Fundamental experimental objects

$\Gamma(a_1 \to b_1 b_2 ... b_n)$ **Decay width = 1/lifetime** (Dimension 1/T=M)

$\sigma(a_1 a_2 \to b_1 b_2 ... b_n)$ **Cross section** (Dimension $L^2=M^{-2}$)

Units "barn"

$$1\,barn = 10^2\,fm^2$$
$$1\,mb = 10^{-1}\,fm^2 \quad \text{"}milli\text{"}$$
$$1\,\mu b = 10^{-4}\,fm^2 \quad \text{"}micro\text{"}$$
$$1\,nb = 10^{-7}\,fm^2 \quad \text{"}nano\text{"}$$
$$1\,pb = 10^{-10}\,fm^2 \quad \text{"}pico\text{"}$$
$$1\,fb = 10^{-13}\,fm^2 \quad \text{"}fempto\text{"}$$

(Natural Units $1 GeV^{-2} = 0.39 mb$)

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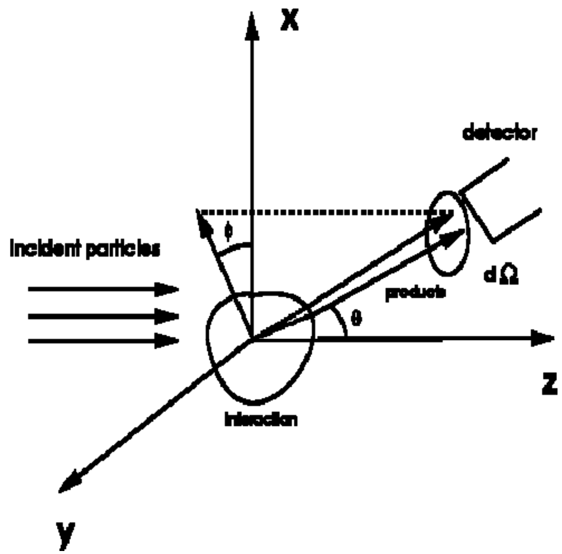

$$\text{Cross section} = \frac{\text{Transition rate x Number of final states}}{\text{Initial flux}}$$

Fundamental experimental objects

$\Gamma(a_1 \to b_1 b_2 \ldots b_n)$ **Decay width = 1/lifetime** (Dimension 1/T=M)

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Momenta of final state forms phase space

Cross section = $\dfrac{\text{Transition rate x Number of final states}}{\text{Initial flux}}$

For a single particle the number of final states in volume V with momenta

in element $d^3 p$ is $\dfrac{Vd^3 p}{(2\pi)^3}$ ⇒ $\prod_{i=1}^{n} \dfrac{Vd^3 p}{(2\pi)^3}$

Fundamental experimental objects

$$\Gamma(a_1 \rightarrow b_1 b_2 \ldots b_n)$$

Decay width = 1/lifetime

$$\sigma(a_1 a_2 \rightarrow b_1 b_2 \ldots b_n)$$

Cross section

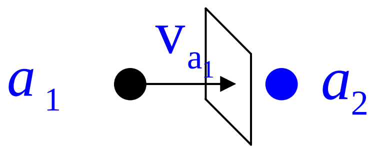
$$\Pi_{i=1}^{n} \frac{V d^3 p}{(2\pi)^3}$$

$$\text{Cross section} = \frac{\text{Transition rate x Number of final states}}{\text{Initial flux}}$$

$$\frac{|\mathrm{v}_{a_1}|}{V} \times \frac{1}{V}$$

particles passing through unit area in unit time

target particles per unit volume



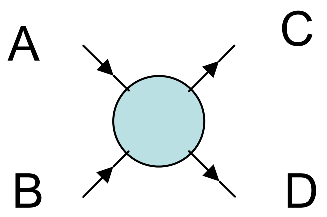
(Lab frame)

The transition rate

$$T_{fi} = -\int d^4x\, \phi_f^*(x) V(x) \phi_i(x) + \ldots$$

$$\phi_{i,f} \rightarrow f_p^{\pm} = e^{\mp ip.x} \frac{1}{\sqrt{2p^0 V}} \equiv \frac{N}{\sqrt{V}} e^{\mp ip.x}$$

e.g.



