UNIVERSITY OF Southampton

RBC–UKQCD Lattice Results for K_{l3} Decays and B_K

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New Opportunities in Kaon Physics
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Contents

Motivation

- **High precision**
  - Unitarity of first row of CKM matrix and quark-lepton universality

$$f_+^{K^0\pi^-}, f_K/f_\pi$$

  - Appearance of right-handed currents, Callan-Treiman point

$$f_0(M_K^2 - M_\pi^2) = f_K/f_\pi + O(m_{ud}/\Lambda_\chi)$$

- **Precision**
  - CP violation in neutral kaon mixing in SM and beyond

B_K and BSM matrix elements

- **Exploratory**
  - $\Delta I = 1/2$ rule
  - Direct CP violation in $K \to \pi\pi$ and Re (ϵ'/ϵ)

RBC–UKQCD lattice simulations

- We use domain wall fermions (DWF) for good chiral symmetry properties with Iwasaki gauge action
- Recent results: 2+1 flavour datasets with lattice spacing $a \approx 0.114\,\mathrm{fm}$

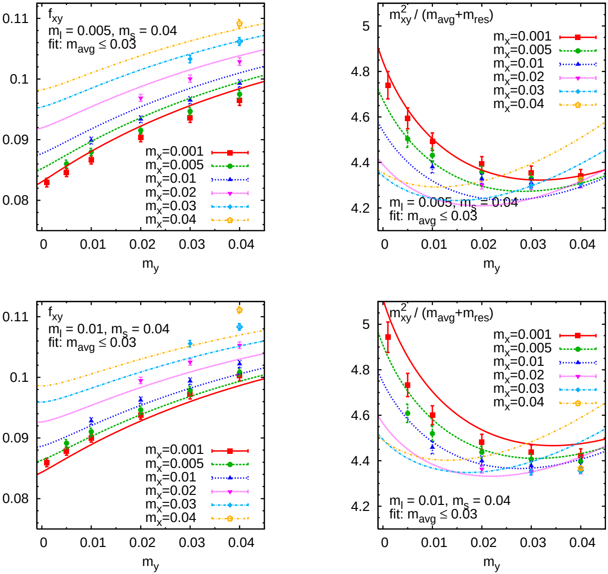
$$24^3 \times 64 \times 16 \qquad L \approx 2.75\,\mathrm{fm}$$
$$16^3 \times 32 \times 16 \qquad L \approx 1.83\,\mathrm{fm}$$

- On the 24^3 lattice:
  - light dynamical quark masses am_l corresponding to pion masses as low as 330 MeV; lightest $m_l \sim m_s/5$
  - $am_h = 0.04$ for sea strange quark: *a posteriori* a little too large
  - lightest valence quark mass about 11% of m_s, corresponds to pion mass 240 MeV (partial quenching)
- Currently generating and analysing $32^3 \times 64 \times 16$ with $a \approx 0.09\,\mathrm{fm}$

RBC–UKQCD lattice simulations (cont)

- Simulate with fixed bare input parameters $g(a)$, am_l (isospin limit) and am_h
- Use three physical quantities (m_π, m_K and m_{Ω^-}) to fix lattice spacing a and masses m_{ud}, m_s
- Simulate with m_l larger than m_{ud} and extrapolate to physical point
- Can work at m_s (after tuning)
- Chiral perturbation theory (ChPT) is used for the extrapolation $m_l \to m_{ud}$
  - How reliable?
  - What are the values of the Low Energy Constants (LECs)?
  - $\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}$ or $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$?
- We do partially quenched simulations with distinct valence and sea masses $\Longrightarrow$ use PQChPT Sharpe & Shoresh PRD62 094503 2000

