu,\tau}]_L & \text{Spin } 0 \\ g & \underbrace{W^\pm, H^\pm}_{\text{Spin } 1} & \underbrace{\gamma, Z, H_1^0, H_2^0}_{\text{Spin } 0} & \text{Spin } 1 / \text{Spin } 0 \\ \tilde{g} & \tilde{\chi}_{1,2}^\pm & \tilde{\chi}_{1,2,3,4}^0 & \text{Spin } \frac{1}{2} \end{array}$$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

$\tilde{t}/\tilde{b}$  sector of the MSSM: (scalar partner of the top/bottom quark)

Stop, sbottom mass matrices ( $X_t = A_t - \mu^*/\tan\beta$ ,  $X_b = A_b - \mu^*\tan\beta$ ):

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_t^2 + DT_{t_1} & m_t X_t^* \\ m_t X_t & M_{\tilde{t}_R}^2 + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix}$$

$$\mathcal{M}_{\tilde{b}}^2 = \begin{pmatrix} M_{\tilde{b}_L}^2 + m_b^2 + DT_{b_1} & m_b X_b^* \\ m_b X_b & M_{\tilde{b}_R}^2 + m_b^2 + DT_{b_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{b}}} \begin{pmatrix} m_{\tilde{b}_1}^2 & 0 \\ 0 & m_{\tilde{b}_2}^2 \end{pmatrix}$$

mixing important in stop sector (also in sbottom sector for large  $\tan\beta$ )

soft SUSY-breaking parameters  $A_t, A_b$  also appear in  $\phi\text{-}\tilde{t}/\tilde{b}$  couplings

$$SU(2) \text{ relation} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$$

$\Rightarrow$  relation between  $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

## Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\ + \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

physical states:  $h^0, H^0, A^0, H^\pm$

Goldstone bosons:  $G^0, G^\pm$

Input parameters:

$$\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta)$$

## Contrary to the SM:

$m_h$  is not a free parameter

MSSM tree-level bound:  $m_h < M_Z$ , excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:

$$\Delta m_h^2 \sim G_\mu m_t^4 \ln \left( \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right)$$

The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Measurement of  $m_h$ , Higgs couplings  $\Rightarrow$  test of the theory

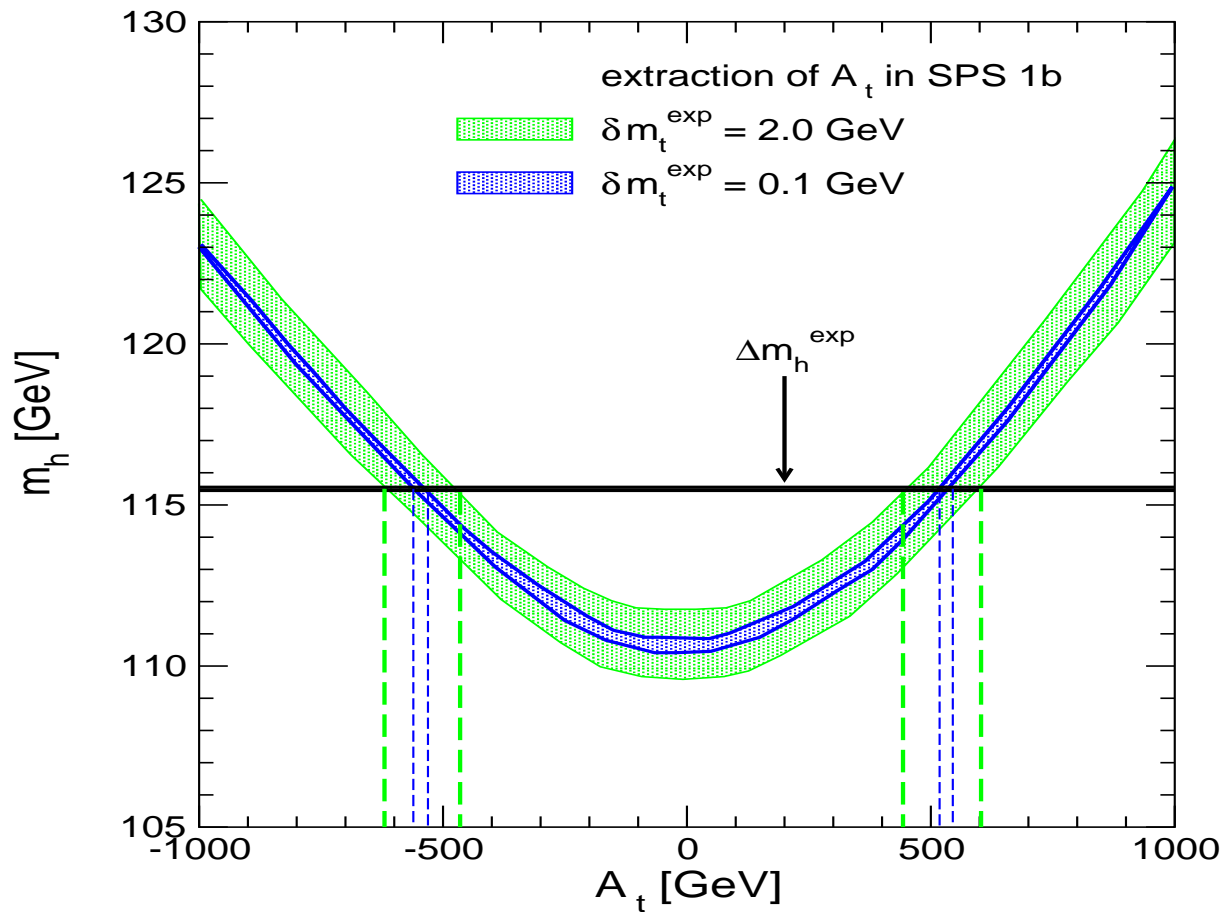
ILC:  $\Delta m_h \approx 0.05$  GeV

$\Rightarrow$  aim for theoretical precision!

( $\Rightarrow m_h$  will be (the best?) electroweak precision observable)

Example of application:  $m_h$  prediction as a function of  $A_t$

[S.H., S. Kraml, W. Porod, G. Weiglein '02]



SPS1b:

$m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$  known,

$A_t$  unknown

$\tan \beta, M_A$  known,

realistic parametric errors assumed

(from SUSY exp. errors)

$\Rightarrow$  extraction of  $A_t$  possible  
Theory error neglected

$\Rightarrow m_h$  is crucial input for SUSY fit programs (Fittino, Sfitter)



## 2. Correction of $\mathcal{O}(\alpha_b\alpha_s)$ in the rMSSM

Evaluation of Higgs boson masses in the MSSM with real parameters:

Two-point vertex function:

$$\Gamma(q^2) = \begin{pmatrix} q^2 - m_{H,\text{tree}}^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m_{h,\text{tree}}^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

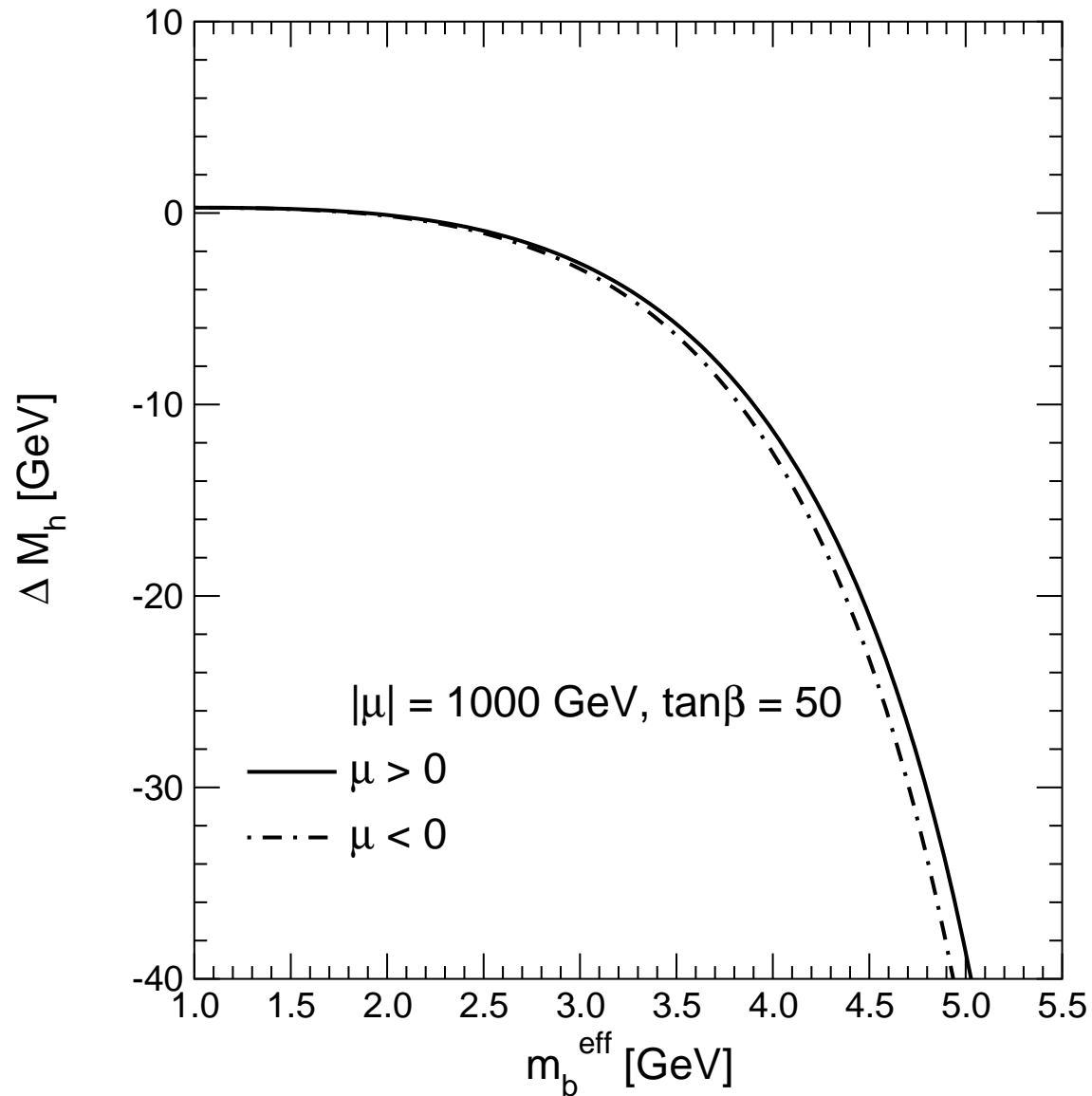
determination of  $\det(\Gamma(q^2)) = 0 \Rightarrow M_h, M_H, \alpha_{\text{eff}}, \dots$