Transition rate per unit volume

$$W_{fi} = \frac{|T_{fi}|^2}{TV}$$

$$\phi_{f,i} = e^{\mp ip.x}$$

$$T_{fi} = -\frac{N_A N_B N_C N_D}{V^2} (2\pi)^4 \delta^4(p_C + p_D - p_A - p_B) \mathrm{M}_{fi}$$

$$W_{fi} = (2\pi)^4 \frac{\delta^4(p_C + p_D - p_A - p_B)|\mathrm{M}|^2}{V^4} \left(\frac{1}{2E_A}\right)\left(\frac{1}{2E_B}\right)\left(\frac{1}{2E_C}\right)\left(\frac{1}{2E_D}\right)$$

The cross section

$$\text{Cross section} = \frac{\text{Transition rate} \times \text{Number of final states}}{\text{Initial flux}}$$

$$d\sigma = \frac{V^2}{|\mathbf{v_A}|2E_A 2E_B}\frac{1}{V^4}|\mathrm{M}|^2\frac{(2\pi)^4}{(2\pi)^6}\delta^4(p_C + p_D - p_A - p_B)\frac{d^3p_C}{2E_C}\frac{d^3p_D}{2E_D}V^2$$

$$d\sigma = \frac{|\mathrm{M}|^2}{F}dQ$$

$$dQ = (2\pi)^4\delta^4(p_C + p_D - p_A - p_B)\frac{d^3p_C}{(2\pi)^3 2E_C}\frac{d^3p_D}{(2\pi)^3 2E_D}$$

Lorentz Invariant Phase space

$$F = |\mathbf{v_A}|2E_A 2E_B$$

$$= 4((p_A.p_B)^2 - m_A^2 m_B^2)^{1/2}$$

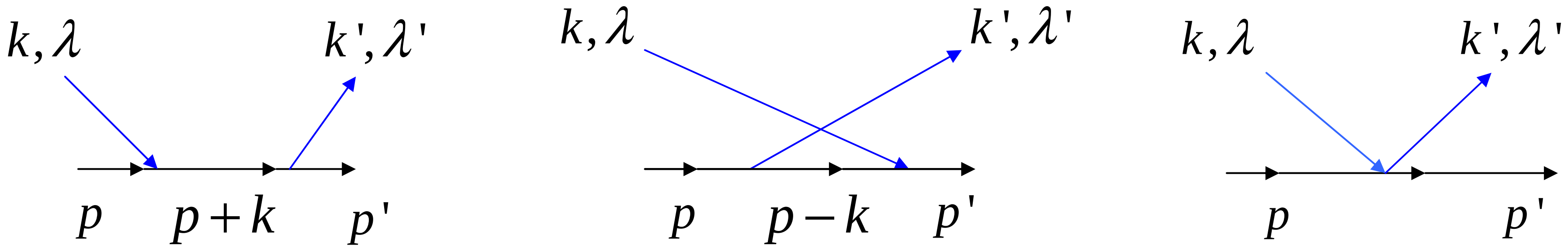
The decay rate

$$d\Gamma = \frac{1}{2E_A} |\mathrm{M}|^2 \, dQ$$

$$dQ = (2\pi)^4 \delta^4 (p_A - p_{B_1} \ldots - p_{B_n}) \frac{d^3 p_{B_1}}{(2\pi)^3 2E_{B_1}} \cdots \frac{d^3 p_{B_n}}{(2\pi)^3 2E_{B_n}}$$

Compton scattering of a π meson

$$\gamma\pi \to \gamma\pi$$

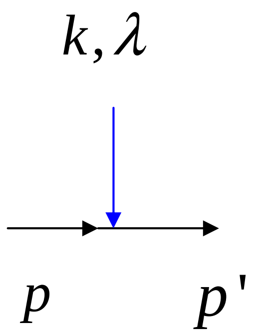


Feynman rules

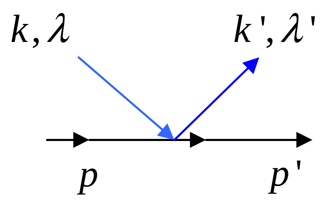
Klein Gordon

$$(\partial_\mu \partial^\mu + m^2)\psi = -V\psi$$

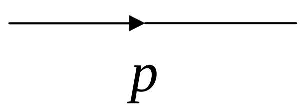
$$V = -ie(\partial_\mu A^\mu + A^\mu \partial_\mu) - e^2 A^2$$



