Lecture V: The History and the Mystery of the Cosmological Constant

Norbert Straumann

Abstract

In the first part of this lecture I shall describe the history of the cosmological constant and then emphasize the profound nature of the vacuum energy problem. In a second part the current evidence for a cosmologically significant nearly homogeneous exotic energy density with negative pressure ('Dark Energy') is reviewed. Special emphasis will be put on the polarization measurements by WMAP and their implications. I shall conclude by addressing the question: Do the current astronomical observations really imply the existence of a dominant dark energy component?

1 Introduction

Several increasingly accurate astronomical observations have strengthened the evidence that the recent ($z < 1$) Universe is dominated by an exotic nearly homogeneous energy density with *negative* pressure. This discovery is of highest interest for particle physicists, gravitational physicists, and cosmologists alike.

The simplest candidate for this so-called *Dark Energy* is a cosmological term in Einstein's field equations, a possibility that has been considered during all the history of relativistic cosmology. Independently of what the nature of this energy is, one thing is clear since a long time: The energy density belonging to the cosmological constant is not larger than the critical cosmological density, and thus incredibly small by particle physics standards. This is a profound mystery, since we expect that all sorts of *vacuum energies* contribute to the effective cosmological constant.

At this point a second puzzle has to be emphasized, because of which it is hard to believe that the vacuum energy constitutes the missing two thirds of the average energy density of the *present* Universe. If this would be the case, we would also be confronted with the following *cosmic coincidence* problem: Since the vacuum energy density is constant in time – at least after the QCD phase transition –, while the matter energy density decreases as the Universe expands, it would be more than surprising if the two would be comparable just at about the present time, while their ratio was tiny in the early Universe and would become very large in the distant future. The goal of so-called *quintessence models* is to avoid such an extreme fine-tuning. In many ways people thereby repeat what has been done in inflationary cosmology. The main motivation there was, as is well-known, to avoid excessive fine tunings of standard big bang cosmology (horizon and flatness problems). – In this talk I am not going to say more on this topical subject. I want to emphasize, however, that the quintessence models do *not* solve the first problem; so far also not the second one.

2 Einstein's original motivation of the Λ-term

One of the contributions in the famous volume *Albert Einstein: Philosopher–Scientist* [1] is an article by George E. Lemaître entitled *"The Cosmological Constant."* In the introduction he says: *"The history of science provides many instances of discoveries which have been made for reasons which are no longer considered satisfactory. It may be that the discovery of the cosmological constant is such a case."* When the book appeared in 1949 – at the occasion of Einstein's seventieth birthday – Lemaître could not be fully aware of how right he was, how profound the cosmological constant problem really is, especially since he was not a quantum physicist.

In the first part of this lecture I shall first review the main aspects of the history of the Λ-term, from its introduction in 1917 up to the point when it became widely clear that we are facing a deep mystery. (See also [2] and [3].) I begin with the classical aspect of the historical development.

As is well-known, Einstein introduced the cosmological term when he applied general relativity the first time to cosmology [4]. Presumably the main reason why Einstein turned so soon after the completion of general relativity to cosmology had much to do with Machian ideas on the origin of inertia, which played in those years an important role in Einstein's thinking. His intention was to eliminate all vestiges of absolute space. He was, in particular, convinced that isolated masses cannot impose a structure on space at infinity. Einstein was actually thinking about the problem regarding the choice of boundary conditions at infinity already in spring 1916. In a letter to Michele Besso from 14 May 1916 he also mentions the possibility of the world being finite. A few month later he expanded on this in letters to Willem de Sitter. It is along these lines that he postulated a Universe that is spatially finite and closed, a Universe in which no boundary conditions are needed. He then believed that this was the only way to satisfy what he later [6] named *Machs principle*, in the sense that the metric field should be determined uniquely by the energy-momentum tensor.

In addition, Einstein assumed that the Universe was *static*. This was not unreasonable at the time, because the relative velocities of the stars as observed were small. (Recall that astronomers only learned later that spiral nebulae are independent star systems outside the Milky Way. This was definitely established when in 1924 Hubble found that there were Cepheid variables in Andromeda and also in other galaxies.)