Main task: calculation of  $\hat{\Sigma}(q^2)$ , including renormalization

Here:

- evaluation of 2-loop corrections of  $\mathcal{O}(\alpha_b\alpha_s)$
- comparison of 4 different renormalization schemes

## Motivation: Why 2-loop corrections in the $b/\tilde{b}$ sector?



1-loop corrections  $\mathcal{O}(\alpha_b)$  to  $M_h$   
can be sizable

Precise  $M_h$  prediction

$\Rightarrow$  2-loop corrections necessary

## The Higgs self-energy at 2-loop:

→  $\alpha_s$  correction to the leading 1-loop term  $\sim m_b^4$

### Approximations:

- only  $m_b^2 (\sim y_b^2)$  terms
- gauge couplings are set to 0
- external momentum is set to 0

$$\Rightarrow \widehat{\Sigma}_{22}^{(2)}(q^2) \approx \Sigma_{22}^{(2)}(0) + \cos^2 \beta \delta M_A^2(2) - \frac{e}{2 M_W s_W} \left( \sin^2 \beta \cos \beta \delta t_1^{(2)} - \sin \beta (1 + \cos^2 \beta) \delta t_2^{(2)} \right)$$

in the  $\phi_1 \phi_2$  basis with

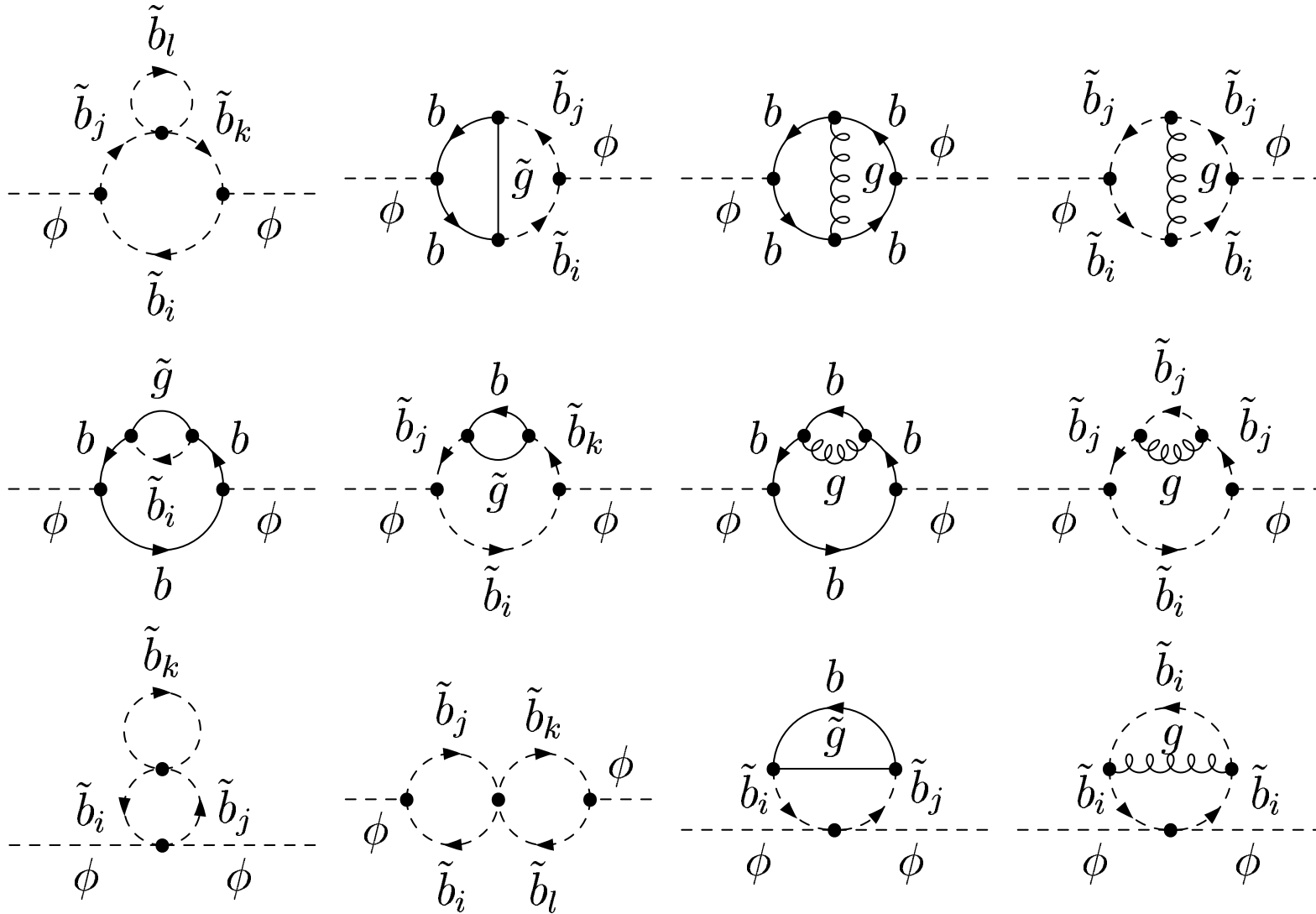
$\Sigma_{22}^{(2)}(0)$  : unrenormalized 2-2 self-energy

$\delta M_A^2(2) = \Sigma_A^{(2)}(0)$ : A mass counter term

$\delta t_i^{(2)} = -T_i^{(2)}$ :  $\phi_i$  tad-pole

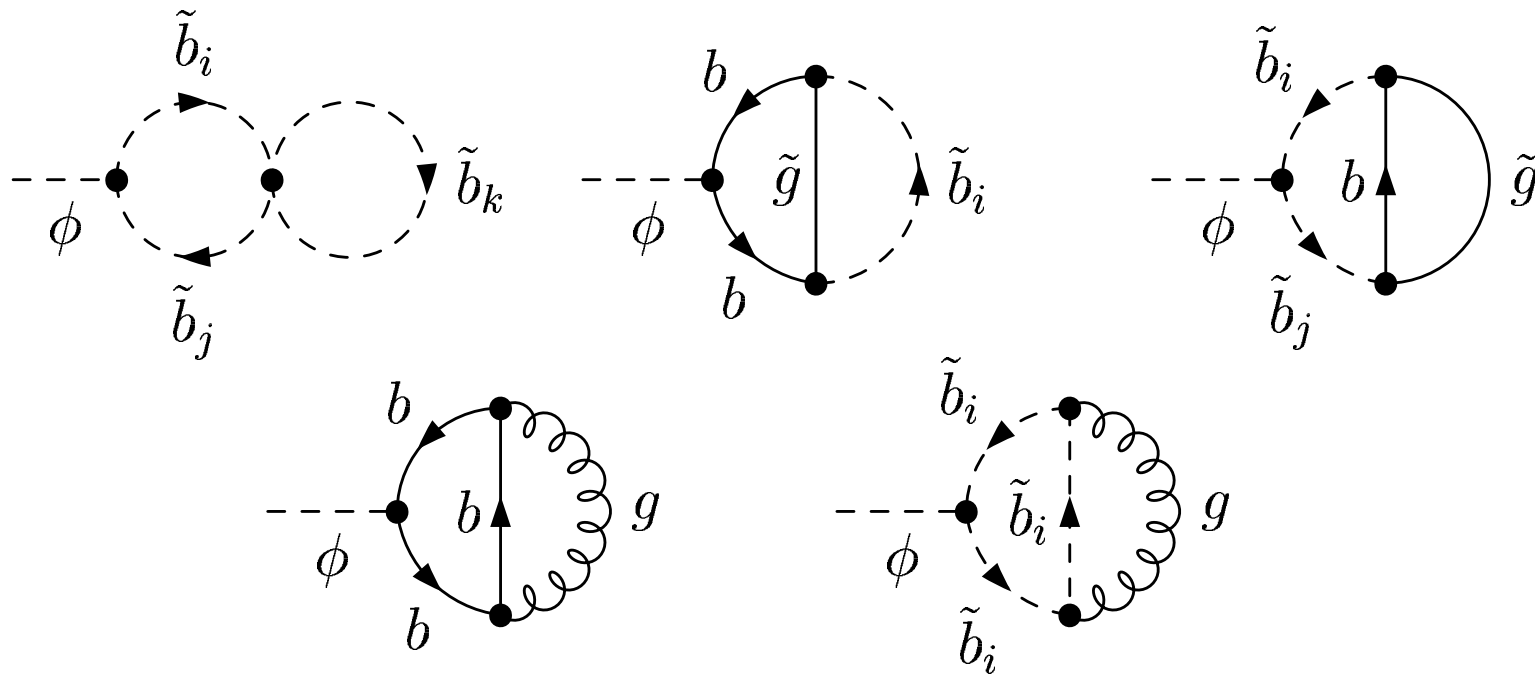
# Contributions to the 2-loop self-energy:

