Accelerators

Lecture III

Oliver Brüning AB/ABP

http://bruening.home.cern.ch/bruening/summer−school/lecture3

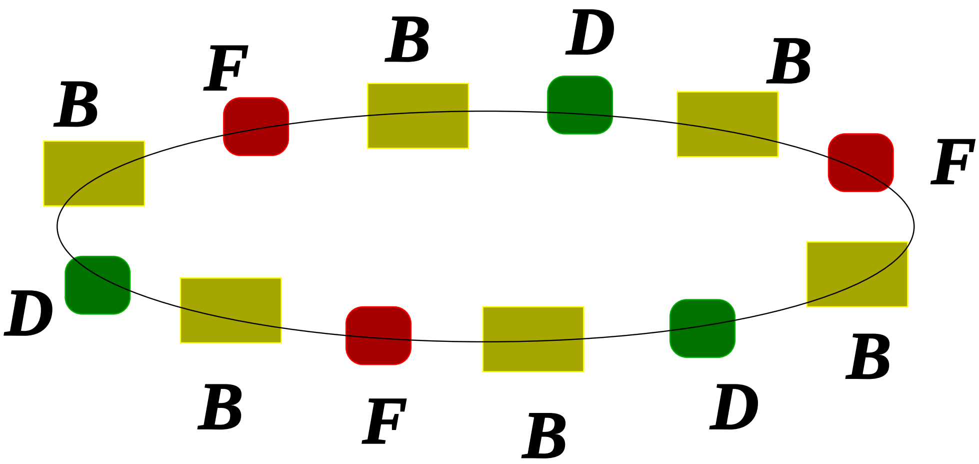
Summary Lecture II

- Collider Concept
- Weak and Strong Focusing
- Beam Optics
- Longitudinal Focusing

III) Orbit Stability + Long Term Stability

- Closed Orbit
- Linear Resonances + Orbit Stability
- Chromaticity + Sextupoles
- Long Term Stability
  - Non-linear resonances
  - Detuning with amplitude
- Summary

Closed Orbit



$$B_x = -g \cdot y$$

$$B_y = -g \cdot x$$

Orbit Offset in Quadrupole:

$$x = x_0 + \tilde{x}$$

quadrupole

$$B_x = -g \cdot \tilde{y}$$

$$B_y = -g \cdot x_0 - g \cdot \tilde{x}$$

dipole component

→ orbit error

Sources for Orbit Errors

- **Alignment:** +/− 0.1 mm

- **Ground motion**
  - slow drift
  - civilisation
  - moon
  - seasons
  - civil engineering

- **Error in dipole strength**
  - power supplies
  - calibration

- **Energy error of particles**

Orbit error:

→

- *x-y coupling*
- *aperture*
- *ernergy error*
- *field imperfections*

Aim:

$\Delta x,\ \Delta y < 4$ mm

rms < 0.5 mm

→ *beam monitors and orbit correctors*

LEP:

784 quadrupoles

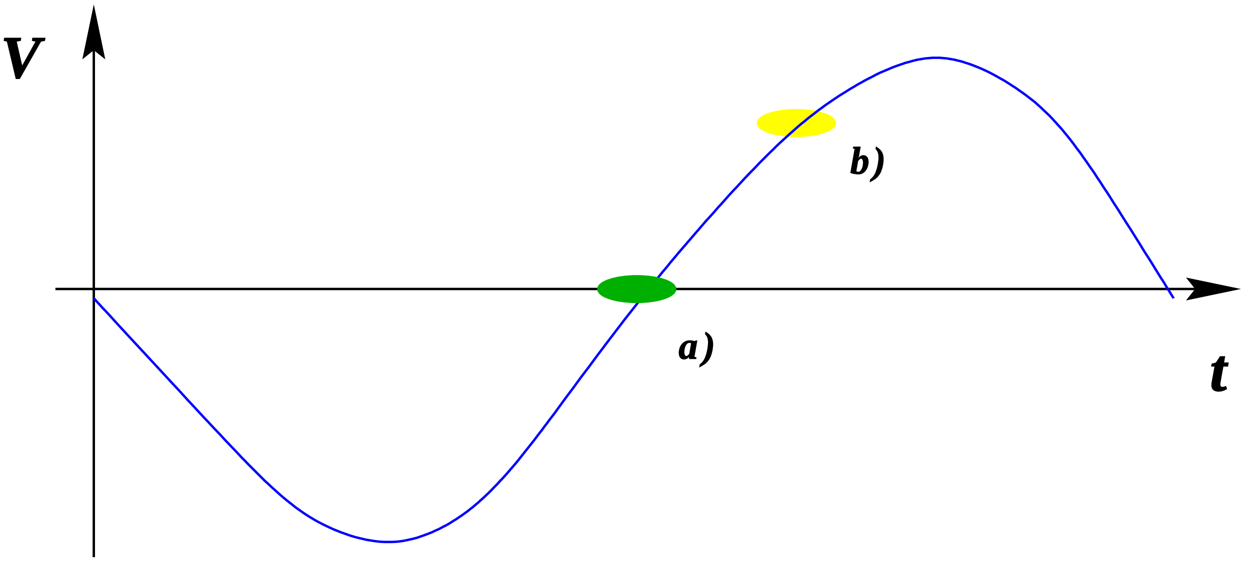
≈ 2 • 540 monitors

≈ 2 • 300 correctors

Synchrotron:

→ **the orbit determines the particle energy!**

assume: L > design orbit



→ **energy increase**

Equilibrium:

$$f_{RF} = h \cdot f_{rev}$$

$$f_{rev} = \frac{1}{2 \cdot \pi} \cdot \frac{q}{m \cdot \gamma} \cdot B$$

→ **E depends on orbit and magnetic field!**

momentum compaction factor:

- ***increase particle energy***

  → *velocity increase*

  shorter revolution time

  → *momentum increase*

  longer revolution time

- ***transition energy***

$$\frac{\Delta R}{R} = \alpha \cdot \frac{\Delta p}{p}$$

$$\alpha = \frac{1}{\gamma_t^2} \qquad \alpha \approx \frac{1}{Q^2}$$

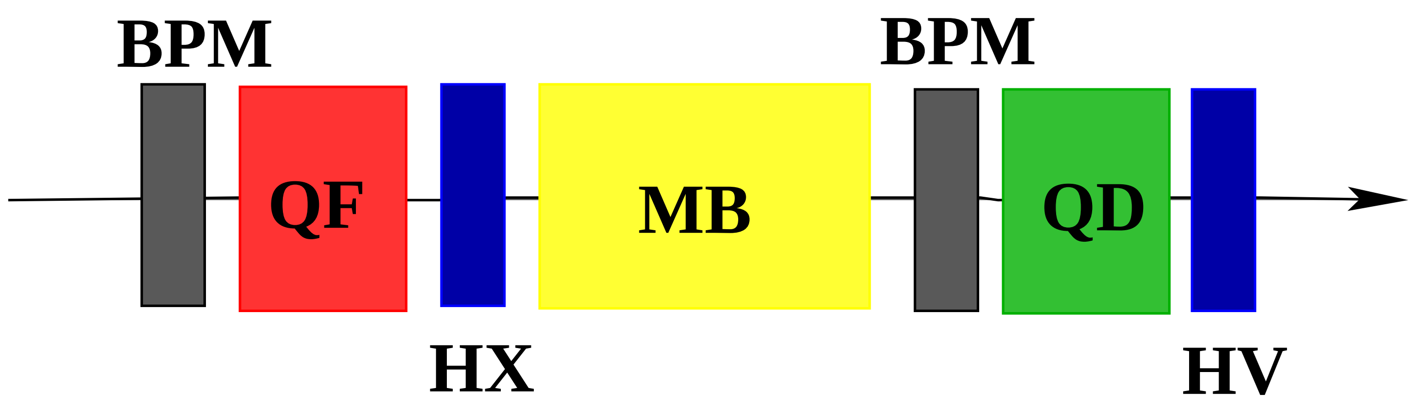
→ ***E error depends on transition energy!***

Orbit Correction

- aim at a local correction of the dipole error due to the quadrupole alignment errors

→ place orbit corrector and BPM next to the main quadrupoles

→ horizontal BPM and corrector next to QF
vertical BPM and corrector next to QD



→

relative alignment of BPM and quadrupole?

Particle Motion:

- ***find closed orbit***

  $x_0(s); y_0(s)$

- ***transverse oscillations around closed orbit***

→ ***complete description of particle motion***

CO: ***periodic:*** $x_0(s + L) = x_0(s)$

common to all particles

ϕ, β: ***periodic:*** $\beta_0(s + L) = \beta_0(s)$

common to all particles

ϕ_0, A: ***individual particle motion***

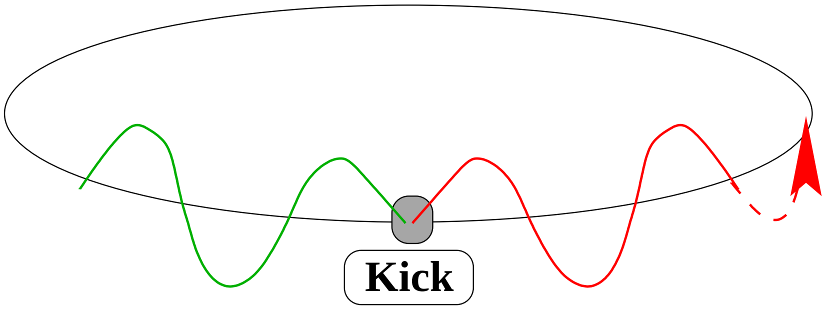
ε: ***beam ensemble***

beam quality

Dipole Error and Orbit Stability

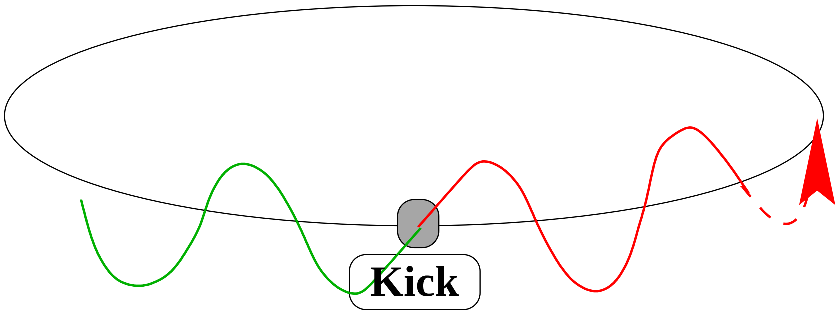
Q: number of β-oscillations per turn

Q = N + 0.5



→ the perturbation cancels after each turn

Q = N



→ the perturbation adds up

watch out for integer tunes!