These two assumptions were, however, not compatible with Einstein's original field equations. For this reason, Einstein added the famous Λ-term, which is compatible with the principles of general relativity, in particular with the energy-momentum law $\nabla_\nu T^{\mu\nu} = 0$ for matter. The modified field equations in standard notation and signature $(+ - - -)$ are

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}. \tag{1}$$

The cosmological term is, in four dimensions, the only possible complication of the field equations if no higher than second order derivatives of the metric are allowed (Lovelock theorem). This remarkable uniqueness is one of the most attractive features of general relativity. (In higher dimensions additional terms satisfying this requirement are allowed.)

For the static Einstein universe the field equations (1) imply the two relations

$$4\pi G\rho = \frac{1}{a^2} = \Lambda, \tag{2}$$

where ρ is the mass density of the dust filled universe (zero pressure) and a is the radius of curvature. (We remark, in passing, that the Einstein universe is the only static dust solution; one does not have to assume isotropy or homogeneity. Its instability was demonstrated by Lemaître in 1927.) Einstein was very pleased by this direct connection between the mass density and geometry, because he thought that this was in accord with Mach's philosophy.

Einstein concludes with the following sentences:

> *"In order to arrive at this consistent view, we admittedly had to introduce an extension of the field equations of gravitation which is not justified by our actual knowledge of gravitation. It has to be emphasized, however, that a positive curvature of space is given by our results, even if the supplementary term is not introduced.*

That term is necessary only for the purpose of making possible a quasi-static distribution of matter, as required by the fact of the small velocities of the stars."

To de Sitter Einstein emphasized in a letter on 12 March 1917, that his cosmological model was intended primarily to settle the question "whether the basic idea of relativity can be followed through its completion, or whether it leads to contradictions". And he adds whether the model corresponds to reality was another matter.

Only later Einstein came to realize that Mach's philosophy is predicated on an antiquated ontology that seeks to reduce the metric field to an epiphenomenon of matter. It became increasingly clear to him that the metric field has an independent existence, and his enthusiasm for what he called Mach's principle later decreased. In a letter to F. Pirani he wrote in 1954: *"As a matter of fact, one should no longer speak of Mach's principle at all."* [7]. GR still preserves some remnant of Newton's absolute space and time.

3 From static to expanding world models

Surprisingly to Einstein, de Sitter discovered in the same year, 1917, a completely different static cosmological model which also incorporated the cosmological constant, but was *anti-Machian*, because it contained no matter [8]. For this reason, Einstein tried to discard it on various grounds (more on this below). The original form of the metric was:

$$g = \Big[1 - (\frac{r}{R})^2\Big]dt^2 - \frac{dr^2}{1-(\frac{r}{R})^2} - r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2).$$

Here, the spatial part is the standard metric of a three-sphere of radius R, with $R = (3/\Lambda)^{1/2}$. The model had one very interesting property: For light sources moving along static world lines there is a gravitational redshift, which became known as the *de Sitter effect*. This was thought to have some bearing on the redshift results obtained by Slipher. Because the fundamental (static) worldlines in this model are not geodesic, a freely- falling object released by any static observer will be seen by him to accelerate away, generating also local velocity (Doppler) redshifts corresponding to *peculiar velocities*. In the second edition of his book [9], published in 1924, Eddington writes about this:

"de Sitter's theory gives a double explanation for this motion of recession; first there is a general tendency to scatter (...); second there is a general displacement of spectral lines to the red in distant objects owing to the slowing

down of atomic vibrations (...), which would erroneously be interpreted as a motion of recession."

I do not want to enter into all the confusion over the de Sitter universe. One source of this was the apparent singularity at $r = R = (3/\Lambda)^{1/2}$. This was at first thoroughly misunderstood even by Einstein and Weyl. ('The Einstein-de Sitter-Weyl-Klein Debate' is now published in Vol. 8 of the *Collected Papers* [5].) At the end, Einstein had to acknowledge that de Sitter's solution is fully regular and matter-free and thus indeed a counter example to Mach's principle. But he still discarded the solution as physically irrelevant because it is not globally static. This is clearly expressed in a letter from Weyl to Klein, after he had discussed the issue during a visit of Einstein in Zurich [10]. An important discussion of the redshift of galaxies in de Sitter's model by H. Weyl in 1923 should be mentioned. Weyl introduced an expanding version of the de Sitter model [11]. For *small* distances his result reduced to what later became known as the Hubble law [1]. Independently of Weyl, Cornelius Lanczos introduced in 1922 also a non-stationary interpretation of de Sitter's solution in the form of a Friedmann spacetime with a positive spatial curvature [12]. In a second paper he also derived the redshift for the non-stationary interpretation [13].