## 2-loop self-energy diagrams:



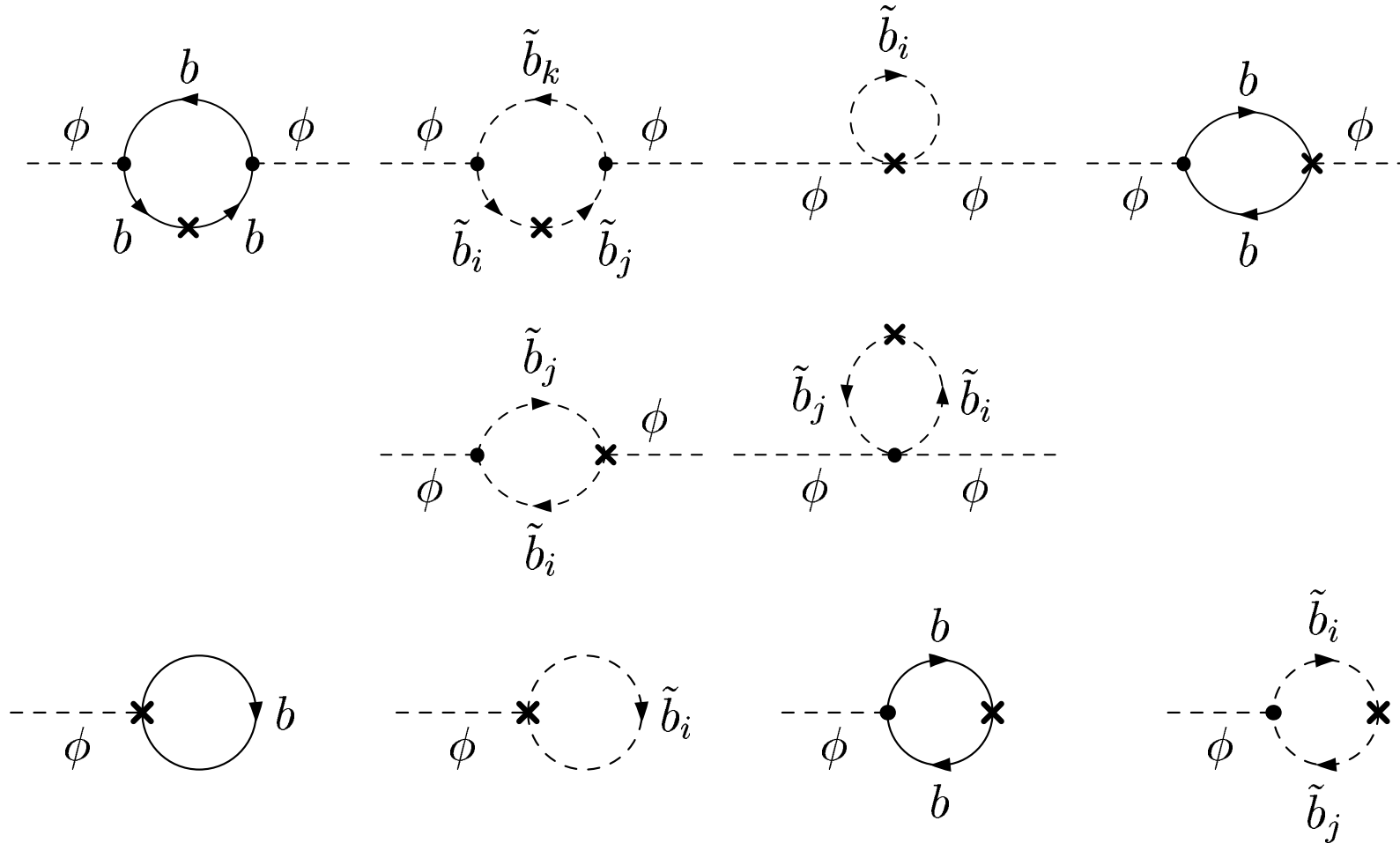
## Contributions to the 2-loop self-energy:

2-loop tad-pole diagrams:



## Contributions to the 2-loop self-energy:

diagrams with counter term insertion:



→ different renormalization schemes enter

## Renormalization:

Calculation of two-loop corrections of  $\mathcal{O}(\alpha_t\alpha_s)$  and  $\mathcal{O}(\alpha_b\alpha_s)$

⇒ parameters of the  $t/\tilde{t}$  and  $b/\tilde{b}$  are defined at the 1-loop level

⇒ different choices of renormalization possible

$t/\tilde{t}$ sector:	one renormalization scheme: 4 independent parameters: $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_t$ on-shell → $A_t$ given in terms of the others
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$b/\tilde{b}$  sector: four schemes analyzed

## Investigation of scheme dependence:

⇒ information about size of missing higher order corrections

⇒ estimate of theory uncertainty

## Renormalization schemes in the $b/\tilde{b}$ sector:

$$SU(2) \text{ relation} \Rightarrow M_{\tilde{t}_L} = M_{\tilde{b}_L}$$

$\Rightarrow$  out of  $m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}, A_b, m_b$  only **3 are independent**

$\Rightarrow$  two parameters (incl. CTs) are given in terms of the others

In all four schemes:  $m_{\tilde{b}_1}$  dep. ( $SU(2)$  relation),  $m_{\tilde{b}_2}$  OS

scheme	b-mass $m_b$	$A_b$	mixing angle $\theta_{\tilde{b}}$
$m_b$ $\overline{DR}$	$\overline{DR}$	$\overline{DR}$	dep.
$A_b, \theta_{\tilde{b}}$ OS	dep.	OS	OS
$A_b, \theta_{\tilde{b}}$ $\overline{DR}$	$\overline{DR}$	dep.	$\overline{DR}$
$m_b$ OS	OS	dep.	OS

$\Rightarrow$  scheme  $m_b$  OS: analogous to the  $t/\tilde{t}$  sector  
 $\rightarrow$  obvious choice ?



## Resummed bottom quark mass:

→ absorb the leading corrections in a resummed form in the bottom quark mass at the 1-loop level

$$m_b^{\overline{\text{DR}}} = \frac{\tilde{m}_b^{\text{pole}} + \sum_b^{\tan \beta \text{ non-enh.}} |_{\text{fin}}}{1 + \Delta m_b}$$

with

$$\Delta m_b = \frac{2\alpha_s}{3\pi} \tan \beta \mu m_{\tilde{g}} I(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2)$$

$$\sum_b^{\tan \beta \text{ non-enh.}} |_{\text{fin}} = \tan \beta \text{ non-enhanced terms in } \Sigma_{b,s}$$