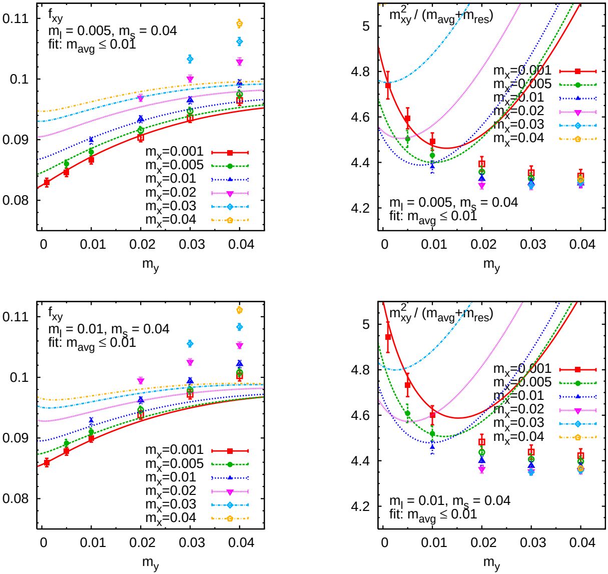
NLO $SU(3)_L \times SU(3)_R$ fit bad for $am_{avg} < 0.03$



Left: f_P

Right: m_P^2

NLO $SU(3)_L \times SU(3)_R$ fit good for $am_{avg} < 0.01$



Left: f_P

Right: m_P^2

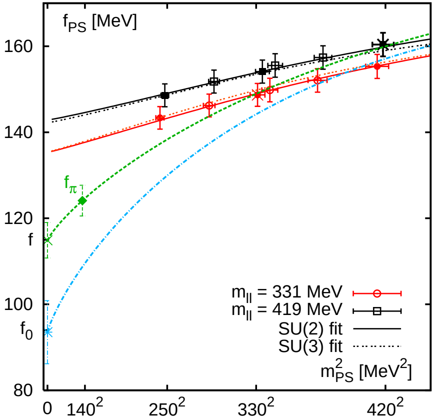
Chiral fits

- NLO SU3 chiral fits to pseudoscalar masses and decay constants work well but only at light masses, below about 400 MeV
  - NLO corrections are very large, up to 50% of LO term for decay constants
  - Fitted decay constant in chiral limit, f_0, very small
- going to NNLO would increase range of good fits
  - number of new LECs too large for the data we have
  - other collaborations use at least the analytical terms
- Fits can also be done using $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ ChPT at light masses
  - NLO corrections smaller:

$$\frac{M_\pi^2}{\Lambda_\chi^2}, \ \frac{M_\pi^2}{M_K^2} \ll \frac{M_K^2}{\Lambda_\chi^2}$$

  - Inclusion of analytic NNLO terms extends range of fit with small NNLO contributions

Results using SU(2) and SU(3) ChPT



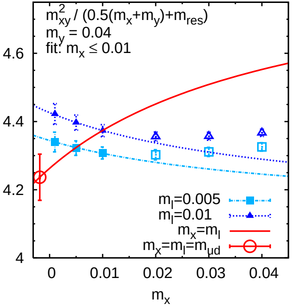
$$f_\pi/f = 1.08$$
$$f/f_0 = 1.23(6)$$

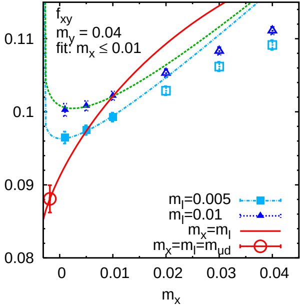
Large value of f_π/f_0 leads us to present results based on $SU(2)_L \times SU(2)_R$ ChPT

$$f_\pi = 124.1\,(3.6)_{stat}(6.9)_{sys}\text{ MeV}$$

SU(3) curve is for three degenerate quarks so does not show f_π

Chiral behaviour of m_K^2 and f_K

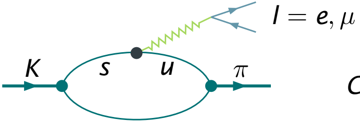




- $SU(2)_L \times SU(2)_R$ ChPT: only u and d transform, include kaon as a matter field $\Longrightarrow$ KChPT
- m_s effects are fully absorbed into the LECs
- Use PQ KChPT with light valence quark $am_{ud} < 0.01$ and $am_s = 0.04$
- Result arXiv:0804.0473

$$f_K/f_\pi = 1.205\,(0.018)_{\rm stat}(0.062)_{\rm sys}$$

K_{l3} decay



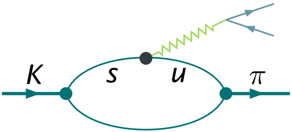
$C_K^2 = 1/2[1]$ for $K^+[K^0]$

$$\Gamma(K_{l3}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} I_{Kl}\, S_{\mathrm{EW}} (1 + \delta_{\mathrm{SU(2)}} + \delta_{\mathrm{EM}})^2 |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2$$

