Accelerators

Lecture IV

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Summary Lecture III

- *Orbit Stability*
- *Linear Resonances*
- *Chromaticity and Sextupoles*
- *Non-Linear Resonances and Long Term Stability*

IV) Synchrotron Radiation + Collective Effects

- Adiabatic Damping
- Synchrotron Radiation
- Radiation Damping
- Collective Effects
- Feedback and Damping
- Summary

Adiabatic Damping

equation of motion (transverse motion):

$$\frac{d^2}{dt^2} x = \frac{F_x}{m}$$

Lorentz Force: $F_x = q \cdot v_{\parallel} \cdot B_y$

accelerator: $B_y = g \cdot x$

with:

$g \propto \gamma$

$v_{\parallel} \approx c$

$m \propto \gamma$

(lecture II)

$\longrightarrow$ $\Omega^2 \propto g/m = \text{const.}$

transverse energy:

$$E = \int F(y)\, dy \longrightarrow E = \frac{1}{2} \cdot g \cdot y_{max}^2$$

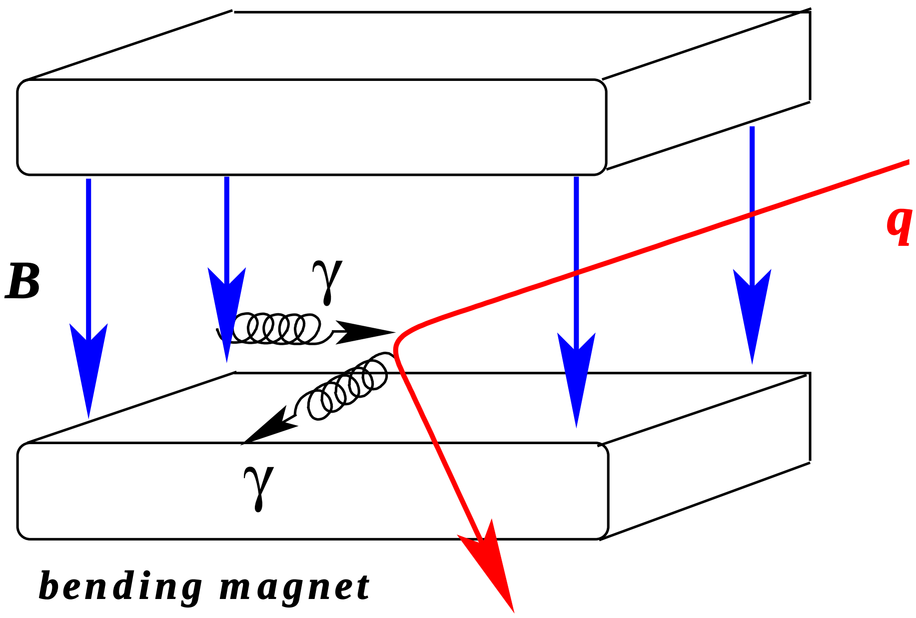
transverse energy does not change during acceleration!

$g \propto \gamma \longrightarrow y_{max} \propto \sqrt{1/\gamma}$

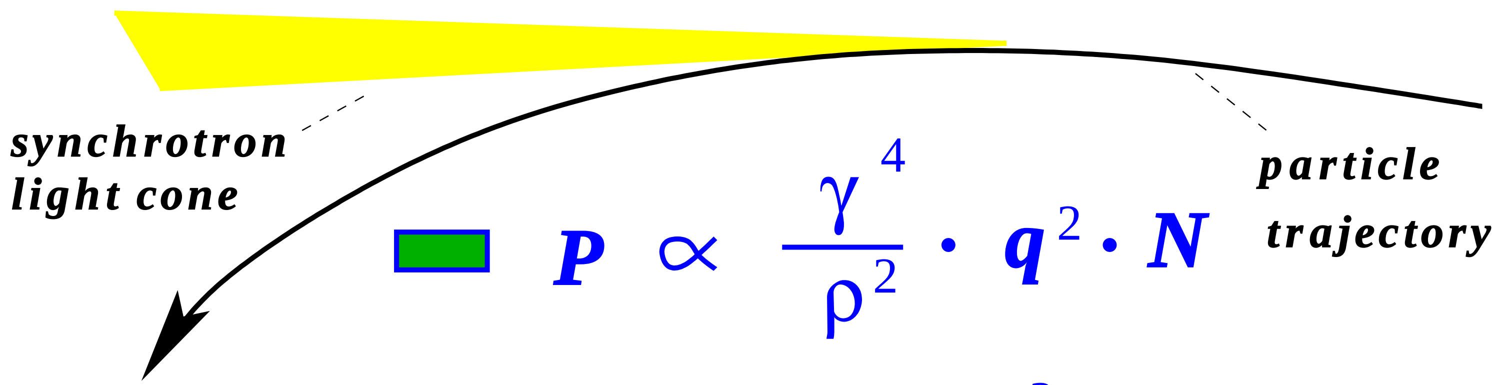
$\longrightarrow$ the beam size shrinks during acceleration!

Synchrotron Radiation

Quantum Picture:



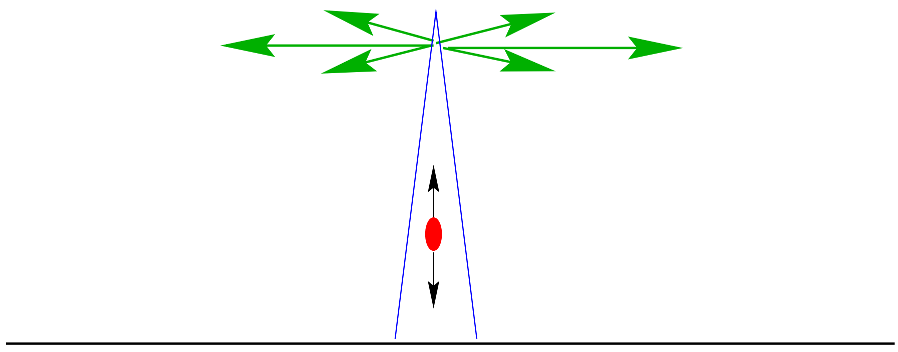
- *radiation fan in bending plane*
- *opening angle* $\propto \frac{1}{\gamma}$



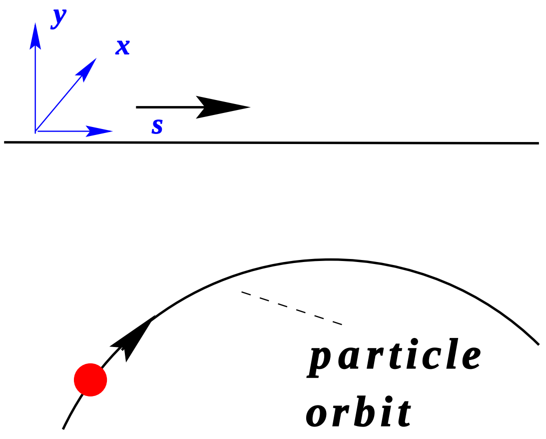
- $P \propto \frac{\gamma^4}{\rho^2} \cdot q^2 \cdot N$
- $\langle E_\gamma \rangle \propto \frac{\gamma^3}{\rho}$
- *polarised*

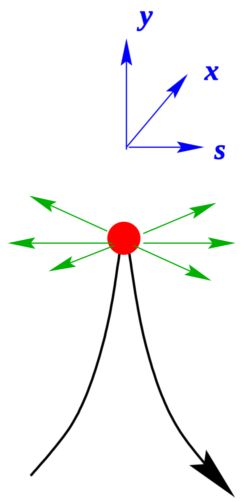
Synchrotron Radiation

Antenna:

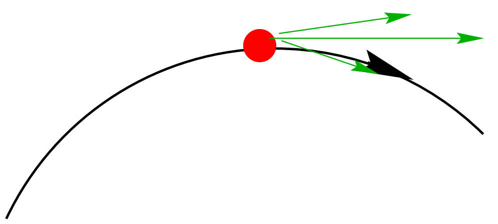


Particle Trajectory:





Lorentz Transformation:



Examples

	E [GeV]	ρ [km]	N [10^{12}]	U [MeV]	P [MW]	E_γ [keV]
LEP 1	45	3.1	4.7	260	2.1	90
LEP 2	100	3.1	4.7	2800	23	715
LHC	7000	3.1	312	0.007	0.005	0.04

- γ-rays: Co_{60} → 1.3 MeV
- X-rays: → keV
- Visible Light: → eV

LEP 1 → X-rays

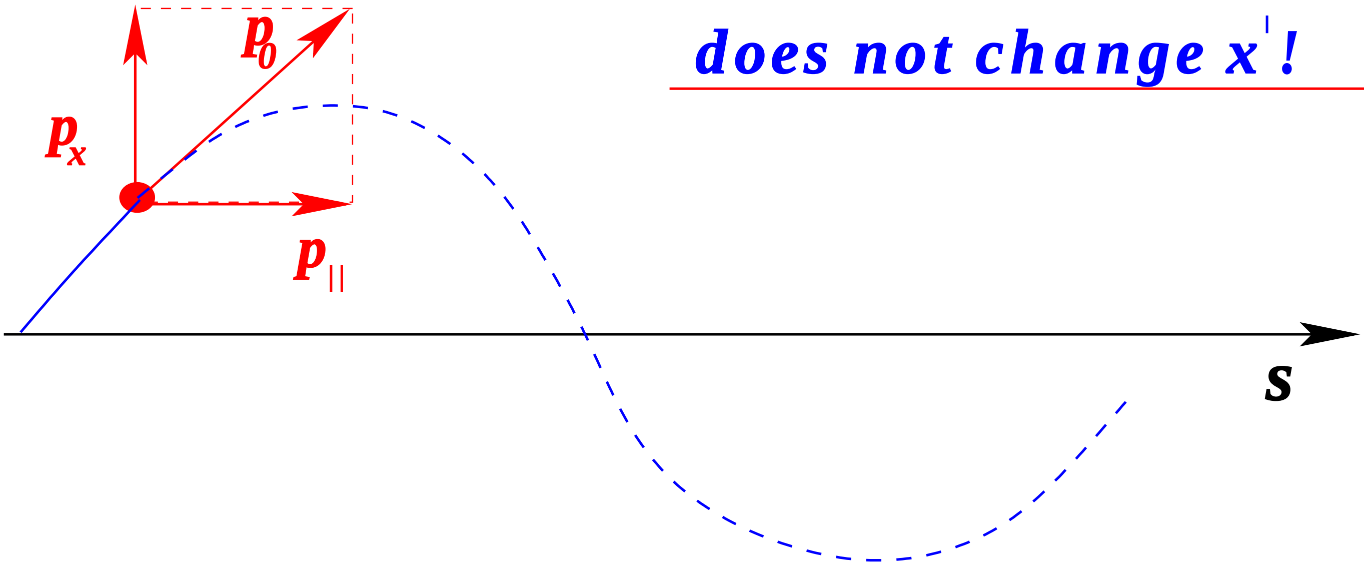
LEP 2 → γ -rays

LHC → UV light

Acceleration Damping I

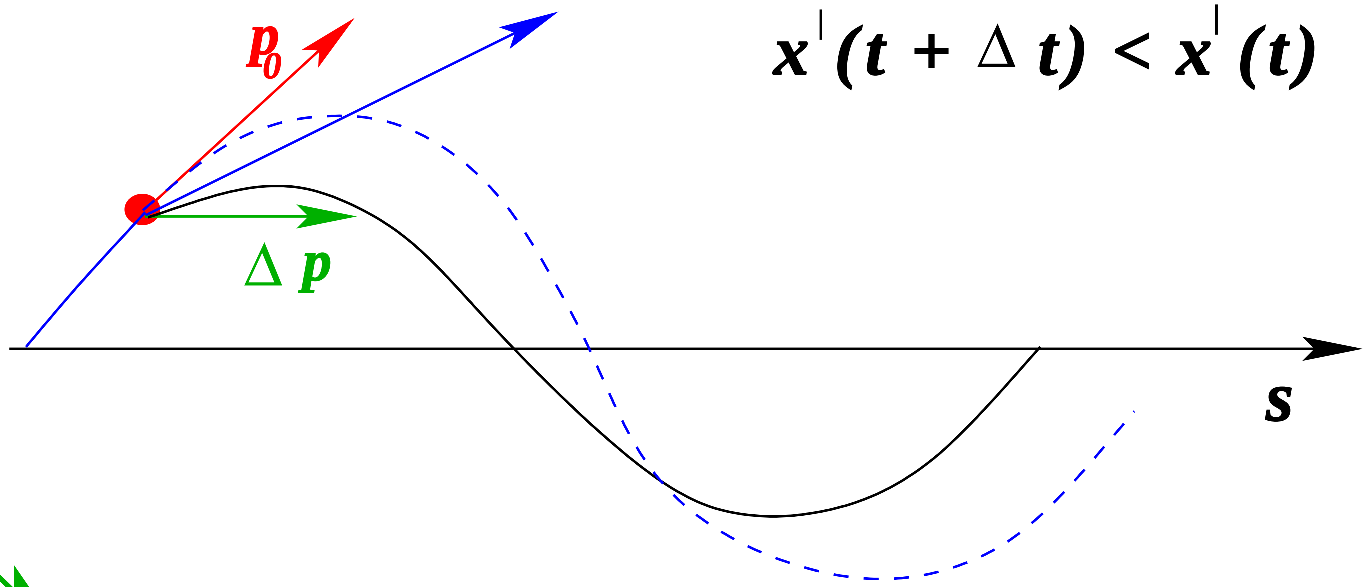
$x' = \frac{p_x}{p_{||}}$

→ *synchrotron radiation does not change x'!*



Acceleration:

$x'(t + \Delta t) < x'(t)$



the beam shrinks as it gets accelerated!