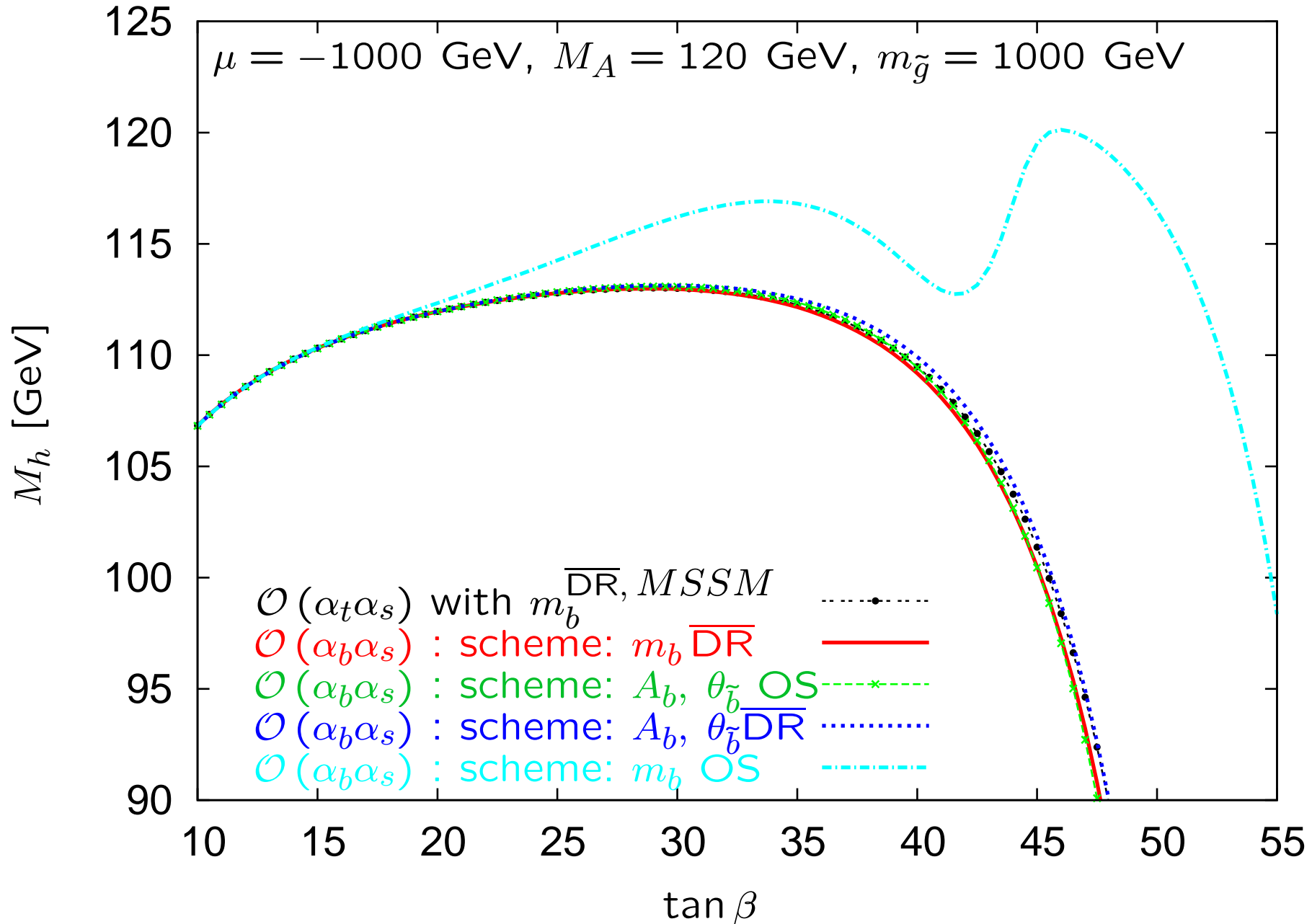
$$\tilde{m}_b^{\text{pole}} = m_b^{\overline{\text{MS}}}(M_Z) \times \left[ 1 + \frac{\alpha_s}{\pi} \left( \frac{4}{3} - \log \frac{(m_b^{\overline{\text{MS}}})^2}{M_Z^2} \right) \right]$$

“formal” pole mass obtained from the  $\overline{\text{MS}}$  mass

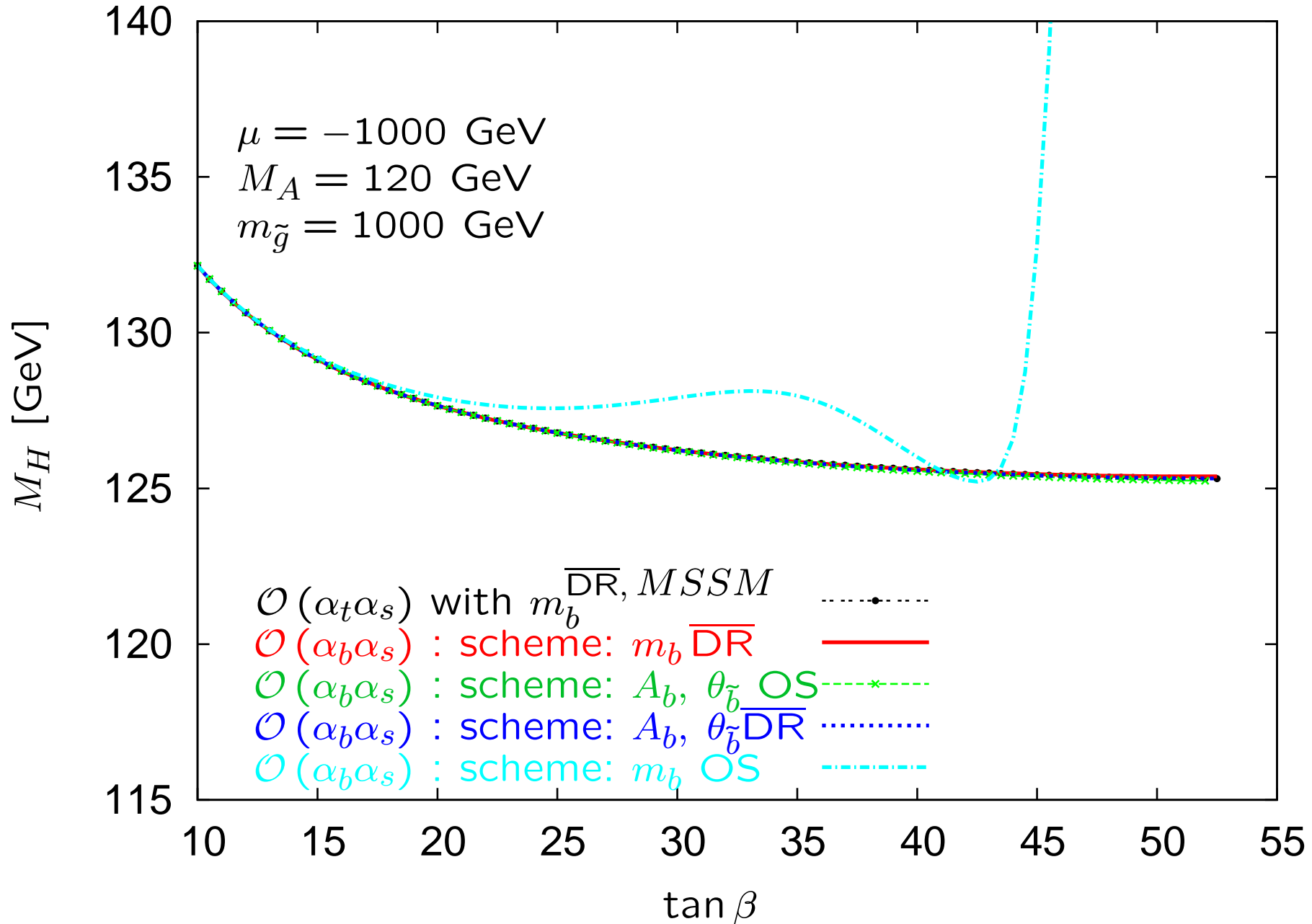
⇒ large higher-order corrections included at the 1-loop level

→ other renormalization schemes by finite shift

$M_h$  as a function of  $\tan \beta$ ,  $\mu < 0$ :



$M_H$  as a function of  $\tan \beta$ ,  $\mu < 0$ :



## Observations:

- Scheme  $m_b$  OS gives very large corrections

Reason:  $A_b$  is a dependent quantity  $\Rightarrow$  large corrections via  $\delta A_b$

$$\begin{aligned}\delta A_b &= \frac{1}{m_b} \left[ -\frac{\delta m_b}{2 m_b} (m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2) \sin 2\theta_{\tilde{b}} + \dots \right] \\ &= \frac{1}{m_b} [-\delta m_b (A_b - \mu \tan \beta) + \dots]\end{aligned}$$

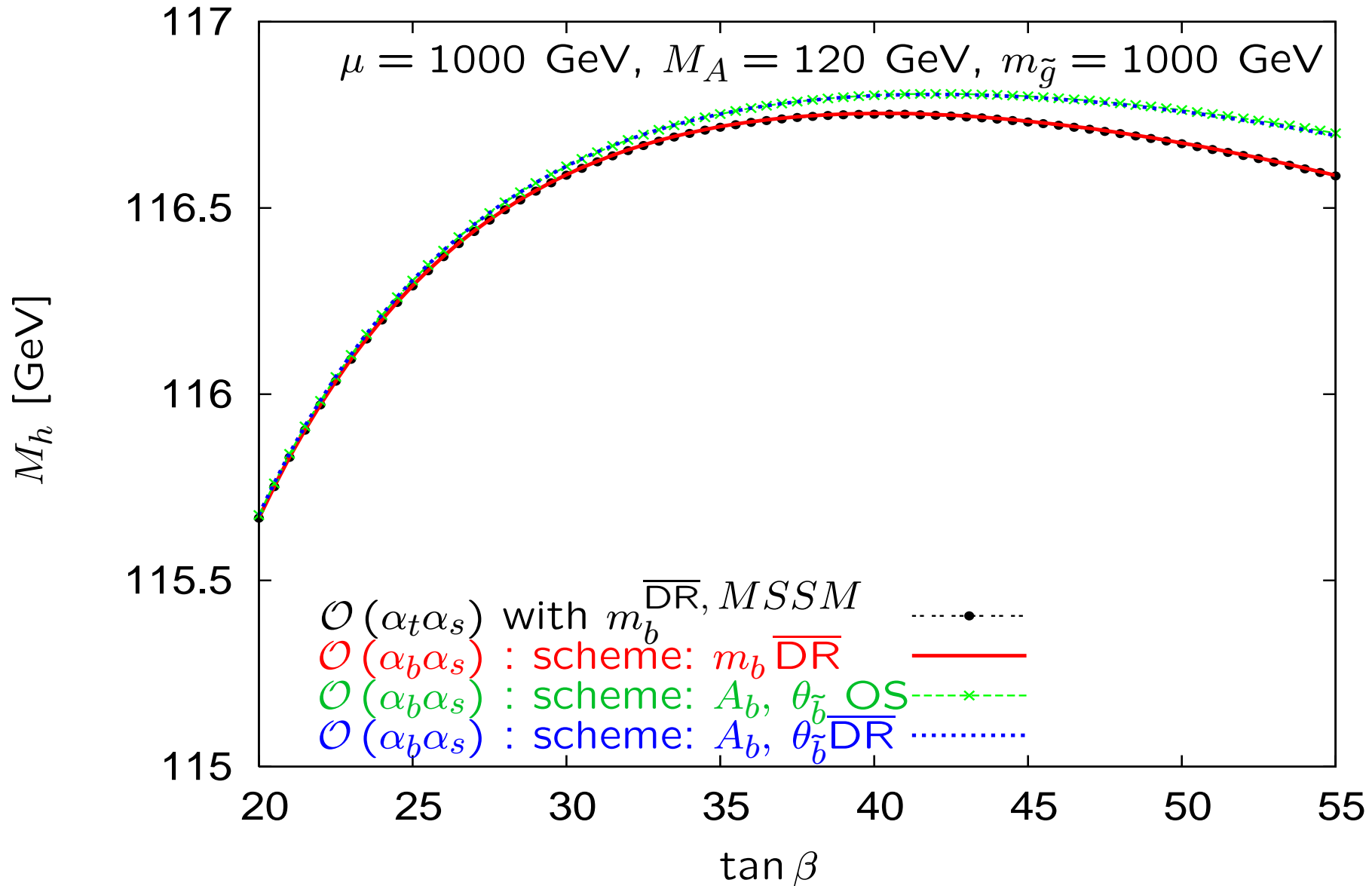
$$\hat{\Sigma}_{HH} \sim (\cos \alpha A_b)^2, \quad \hat{\Sigma}_{hh} \sim (\sin \alpha A_b)^2$$