$$-ie(p_\lambda + p'_\lambda)$$



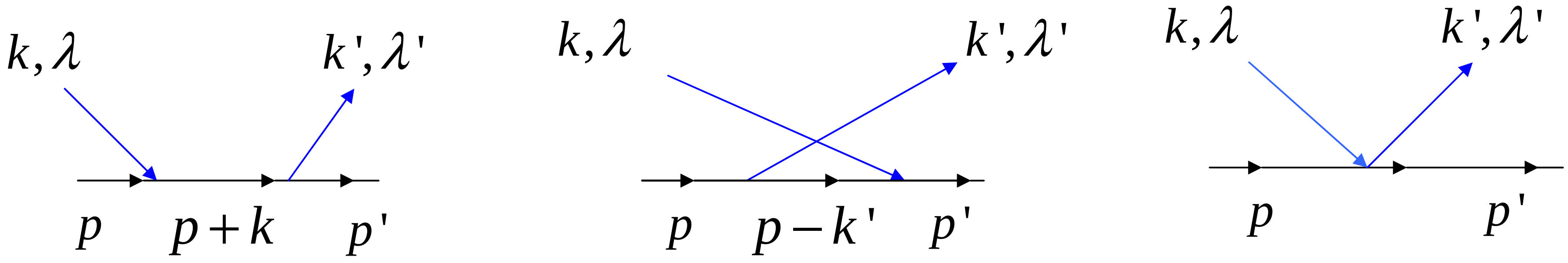
$$ie^2$$



$$\frac{i}{p^2 - m^2}$$

External photon ε^λ

Compton scattering of a π meson

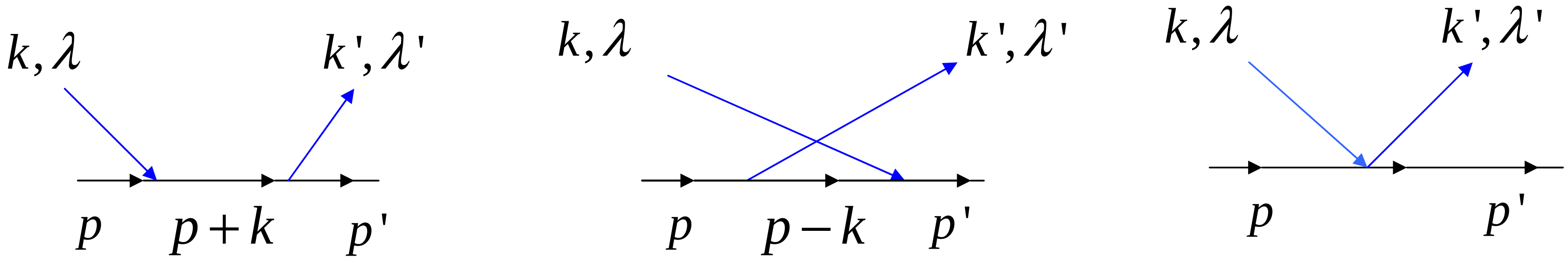


$$iM_{fi} = (-ie)^2 [\varepsilon.(2p+k)\frac{i}{(p+k)^2 - m^2}\varepsilon'.(2p'+k')$$

$$+\varepsilon.(2p'-k)\frac{i}{(p-k')^2 - m^2}\varepsilon'.(2p-k') - 2i\varepsilon.\varepsilon']$$

$$d\sigma = \frac{V^2}{|\mathbf{v_A}| 2E_A 2E_B}\frac{1}{V^4}|M|^2 \frac{(2\pi)^4}{(2\pi)^6}\delta^4(p_C + p_D - p_A - p_B)\frac{d^3 p_C}{2E_C}\frac{d^3 p_D}{2E_D}V^2$$

Compton scattering of a π meson



$$\mathrm{M}_{fi} = \varepsilon.(2p+k)\frac{i}{(p+k)^2-m^2}\varepsilon'.(2p'+k')$$

$$+\varepsilon.(2p'-k')\frac{i}{(p+k)^2-m^2}\varepsilon'.(2p'-k') - 2i\varepsilon.\varepsilon'$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{\alpha^2}{m^2}\frac{(\varepsilon.\varepsilon')^2}{\left[1+\frac{k}{m}(1-\cos\theta)\right]^2}$$

$(\varepsilon.p = \varepsilon'.p = 0 \text{ gauge})$

$$\sigma_{total}\big|_{k=0} = \int \frac{d\sigma}{d\Omega}.d\Omega = \frac{8\pi\alpha^2}{3m_\pi^2} \approx 8.10^{-2}\,GeV^{-2} = 3.10^{-2}\,mb$$

$$\sigma_{total}\big|_{k/m>>1} \approx \frac{2\pi\alpha^2}{mk}$$

- **The Lorentz transformations form a group, G** $(g_1 g_2 \in G \quad if \quad g_1, g_2 \in G)$