damping $\propto \sqrt{\frac{1}{\gamma}}$

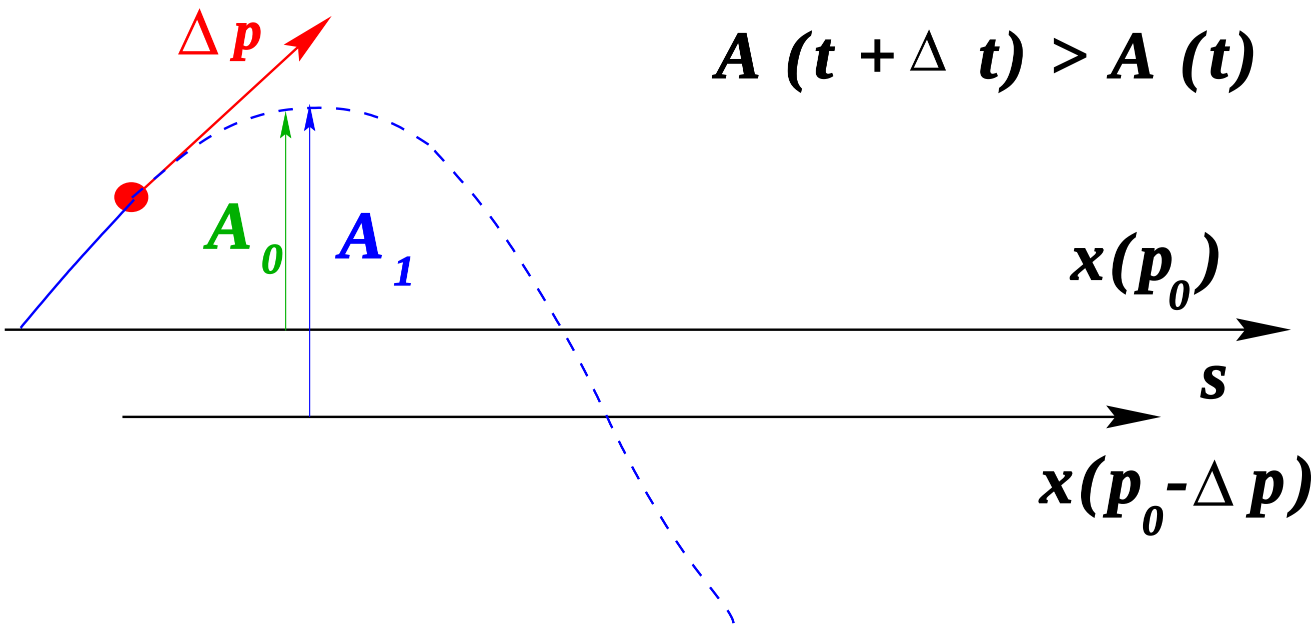
Acceleration Damping II

Synchrotron radiation + acceleration

→ continuous damping

- **Limits:**
  - quantum excitations
    → small but finite beam size
  - dispersion
  - orbit curvature
    → synchrotron radiation increases beam size

$$A\,(t+\Delta\,t) > A\,(t)$$



synchrotron radiation + dispersion

→ excitation

Summary Synchrotron Radiation

- **Pro:**
  - synchrotron radiation
    → dedicated light sources
  - vertical damping
    → flat beams
  - damped motion
    → not sensitive to errors

- **Contra:**
  - power loss
    → large storage ring
    energy limit
  - radiation excitation
    → the horizontal beam size
    increases with γ !

Collective Effects

- Communication of particles with each other via:

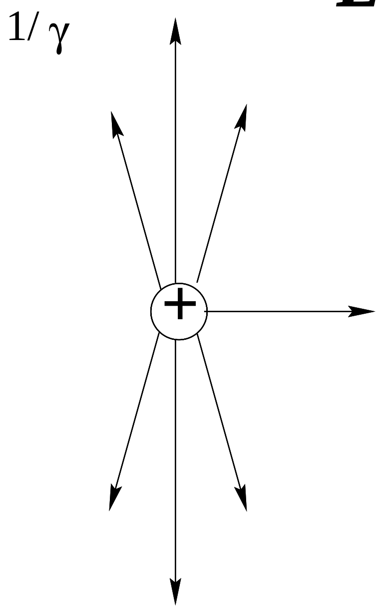
  - image charges on the vacuum chamber
  - direct coulomb interaction
  - residual gas ionisation
  - synchrotron radiation

→ change of individual particle motion

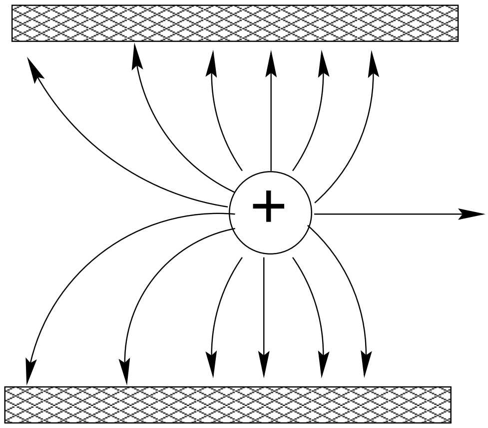
→ collective motion

→ stability? { beam size, oscillation modes }

Collective Effects



E-Field of point charge



- electro-magnetic field of the bunch
- image charge currents

change in bunch distribution

force changes

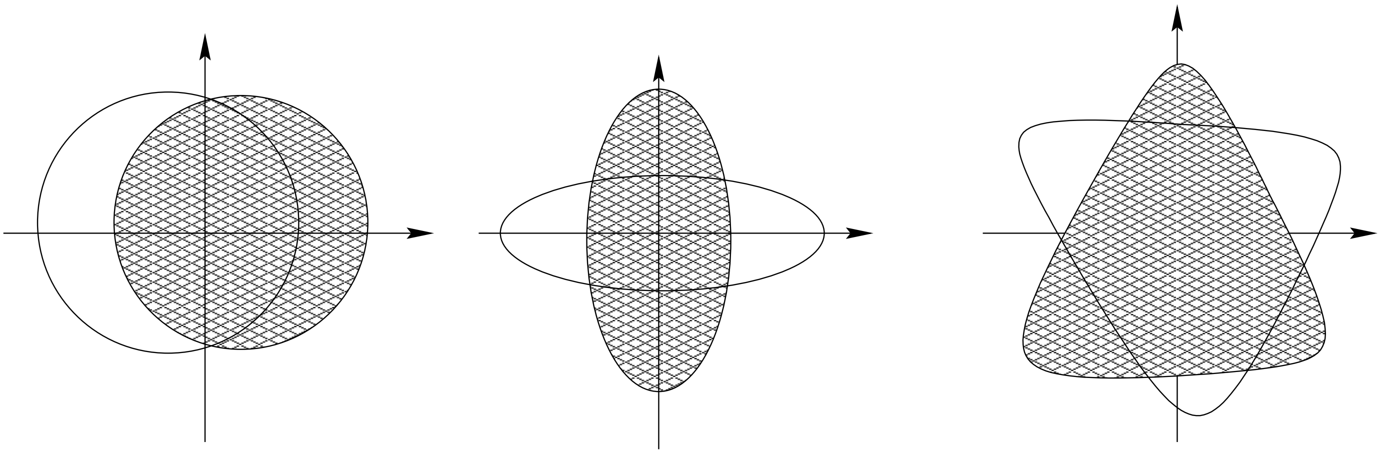
→ equilibrium?

- wakefields

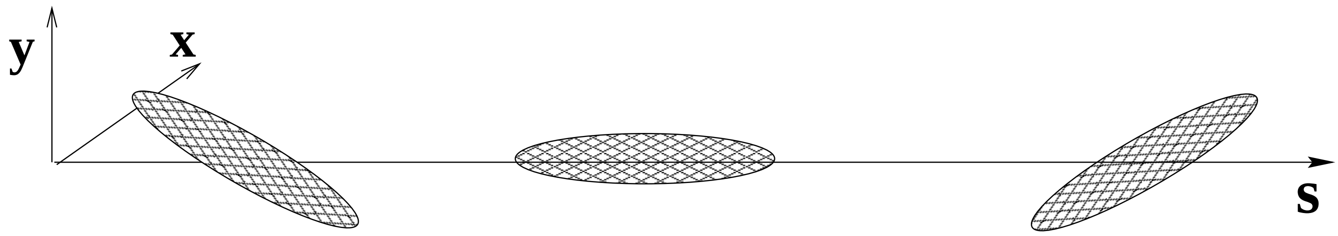
→ particle-environment interaction

Oscillation Modes

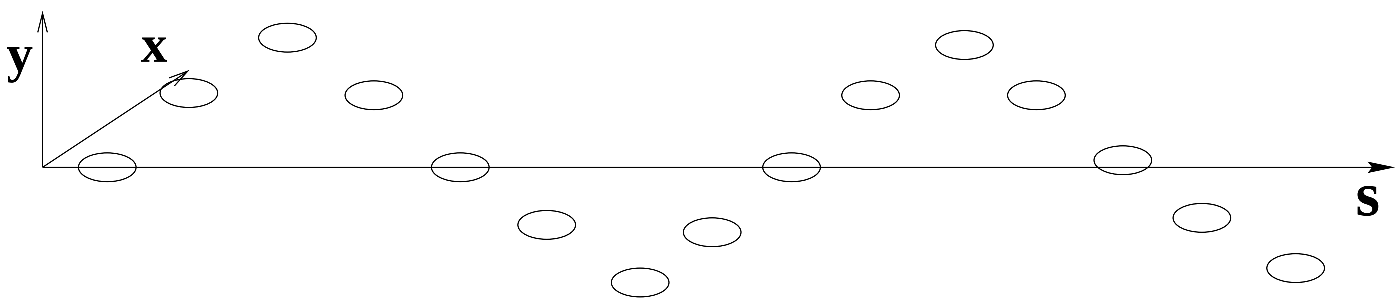
Single Bunch: longitudinal or transverse