$\Rightarrow$  effect more pronounced for  $M_H$

$\Rightarrow$  Scheme  $m_b$  OS is discarded as a useful renormalization scheme

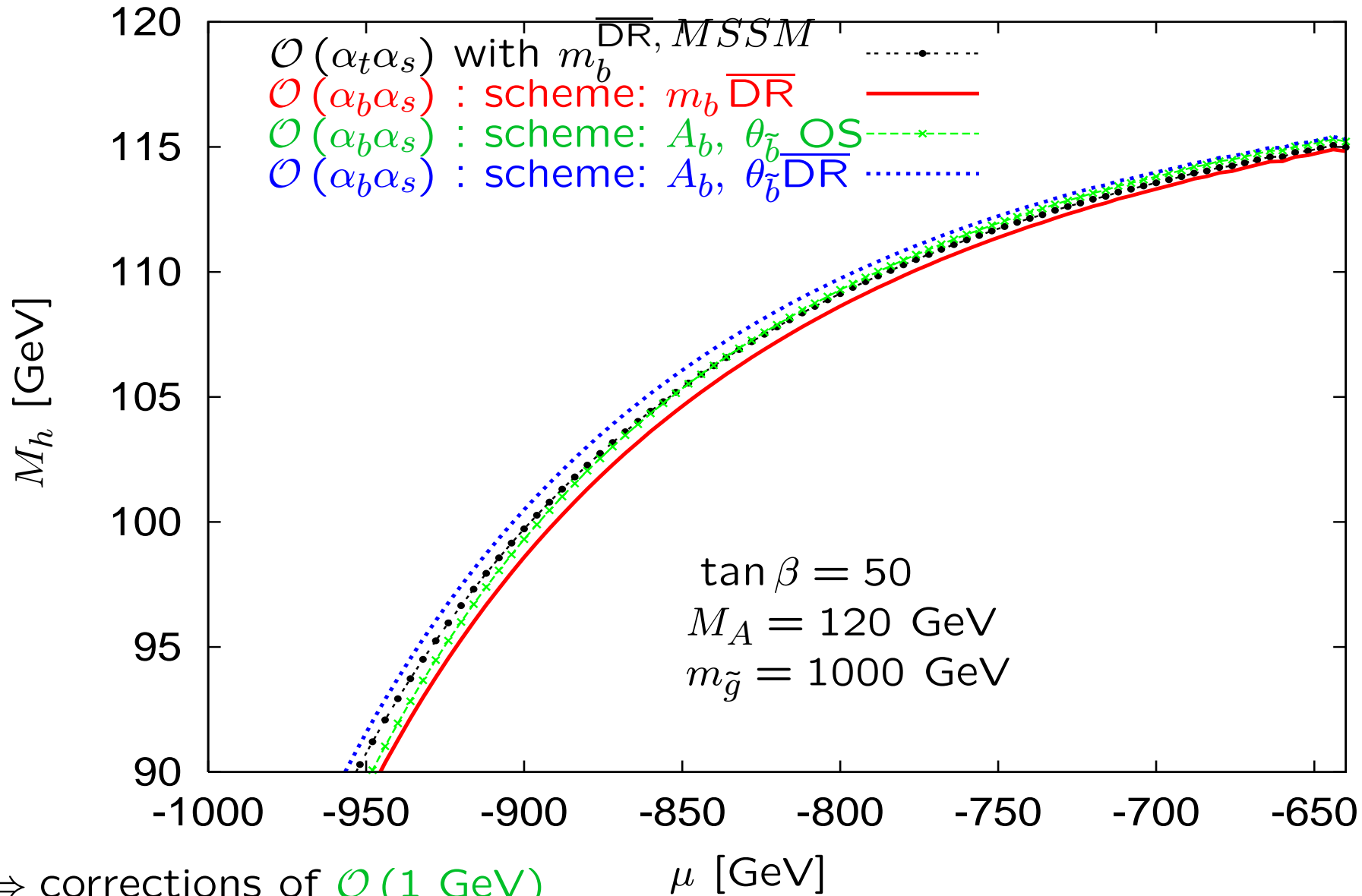
- Other schemes: differences of  $\mathcal{O}(1 \text{ GeV})$  for large  $\tan \beta$   
 $\Rightarrow$  non-negligible

$M_h$  as a function of  $\tan \beta$ ,  $\mu > 0$ :



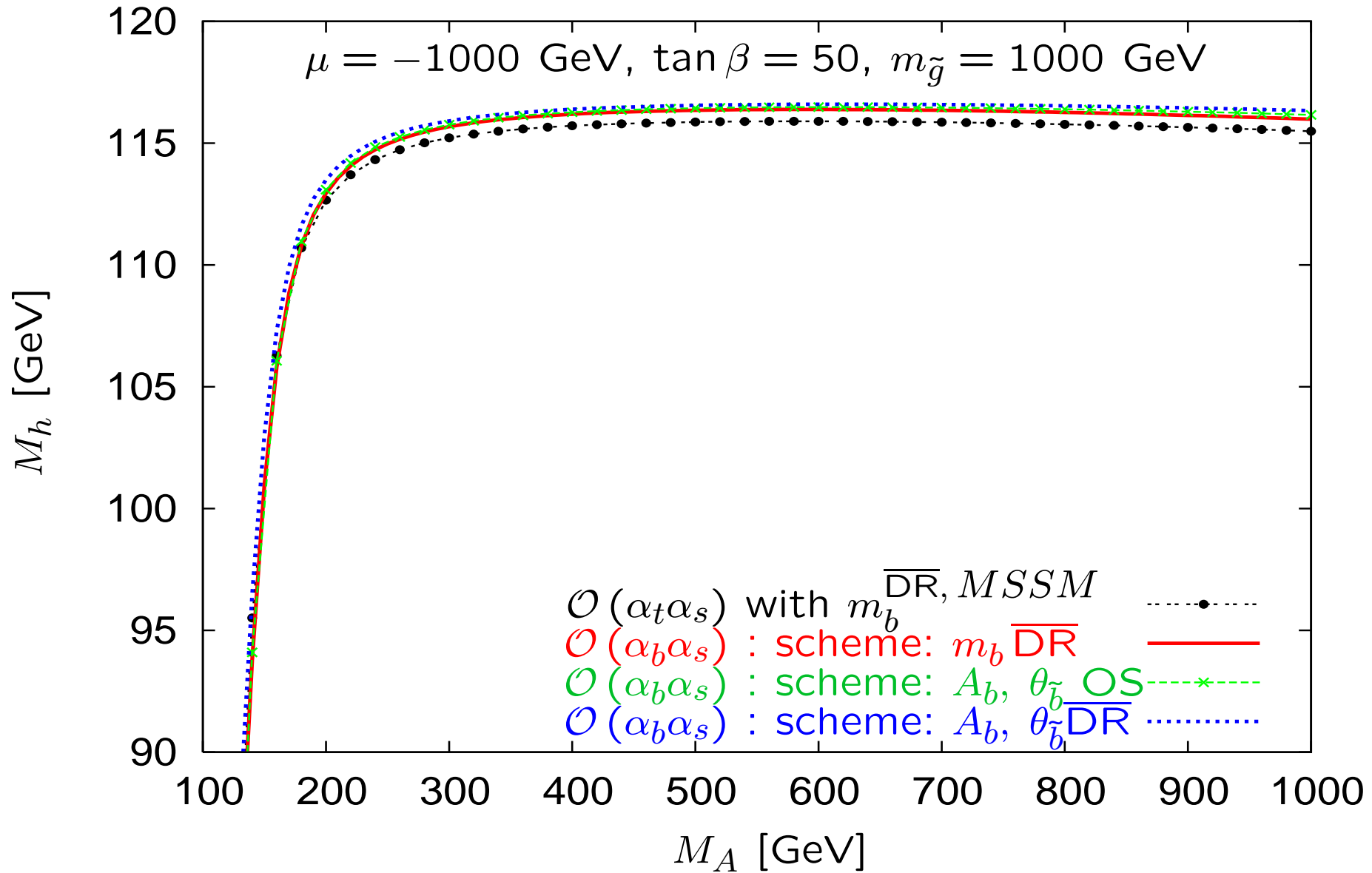
$\Rightarrow$  small corrections, **scheme  $m_b^{\overline{\text{DR}}}$ : “no” correction**