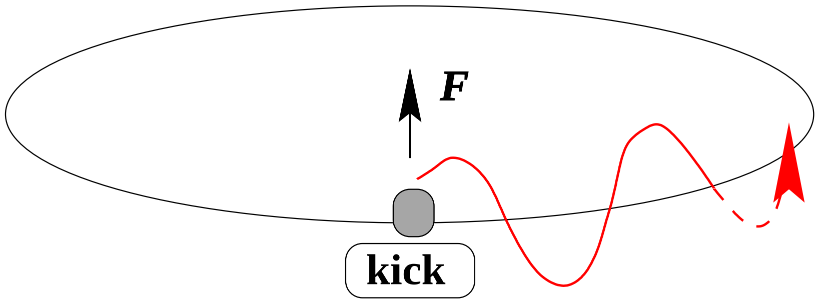
Quadrupole Error and Orbit Stability

Quadrupole Error:

→ orbit kick proportional to beam offset in quadrupole

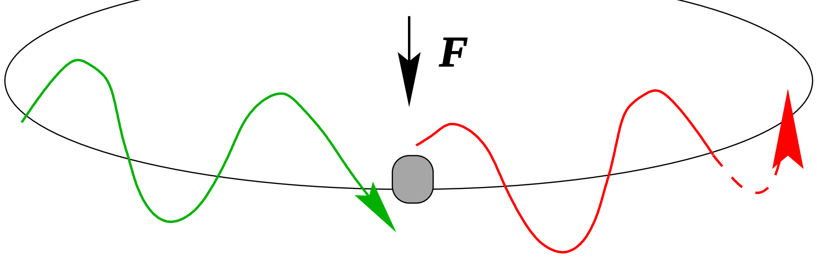
$Q = N + 0.5$

1. Turn: $x > 0$



→ amplitude increase

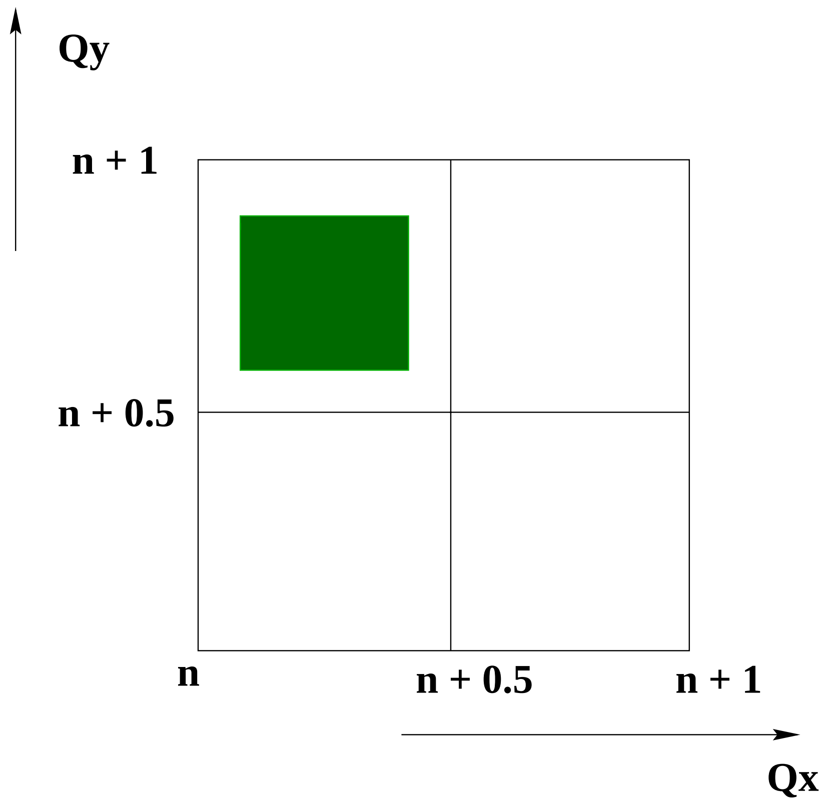
2. Turn: $x < 0$



→ amplitude increase

→ **watch out for half integer tunes!**

Tune Diagram



Problem:

$$K\ (quadrupole) = \frac{e \cdot g}{p} \quad \text{(lecture II)}$$

$$\longrightarrow \quad Q = Q_0 + \xi \cdot \frac{\Delta p}{p_0}$$

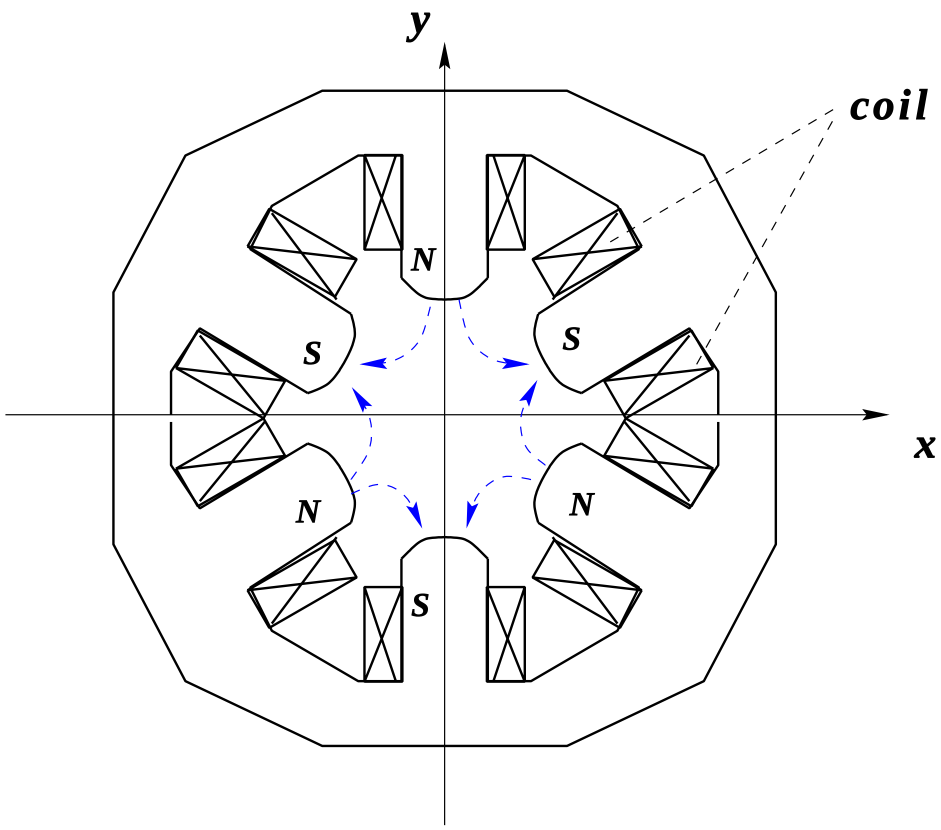
Large Machine (LEP / LHC):

$$\xi \approx 100 \div 500$$

$$\frac{\Delta p}{p_0} \approx 10^{-3}$$

→ **requires correction!**

Sextupole Magnet



$$B_x = \tilde{g} \cdot x \cdot y$$

$$B_y = \frac{1}{2} \cdot \tilde{g} \cdot (x^2 - y^2)$$

$$[\tilde{g}] = T / m^2$$

Orbit Offset:

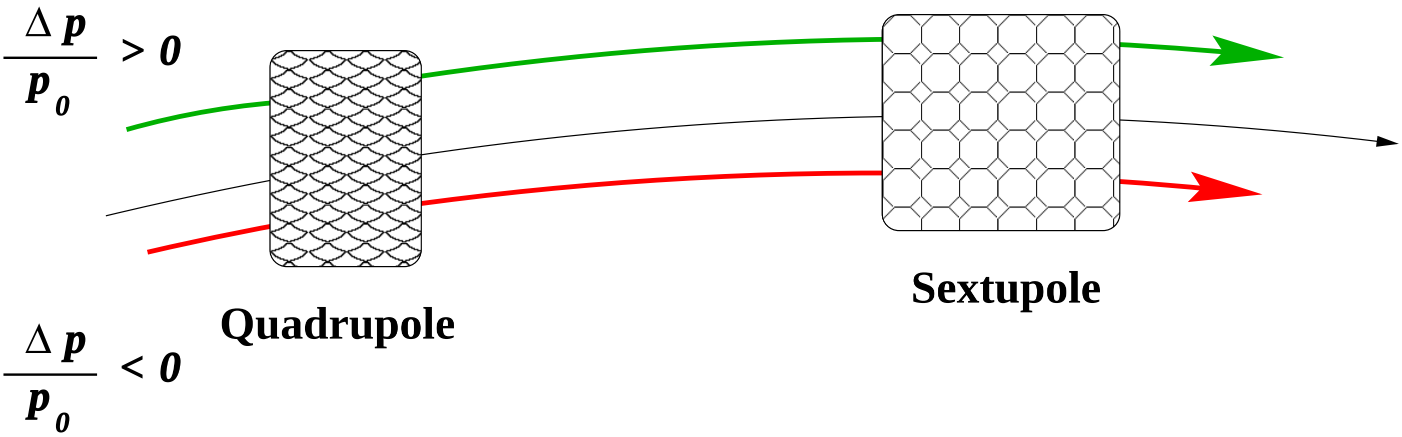
$$y = y_0 + \tilde{y}$$

$$B = \tilde{g} \cdot x \cdot y_0 + \tilde{g} \cdot x \cdot \tilde{y}$$

quadrupole component

$$B = \frac{1}{2} \cdot \tilde{g} \cdot (x^2 - \tilde{y}^2) - \tilde{g} \cdot y_0 \cdot \tilde{y}$$

