U(1) local gauge invariance and QED

$$\phi(x) \to e^{i\alpha(x)Q}\phi(x)$$

$$L = \left(\partial_\mu \phi(x)\right)^\dagger \partial^\mu \phi(x) - m^2 \phi(x)^\dagger \phi(x)$$ not invariant due to derivatives

$$\partial_\mu \phi \to \partial_\mu e^{i\alpha(x)Q}\phi = e^{i\alpha(x)Q}\partial_\mu \phi + iQe^{i\alpha(x)Q}\phi\, \partial_\mu \alpha(x)$$

To obtain invariant Lagrangian look for a modified derivative transforming covariantly

$$D_\mu \phi \to e^{i\alpha(x)} D_\mu \phi$$

Need to introduce a new vector field $A_\mu \to A_\mu + \partial_\mu \alpha$

$$D_\mu = \partial_\mu - iQA_\mu$$

Extension to non-Abelian symmetry

(The Standard Model $SU(3)\otimes SU(2)\otimes U(1)$)

$SU(2)$ local gauge invariance

$$Q=\begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R$$

$$Q\rightarrow e^{ig_2\boldsymbol{\alpha}(x).\frac{\boldsymbol{\sigma}}{2}}Q$$

$$\sigma_1=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2=\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \quad \sigma_3=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$L = i\overline{Q}D_\mu\gamma^\mu Q+i\overline{u}_R\partial_\mu\gamma^\mu u_R+i\overline{d}_R\partial_\mu\gamma^\mu d_R$$

$$D_\mu=\partial_\mu+ig_2\frac{\sigma_i}{2}W_\mu^i$$

Need 3 gauge bosons

W^+, W^-, W^3

where

$$W_{\mu,i}\rightarrow W_{\mu,i}-\partial_\mu\alpha_i-g_2\varepsilon_{ijk}\alpha_j W_{\mu,k}$$

$$\left(\left[\frac{\sigma_i}{2},\frac{\sigma_j}{2}\right]=i\varepsilon_{ijk}\frac{\sigma_k}{2}\right)$$

$SU(3)$ local gauge invariance

$$\begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_8 = \sqrt{\frac{1}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots$$

Symmetry :

Local conservation of
3 strong colour charges

$$\Psi_a \to \left(e^{ig_3\boldsymbol{\alpha}(x).\boldsymbol{\lambda}}\right)_b^a \Psi_a$$

$$G_\mu^r \to G_\mu^r - \partial_\mu \alpha^r - g_3 f^{rst} \alpha^s G_\mu^t$$

$$\overline{\Psi}(\partial_\mu - ig_3\lambda_r G_\mu^r)\gamma^\mu \Psi$$

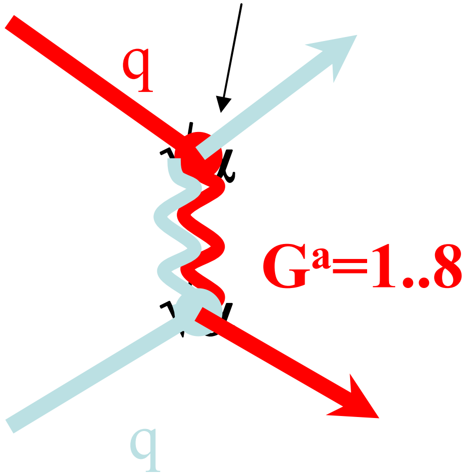
QCD : a non-Abelian (SU(3))
local gauge field theory

The strong interactions

QCD Quantum Chromodynamics

SU(3) $\begin{pmatrix} q \\ q \\ q \end{pmatrix}$

Strong coupling, α_3



Gauge boson
(J=1)
"Gluons"

Symmetry :