$M_h$  as a function of  $\mu$ ,  $\mu < 0$ :



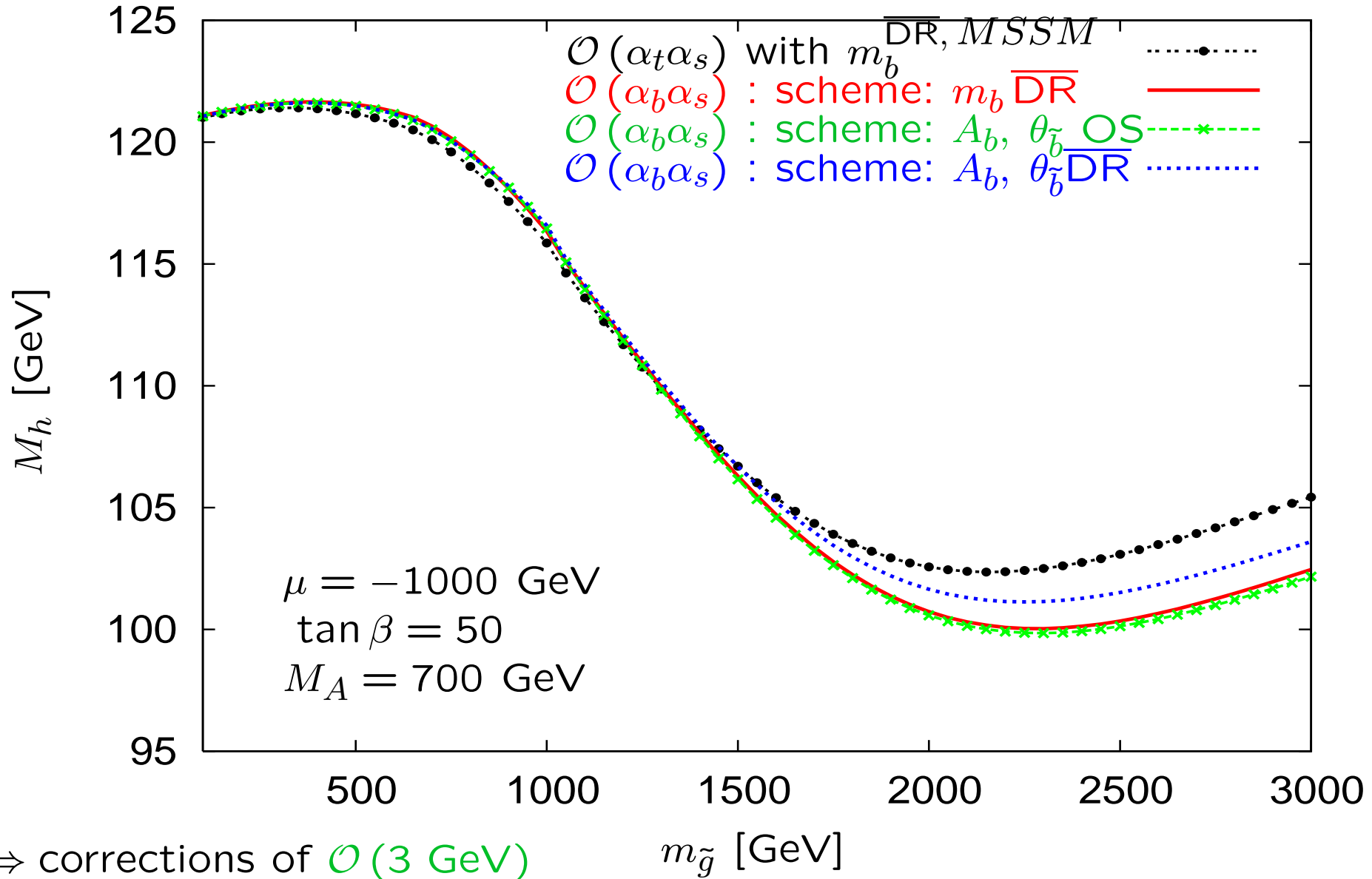
$\Rightarrow$  corrections of  $\mathcal{O}(1 \text{ GeV})$   
 scheme difference similar

$M_h$  as a function of  $M_A$ ,  $\mu < 0$ :



$\Rightarrow$  subleading corrections of  $\mathcal{O}(1 \text{ GeV})$

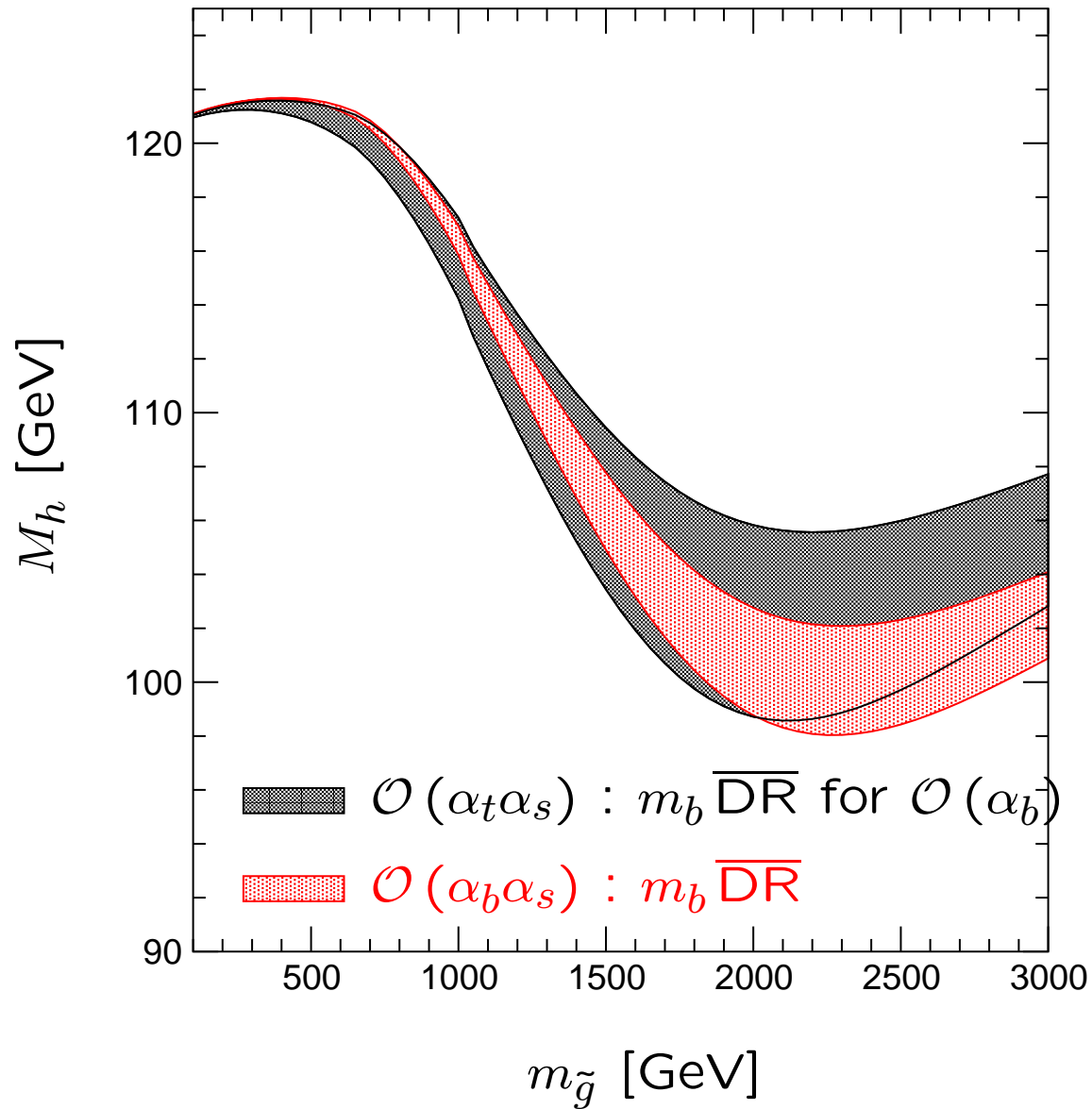
$M_h$  as a function of  $m_{\tilde{g}}$ ,  $\mu < 0$ :



$\Rightarrow$  corrections of  $\mathcal{O}(3 \text{ GeV})$   
 scheme difference  $\mathcal{O}(2 \text{ GeV})$