Chromaticity Correction



$$x(s) = x_0(s) + D(s) \cdot \frac{\Delta p}{p_0}$$

→ *offset in sextupole*

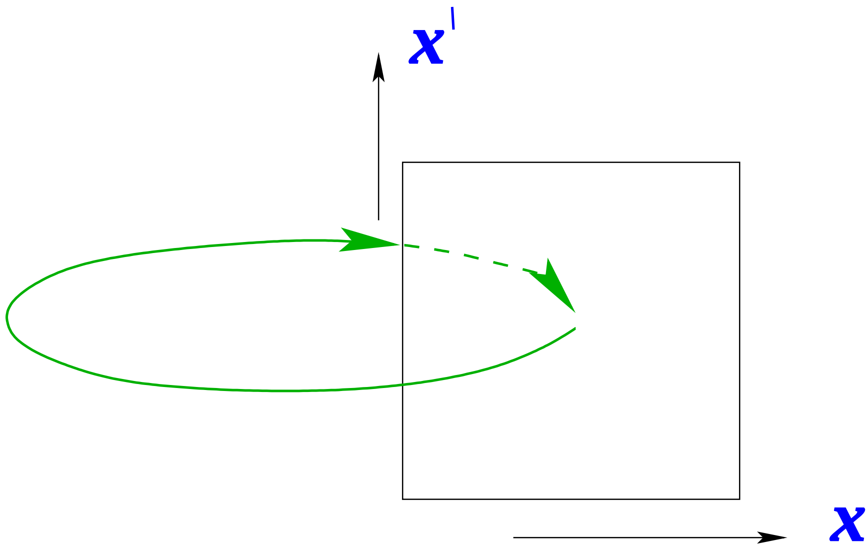
→ $$Q = Q_0 + \underbrace{\Delta Q_Q \left(\frac{\Delta p}{p_0}\right) + \Delta Q_S \left(\frac{\Delta p}{p_0}\right)}_{\approx\, 0}$$

- ***Problem:***

non-linear resonances

Poincare Section

Display coordinates after each turn:

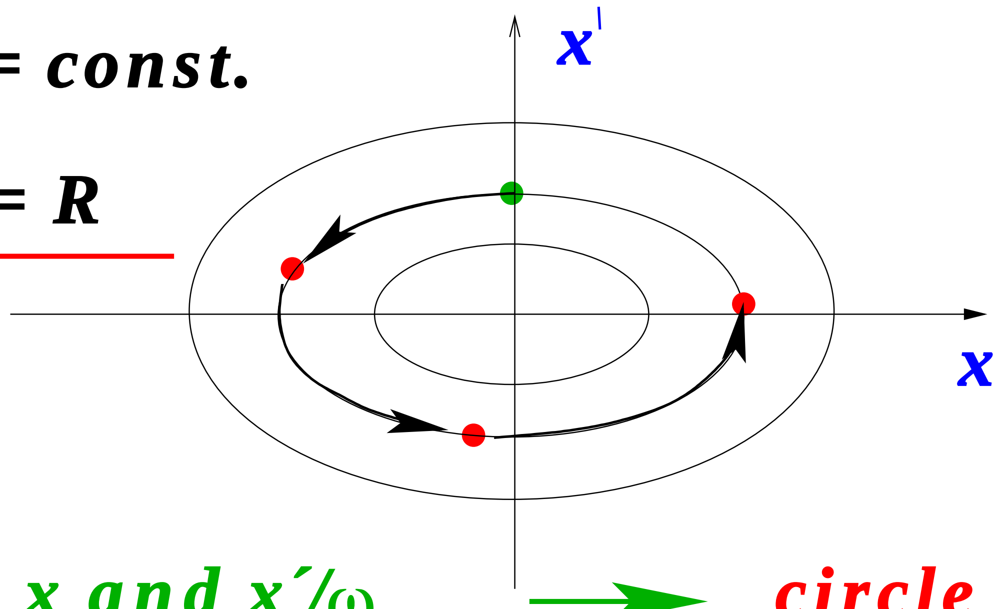


Linear β − motion:

$$x = \sqrt{R} \cdot \sin\left(\omega_0 \cdot s + \phi_0\right) \qquad (\beta = const)$$

$$x' = \omega_0 \cdot \sqrt{R} \cdot \cos\left(\omega_0 \cdot s + \phi_0\right) \qquad \left(\omega_0 = \frac{2\pi}{L} \cdot Q\right)$$

$$\rightarrow \quad x^2 + \frac{x'^2}{\omega_0^2} = const. = R$$



→ **ellipse**

display x and x'/ω_0 → **circle**

Sextupole Perturbation

Lorentz Force:

$$\frac{d}{ds}x' = -\omega^2 \cdot x + \frac{F_x}{v \cdot p}$$

$$F_x = q \cdot v_s \cdot B_y \longrightarrow \frac{d}{ds}x' = -\omega^2 \cdot x + q \cdot \frac{B_y}{p}$$