Until about 1930 almost everybody believed that the Universe was static, in spite of the two fundamental papers by Friedmann [14] in 1922 and 1924 and Lemaître's independent work [15] in 1927. These path breaking papers were in fact largely ignored. The history of this early period has – as is often the case – been distorted by some widely read documents. Einstein too accepted the idea of an expanding Universe only much later. After the first paper of Friedmann, he published a brief note claiming an error in Friedmann's work; when it was pointed out to him that it was his error, Einstein published a retraction of his comment, with a sentence that luckily was deleted before publication: *"[Friedmann's paper] while mathematically correct is of no physical significance"*. In comments to Lemaître during the Solvay meeting in 1927, Einstein again rejected the expanding universe solutions as physically unacceptable. According to Lemaître, Einstein was telling him: *"Vos calculs sont corrects, mais votre physique est abominable"*. It appears astonishing that Einstein – after having studied carefully Friedmann's papers – did not realize that his static model is unstable, and hence that the Universe has to be expanding or contracting. On the other hand, I found in the archive of the ETH many years ago a postcard of Einstein to Weyl from 1923, related to Weyl's reinterpretation of de Sitter's solution, with the

[1] I recall that the de Sitter model has many different interpretations, depending on the class of fundamental observers that is singled out.

following interesting sentence: *"If there is no quasi-static world, then away with the cosmological term"*.

It also is not well-known that Hubble interpreted his famous results on the redshift of the radiation emitted by distant 'nebulae' in the framework of the de Sitter model, as was suggested by Eddington.

The general attitude is well illustrated by the following remark of Eddington at a Royal Society meeting in January, 1930: *"One puzzling question is why there should be only two solutions. I suppose the trouble is that people look for static solutions."*

Lemaître, who had been for a short time a post-doctoral student of Eddington, read this remark in a report to the meeting published in *Observatory*, and wrote to Eddington pointing out his 1927 paper. Eddington had seen that paper, but had completely forgotten about it. But now he was greatly impressed and recommended Lemaître's work in a letter to *Nature*. He also arranged for a translation which appeared in MNRAS [16]. Eddington also "pointed out that it was immediately deducible from his [Lemaître's] formulae that Einstein's world is unstable, so that an expanding or a contracting universe is an inevitable result of Einstein's law of gravitation."

Lemaître's successful explanation of Hubble's discovery finally changed the viewpoint of the majority of workers in the field. At this point Einstein *rejected the cosmological term as superfluous and no longer justified* [17]. At the end of the paper, in which he published his new view, Einstein adds some remarks about the age problem which was quite severe without the Λ-term, since Hubble's value of the Hubble parameter was then about seven times too large. Einstein is, however, not very worried and suggests two ways out. First he says that the matter distribution is in reality inhomogeneous and that the approximate treatment may be illusionary. Then he adds that in astronomy one should be cautious with large extrapolations in time.

Einstein repeated his new standpoint also much later [18], and this was adopted by many other influential workers, e.g., by Pauli [19]. Whether Einstein really considered the introduction of the Λ-term as "the biggest blunder of his life" appears doubtful to me. In his published work and letters I never found such a strong statement. Einstein discarded the cosmological term just for simplicity reasons. For a minority of cosmologists (O.Heckmann, for example [20]), this was not sufficient reason. Paraphrasing Rabi, one might ask: 'who ordered it away'?

Einstein published his new view in the *Sitzungsberichte der Preussischen Akademie der Wissenschaften.* The correct citation is:

Einstein. A. (1931). Sitzungsber. Preuss. Akad. Wiss. 235-37.

Many authors have quoted this paper but never read it. As a result, the quotations gradually changed in an interesting, quite systematic fashion. Some steps are shown in the following sequence:

- A. Einstein. 1931. Sitzsber. Preuss. Akad. Wiss. ...
- A. Einstein. Sitzber. Preuss. Akad. Wiss. ... (1931)
- A. Einstein (1931). Sber. preuss. Akad. Wiss. ...
- Einstein. A .. 1931. Sb. Preuss. Akad. Wiss. ...
- A. Einstein. S.-B. Preuss. Akad. Wis. ...1931
- A. Einstein. S.B. Preuss. Akad. Wiss. (1931) ...
- Einstein, A., and Preuss, S.B. (1931). Akad. Wiss. **235**

Presumably, one day some historian of science will try to find out what happened with the young physicist S.B. Preuss, who apparently wrote just one important paper and then disappeared from the scene.