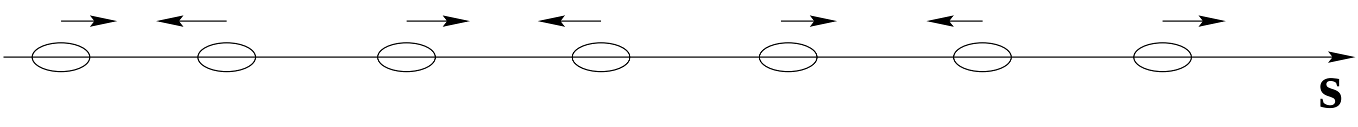
Single Bunch: longitudinal and transverse



Multibunch: transverse



Multibunch: longitudinal



→ *beam stability depends*

on the surface properties

+

geometry of the vacuum chamber

→ *careful design of all vacuum equipement*

- ***General Rules:***
  - *smooth transitions*
  - *shielded discontinuities*

- ***Quantitative Analysis:***
  - *evaluate E and B for a given test distribution*
  - *study the beam stability by super-imposition*

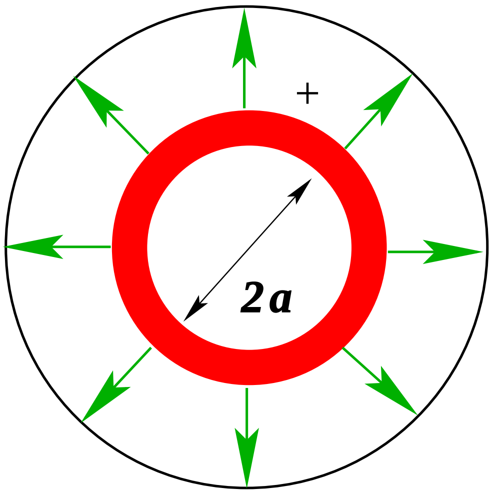
Multipole Moments

$$\rho = \sum_{m=0}^{\infty} \rho_m \qquad \vec{j} = \sum_{m=0}^{\infty} \vec{j}_m$$

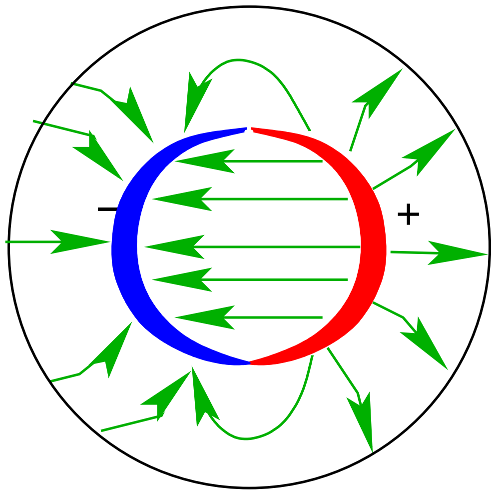
$$\rho_m = \frac{I_m}{\pi \cdot a^{m+1} \cdot (1+\delta_{m0})} \cdot \delta(s-ct) \cdot \delta(r-a) \cdot \cos(m\,\theta)$$