Single Sextupole Magnet: $B_y = \frac{1}{2} \cdot \tilde{g} \cdot x^2$

$$\longrightarrow \Delta x' = \int_l \frac{F}{v \cdot p}\, ds$$

$$= \frac{1}{2} \cdot l \cdot k_2 \cdot x^2$$

with: $y = 0$ **and:** $k_2 = \frac{q}{p} \cdot \tilde{g} \cdot x^2$

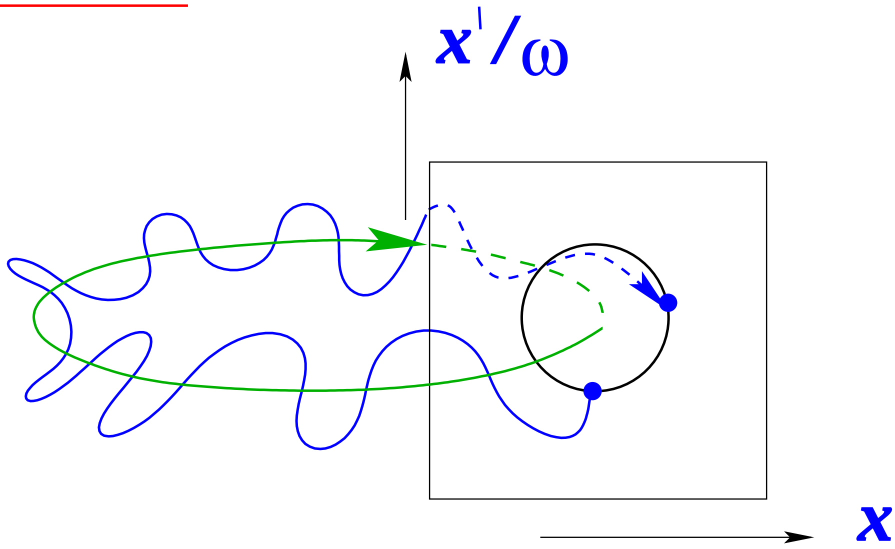
$$k_2 = 0.3 \cdot \frac{B[T/m]}{p[GeV]} \qquad [k_2] = m^{-3}$$

→ *is the particle motion still stable?*

Amplitude Growth

Poincare Section:

$$R = x^2 + x'^2/\omega^2$$



$$\frac{dR}{ds} = 2 \cdot x \cdot x' + 2 \cdot x' \cdot x''/\omega^2$$

with: $x'' = -\omega^2 \cdot x + \frac{F_x}{v \cdot p}$

Sextupole Kick:

$$\frac{\Delta R}{Turn} = \frac{1}{4} \cdot l \cdot k_2 \cdot R^{3/2} \cdot \left[3\cos(\phi) + \cos(3\phi)\right]/\omega$$

$(\Delta\phi / Turn = 2\pi Q)$

Many Turns:

$\Delta R = 0$ **unless:** $Q, 3 \cdot Q = n$

Detuning with Amplitude

Non-linear Perturbation:

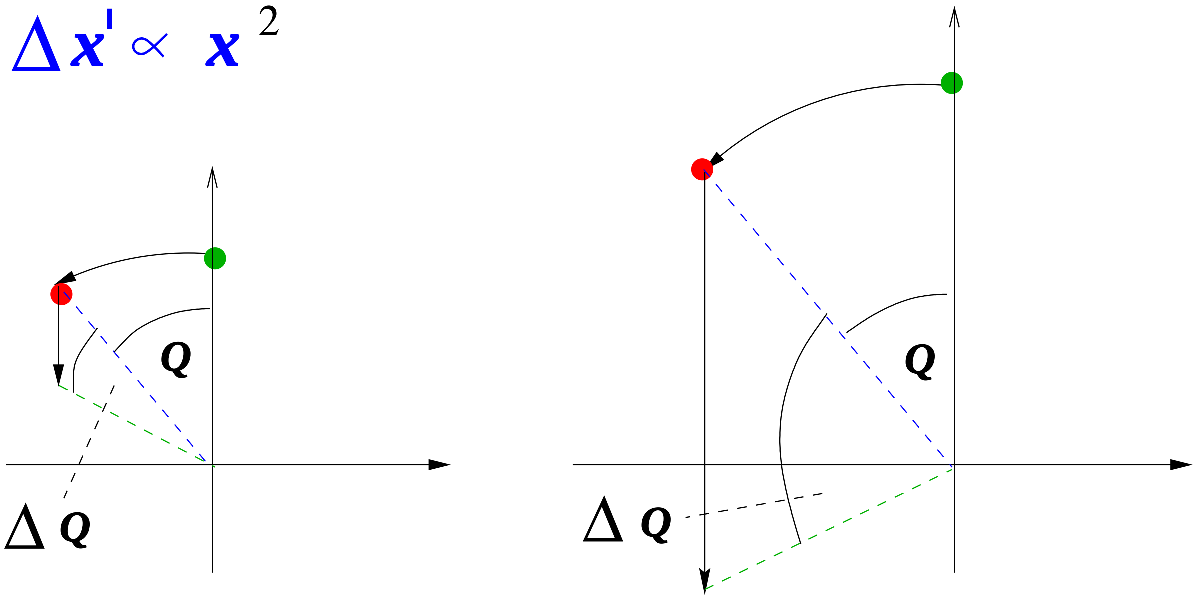
avoid resonances: $r \cdot Q = n$!

Problem:

there are resonances everywhere!

