Einstein's Impact on the Physics of the Twentieth Century

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Program

Lectures I and II:

Einstein's Contributions to Statistical Mechanics and Quantum Theory

Lecture III:

Einstein's Thesis at the University of Zürich

Lecture IV:

From Special to General Relativity

Lecture V:

The History and the Mystery of the Cosmological Constant

Some papers of NS connected with the lectures

1. Domenico Giulini and NS: *Einstein's Impact on the Physics of the Twentieth Century*, physics/0507107.
2. NS: *Einstein's Contributions to Quantum Theory*, hep-ph/0508131.
3. NS (in german): *Light Quanta and Molecules, A Contribution to the Annus Mirabilis*, physics/0507118.
4. NS: *On Einstein's Doctoral Thesis*, physics/0504201.
5. NS: *Reflections on Gravity*, astro-ph/0006423.
6. NS: *General Relativity, With Applications to Astrophysics*, Texts and Monographs in Physics, Springer-Verlag, 2004.

7. NS: *On the Cosmological Constant Problems and the Astronomical Evidence for a Homogeneous Energy Density with Negative Pressure*.
In *Poincaré Seminar 2002, Vacuum Energy – Renormalization*, 7-51. B. Duplantier, and V. Rivasseau, eds.; (Birkhäuser-Verlag); astro-ph/0203330.
8. NS: *The History of the Cosmological Constant Problem*. In *On the Nature of Dark Energy*, IAP Astrophysics Colloquium 2002, Frontier Group, 2003, p.17; gr-qc/0208027.
9. NS: *Dark Energy*. In *Relativity Today*, Proceedings of the Seventh Hungarian Relativity Workshop, 127. I. Racz, ed.; (Akadmiai Kiado, Budapest, 2004); gr-qc/0311083.
10. NS: *From Primordial Quantum Fluctuations to the Anisotropies of the Cosmic Microwave Background Radiation*, hep-ph/0505249.

Einstein's five papers of 1905

On a Heuristic Point of View Concerning the Production and Transformation of Light,
17 March 1905.

A New Determination of Molecular Dimensions,
30 April 1905.

On the Movement of Small Particles Suspended in Stationary Liquids Required by the Molecular-Kinetic Theory of Heat,
11 May 1905.

On the Electrodynamics of Moving Bodies,
30 June 1905.

Does the Inertia of a Body Depend upon its Energy Content?,
27 September 1905.

Einstein's Contributions to Statistical Mechanics

Einstein (1949):

> *"Not acquainted with the earlier investigations of Boltzmann and Gibbs, which had appeared earlier and actually exhausted the subject, I developed the statistical mechanics and the molecular-kinetic theory of thermodynamics which was based on the former. My major aim in this was to find facts which would guarantee as much as possible the existence of atoms of definite finite size. [...]."*

Already as a student Einstein was very interested in thermodynamics and kinetic theory, and he studied intensively some of Boltzmann's work.

September 13th 1900 to Mileva (CPAE, Vol. 2, Doc. 75):

> "*The Boltzmann is absolutely magnificent. I'am almost finished with it. He's a masterful writer. I am firmly convinced of the correctness of the principles of the theory, i.e., I am convinced that in the case of gases, we are really dealing with discrete mass points of definite finite size which move according to certain conditions. Boltzmann quite correctly emphasizes that the hypothetical forces between molecules are not essential components of the theory, as the whole energy is essentially kinetic in character. This is a step forward in the dynamic explanation of physical phenomena.*"

June 1902: "Kinetic Theory of Thermal Equilibrium and the Second Law of Thermodynamics"

from Introduction:

> "*Great as the achievements of the kinetic theory of heat have been in the domain of gas theory, the science of mechanics has not yet been able to produce an adequate foundation for the general theory of heat, for one has not yet succeeded in deriving the laws of thermal equilibrium and the second law of thermodynamics using only the equations of mechanics and the probability calculus, though Maxwell's and Boltzmann's theories came close to this goal. The purpose of the following considerations is to close this gap. At the same time, they will yield an extension of the second law that is of importance for the application of thermodynamics. They will also yield the mathematical expression for entropy from the standpoint of mechanics.*"

In the third paper "On the General Molecular Theory of Heat", Einstein derives the energy fluctuation formula in the canonical ensemble. Recall:

$$\boxed{\left\langle (E - \langle E \rangle)^2 \right\rangle = kT^2 \frac{\partial \langle E \rangle}{\partial T}.}$$

> "*Thus the absolute constant k determines the thermal stability of the system. The relationship just found is interesting because it no longer contains any quantity reminiscent of the assumption on which the theory is based.*"

Applications of the classical theory

In his thesis "*A new Determination of molecular Dimensions*" Einstein derived a novel formula for the diffusion constant D of suspended microscopic particles. (Lecture III.) Result:

$$D = \frac{kT}{6\pi\eta a};$$

η = viscosity of the fluid and a = radius of the particles (assumed to be spherical).