Local conservation of
3 strong colour charges

$$\Psi_a \to \left(e^{i\alpha(x).\lambda}\right)^a_b \Psi_a$$

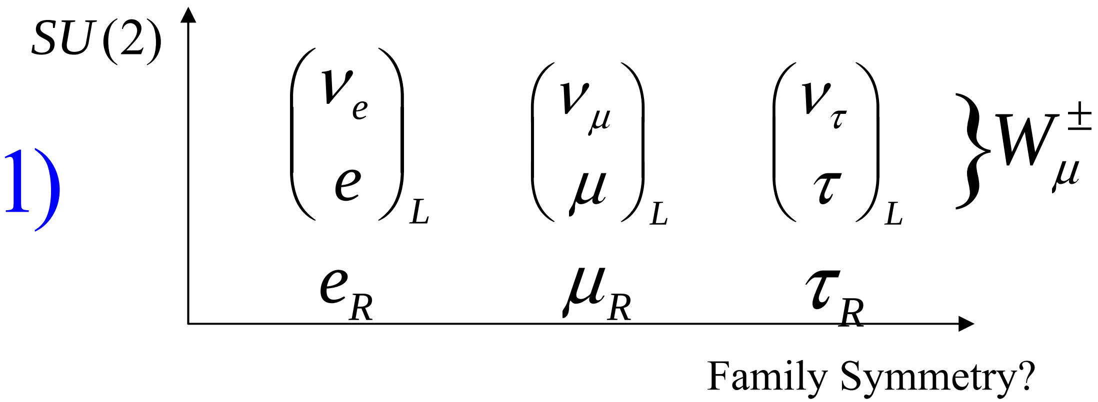
$$G^r_\mu \to G^r_\mu - \tfrac{1}{g_3}\partial_\mu \alpha^r - f^{rst}\alpha^s G^t_\mu$$

$$\overline{\Psi}(\partial_\mu - i g_3 \lambda_r G^r_\mu)\gamma^\mu \Psi$$

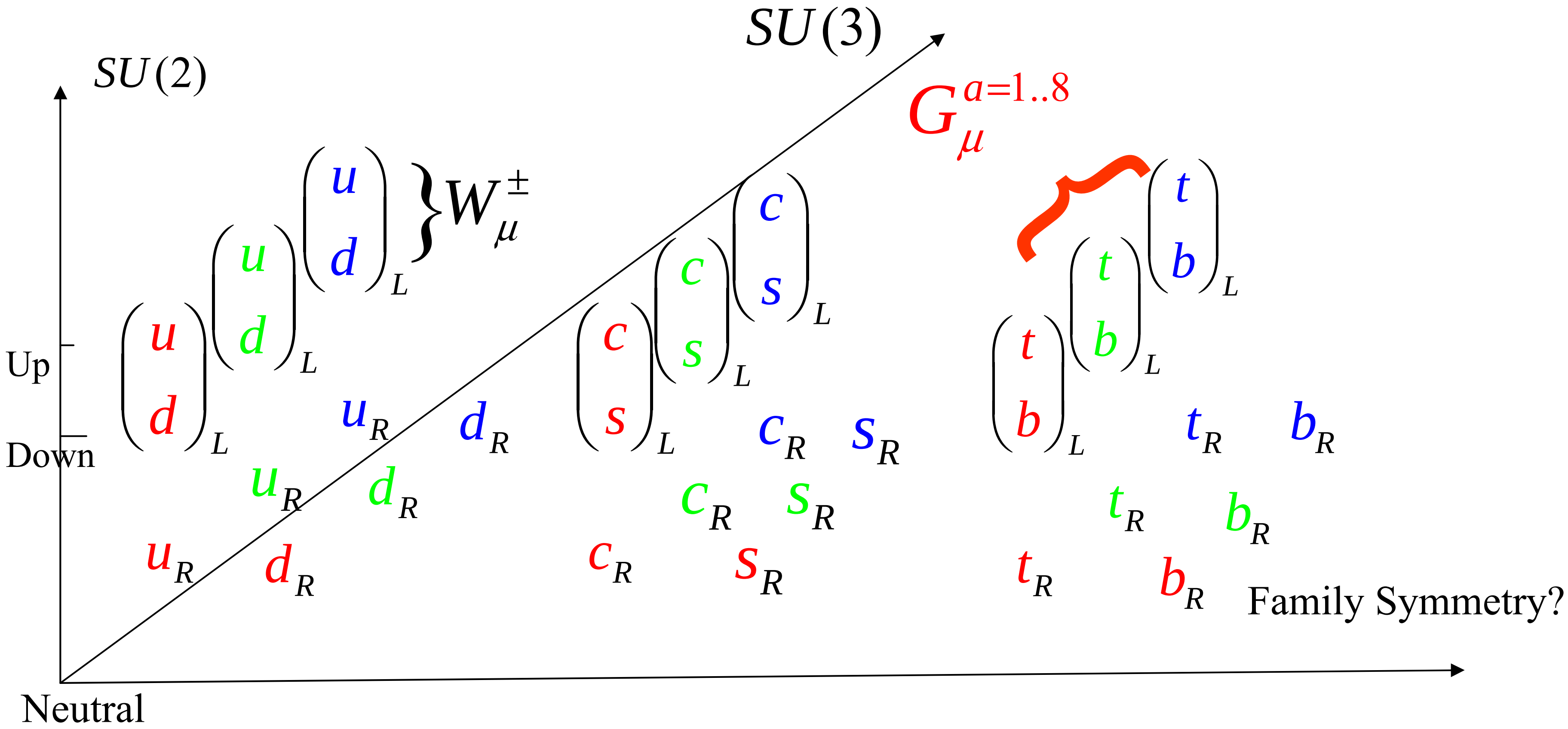
QCD : a non-Abelian (SU(3))
local gauge field theory