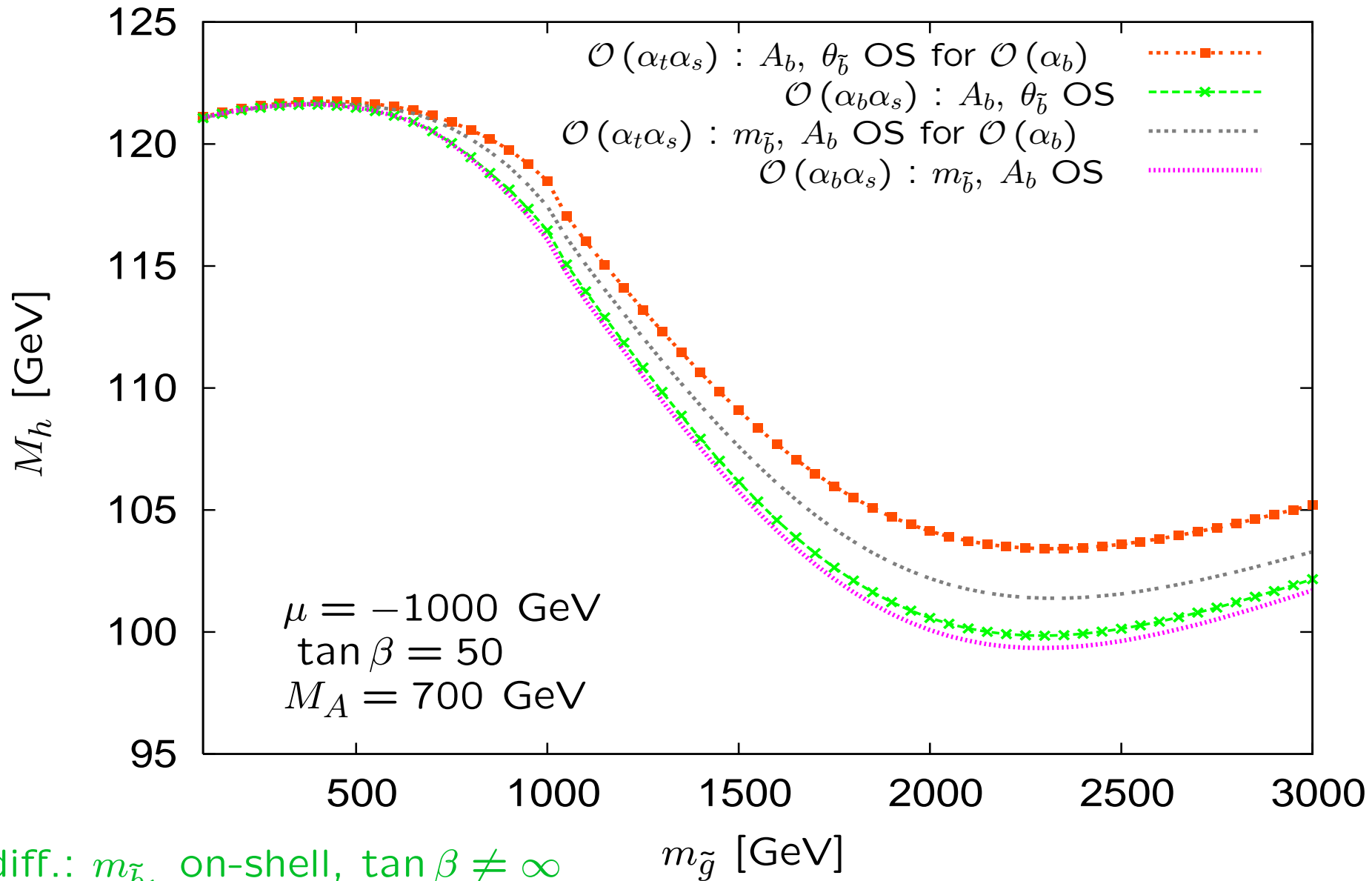
## Dependence on renormalization scale $\mu^{\overline{\text{DR}}}$ :



$M_A = 700$  GeV  
 $\mu = -1000$  GeV  
 $\tan \beta = 50$   
 $m_t/2 \leq \mu^{\overline{\text{DR}}} \leq 2 m_t$

$\Rightarrow$  scale dependence  
 $\mathcal{O}(\pm 2$  GeV) for large  $m_{\tilde{g}}$

Comparison with existing calculation: [A. Brignole et al. '02]



diff.:  $m_{\tilde{b}_1}$  on-shell,  $\tan \beta \neq \infty$   
 $\Rightarrow$  2-loop: differences  $\mathcal{O}(0.5 \text{ GeV})$

### 3. Corrections of $\mathcal{O}(\alpha_t\alpha_s)$ in the cMSSM

Higgs potential of the cMSSM contains two Higgs doublets:

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$
$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$
$$+ \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

Five physical states:  $h^0, H^0, A^0, H^\pm$  (no CPV at tree-level)

2 CP-violating phases:  $\xi, \arg(m_{12}) \Rightarrow$  can compensate each other

Input parameters:  $\tan\beta = \frac{v_2}{v_1}, M_A$  or  $M_{H^\pm}$

## Effects of complex parameters in the Higgs sector:

Complex parameters enter via loop corrections:

- $\mu$  : Higgsino mass parameter
- $A_{t,b,\tau}$  : trilinear couplings  $\Rightarrow X_{t,b,\tau} = A_{t,b} - \mu^* \{\cot \beta, \tan \beta\}$  complex
- $M_{1,2}$  : gaugino mass parameter (one phase can be eliminated)
- $m_{\tilde{g}}$  : gluino mass

$\Rightarrow$  can induce  $\mathcal{CP}$ -violating effects

Result:

$$(A, H, h) \rightarrow (h_3, h_2, h_1)$$

with

$$m_{h_3} > m_{h_2} > m_{h_1}$$

## Inclusion of higher-order corrections:

(→ Feynman-diagrammatic approach)

Propagator / mass matrix with higher-order corrections:

$$\begin{pmatrix} q^2 - M_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$  ( $i, j = h, H, A$ ) : renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0 \Rightarrow \mathcal{CPV}$ ,  $\mathcal{CP}$ -even and  $\mathcal{CP}$ -odd fields can mix

Our result for  $\hat{\Sigma}_{ij}$ :

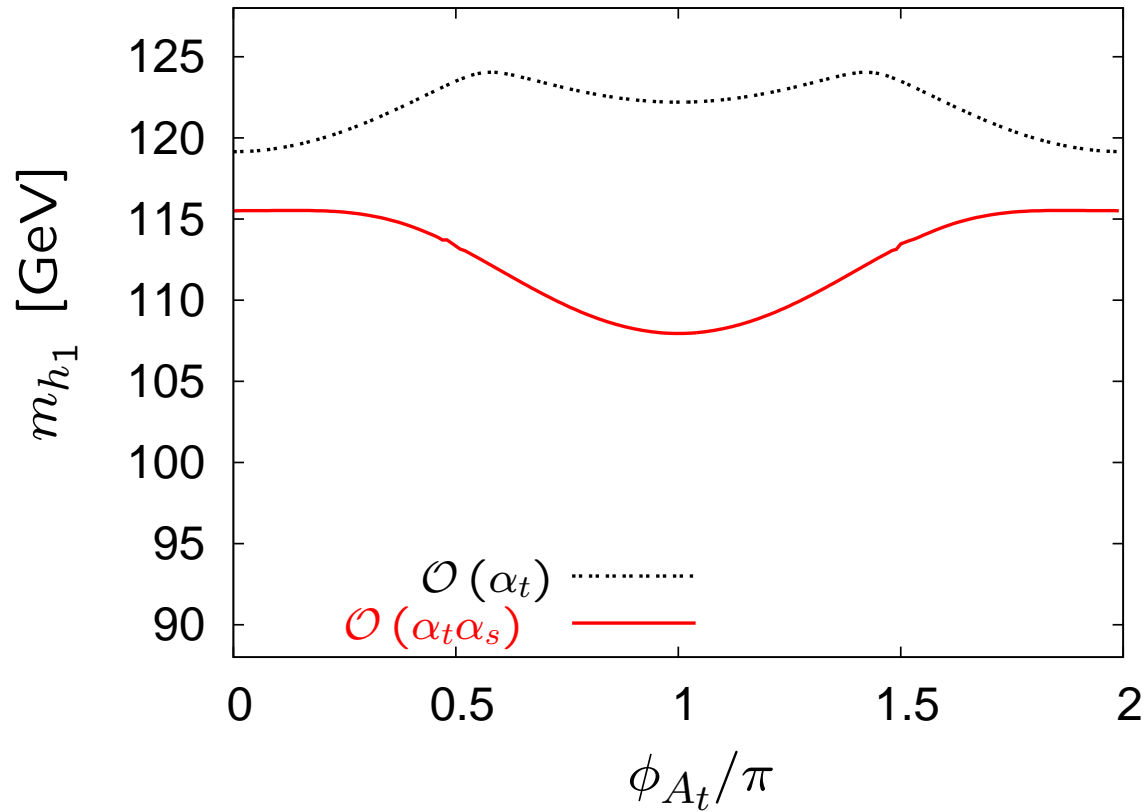
- full 1-loop evaluation: dependence on all possible phases included
- New:  $\mathcal{O}(\alpha_t \alpha_s)$  corrections in the FD approach
  - rMSSM: difference between FD and RGiEP approach  $\mathcal{O}$  (few GeV)

## Differences to the real case:

- use  $M_{H^\pm}$  as on-shell mass, since  $M_A$  receives loop corrections  
⇒  $\tilde{b}$  sector enters via  $\Sigma_{H^\pm}$   
⇒ renormalization of the  $\tilde{b}$  sector
- $A_t$  complex ⇒ renormalization of  $|A_t|$  and  $\phi_{A_t}$   
(no renormalization of  $\mu$ , no  $\mathcal{O}(\alpha_s)$  corrections)
- $M_3$  complex, but  $m_{\tilde{g}}$  is real (and positive)  
⇒ phase of  $M_3$  enters gluino vertices
- $T_A \neq 0$  ⇒ renormalized to zero  
⇒  $\delta t_A$  enters renormalized self-energies  $\hat{\Sigma}_{hA}$ ,  $\hat{\Sigma}_{HA}$

→ so far all results preliminary!