After the Λ-force was rejected by its inventor, other cosmologists, like Eddington, retained it. One major reason was that it solved the problem of the age of the Universe when the Hubble time scale was thought to be only 2 billion years (corresponding to the value $H_0 \sim 500\ km\ s^{-1} Mpc^{-1}$ of the Hubble constant). This was even shorter than the age of the Earth. In addition, Eddington and others overestimated the age of stars and stellar systems.

For this reason, the Λ-term was employed again and a model was revived which Lemaître had singled out from the many solutions of the Friedmann-Lemaître equations[2]. This so-called Lemaître hesitation universe is closed and has a repulsive Λ-force ($\Lambda > 0$), which is slightly greater than the value chosen by Einstein. It begins with a big bang and has the following two stages of expansion. In the first the Λ-force is not important, the expansion is decelerated due to gravity and slowly approaches the radius of the Einstein universe. At about the same time, the repulsion becomes stronger than gravity and a second stage of expansion begins which eventually inflates. In this way a positive Λ was employed to reconcile the expansion of the Universe with the age of stars.

[2]I recall that Friedmann included the Λ-term in his basic equations. I find it remarkable that for the negatively curved solutions he pointed out that these may be open or compact (but not simply connected).

The *repulsive* effect of a positive cosmological constant can be seen from the following consequence of Einstein's field equations for the time-dependent scale factor $a(t)$:

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)a + \frac{\Lambda}{3}a, \tag{3}$$

where p is the pressure of all forms of matter.

Historically, the Newtonian analog of the cosmological term was regarded by Einstein, Weyl, Pauli, and others as a *Yukawa term*. This is not correct, as I now show.

For a better understanding of the action of the Λ-term it may be helpful to consider a general static spacetime with the metric (in adapted coordinates)

$$ds^2 = \varphi^2 dt^2 + g_{ik} dx^i dx^k, \tag{4}$$

where φ and g_{ik} depend only on the spatial coordinates x^i. The component R_{00} of the Ricci tensor is given by $R_{00} = \bar{\Delta}\varphi/\varphi$, where $\bar{\Delta}$ is the three-dimensional Laplace operator for the spatial metric $-g_{ik}$ in (4) (see, e.g., [21]). Let us write Eq. (1) in the form

$$G_{\mu\nu} = \kappa(T_{\mu\nu} + T^{\Lambda}_{\mu\nu}) \qquad (\kappa = 8\pi G), \tag{5}$$

with

$$T^{\Lambda}_{\mu\nu} = \frac{\Lambda}{8\pi G} g_{\mu\nu}. \tag{6}$$

This has the form of the energy-momentum tensor of an ideal fluid, with energy density $\rho_\Lambda = \Lambda/8\pi G$ and pressure $p_\Lambda = -\rho_\Lambda$. [3] For an ideal fluid at rest Einstein's field equation implies

$$\frac{1}{\varphi}\bar{\Delta}\varphi = 4\pi G\Big[(\rho + 3p) + \underbrace{(\rho_\Lambda + 3p_\Lambda)}_{-2\rho_\Lambda}\Big]. \tag{7}$$

Since the energy density and the pressure appear in the combination $\rho + 3p$, we understand that a positive ρ_Λ leads to a repulsion (as in (3)). In the Newtonian limit we have $\varphi \simeq 1 + \phi$ (ϕ : Newtonian potential) and $p \ll \rho$, hence we obtain the modified Poisson equation

$$\Delta\phi = 4\pi G(\rho - 2\rho_\Lambda). \tag{8}$$

This is the correct Newtonian limit.

As a result of revised values of the Hubble parameter and the development of the modern theory of stellar evolution in the 1950s, the controversy over

[3] This way of looking at the cosmological term was soon (in 1918) emphasized by Schrödinger and also by F. Klein.

ages was resolved and the Λ-term became again unnecessary. (Some tension remained for values of the Hubble parameter at the higher end of published values.)