Partial Unification

$SU(3) \otimes SU(2) \otimes U(1)$



Matter Sector "chiral"



Fermions and the weak interactions

Fermi theory of β decay $\quad n \rightarrow pe\,\bar{\nu}_e$

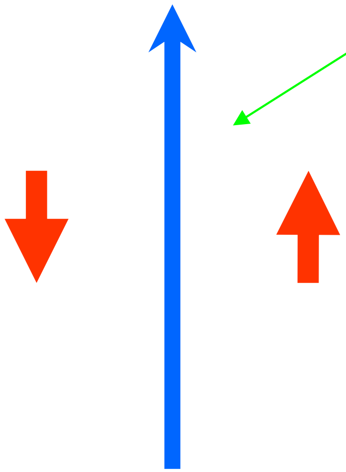
$$\frac{G_F}{\sqrt{2}}\left[\bar{u}_n \gamma^\sigma (1-\gamma_5) u_p\right]\left[\bar{u}_{\nu_e} \gamma_\sigma (1-\underbrace{\gamma_5) u_e}\right]$$

$$u_{e^-_L} \equiv u^c_{e^+_R}$$

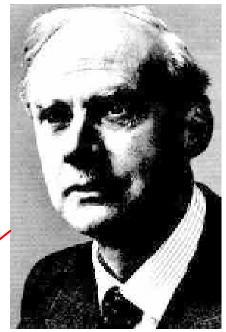
velocity

Left-handed

m=0



Right-handed

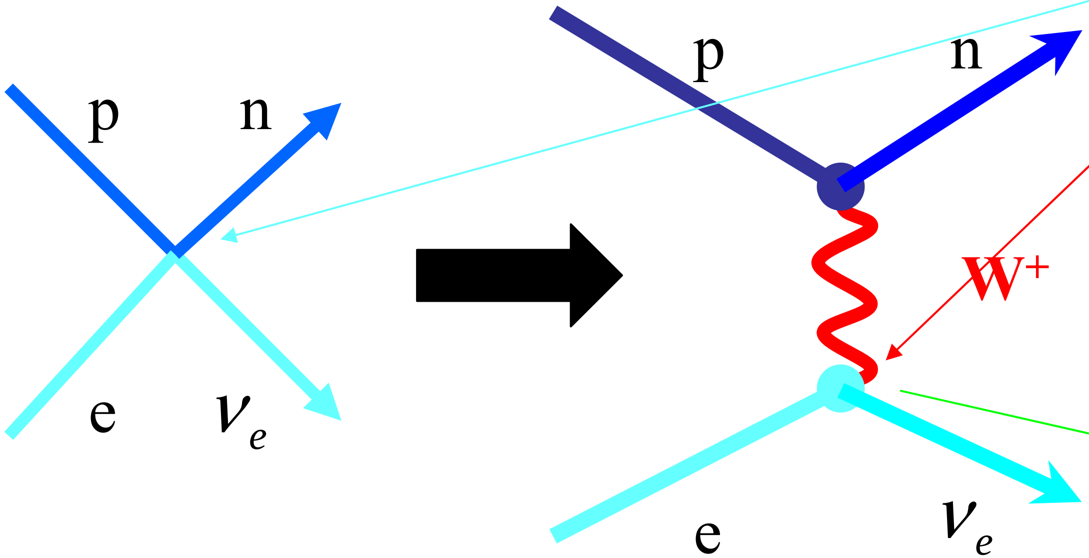


Fermions and the weak interactions

Fermi theory of β decay $\quad n \rightarrow p e \bar{\nu}_e$

$$L = \frac{G_F}{\sqrt{2}}\left[\bar{u}_n \gamma^\sigma (1-\gamma_5) u_p\right]\left[\bar{u}_{\nu_e} \gamma_\sigma (1-\gamma_5) u_e\right]$$

V-A $\quad \not{P}, \not{C}$



$$\frac{G_F}{\sqrt{2}} = \frac{g_2^{\,2}}{8M_W^{\,2}}$$

g_2

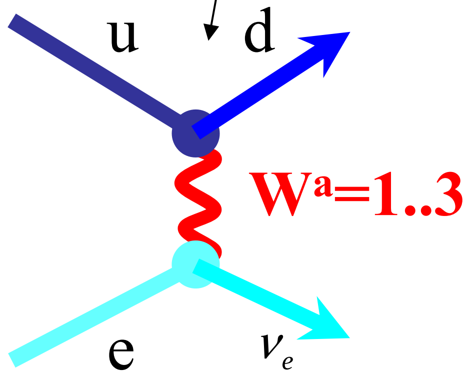
Weak Interactions

SU(2) local gauge theory

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$u_R, d_R, e_R$$

Weak coupling, α_2



Gauge boson
(J=1)

$$W^{\pm}, Z$$

Neutral currents

Symmetry :

Local conservation of
2 weak isospin charges