Brownian motion

E. repeats derivation of D (gives first a statistical mechanical derivation of the osmotic pressure).
Short novel part:
E. considers the diffusion alternatively as the result of a highly irregular random motion, caused by the bombardment of an enormously large number of molecules. On the basis of some idealizing assumptions, he shows that the random walks of the suspended particles can be described by a Gaussian process, "which was to be expected". Moreover,

the width of the probability distribution for the position of a particle is determined by the diffusion constant. Therefore, the one-dimensional variance of the position is given by the famous formula

$$\boxed{\left\langle (\Delta x)^2 \right\rangle = 2Dt = \frac{kT}{3\pi\eta_0 a}\ t.}$$

> *"If it is really possible to observe the motion to be discussed here, along with the laws it is expected to obey, then classical thermodynamics can no longer be viewed as strictly valid even for microscopically distinguishable spaces, and an exact determination of the real size of atoms becomes possible. Conversely, if the prediction of this motion were to be proven wrong, this fact would provide a weighty argument against the molecular-kinetic conception of heat."*

Critical opalescence

Since about 1874 it was known that the scattering and attenuation of light passing through gas becomes very large near the critical point. In 1908 Marian von Smoluchowski pointed out that this phenomenon is the result of density fluctuations of the medium, but he did not derive a quantitative formula for the scattering or extinction coefficient. It was Einstein who closed this gap.

Before he approaches this task, Einstein gives a lengthy introduction to the theory of statistical fluctuations, based on Boltzmann's principle. He then applies the general theory to density fluctuations of fluids and mixtures of fluids. This opening section has to be regarded as a major and influential contribution to statistical thermodynamics.

If the refraction index n is close to 1, this reduces to

$$\alpha(\omega) = \frac{1}{6\pi}\left(\frac{\omega}{c}\right)^4 (n^2-1)^2 \frac{kT}{-V(\partial p/\partial V)_T}$$

($\omega=$ angular frequency of the light). With this formula Einstein found a quantitative relationship between Rayleigh scattering and critical opalescence.

At the critical point this expression diverges, because the correlation length for the density fluctuations diverges. As was first pointed out by Ornstein and Zernicke, Einstein's implicit assumption of statistical independence in separated volume elements is then no longer valid. In this sense, Einstein's work on critical opalescence became the starting point of several research directions of the twentieth-century.

Post-Einstein developments

Einstein did not have a dynamical theory of Brownian motion; he determined the nature of the motion on the basis of some assumptions. Einstein's heuristic considerations, that have been criticized by many people (including Einstein himself), are tantamount to the assumption (iii) of the following theorem:

Theorem. *Let* X_t $(0 \leq t < \infty)$ *be a stochastic process, satisfying the properties:*

(i) Independence: Each increment $X_{t+\Delta t} - X_t$ *is independent of* $\{X_\tau,\ \tau \leq t\}$.

(ii) Stationarity: The distribution of $X_{t+\Delta t} - X_t$ *does not depend on* t.

(iii) Continuity: If P *denotes the probability measure belonging to the stochastic process, then*

$$\lim_{\Delta t \downarrow 0} \frac{P(\{|X_{t+\Delta t} - X_t| \geq \delta\})}{\Delta t} = 0, \quad \textit{for all } \delta > 0\,.$$

(iv) $X_{t=0} = 0$.

Then X_t *has a normal distribution with* $\langle X_t \rangle = 0$ *and* $\langle X_t^2 \rangle = \sigma^2 t$, *where* σ *is a numerical constant.*

The theory of Brownian motion of Einstein is highly idealized, since for example the velocity of a particle is not defined. Langevin's approach, perfected by L.S. Ornstein and G.E. Uhlenbeck in 1930, is closer to Newtonian mechanics of particles and is thus truly dynamical. In practice, for 'ordinary' Brownian motion, the predictions of the two theories are, however, numerically indistinguishable.