Theory inputs	
S_{EW}	universal short distance correction
$\delta_{\mathrm{SU(2)}}$	isospin breaking correction to form factor
δ_{EM}	long distance EM effects
$f_+^{K^0\pi^-}(0)$	hadronic matrix element at zero mom transfer

Experimental inputs	
$\Gamma(K_{l3})$	Branching ratios and lifetimes
I_{Kl}	phase space integral with parametrized form factor

K_{l3} decay



$$\langle \pi^+(k) | \bar{u}\gamma_\mu s | \bar{K}^0(p) \rangle$$
$$= f_+(q^2)\left(p_\mu + k_\mu - \frac{m_K^2 - m_\pi^2}{q^2} q_\mu\right) + f_0(q^2)\frac{m_K^2 - m_\pi^2}{q^2} q_\mu$$

- Require $f_0(0) = f_+(0)$ to better than 1% precision
- ChPT $\Longrightarrow$

$$f_+(0) = 1 + f_2 + f_4 + \cdots \qquad \text{where } f_n = O(M^n_{\pi,K,\eta})$$

- Reference value is $f_+(0) = 0.961(8)$ where $f_2 = -0.023$ relatively well known from ChPT and $f_4, f_6, \ldots$ are found from models Leutwyler & Roos 1984

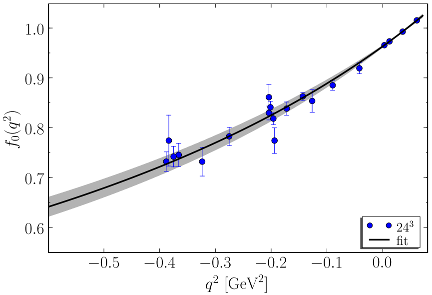
K_{l3} "standard method"

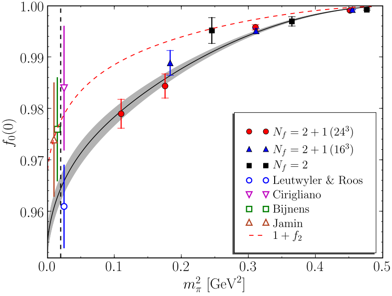
- 1% precision attainable because $1 - f_+(0)$ is calculated Becirevic et al NPB705 339 2005 based on Okamoto et al PRD61 014502 2000
- Evaluate $f_0(q^2_{\rm max})$ with excellent precision for varying m_l using a double ratio

$$\frac{\langle\pi|\bar{s}\gamma_4 u|K\rangle\langle K|\bar{u}\gamma_4 s|\pi\rangle}{\langle\pi|\bar{u}\gamma_4 u|\pi\rangle\langle K|\bar{s}\gamma_4 s|K\rangle} = \left[f_0(q^2_{\rm max})\right]^2 \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}$$

- Evaluate somewhat less precisely at other values of q^2 for varying m_l
- Extrapolate in q^2 and do chiral extrapolation $m_l \to m_{ud}$

K_{l3}: q^2 and chiral extrapolations



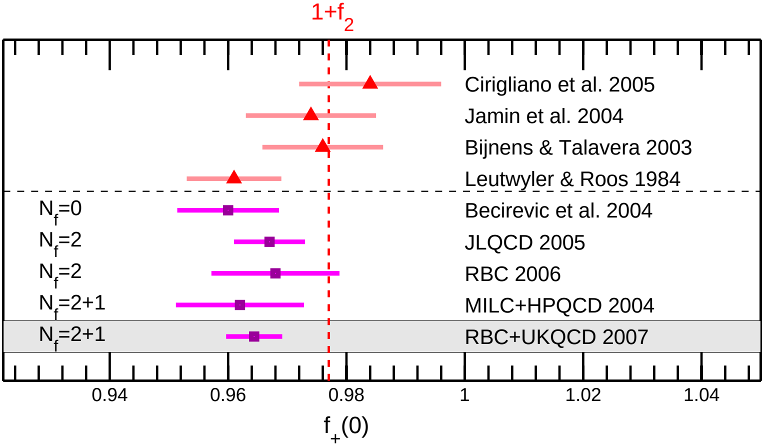


- Central values from combined pole fit to q^2 and $m_l \to m_{ud}$ dependence
- $f_0(0)$ as function of pion mass