$$\Psi_a \rightarrow \left(e^{ig_2 \boldsymbol{\alpha}(x).\tau}\right)^a_b \Psi_a$$

$$W^r_\mu \rightarrow W^r_\mu - \partial_\mu \alpha^r - f^{rst} \alpha^s W^t_\mu$$

$$\overline{\Psi}(\partial_\mu - i g_2 \lambda_r W^r_\mu)\gamma^\mu \Psi$$

A non-Abelian (SU(2))
local gauge field theory

Massive vector propagator (W, Z bosons)

$$\left(g^{\nu\mu}(\partial^2 + M^2) - \partial^\nu \partial_\mu\right) B^\mu = j^\nu$$

$$(-g^{\mu\nu}(-p^2 + M^2) + p_\mu p_\nu)^{-1} = \frac{i(-g^{\mu\nu} + p^\mu p^\nu / M^2)}{p^2 - M^2}$$

$$B_\mu = \varepsilon_\mu e^{ip.x}$$

Free particle solution

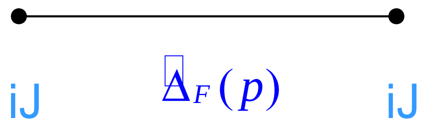
$$\varepsilon^{(\lambda=\pm 1)} = \mp(0, 1, \pm i, 0)/\sqrt{2}$$

$$\varepsilon^{(\lambda=0)} = (|\mathbf{p}|, 0, 0, E)/M$$

Helicity polarisation vectors

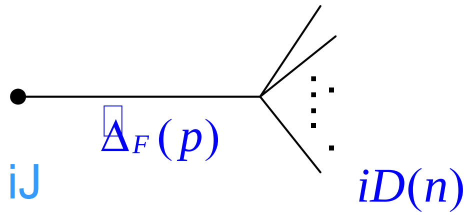
$$\sum_\lambda \varepsilon_\mu^{(\lambda)*} \varepsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2}$$

Propagation of unstable scalar particle



$$= -iJ^2 \tilde{\Delta}_F(p)$$

No decay



$$= -iJ \tilde{\Delta}_F(p) D(n)$$

Particle decays into final state n

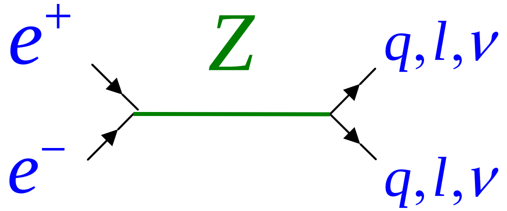
Optical theorem – conservation of probability, time evolution is unitary