In the Ornstein-Uhlenbeck theory the velocity process V_t is described in terms of the stochastic differential equation (Langevin equation)

$$\dot{V}_t = -\alpha V_t + \sigma \xi_t;$$

ξ_t denotes 'white noise'.

The theory of stochastic differential equations has expanded into a huge field of stochastic analysis, with rich applications in physics, engineering, and mathematical finance. In quantum physics (generalized) stochastic processes have become very important through Feynman-Kac path integral representations.

Properties of Ornstein-Uhlenbeck process

• The distribution of V_t converges for large t to a Gaussian distribution with mean zero and variance $\sigma^2/2\alpha$ (with equipartition $\frac{1}{2}m\frac{\sigma^2}{2\alpha} = \frac{1}{2}kT$); dissipation α induces thus a fluctuation

$$\sigma^2 = \frac{2\alpha}{m}kT.$$

• The distributions of the positions X_t converge for large t to those of the Gaussian process

$$\tilde{B}_t = X_0 + \sqrt{2D}B_t\,,$$

B_t = Brownian (Wiener) process with variance 1, X_0 = initial position of the particle, and

$$D = \frac{\sigma^2}{2\alpha} = \frac{kT}{m\alpha}.$$

The distribution function of X_t is thus,

$$p_t(x) = \frac{1}{\sqrt{4\pi Dt}}e^{-x^2/4Dt} \quad ;$$

satisfies the diffusion equation

$$\partial_t p_t - D\partial_x^2 p_t = 0.$$

Therefore, D is the diffusion constant; given by the Einstein value, if we also use Stokes' law for α.

Einstein's Contributions to Quantum Theory

Einstein to O. Stern:

> "*Ich habe hundertmal mehr über Quantenprobleme nachgedacht, als über die allgemeine Relativitätstheorie.*"

NS: hep-ph/0508131

Einstein's first paper from 1905

Einstein to Habicht, May 1905:

" *I promise to send you four papers [...]. The first deals with the energetic properties of light and is very revolutionary, as you will see. [...] The forth paper deals with the electrodynamics of moving bodies. The kinematic part of it will interest you.*"

The significance and originality
of the paper

"*On a heuristic point of view concerning the production and transformation of light*"

can hardly be overestimated.

Opening: Classical physics implies a nonsensical radiation law; first correspondence argument

classical physics $\Rightarrow$

$$\rho(\nu, T) = (8\pi\nu^2/c^3)kT$$

: **UV divergence**!!!
but approximately satisfied for
large wavelengths and radiation densities;
satisfied by Planck distribution, if

$$\boxed{N_A = 6.17 \times 10^{23}.}$$

> "*The greater the energy density and the wavelength of the radiation, the more useful the theoretical principles we have been using prove to be; however, these principles fail completely in the case of small wavelengths and small radiation densities.*"

Einstein's statistical analysis of Wien's law:

$$\rho(T,\nu) = \frac{8\pi\nu^2}{c^3} h\nu e^{-h\nu/kT}$$

$E_V(T,\nu)$: energy of radiation in volumen V and small frequency interval $[\nu, \nu+\Delta\nu]$,

$$E_V = \rho(T,\nu)V\Delta\nu; \quad S_V = \sigma(T,\nu)V\Delta\nu.$$

Thermodynamics $\Rightarrow$

$$\frac{\partial\sigma}{\partial\rho} = \frac{1}{T}.$$

$\Rightarrow$

$$\frac{\partial\sigma}{\partial\rho} = -\frac{k}{h\nu}\ln\left[\frac{\rho}{8\pi h\nu^3/c^3}\right].$$

$\Rightarrow$

$$S_V = -k\frac{E_V}{h\nu}\left\{\ln\left[\frac{E_V}{V\Delta\nu 8\pi h\nu^3/c^3}\right] - 1\right\}.$$

Einstein is interested in the V-dependence:

$$S_V - S_{V_0} = k\frac{E}{h\nu}\ln\left(\frac{V}{V_0}\right) = k\ln\left(\frac{V}{V_0}\right)^{E/h\nu}.$$

$$S_V - S_{V_0} = k \ln \left(\frac{V}{V_0}\right)^{E/h\nu}$$

'**Boltzmann's principle**':

$$\boxed{S = k \ln W}$$

$\Rightarrow$

$$W = \left(\frac{V}{V_0}\right)^{E/h\nu} \quad ;$$

comparison with ideal gas (N particles):

$$W = \left(\frac{V}{V_0}\right)^{N}$$

"*Monochromatic radiation of low density (within the range of Wien's radiation formula) behaves thermodynamically as if it consisted of mutually independent energy quanta of magnitude* $h\nu$."