Rotations

$$R(\theta) = e^{-i\mathbf{J}.\theta/\hbar}, \quad J_z = i\hbar(x\tfrac{\partial}{\partial y} - y\tfrac{\partial}{\partial x})..$$

Angular momentum operator

$$(c.f.\ \mathbf{J} = \mathbf{r} \times \mathbf{p})$$

$$[J_i, J_j] = i\hbar \sum_{k=1}^{3} \varepsilon_{ijk} J_k$$

$$SO(3) \quad (SU(2))$$

$$e.g. \quad Spin \quad R(\theta)\begin{pmatrix}1\\0\end{pmatrix} = e^{-i\boldsymbol{\sigma}.\boldsymbol{\theta}/2\hbar}\begin{pmatrix}1\\0\end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \quad \sigma_2 = \begin{pmatrix}0 & i\\-i & 0\end{pmatrix} \quad \sigma_3 = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

The Lorentz group

Rotations J_i Boosts K_i

$$[J_i, J_j] = i\varepsilon_{ijk} J_k$$
$$[J_i, K_j] = i\varepsilon_{ijk} K_k$$
$$[K_i, K_j] = -i\varepsilon_{ijk} J_k$$

} Generate the group SO(3,1)

$$\left(M_{\rho\sigma} = i\left(x_\rho \tfrac{\partial}{\partial x^\sigma} - x_\sigma \tfrac{\partial}{\partial x^\rho}\right) \qquad J_i = \tfrac{1}{2}\varepsilon_{ijk} M_{jk} \qquad K_i = M_{0i}\right)$$

To construct representations a more convenient (non-Hermitian) basis is

$$N_i = \tfrac{1}{2}(J_i + iK_i)$$

$$[N_i, N_j] = i\varepsilon_{ijk} N_k$$
$$[N_i^\dagger, N_j^\dagger] = i\varepsilon_{ijk} N_k^\dagger$$
$$[N_i, N_j^\dagger] = 0$$

} $SU(2) \otimes SU(2)$ *representation* (n, m)

The Lorentz group

Rotations J_i Boosts K_i

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Representations $\quad J_i = N_i + N_i^\dagger$

$(n, m) \quad J = n + m$

$(0,0)$ scalar $J=0$

$(\frac{1}{2},0), (0,\frac{1}{2})$ LH and RH spinors $J=\frac{1}{2}$

$(\frac{1}{2},\frac{1}{2})$ vector $J=1$, etc

Weyl spinors

$(\frac{1}{2},0) \quad (0,\frac{1}{2})$

$\psi_L \qquad \psi_R$

2-component spinors of SU(2)

Rotations and Boosts $\quad \psi_{L(R)} \rightarrow S_{L(R)}\, \psi_{L(R)}$

$S_{L(R)} = e^{i\frac{\boldsymbol{\sigma}}{2}.\boldsymbol{\omega}}$: Rotations

$S_{L(R)} = e^{\pm\frac{\boldsymbol{\sigma}}{2}.\mathbf{v}}$: Boosts

†

Dirac spinor

Can combine $\psi_L,\ \psi_R$ to form a 4-component "Dirac" spinor $\psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}$

Lorentz transformations $\quad \psi \rightarrow e^{i\omega\sigma}\psi, \qquad \omega\sigma = \omega^{\mu\nu}\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]\omega^{\mu\nu}$

where $\quad \gamma_0 = \begin{bmatrix} 0 & -I \\ -I & 0 \end{bmatrix}, \quad \gamma_i = \begin{bmatrix} 0 & \boldsymbol{\sigma}_i \\ -\boldsymbol{\sigma}_i & o \end{bmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$

Weyl basis

†

$\omega^{0i} \rightarrow boosts, \quad \omega^{ij} \rightarrow rotations \qquad i, j = 1, 2, 3$

Weyl spinors

$(\tfrac{1}{2},0) \quad (0,\tfrac{1}{2})$

$\psi_L \qquad \psi_R$

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(Dirac gamma matrices, …new 4-vector γ_μ)

$$\left\{\gamma^\mu, \gamma^\nu\right\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$$

Note : $\quad \psi_{L(R)} = \tfrac{1}{2}(1 \mp \gamma_5)\psi$