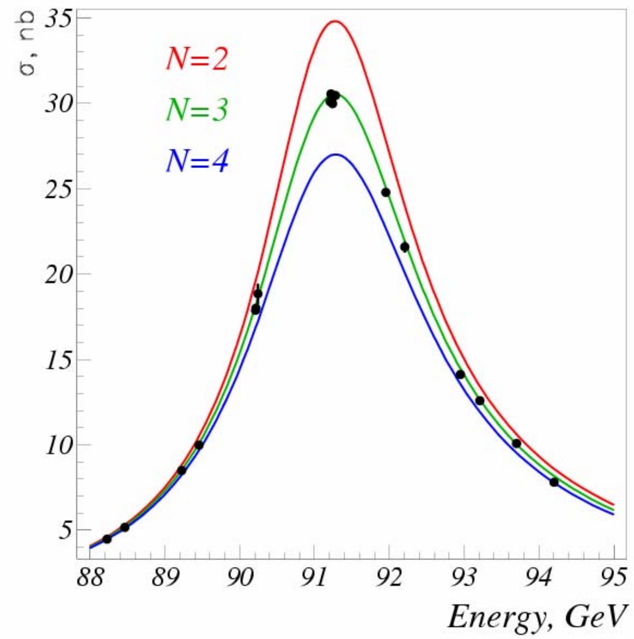
$$S^\dagger S = S S^\dagger = 1 \qquad S_{fi} = \delta_{fi} + i T_{fi}$$

$$\mathrm{Im}(T_{kk}) = \tfrac{1}{2} \sum_n \left| T_{nk} \right|^2$$

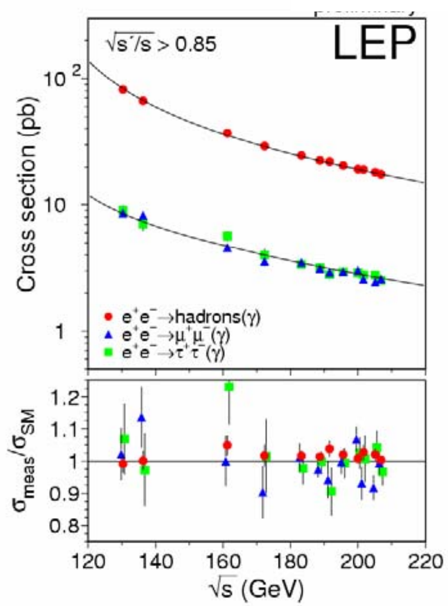
$$-J^2 \, \mathrm{Im}(\tilde{\Delta}_F(p)) = \tfrac{1}{2} \sum_n \left| -iJ \tilde{\Delta}_F(p) D(n) \right|^2 = J^2 \left| \tilde{\Delta}_F(p) \right|^2 \underbrace{\int \tfrac{1}{2} \sum_n \left| D(n) \right|^2 dQ}_{m\Gamma_{tot}}$$

$$\Rightarrow \quad \boxed{\tilde{\Delta}_F(p) = \frac{1}{p^2 - m^2 + i m \Gamma_{tot}}} \qquad (\Delta_F(x) \propto e^{-m \Gamma_{tot} t})$$