Light quantum hypothesis

So far no revolutionary statement has been made. The famous sentences express the result of a statistical mechanical analysis; lead Einstein to the hypothesis:

> *"If, with regard to the dependence of its entropy on volume, a monochromatic radiation (of sufficient low density) behaves like a discontinuous medium consisting of energy quanta of magnitude $h\nu$, then it seems reasonable to investigate whether the laws of generation and conversion of light are so constituted as if light consisted of such energy quanta."*

Application:

$$\boxed{E_{max} = h\nu - P.}$$

Einstein's bold light quantum hypothesis was very far from Planck's conception. Planck neither envisaged a quantization of the free radiation field, nor did he, as it is often stated, quantize the energy of a material oszillator per se.

It was Einstein in 1906 who interpreted Planck's result as follows:

> *"Hence, we must view the following proposition as the basis underlying Planck's theory of radiation: The energy of an elementary resonator can only assume values that are integral multiples of $h\nu$; by emission and absorption, the energy of a resonator changes by jumps of integral multiples of $h\nu$."*

Energie and momentum fluctuations of the radiation field

Einstein 1909: "*On the present status of the radiation problem*"

variance of E_V:

$$\left\langle (E_V - \langle E_V \rangle)^2 \right\rangle = kT^2 \frac{\partial \langle E_V \rangle}{\partial T} = kT^2 V \Delta\nu \frac{\partial \rho}{\partial T};$$

for Planck distribution:

$$\boxed{\left\langle (E_V - \langle E_V \rangle)^2 \right\rangle = \left(h\nu\rho + \frac{c^3}{8\pi\nu^2}\rho^2 \right) V \Delta\nu};$$

interpretation: Particle-Wave Duality.

1909: Salzburg Lecture
"turning point in the development of theoretical physics" (W. Pauli):

> *"It is therefore my opinion that the next stage in the development of theoretical physics will bring us a theory of light that can be understood as a kind of fusion of the wave and emission theories of light."*

Reactions:

Planck, Nernst, Rubens, Wartburg in 1913:

> "*In sum, one can say that there is hardly one among the great problems in which modern physics is so rich to which Einstein has not made a remarkable contribution. That he may sometimes have missed the target in his speculations, as, for example, in his hypothesis of light-quanta, cannot really be held to much against him, for it is not possible to introduce really new ideas even in the most exact sciences without sometimes taking a risk.*"

Millikan 1915:

"Despite the apparently complete success of the Einstein equation, the physical theory of which it was designed to be the symbolic expression is found so untenable that Einstein himself, I believe, no longer holds to it."

1916: "On the Quantum Theory of Radiation"

- "An amazingly simple derivation of Planck's formula, I should like to say *the* derivation".

- momentum transfer in random direction for each elementary process = $h\nu/c$.

> "*The weakness of the theory lies, on the one hand, in the fact that it does not bring us any closer to a merger with the undulatory theory, and, on the other hand, in the fact that it leaves the time and direction of elementary processes to 'chance'; in spite of this I harbor full confidence in the trustworthiness of the path entered upon.*"

Einstein studies the Brownian motion of molecules in thermodynamic radiation field;

molecule experiences:

• a systematic drag force Rv; leads in a small time intervall $(t, t+\tau)$ to the momentum change $Rv\tau$;

• an irregular change of momentum Δ in the time τ, due to fluctuations of the radiation pressure.

In thermal equilibrium

$$\langle (Mv - Rv\tau + \Delta)^2 \rangle = \langle (Mv)^2 \rangle$$

assuming $\langle v \cdot \Delta \rangle = 0$ implies

$$\langle \Delta^2 \rangle = 2R \langle M^2 v^2 \rangle \tau = 2RkT\tau.$$

“**Nadelstrahlung**”

recall

$$\langle\Delta^2\rangle = 2RkT\tau;$$

absorption and induced emission $\to$

$$R = \frac{1}{3}\left(\frac{h\nu}{c}\right)^2 \frac{1}{2kT} Z,$$

Z = number of elementary processes per sec.

Interpretation: corresponding

$$\langle\Delta^2\rangle = \left(\frac{h\nu}{c}\right)^2 \langle\cos^2\vartheta\rangle_{S^2} Z\tau,$$

hence *directed* recoil $= h\nu/c$ also for **spontaneous** emission.