Final answer Boyle et al PRL100 141601 2008

$$f_+(0) = 0.9644(33)(34)(14) = 0.964(5)$$

K_{l3} result from "standard method"



Final answer Boyle et al PRL100 141601 2008

$$f_+(0) = 0.964(5)$$

K_{l3}: chiral extrapolation at $q^2_{\rm max}$

- Callan–Treiman relation in SU(2) chiral limit

$$f_0(q^2_{\rm max}) \xrightarrow{m_\pi^2\to 0} \frac{f^{(K)}}{f} \approx 1.28 \quad \text{for our simulated } m_s$$

- Lattice results at simulated masses about 25% below $f^{(K)}/f$ and increasing very slowly as $m_\pi^2 \to 0$
- Chiral behaviour in KChPT

$$f_0(q^2_{\rm max}) = \frac{f^{(K)}}{f}\left[1-\frac{11}{4}\frac{m_\pi^2}{(4\pi f)^2}\log\frac{m_\pi^2}{\mu^2}+\frac{\lambda_1}{4\pi f}m_\pi+\frac{\lambda_2}{(4\pi f)^2}m_\pi^2+\cdots\right]$$

- Log term large but wrong sign
- converting from SU(3) provides semi-quantitative agreement
- Similar issues for $f_0^{B\to\pi}(q^2_{\rm max})$ vs f_B/f_π

JMF & Sachrajda arXiv:0809.1229

Eliminating the q^2 interpolation

- Momentum resolution with conventional methods is poor on the lattice:

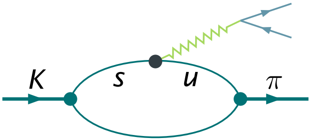
$$L = 24a \quad \text{with} \quad a^{-1} = 1.73\,\text{GeV} \quad \Rightarrow \quad 2\pi/L = 0.45\,\text{GeV}$$

- Modify momentum spectrum (relative to periodic BCs) using twisted BCs

$$q(x_i + L) = e^{i\theta_i} q(x_i) \quad \longrightarrow \quad p_i = \frac{2\pi}{L} n_i + \frac{\theta_i}{L}$$

- Partial twisting: it's possible to twist the valence quarks only and keep periodic BCs for the sea quarks (FV effects remain exponentially small) Sachrajda & Villadoro 2004
- No need to generate new ensembles for every choice of twists

K_{l3} directly at $q^2 = 0$



- Tune twists to calculate matrix element at $q^2 = 0$ (or any chosen value of q^2) Boyle et al JHEP 0705:016 2007

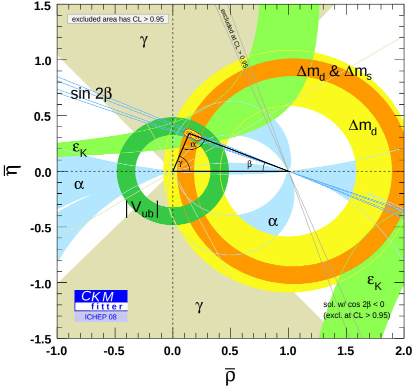
$$\langle \pi(\vec{0}) | V_4 | K(\vec{\theta}_K) \rangle \quad \text{with} \quad |\vec{\theta}_K| = L\sqrt{\left(\frac{m_K^2 + m_\pi^2}{2m_\pi}\right)^2 - m_K^2}$$

$$\langle \pi(\vec{\theta}_\pi) | V_4 | K(\vec{0}) \rangle \quad \text{with} \quad |\vec{\theta}_\pi| = L\sqrt{\left(\frac{m_K^2 + m_\pi^2}{2m_\pi}\right)^2 - m_\pi^2}$$

K_{l3} directly at $q^2 = 0$

- Feasibility demonstrated on a $16^3 \times 32$ lattice at two values of m_{ud} Boyle et al JHEP 0705:016 2007
- Currently using partial twisting to get $f_0(0)$ at our lightest quark mass ($am = 0.005$) on $24^3 \times 64$ lattice
- Preliminary results suggest we can get the same accuracy with this direct method as with the “traditional” one
- We have also used partial twisting to study the electromagnetic form factor of a pion with mass 330 MeV at small momentum transfers, using NLO ChPT to determine results for a physical pion