However, in 1967 it was revived again in order to explain why quasars appeared to have redshifts that concentrated near the value $z = 2$. The idea was that quasars were born in the hesitation era [22]. Then quasars at greatly different distances can have almost the same redshift, because the universe was almost static during that period. Other arguments in favor of this interpretation were based on the following peculiarity. When the redshifts of emission lines in quasar spectra exceed 1.95, then redshifts of absorption lines in the same spectra were, as a rule, equal to 1.95. This was then quite understandable, because quasar light would most likely have crossed intervening galaxies during the epoch of suspended expansion, which would result in almost identical redshifts of the absorption lines. However, with more observational data evidence for the Λ-term dispersed for the third time.

4 Quantum aspects of the Λ-problem

At this point I want to leave the classical discussion of the Λ-term, and turn to the quantum aspect of the Λ-problem, where it really becomes very serious. Since quantum physicists had so many other problems, it is not astonishing that in the early years they did not worry about this subject. An exception was Pauli, who wondered in the early 1920s whether the zero-point energy of the radiation field could be gravitationally effective.

As background I recall that Planck had introduced the zero-point energy with somewhat strange arguments in 1911. The physical role of the zero-point energy was much discussed in the early years of quantum theory. There was, for instance, a paper by Einstein and Stern in 1913 [Collected Papers, Vol. 4, Doc. 11; see also the Editorial Note, p. 270-] that aroused widespread interest. In this two arguments in favor of the zero-point energy were given. The first had to do with the specific heat of rotating (diatomic) molecules. The authors developed an approximate theory of the energy of rotating molecules and came to the conclusion that the resulting specific heat agreed much better with recent experimental results by Arnold Eucken, if they included the zero-point energy. The second argument was based on a new derivation of Planck's radiation formula. In both arguments Einstein and Stern made a number of problematic assumptions, and in fall 1913 Einstein retracted their results. At the second Solvay Congress in late October 1913 Einstein said that he no longer believed in the zero-point energy, and in a letter to Ehrenfest [Vol. 5, Doc. 481] he wrote that the zero-point energy was "dead as a doornail".

From Charly Enz and Armin Thellung – Pauli's last two assistants – I have learned that Pauli had discussed this issue extensively with O.Stern in Hamburg. Stern had calculated, but never published, the vapor pressure difference between the isotopes 20 and 22 of Neon (using Debye theory). He came to the conclusion that without zero-point energy this difference would be large enough for easy separation of the isotopes, which is not the case in reality. These considerations penetrated into Pauli's lectures on statistical mechanics [23] (which I attended). The theme was taken up in an article by Enz and Thellung [24]. This was originally written as a birthday gift for Pauli, but because of Pauli's early death, appeared in a memorial volume of Helv.Phys.Acta.

From Pauli's discussions with Enz and Thellung we know that Pauli estimated the influence of the zero-point energy of the radiation field – cut off at the classical electron radius – on the radius of the universe, and came to the conclusion that it *"could not even reach to the moon"*.

When, as a student, I heard about this, I checked Pauli's unpublished[4] remark by doing the following little calculation (which Pauli most have done):

In units with $\hbar = c = 1$ the vacuum energy density of the radiation field is

$$< \rho >_{vac} = \frac{8\pi}{(2\pi)^3} \int_0^{\omega_{max}} \frac{\omega}{2} \omega^2 d\omega = \frac{1}{8\pi^2} \omega_{max}^4,$$

with

$$\omega_{max} = \frac{2\pi}{\lambda_{max}} = \frac{2\pi m_e}{\alpha}.$$

The corresponding radius of the Einstein universe in Eq.(2) would then be ($M_{pl} \equiv 1/\sqrt{G}$)

$$a = \frac{\alpha^2}{(2\pi)^{\frac{2}{3}}} \frac{M_{pl}}{m_e} \frac{1}{m_e} \sim 31 km.$$

This is indeed less than the distance to the moon. (It would be more consistent to use the curvature radius of the static de Sitter solution; the result is the same, up to the factor $\sqrt{3/2}$.)