Calculation of resistive force R

K: rest system of radiation; K': rest system of atom; sees anisotropic radiation field. Partition sum:

$$S := g_n e^{-E_n/kT} + g_m e^{-E_m/kT} + \cdots .$$

Fraction of time in state $E_n = g_n e^{-E_n/kT}/S$; correspondingly for state m. Number of absorptions $n \to m$ per unit time from solid angle $d\Omega'$

$$= \frac{1}{S} g_n e^{-E_n/kT} B_m^n \rho_{\nu_0} \frac{d\Omega'}{4\pi}; \quad \nu_0 = \frac{E_m - E_n}{h};$$

correspondingly for induced emission $m \to n$

$$= \frac{1}{S} g_m e^{-E_m/kT} B_n^m \rho_{\nu_0} \frac{d\Omega'}{4\pi}.$$

With Einstein relation $g_m B_n^m = g_n B_m^n$ the momentum transfer per unit time in the x-direction is

$$\begin{aligned} -Rv &= \frac{h\nu_0}{c} \frac{1}{S} g_n B_m^n \left(e^{-E_n/kT} - e^{-E_m/kT} \right) \\ &\times \int \rho_{\nu_0}(\theta', \varphi') \cos\theta' \frac{d\Omega'}{4\pi}. \end{aligned}$$

Calculation of $\rho_{\nu_0}(\theta', \varphi')$

Use (Exercise)

$$\frac{\rho'_{\nu'}}{\nu'^3} = \frac{\rho_\nu}{\nu^3}$$

and the Doppler shift in first order:
$\nu = \nu' \left(1 + \frac{v}{c} \cos\theta'\right)$. This gives

$$\rho'_{\nu'} = \left(1 + \frac{v}{c}\right)^{-3} \rho_{[1+\frac{v}{c}\cos\theta']\nu'} \ ,$$

$$\begin{aligned}
\rho'_{\nu_0} &\simeq \left[\rho_{\nu_0} + \left(\frac{\partial \rho_\nu}{\partial \nu}\right)_{\nu_0} \nu_0 \frac{v}{c} \cos\theta'\right] \left(1 - 3\frac{v}{c}\cos\theta'\right) \\
&\simeq \rho_{\nu_0} + \frac{v}{c}\cos\theta' \left[\nu_0 \left(\frac{\partial \rho_\nu}{\partial \nu}\right)_{\nu_0} - 3\rho_{\nu_0}\right],
\end{aligned}$$

thus

$$\begin{aligned}
\int \rho'_{\nu_0} \cos\theta' \frac{d\Omega'}{4\pi} &= -\frac{v}{c}\left[\rho_{\nu_0} - \frac{1}{3}\nu_0 \left(\frac{\partial \rho_\nu}{\partial \nu}\right)_{\nu_0}\right] \\
&= \frac{v}{c}\left\{\frac{\nu^4}{3}\frac{\partial}{\partial \nu}\left(\frac{\rho_\nu}{\nu^3}\right)\right\}_{\nu=\nu_0} .
\end{aligned}$$

$\longrightarrow$ (writing ν instead of ν_0)

$$\begin{aligned} R &= \frac{h\nu}{c^2}\frac{1}{S} g_n B_m^n e^{-E_n/kT}\left(1 - e^{-h\nu/kT}\right) \\ &\times \left[-\frac{\nu^4}{3}\frac{\partial}{\partial\nu}\left(\frac{\rho_\nu}{\nu^3}\right)\right]. \end{aligned}$$

For the Planck distribution

$$R = \frac{1}{3}\left(\frac{h\nu}{c}\right)^2 \frac{1}{kT}\frac{g_n}{S} e^{-E_n/kT} B_m^n \rho_\nu.$$

Note that $\frac{g_n}{S} e^{-E_n/kT} B_m^n \rho_\nu$ is the number of absorptions per unit time. So if Z denotes the number of elementary processes per unit time, we indeed find

$$R = \frac{1}{3}\left(\frac{h\nu}{c}\right)^2 \frac{1}{2kT} Z.$$

Einstein to Besso (1916):

"*With that, [the existence of] light-quanta is practically certain.*"

Einstein to Besso (1918):

"*I do not doubt anymore the* reality *of radiation quanta, although I still stand quite alone in this conviction.*"

Einstein to O. Stern:

"*Ich habe hundertmal mehr über Quantenprobleme nachgedacht, als über die allgemeine Relativitätstheorie.*"