Neutral Kaon mixing: B_K



- $|\epsilon_K| = (2.232 \pm 0.007) \times 10^{-3}$ known to 0.3% precision
- $\delta B_K > 5\%$ from lattice calculations

In the standard analysis (Buras et al 1998)

$$|\epsilon_K| \propto B_K |V_{cb}|^2 \left[|V_{cb}|^2 (1 - \bar{\rho}) \eta_{tt} S(x_t) + \eta_{ct} S(x_c, x_t) - \eta_{cc} x_c \right]$$

... but $\delta |V_{cb}| \approx 2.5\%$ so $\delta |V_{cb}|^4$ also relevant

RBC–UKQCD calculation of B_K

$$\langle K^0 | O_{VV+AA} | \bar{K}^0 \rangle = \frac{8}{3} B_K f_K^2 M_K^2$$

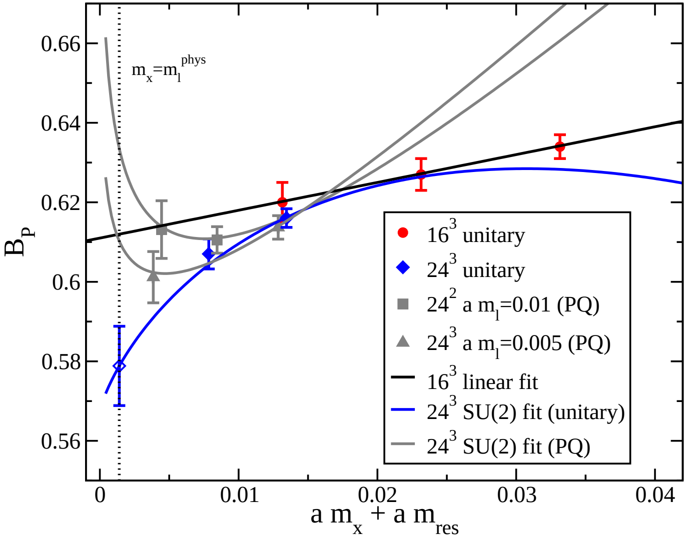
Flavour and chiral symmetry properties of DWF well-suited to this calculation

- $2+1$ dynamical flavours: correct light flavour content with pion masses down to 243 MeV
- reduced discretization errors ($O(a)$ improved)
- good chiral symmetry $\Rightarrow \Delta S{=}2$ operator is multiplicatively renormalised
- nonperturbative renormalisation
- use $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ (PQ)ChPT

$$B_K = B_0 \left[1 + \frac{b_1 \chi_l}{f^2} + \frac{b_2 \chi_x}{f^2} - \frac{\chi_l}{32\pi^2 f^2} \log \frac{\chi_x}{\Lambda_\chi^2} \right]$$

Use kaons with $m_s \neq m_d$ and fit chiral behaviour in m_d

B_K chiral extrapolation



B_K result

$$B_K^{\overline{MS}}(2\,\mathrm{GeV}) = 0.524(10)(28) \qquad \hat{B}_K = 0.720(13)(37)$$

- Systematic error includes estimates for finite-volume effects, discretization errors, interpolation to physical strange quark mass and ChPT
- Evaluation at a second, finer, lattice spacing is in progress: will reduce the estimated 4% discretization error, currently our largest systematic error

RBC–UKQCD prl 100 (2008) 032001

Conclusions/Outlook

- Presented selected phenomenological results in kaon physics from $2+1$ flavour dynamical lattice simulations using action with good chiral properties
- Lattice community beginning to make strong contact with ChPT community and to determine LECs with precision
- RBC–UKQCD now moving on to finer lattices (will gain information on continuum extrapolation)
- Will continue to extend range of quantities calculated (eg $K \to \pi\pi$ decays)
- Medium term aim is target simulation with $a = 0.06\,\mathrm{fm}$, $L = 4\,\mathrm{fm}$, $m_\pi = 195\,\mathrm{MeV}$