For decades nobody else seems to have worried about contributions of quantum fluctuations to the cosmological constant, although physicists learned after Dirac's hole theory that the vacuum state in quantum field theory is not an empty medium, but has interesting physical properties. As an important example I mention the papers by Heisenberg and Euler [26] in which

[4]A trace of this is in Pauli's Handbuch article [25] on wave mechanics in the section where he discusses the meaning of the zero-point energy of the quantized radiation field.

they calculated the modifications of Maxwell's equations due to the polarization of the vacuum. Shortly afterwards, Weisskopf [27] not only simplified their calculations but also gave a thorough discussion of the physics involved in charge renormalization. Weisskopf related the modification of Maxwell's Lagrangian to the change of the energy of the Dirac sea as a function of slowly varying external electromagnetic fields. (Avoiding the old fashioned Dirac sea, this effective Lagrangian is due to the interaction of a classical electromagnetic field with the vacuum fluctuations of the electron positron field.) After a charge renormalization this change is finite and gives rise to electric and magnetic polarization vectors of the vacuum. In particular, the refraction index for light propagating perpendicular to a static homogeneous magnetic field depends on the polarization direction. This is the vacuum analog of the well-known Cotton-Mouton effect in optics. As a result, an initially linearly polarized light beam becomes elliptic. (In spite of great efforts it has not yet been possible to observe this effect.)

Another beautiful example for the importance of vacuum energies as a function of varying external conditions is the **Casimir effect**. This is the most widely cited example of how vacuum fluctuations can have observable consequences.

The presence of conducting plates modifies the vacuum energy density in a manner which depends on the separation of the plates. This leads to an attractive force between the two plates.

Historically, this was a byproduct of some applied industrial research in the stability of colloidal suspensions used to deposit films in the manufacture of lamps and cathode tubes. This lead Casimir and Polder to reconsider the theory of van der Waals interaction with *retardation* included. They found that this causes the interaction to vary at large intermolecular separations as r^{-7}. Casimir mentioned his result to Niels Bohr during a walk, and told him that he was puzzled by the extreme simplicity of the result at large distance. According to Casimir, Bohr mumbled something about zero-point energy. That was all, but it put him on the right track.

Precision experiments have recently confirmed the theoretical prediction to about 1 percent. By now the literature related to the Casimir effect is enormous. For further information we refer to the recent book [28].

5 Vacuum energy and gravity

When we consider the coupling to gravity, the vacuum energy density acts like a cosmological constant. In order to see this, first consider the vacuum expectation value of the energy-momentum tensor in Minkowski spacetime.

Since the vacuum state is Lorentz invariant, this expectation value is an invariant symmetric tensor, hence proportional to the metric tensor. For a curved metric this is still the case, up to higher curvature terms:

$$< T_{\mu\nu} >_{vac}= g_{\mu\nu}\rho_{vac} + higher\ curvature\ terms. \tag{9}$$

The *effective* cosmological constant, which controls the large scale behavior of the Universe, is given by

$$\Lambda = 8\pi G\rho_{vac} + \Lambda_0, \tag{10}$$

where Λ_0 is a bare cosmological constant in Einstein's field equations.

We know from astronomical observations that $\rho_\Lambda \equiv \Lambda/8\pi G$ can not be larger than about the critical density:

$$\begin{aligned} \rho_{crit} &= \frac{3H_0^2}{8\pi G} \\ &= 1.88 \times 10^{-29} h_0^2 gcm^{-3} \\ &\simeq (3 \times 10^{-3} eV)^4, \end{aligned} \tag{11}$$

where h_0 is the *reduced Hubble parameter*

$$h_0 = H_0/(100kms^{-1}Mpc^{-1}) \tag{12}$$

and is close to 0.7.

It is a complete mystery as to why the two terms in (29) should almost exactly cancel. This is – more precisely stated – the famous Λ-problem.

As far as I know, the first who came back to possible contributions of the vacuum energy density to the cosmological constant was Zel'dovich. He discussed this issue in two papers [29] during the third renaissance period of the Λ-term, but before the advent of spontaneously broken gauge theories. The following remark by him is particularly interesting. Even if one assumes completely ad hoc that the zero-point contributions to the vacuum energy density are exactly cancelled by a bare term, there still remain higher-order effects. In particular, *gravitational* interactions between the particles in the vacuum fluctuations are expected on dimensional grounds to lead to a gravitational self-energy density of order $G\mu^6$, where μ is some cut-off scale. Even for μ as low as 1 GeV (for no good reason) this is about 9 orders of magnitude larger than the observational bound.