$$\vec{j} = c \cdot \rho_m \cdot \vec{s}$$

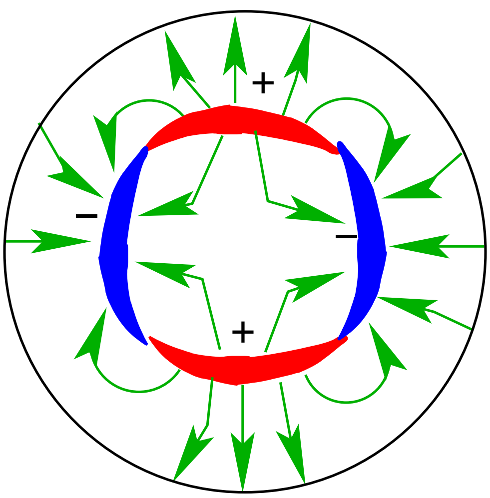
m = 0

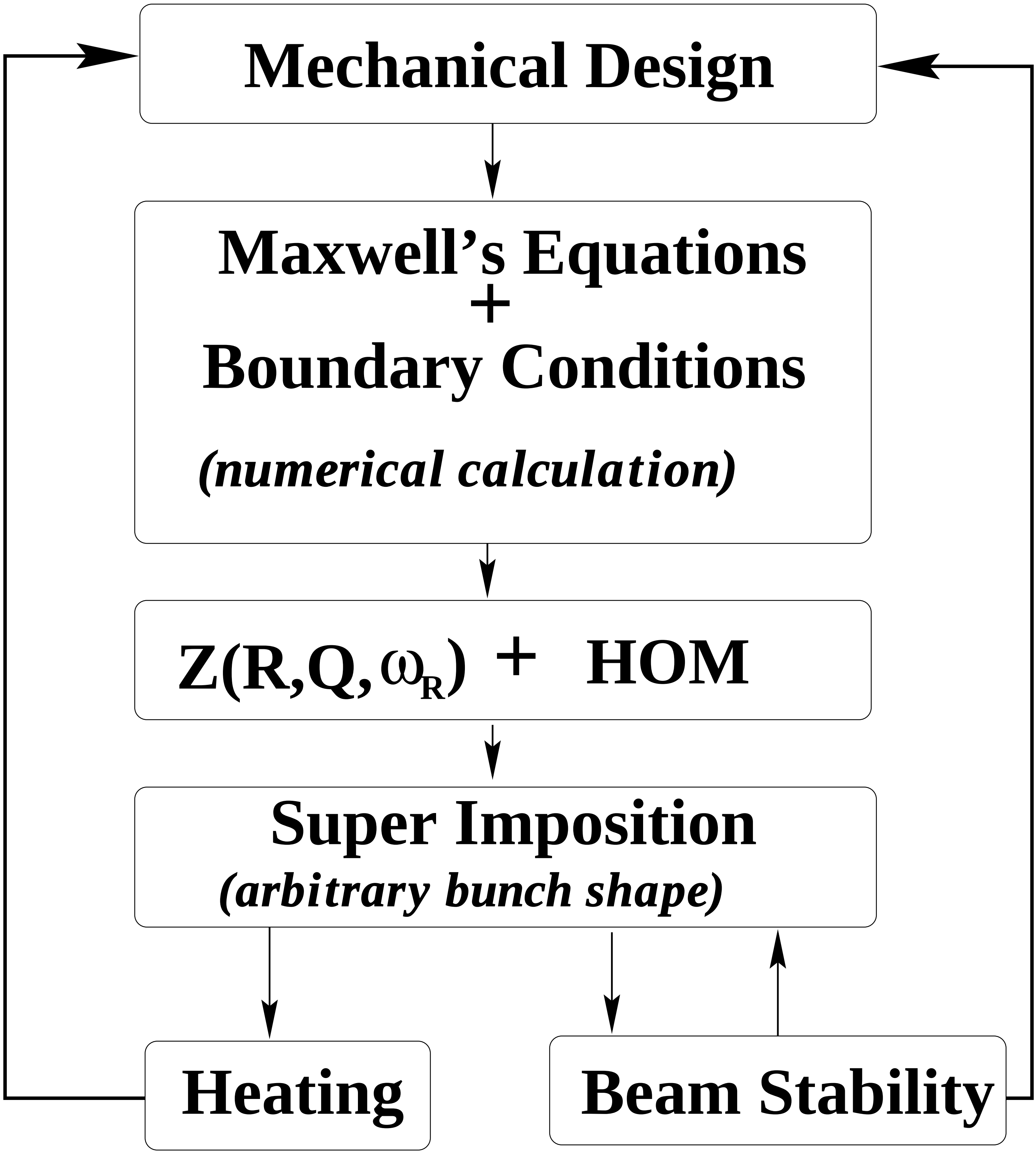


m = 1



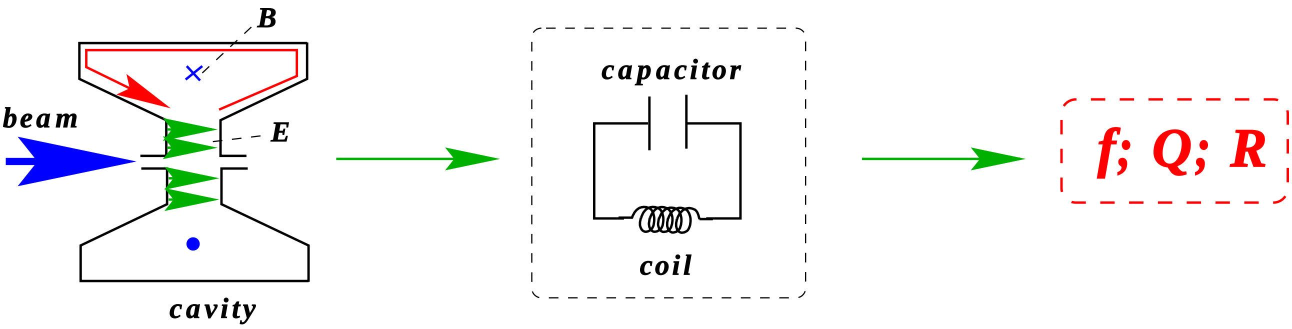
m = 2

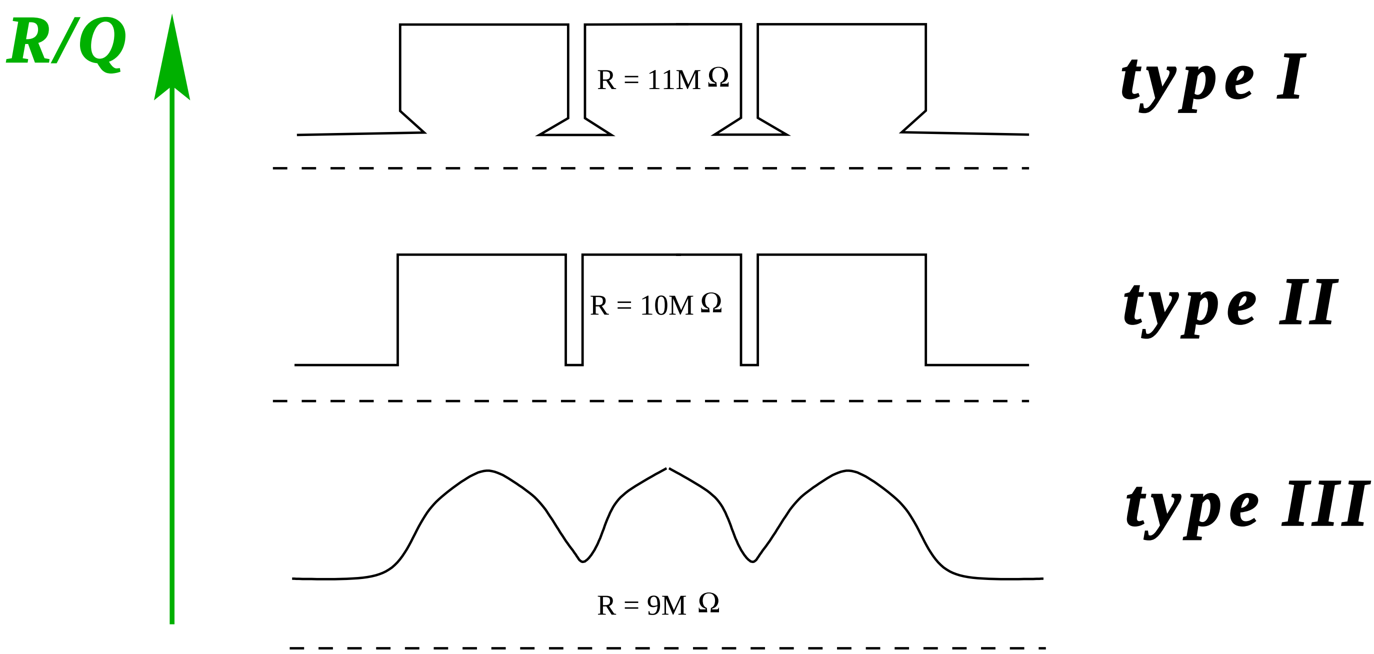



Mechanical Design
Maxwell's Equations
+
Boundary Conditions
(numerical calculation)
$Z(R, Q, \omega_R)$ + HOM
Super Imposition
(arbitrary bunch shape)
Heating
Beam Stability

Cavity Design I

- $P = \frac{U^2}{R}$ → maximise R
- R/Q is determined by cavity geometry:





- Q is given by the surface conductivity

→ LEP I:

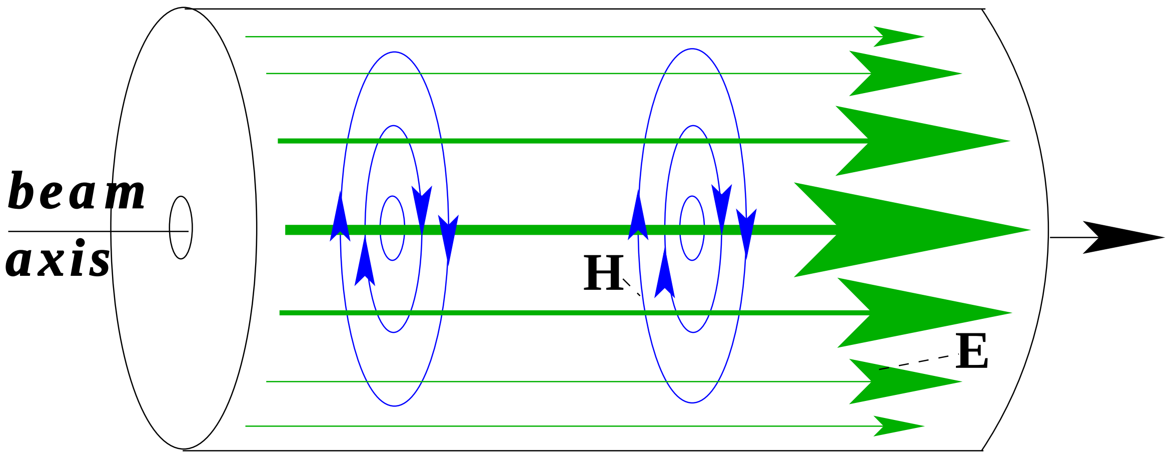
copper cavities of type I + extra resonator