Stabilisation Mechanism:

$\Delta x' \propto x^2$



Poincare section

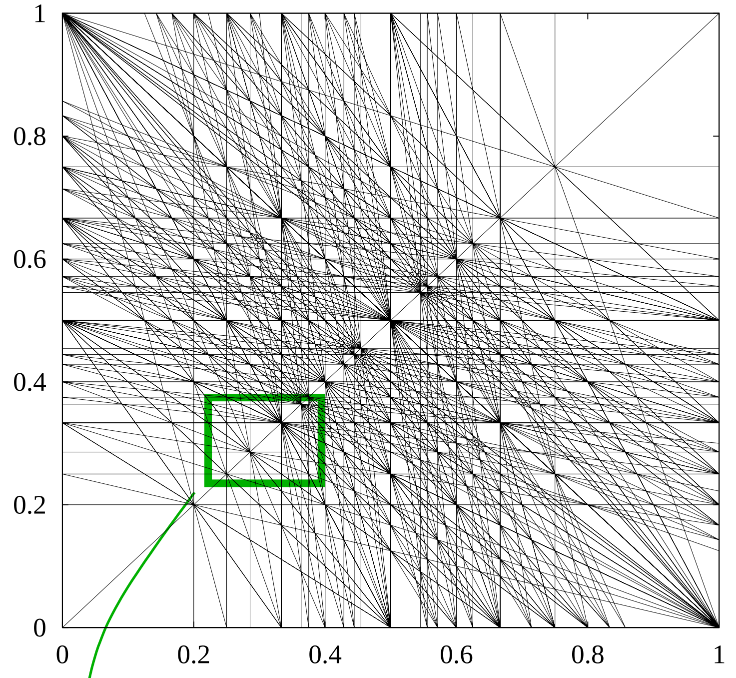
Tune Diagram

resonances: $n \cdot Q_x + m \cdot Q_y + r \cdot Q_s = p$

strength: $h \propto A^{n+m+s}$

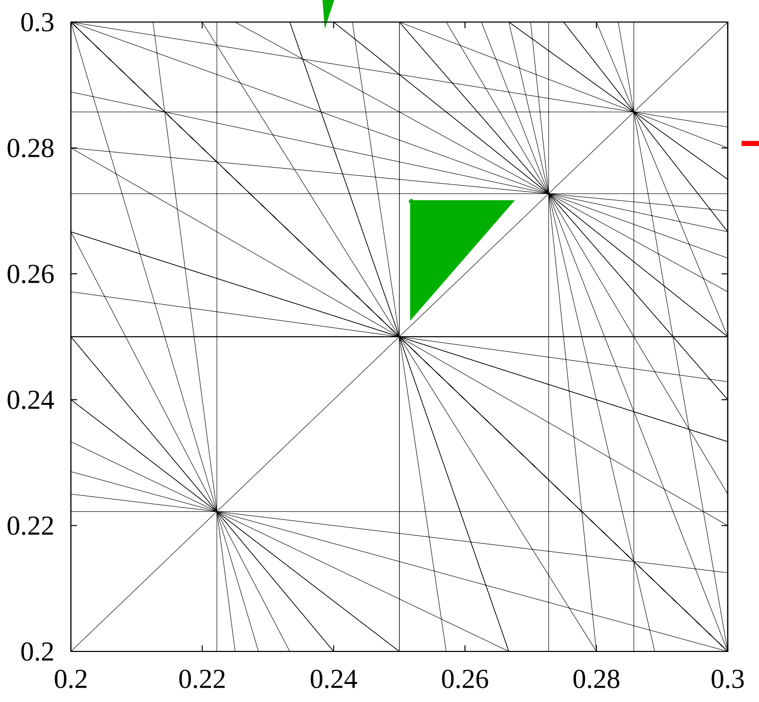
→ **avoid low order resonances!**

Q_x



Q_y

Q_x



Q_y

limits for b_n and tune changes

Sources for Non-Linear Fields

- **Sextupoles**

- **Magnet errors:**

  pole face accuracy

  geometry errors

  eddy currents

  edge effects

- **Vacuum chamber:**

  LEP I welding

- **Beam-beam interaction**

→ careful analysis of all components

Long Term Stability

Non-linear Perturbation:

- *amplitude growth*
- *detuning with amplitude*
- *coupling*

sextupole: $(B_y = g \cdot [x^2 - y^2])$

→ **Complex dynamics:**

3 degrees of freedom

\+ *1 invariant of the motion*

\+ *non-linear dynamics*

→ ***no analytical solution!***

→ *analysis of long term stability relies on numerical simulations*

Summary Resonances

- **Linear Optics:**

$$Q = n \cdot \pi \; ; \; n \cdot \pi + \frac{1}{2}$$

- **Chromaticity:**

  → *sextupoles*

  → *resonance driving terms*

- **Non-Linear Resonances:**

  → *amplitude growth*

  → *detuning with amplitude*

long term stability?