$m_{h_1}$  as a function of  $\phi_{A_t}$ :



$$M_{\text{SUSY}} = 1000 \text{ GeV}$$

$$|A_t| = 2000 \text{ GeV}$$

$$\tan \beta = 10$$

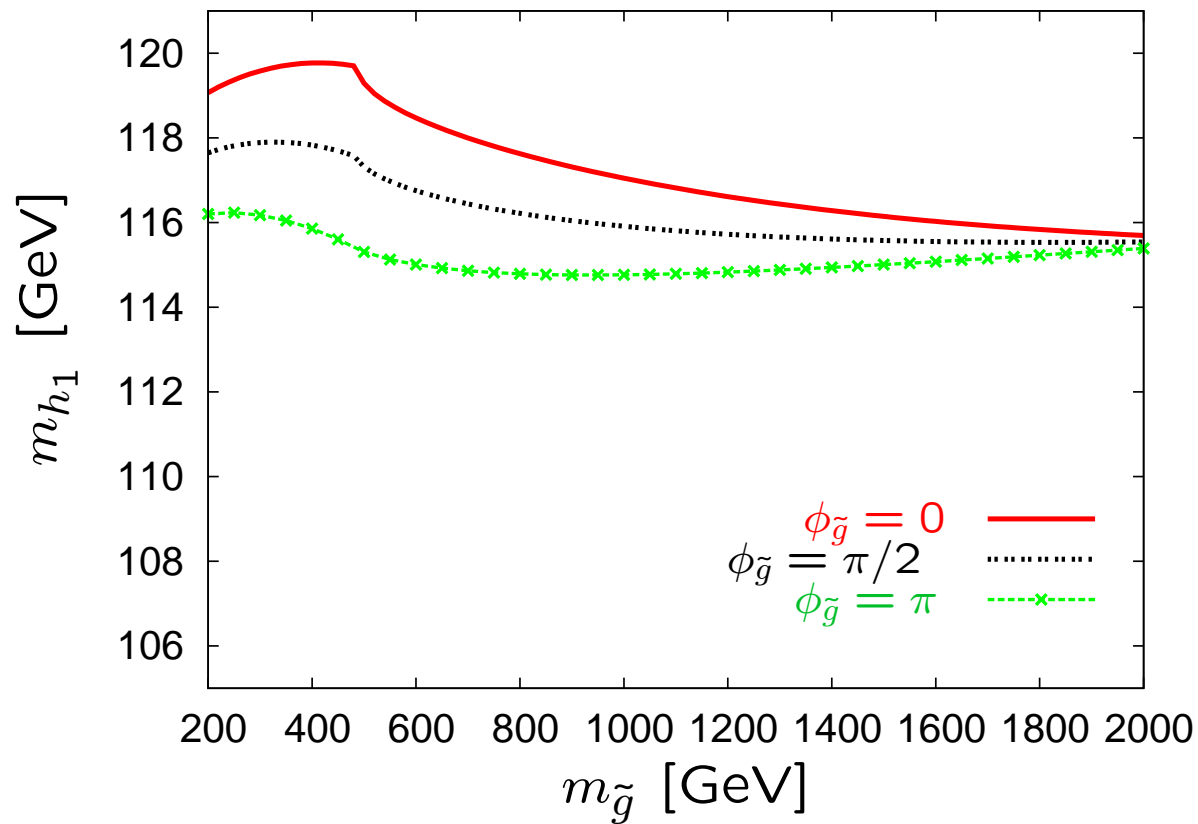
$$M_{H^\pm} = 150 \text{ GeV}$$

OS renormalization

$\Rightarrow$  modified dependence

on  $\phi_{A_t}$  at the 2-loop level

$m_{h_1}$  as a function of  $\phi_{\tilde{g}}$ :



$M_{\text{SUSY}} = 500 \text{ GeV}$

$A_t = 1000 \text{ GeV}$

$\tan \beta = 10$

$M_{H^\pm} = 500 \text{ GeV}$

OS renormalization

$\Rightarrow$  threshold at  $m_{\tilde{g}} = m_{\tilde{t}} + m_t$

$\Rightarrow$  large effects around threshold

$\Rightarrow$  phase dependence has to be taken into account



## 4. Conclusinos

- The ILC will provide high precision results for a light r/cMSSM Higgs
- MSSM Higgs masses and couplings is connected via radiative corrections to all other sectors
- Evaluation of  $\mathcal{O}(\alpha_b\alpha_s)$  corrections in the rMSSM:
  - new result for  $\tan\beta \neq \infty$
  - investigation of different renormalization schemes  
⇒ error estimate from scheme and scale dependence
  - $\mu > 0$ : corrections  $\mathcal{O}(100 \text{ MeV}) \Rightarrow$  under control
  - $\mu < 0$ : corrections  $\mathcal{O}(2 - 3 \text{ GeV})$   
error estimate  $\mathcal{O}(2 \text{ GeV}) \Rightarrow$  not under control
- Evaluation of  $\mathcal{O}(\alpha_s\alpha_t)$  corrections in the cMSSM:
  - new renormalization for complex parameters
  - $\tilde{b}$  sector enters
  - $\phi_{A_t}$  dependence modified
  - $\phi_{M_3}$  dependence  $\mathcal{O}(1 \text{ GeV})$
- Results will be implemented into *FeynHiggs* ([www.feynhiggs.de](http://www.feynhiggs.de))

# Backup slides

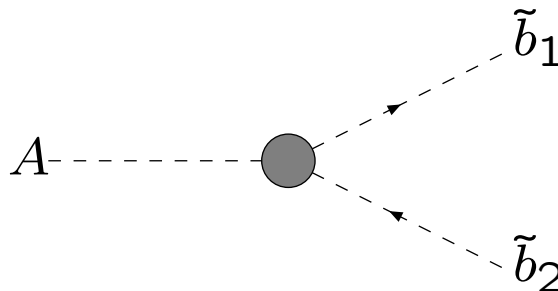
## Evaluation of 2-loop diagrams:

1. Generation of diagrams and amplitudes with **FeynArts**  
*[Küblbeck, Böhm, Denner '90] [Hahn '00 - '03]*
2. Algebraic evaluation and tensor integral reduction to scalar integrals:  
**TwoCalc**  
(works for two-loop self-energies)  
*[G. Weiglein '92] [G. Weiglein, R. Scharf, M. Böhm '94]*
3. Further evaluation: insertion of integrals, expansion in  $\delta = \frac{1}{2}(4 - D)$   
→ **algebraical check**: cancellation of divergencies
4. **Result**:
  - algebraic **Mathematica** code
  - Fortran code (planned: **implementation into FeynHiggs** )

## Some more details:

- scheme  $m_{\tilde{b}}$  OS: analogous to the  $t/\tilde{t}$  sector  
→ obvious choice ?

- $A_{\tilde{b}}$  OS: determined via



analogous to [A. Brignole, G. Degrassi, P. Slavich and F. Zwirner '02]

- $\theta_{\tilde{t}}$  OS: 
$$\delta\theta_{\tilde{b}} = \frac{\text{Re}\Sigma_{\tilde{b}_{12}}(m_{\tilde{b}_1}^2) + \text{Re}\Sigma_{\tilde{b}_{12}}(m_{\tilde{b}_2}^2)}{m_{\tilde{b}_1}^2 + m_{\tilde{b}_2}^2}$$

## The Higgs self-energy at 2-loop:

→  $\alpha_s$  correction to the leading 1-loop term  $\sim m_t^4$

Approximations:

- only  $m_t^2$  terms
- gauge couplings are set to 0
- external momentum is set to 0

$$\Rightarrow \hat{\Sigma}_{hh}^{(2)}(0) = \Sigma_{hh}^{(2)}(0) + (\cos\alpha \cos\beta + \sin\alpha \sin\beta)^2 \delta M_{H^\pm}^{2(2)} - \frac{e}{2 M_W s_W} \left( f_1(\alpha, \beta) \delta t_1^{(2)} + f_2(\alpha, \beta) \delta t_2^{(2)} \right)$$

in the  $hH$  basis with

$\Sigma_{hh}^{(2)}(0)$  : unrenormalized  $hh$  self-energy

$\delta M_{H^\pm}^{2(2)} = \Sigma_{H^\pm}^{(2)}(0)$ :  $H^\pm$  mass counter term

$\delta t_i^{(2)} = -T_i^{(2)}$ :  $\phi_i$  tad-pole

$\hat{\Sigma}_{hA}$ ,  $\hat{\Sigma}_{HA}$ :  $\delta t_A = -T_A$  enters