$R/Q = 230\ \Omega$

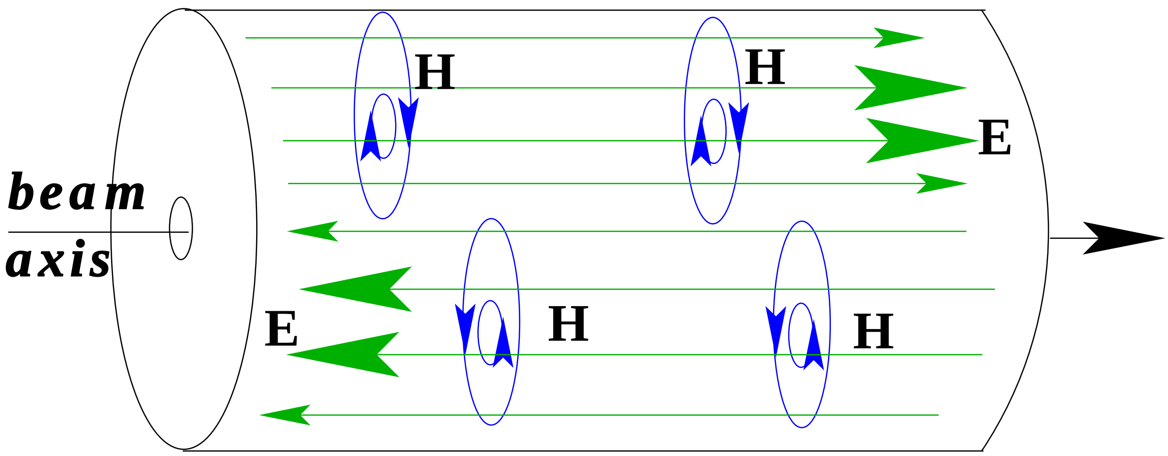
Higher Order Modes

Cavity Resonator:

Fundamental TM mode;

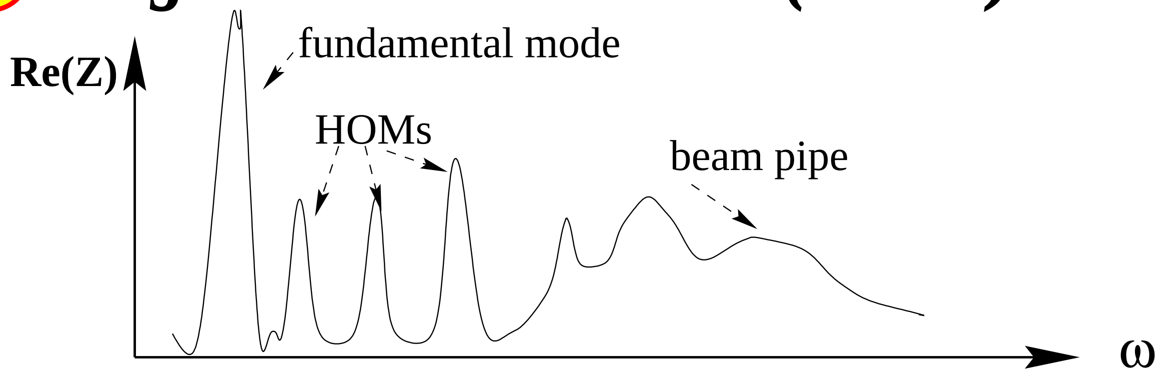


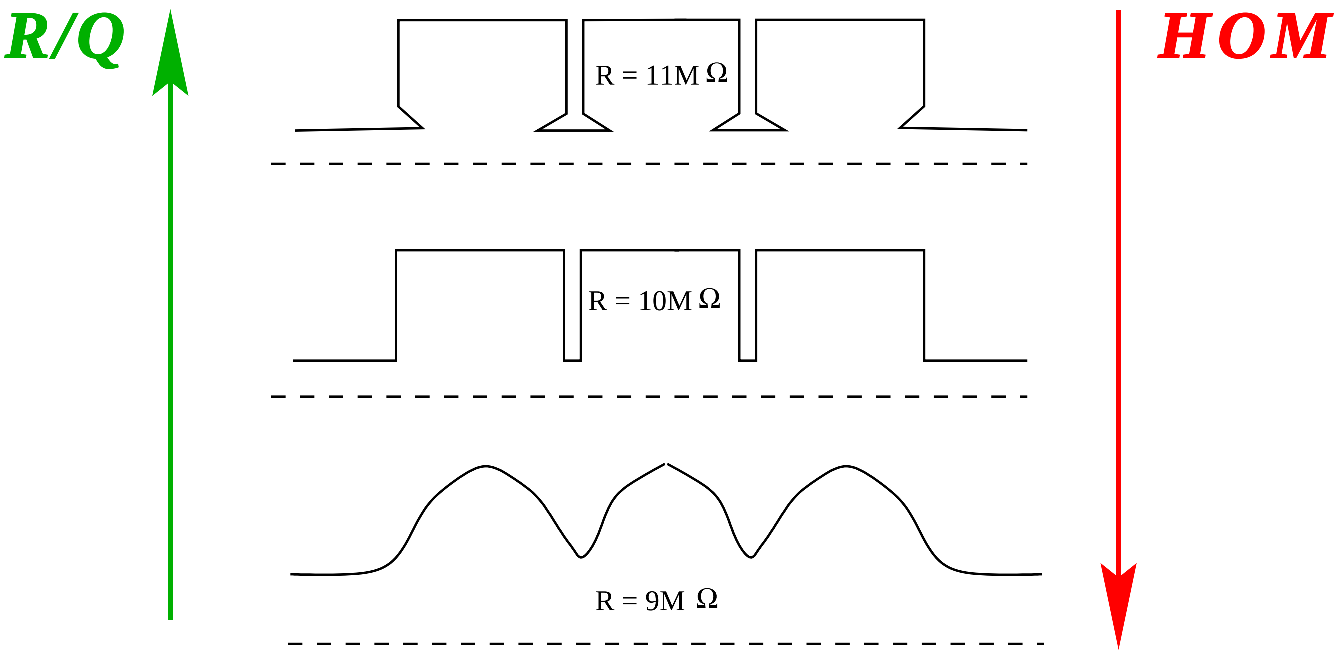
Example for Higher Order Mode:



Cavity Design II

Higher Order Modes (HOM):





HOMs drive beam instabilities!!

LEP:

superconducting cavities of type III

Cu with Nb coating

→ $f = 350$ MHz, $Q = 5*10^9$, $V = 6$ MV/m

$R/Q = 230\ \Omega$

→ ***beam stability depends***

on the surface properties

+

geometry of the vacuum chamber

→ ***careful design of all vacuum equipement***

- ***General Rules:***
  - ***smooth transitions***
  - ***shielded discontinuities***

- ***Quantitative Analysis:***
  - ***evaluate E and B for a given test distribution***
  - ***study the beam stability by super-imposition***