→ *classical mechanics*
+
chaos theory

Accelerator Model

- **Toy Model:** → simple

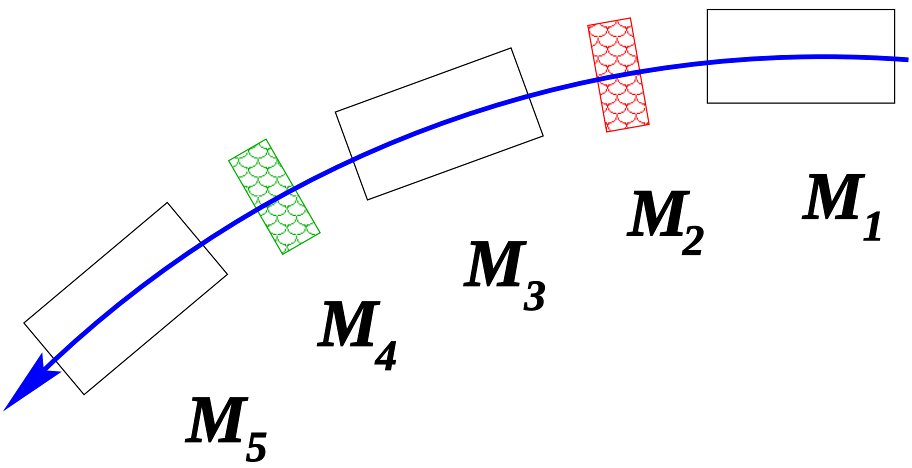
  HO + perturbation

  → Hamilton Function

  # > 1000 elements!

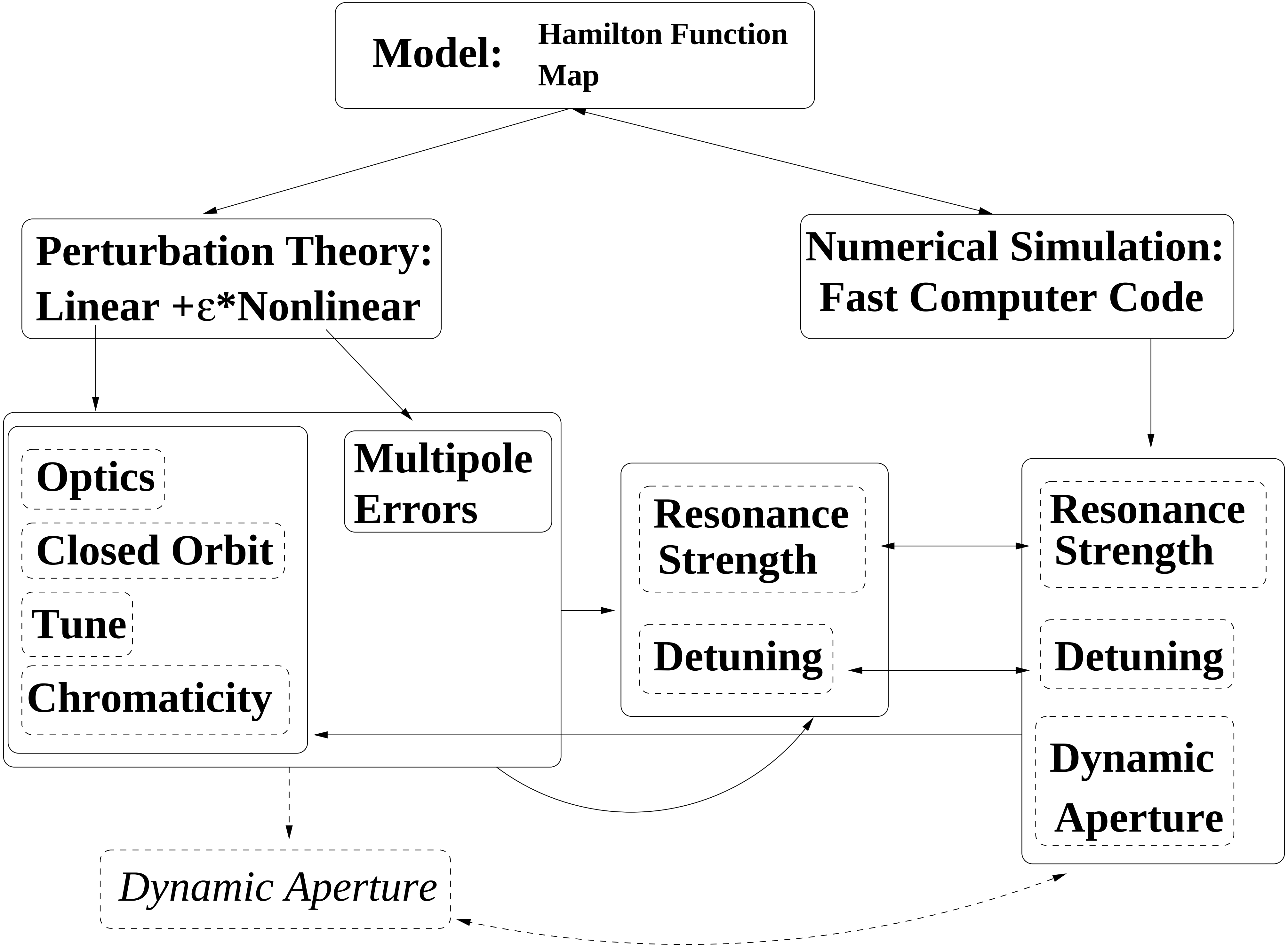
- **Element by Element Tracking**

  → numberical analysis



→ One Turn Map (Taylor Series)

→ Hamilton Function


Model:
Hamilton Function
Map
Perturbation Theory:
Linear +ε*Nonlinear
Numerical Simulation:
Fast Computer Code
Optics
Closed Orbit
Tune
Chromaticity
Multipole
Errors
Resonance
Strength
Detuning
Resonance
Strength
Detuning
Dynamic
Aperture
Dynamic Aperture