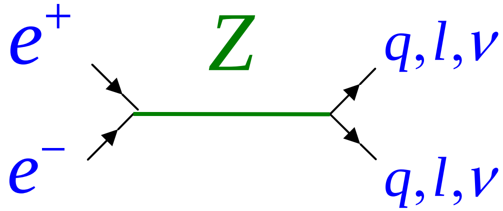


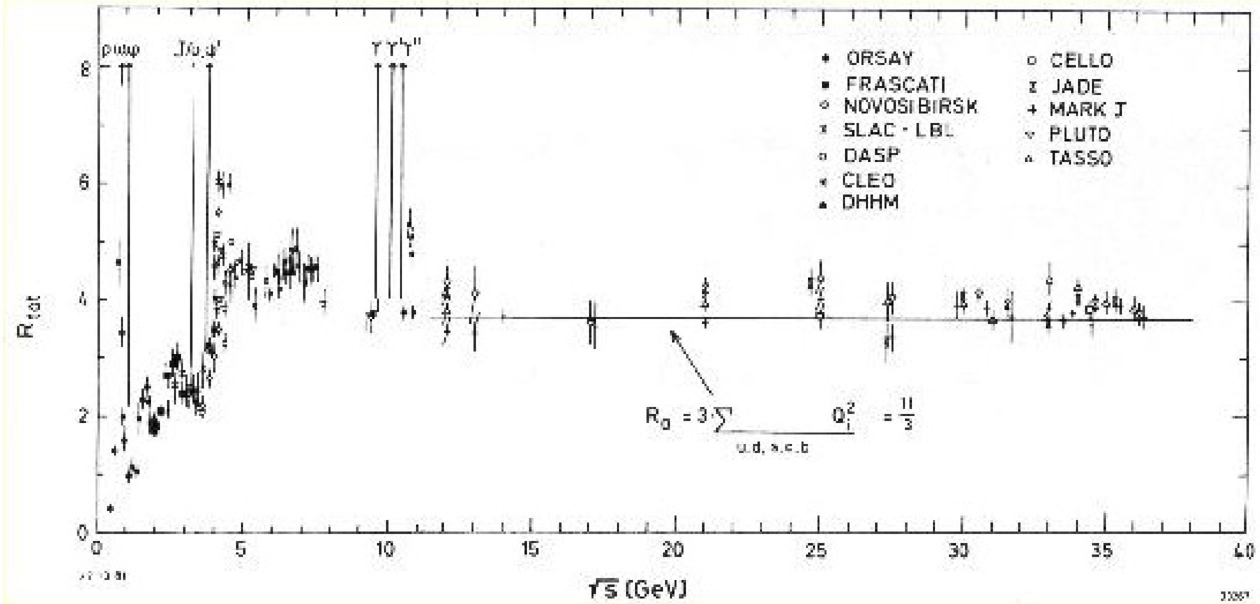


(N is no. of light - wrt M_Z - neutrinos)



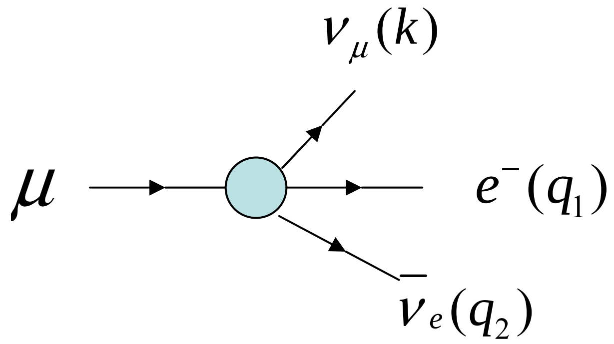
$$\sigma(e^+e^- \to hadrons) = \frac{12\pi\, \Gamma(Z \to ee)\Gamma(Z \to hadrons)}{(E^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$





$$\frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_{f=1}^{n_f} Q_f^2$$

μ decay



Fermi theory ('40s)

$$M = \frac{G_F}{\sqrt{2}} \bar{u}(k)\gamma^\mu(1-\gamma_5)u(p)\,\bar{u}(q\ \ )\gamma_\mu(1-\gamma_5)v(p)$$