Beam Size

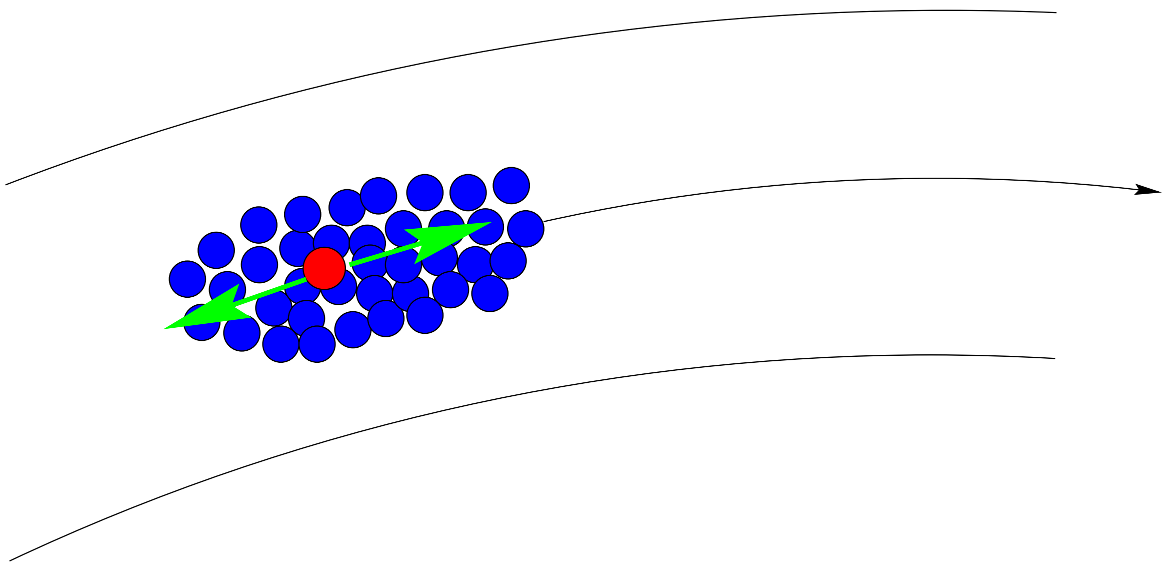
Intra Beam Scattering:

- *each particle performs longitudinal*

  + *transverse oscillations*

  → *uncorrelated motion!*

Coulomb Scattering → Emittance Growth



- ***Emittance blow-up:***

$$\varepsilon\,(t+\Delta t) = \left(1+\frac{\Delta t}{\tau}\right)\cdot\varepsilon\,(t)$$

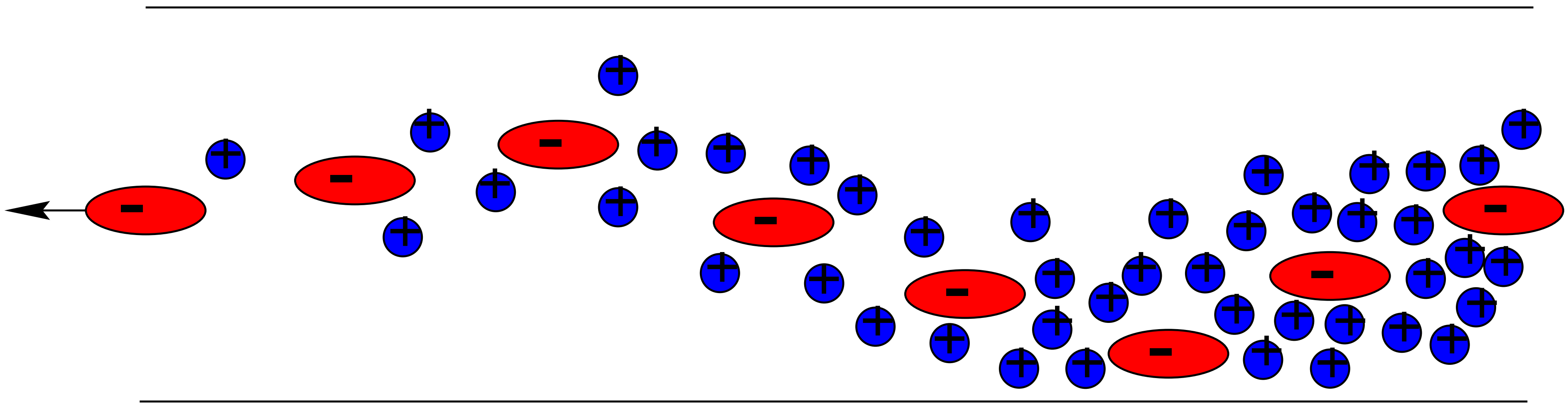
- ***Growth rate depends on beam size:***

$$1/\tau \propto \frac{N}{\varepsilon_t^2 \bullet \varepsilon_l} \cdot A \cdot Z$$

Fast Ion Instability

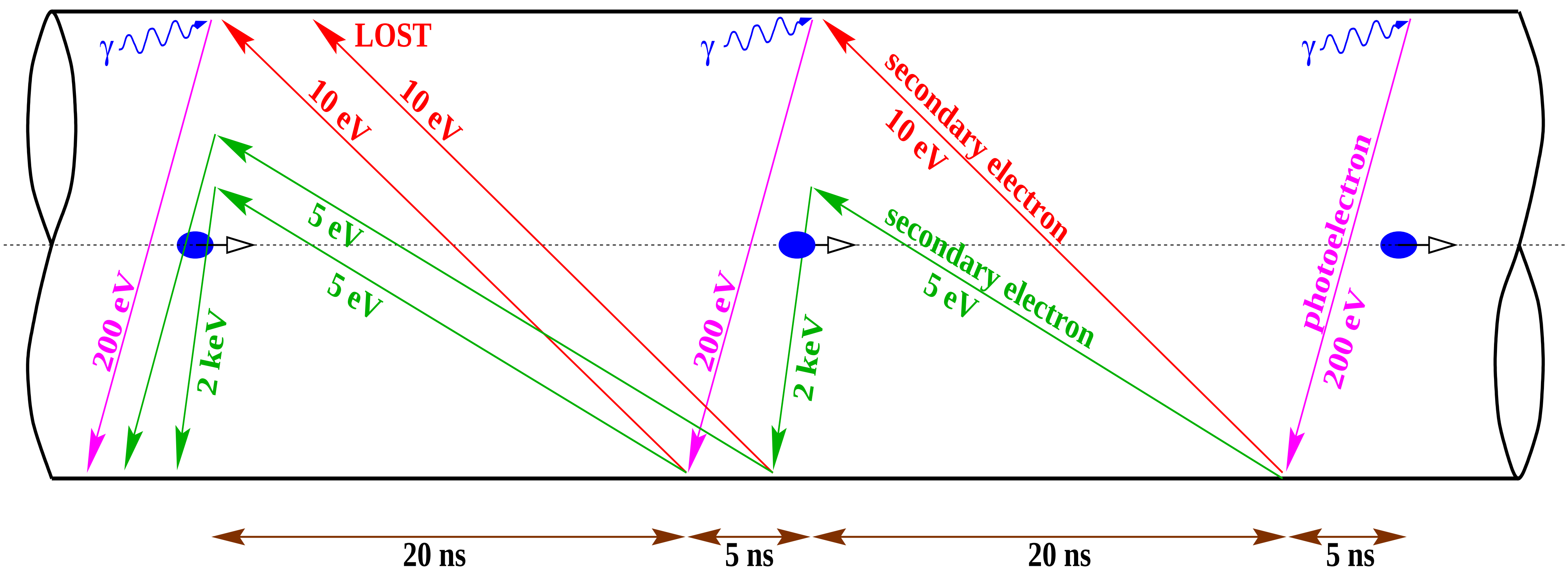
Restgas Ionisation: → **Ion Cloud + Wakefield**

Instability!



Electron Cloud Instability

- Synchrotron light removes electrons from chamber wall
- Electrons are accelerated by the beam
- Electrons hit vacuum chamber and generate more electrons
- Electron cloud → instability and heat losses!



Synchrotron Radiation

power and critical photon energy:

- $P \propto \frac{\gamma^4}{\rho^2} \cdot q^2 \cdot N \longrightarrow P \propto N$

- $\langle E_\gamma \rangle \propto \frac{\gamma^3}{\rho} \longrightarrow \omega_\gamma$

$$c = \frac{\omega}{2\pi} \cdot \lambda \longrightarrow \lambda_\gamma \propto \frac{\rho}{\gamma^3}$$

→ if the bunch dimensions become comparable or smaller than the radiation wave length a bunch can radiate like a single macro particle!