This illustrates that there is something profound that we do not understand at all, certainly not in quantum field theory (so far also not in string theory). We are unable to calculate the vacuum energy density in quantum field theories, like the Standard Model of particle physics. But we can attempt to make what appear to be reasonable order-of-magnitude estimates for the various contributions. All expectations are **in gigantic conflict with the facts** (see below). Trying to arrange the cosmological constant to be zero is unnatural in a technical sense. It is like enforcing a particle to be massless, by fine-tuning the parameters of the theory when there is no symmetry principle which implies a vanishing mass. The vacuum energy density is unprotected from large quantum corrections. This problem is particularly severe in field theories with spontaneous symmetry breaking. In such models there are usually several possible vacuum states with different energy densities. Furthermore, the energy density is determined by what is called the effective potential, and this is a *dynamical* object. Nobody can see any reason why the vacuum of the Standard Model we ended up as the Universe cooled, has – for particle physics standards – an almost vanishing energy density. Most probably, we will only have a satisfactory answer once we shall have a theory which successfully combines the concepts and laws of general relativity about gravity and spacetime structure with those of quantum theory.

Simple estimates of vacuum energy contributions

If we take into account the contributions to the vacuum energy from vacuum fluctuations in the fields of the Standard Model up to the currently explored energy, i.e., about the electroweak scale $M_F = G_F^{-1/2} \approx 300 GeV (G_F$: Fermi coupling constant), we cannot expect an almost complete cancellation, because there is *no symmetry principle* in this energy range that could require this. The only symmetry principle which would imply this is *supersymmetry*, but supersymmetry is broken (if it is realized in nature). Hence we can at best expect a very imperfect cancellation below the electroweak scale, leaving a contribution of the order of M_F^4 . (The contributions at higher energies may largely cancel if supersymmetry holds in the real world.)

We would reasonably expect that the vacuum energy density is at least as large as the condensation energy density of the QCD phase transition to the broken phase of chiral symmetry. Already this is far too large: $\sim \Lambda_{QCD}^4/16\pi^2 \sim 10^{-4} GeV^4$; this is *more than 40 orders of magnitude larger* than ρ_{crit}. Beside the formation of quark condensates $< \bar{q}q >$ in the QCD vacuum which break chirality, one also expects a gluon condensate $< G_a^{\mu\nu} G_{a\mu\nu} >$ $\sim \Lambda_{QCD}^4$. This produces a significant vacuum energy density as a result of a dilatation anomaly: If Θ_μ^μ denotes the "classical" trace of the energy-momentum tensor, we have [30]

$$T^{\mu}_{\mu} = \Theta^{\mu}_{\mu} + \frac{\beta(g_3)}{2g_3} G^{\mu\nu}_{a} G_{a\mu\nu}, \tag{13}$$

where the second term is the QCD piece of the trace anomaly ($\beta(g_3)$ is the β-function of QCD that determines the running of the strong coupling constant). I recall that this arises because a scale transformation is no more a symmetry if quantum corrections are included. Taking the vacuum expectation value of (32), we would again naively expect that $< \Theta^{\mu}_{\mu} >$ is of the order M_F^4. Even if this should vanish for some unknown reason, the anomalous piece is cosmologically gigantic. The expectation value $< G^{\mu\nu}_{a} G_{a\mu\nu} >$ can be estimated with QCD sum rules [31], and gives

$$< T^{\mu}_{\mu} >^{anom} \sim (350 MeV)^4, \tag{14}$$

about 45 orders of magnitude larger than ρ_{crit}. This reasoning should show convincingly that the cosmological constant problem is indeed a profound one. (Note that there is some analogy with the (much milder) strong CP problem of QCD. However, in contrast to the Λ-problem, Peccei and Quinn [32] have shown that in this case there is a way to resolve the conundrum.)

Let us also have a look at the Higgs condensate of the electroweak theory. Recall that in the Standard Model we have for the Higgs doublet Φ in the broken phase for $< \Phi^* \Phi > \equiv \frac{1}{2}\phi^2$ the potential

$$V(\phi) = -\frac{1}{2} m^2 \phi^2 + \frac{\lambda}{8}\phi^4. \tag{15}$$

Setting as usual $\phi = v + H$, where v is the value of ϕ where V has its minimum,

$$v = \sqrt{\frac{2m^2}{\lambda}} = 2^{-1/4} G_F^{-1/2} \sim 246 GeV, \tag{16}$$

we find that the Higgs mass is related to λ by $\lambda = M_H^2/v^2$. For $\phi = v$ we obtain the energy density of the Higgs condensate

$$V(\phi = v) = -\frac{m^4}{2\lambda} = -\frac{1}{8\sqrt{2}} M_F^2 M_H^2 = \mathcal{O}(M_F^4). \tag{17}$$

We can, of course, add a constant V_0 to the potential (34) such that it cancels the Higgs vacuum energy in the broken phase – including higher order corrections. This again requires an extreme fine tuning. A remainder of only $\mathcal{O}(m_e^4)$, say, would be catastrophic. This remark is also highly relevant for models of inflation and quintessence.