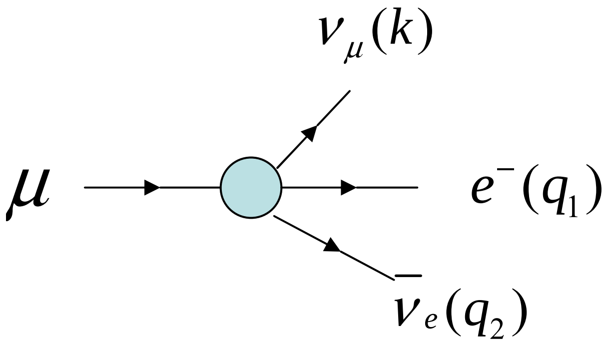
Dimensional analysis

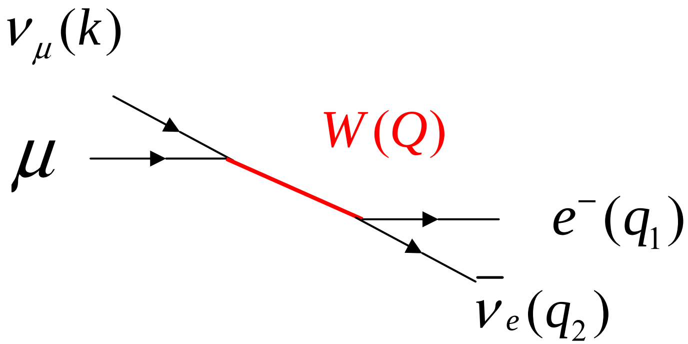
$$\Gamma_{tot} = \frac{1}{192\pi^3} m_\mu^5 G_F^2$$

The hard part!

$$\tau_\mu^{\mathrm{expt}} = \frac{1}{\Gamma} = 2.19703(4)\,10^{-6}\ \mathrm{sec} \quad \Rightarrow \quad G_F = 1.16637(1)\,10^{-5}\ GeV^{-2}$$

μ decay





$$\frac{Q^\mu Q^\nu}{M_W^2} \sim \frac{m_\mu m_e}{M_W^2} \sim 0$$

$$M = i g_W \, \bar{u}(k)\gamma^\mu(1-\gamma_5)u(p) \quad \frac{g_{\mu\nu} - \frac{Q_\mu Q_\nu}{M_W^2}}{Q^2 - M_W^2 + i\varepsilon} \quad g_W \, \bar{u}(q\ )\gamma_\mu(1-\gamma_5)v(p)$$

In μ decay $\quad Q^2 \le O(m_\mu^2) << M_W^2$

$$\Rightarrow \quad \frac{g_W^2}{Q^2 - M_W^2} \sim \frac{-g_W^2}{M_W^2} \quad \Rightarrow \quad \frac{g_W^2}{M_W^2} = \frac{G_F}{\sqrt{2}}$$

$G_F = 1.16637(1)\,10^{-5}\ GeV^{-2}$

$\sim 80 GeV$

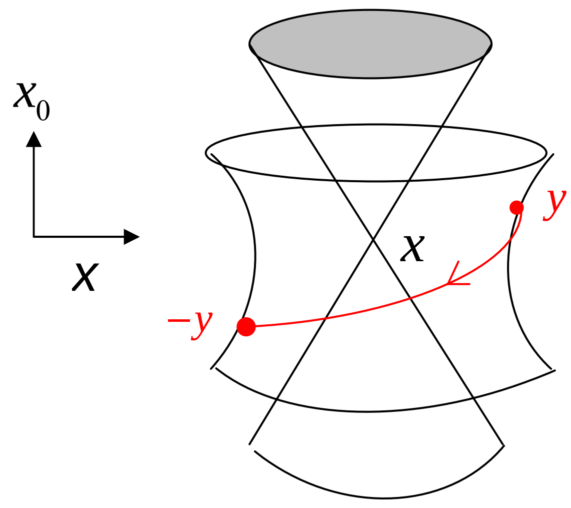
Causality?

QM : $U(x'-x) \propto e^{-m\sqrt{(\mathbf{x}'-\mathbf{x})^2-(t'-t)^2}}$ ✗

Field theory :

$$\Delta_F(x'-x) = -i\int \frac{d^3p}{(2\pi)^3 2\omega_p} e^{-i\omega_p|t'-t|-i\mathbf{p}.(\mathbf{x}'-\mathbf{x})}$$

$$= D(x-y) - D(y-x)$$



When $(x-y)^2 < 0$, we can perform a Lorentz transformation taking $(x-y) \to -(x-y)$
...causality preserved $(e^{-m|r|} - e^{-m|r|})$

No (continuous) transformation possible for $(x\text{-}y)^2 > 0$
...and amplitude nonvanishing $(e^{-imt} - e^{imt})$

Fundamental principles of particle physics

- Introduction - Fundamental particles and interactions
- Symmetries I - Relativity
- Quantum field theory - Quantum Mechanics + relativity
- Theory confronts experiment - Cross sections and decay rates
- Symmetries II – Gauge symmetries, the Standard Model
- Fermions and the weak interactions

⇒ The Standard Model and Beyond

Have Fun!