→ $q = N \cdot e \longrightarrow P \propto N^2$ $(N = 10^{11})$

→ Free Electron Laser (FEL) machines

Summary Instabilities

- Particles interact within each bunch

- Bunches interact with each other
  - surface properties
  - chamber geometry

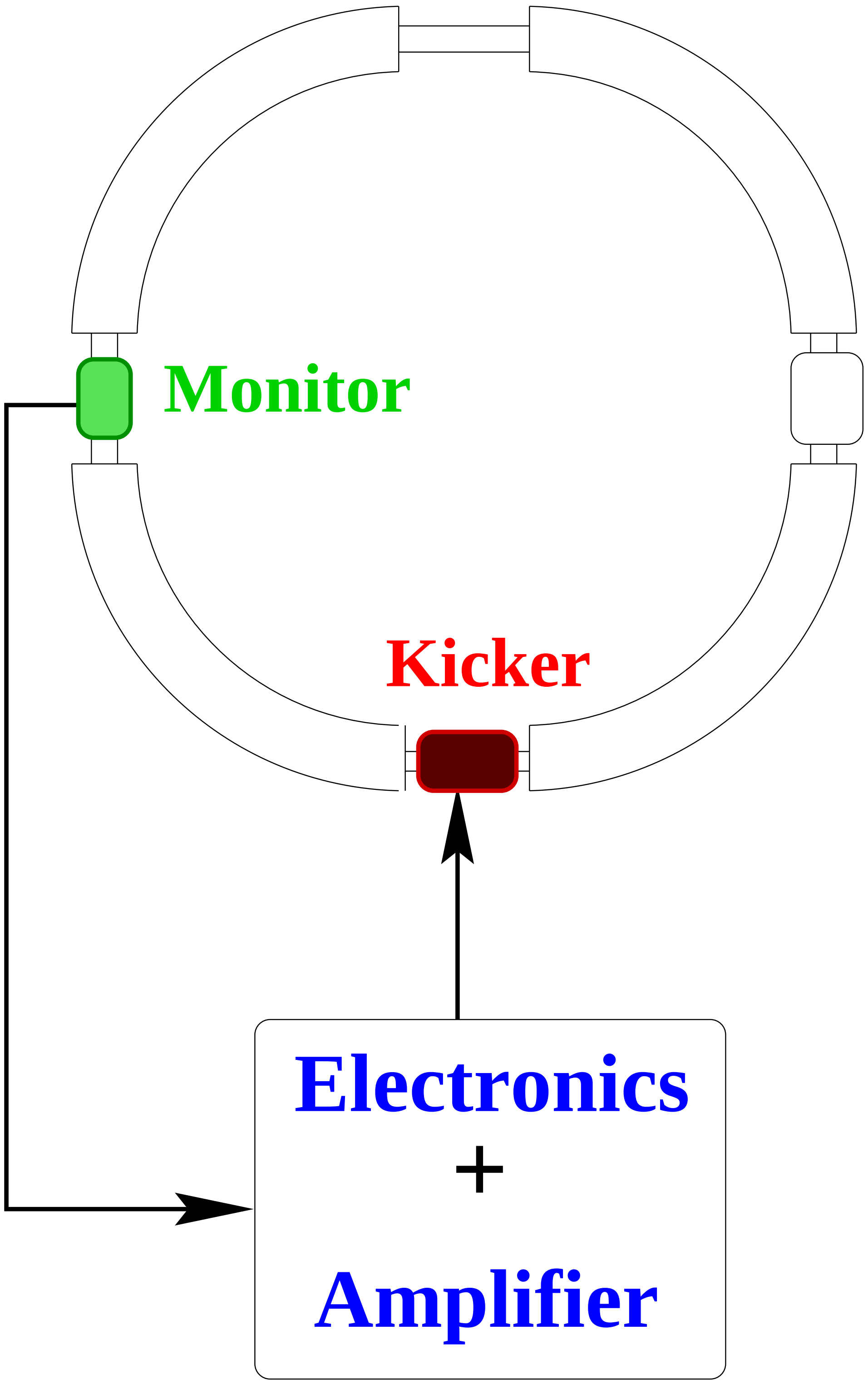
- E and B fields depend on chamber and beam distribution
  - super imposition of multipole moments

→ design of chamber geometry

→ instability estimates

→ threshold currents

Feedback System



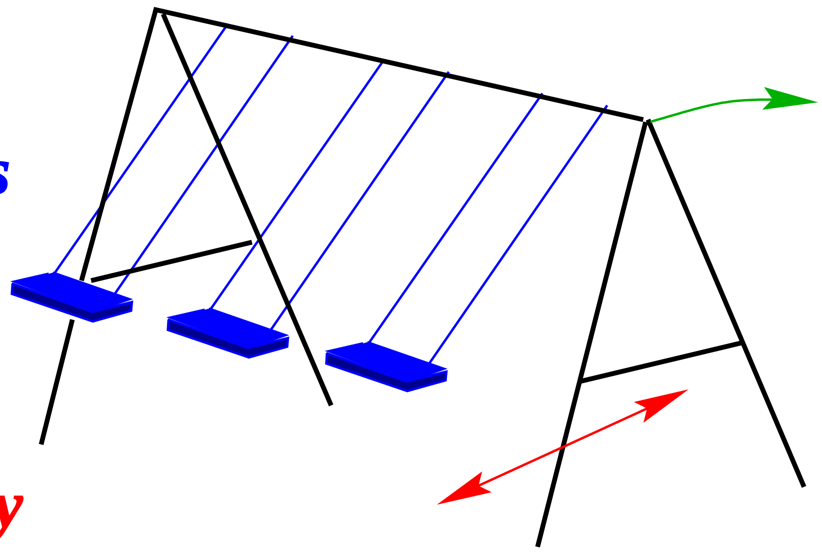
- **Limit:**

power and bandwidth

Landau Damping

Three Coupled Oscillators:

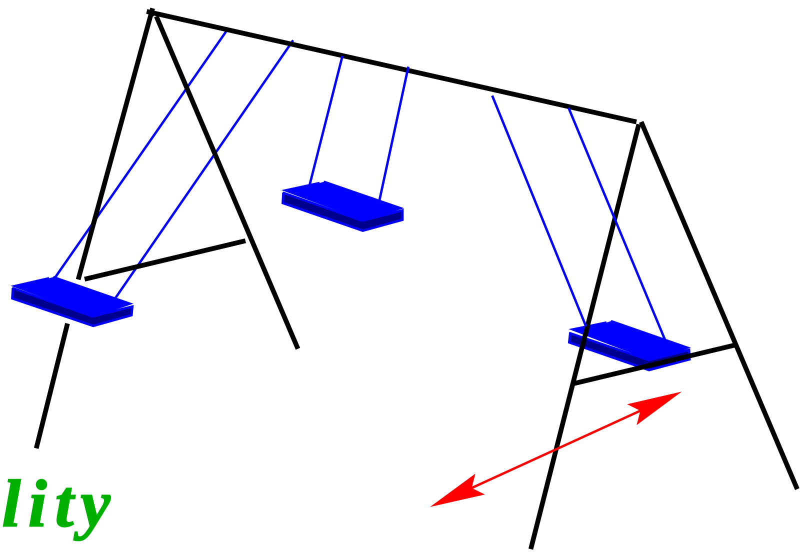
- **equal frequencies**



→ **instability**

Three Coupled Oscillators:

- **different frequencies**



→ **no instability**

Limit:

frequency spread (tune spread)

single particle resonances