In attempts beyond the Standard Model the vacuum energy problem so far remains, and often becomes even worse. For instance, in supergravity theories with spontaneously broken supersymmetry there is the following simple relation between the gravitino mass m_g and the vacuum energy density

$$\rho_{vac} = \frac{3}{8\pi G} m_g^2.$$

Comparing this with eq.(30) we find

$$\frac{\rho_{vac}}{\rho_{crit}} \simeq 10^{122} \Big(\frac{m_g}{m_{Pl}}\Big)^2.$$

Even for $m_g \sim 1\ eV$ this ratio becomes 10^{66}. (m_g is related to the parameter F characterizing the strength of the supersymmetry breaking by $m_g = (4\pi G/3)^{1/2} F$, so $m_g \sim 1\ eV$ corresponds to $F^{1/2} \sim 100\ TeV$.)

Also string theory has not yet offered convincing clues why the cosmological constant is so extremely small. The main reason is that a *low energy mechanism* is required, and since supersymmetry is broken, one again expects a magnitude of order M_F^4, which is *at least 50 orders of magnitude too large* (see also [33]). However, non-supersymmetric physics in string theory is at the very beginning and workers in the field hope that further progress might eventually lead to an understanding of the cosmological constant problem.

I hope I have convinced you, that there is something profound that we do not understand at all, certainly not in quantum field theory, but so far also not in string theory. (For other recent reviews, see also [34], [35], and [36]. These contain more extended lists of references.)

6 Microwave background anisotropies

Investigations of the cosmic microwave background have presumably contributed most to the remarkable progress in cosmology during recent years (For a recent review, see [37]). Beside its spectrum, which is Planckian to an incredible degree, we also can study the temperature fluctuations over the "cosmic photosphere" at a redshift $z \approx 1100$. Through these we get access to crucial cosmological information (primordial density spectrum, cosmological parameters, etc). A major reason for why this is possible relies on the fortunate circumstance that the fluctuations are tiny ($\sim 10^{-5}$) at the time of recombination. This allows us to treat the deviations from homogeneity and isotropy for an extended period of time perturbatively, i.e., by linearizing the Einstein and matter equations about solutions of the idealized Friedmann-Lemaître models. Since the physics is effectively *linear*, we can accurately work out the *evolution* of the perturbations during the early phases of the Universe, given a set of cosmological parameters. Confronting this with observations, tells us a lot about the cosmological parameters as well as the initial conditions, and thus about the physics of the very early Universe. Through this window to the earliest phases of cosmic evolution we can, for instance, test general ideas and specific models of inflation.

6.1 Qualitative remarks

Let me begin with some qualitative remarks, before I go into more technical details. Long before recombination (at temperatures $T > 6000K$, say) photons, electrons and baryons were so strongly coupled that these components may be treated together as a single fluid. In addition to this there is also a dark matter component. For all practical purposes the two interact only gravitationally. The investigation of such a two-component fluid for small deviations from an idealized Friedmann behavior is a well-studied application of cosmological perturbation theory.

At a later stage, when decoupling is approached, this approximate treatment breaks down because the mean free path of the photons becomes longer (and finally 'infinite' after recombination). While the electrons and baryons can still be treated as a single fluid, the photons and their coupling to the electrons have to be described by the general relativistic Boltzmann equation. The latter is, of course, again linearized about the idealized Friedmann solution. Together with the linearized fluid equations (for baryons and cold dark matter, say), and the linearized Einstein equations one arrives at a complete system of equations for the various perturbation amplitudes of the metric and matter variables. There exist widely used codes e.g. CMBFAST [38], that provide the CMB anisotropies – for given initial conditions – to a precision of about 1%. A lot of qualitative and semi-quantitative insight into the relevant physics can, however, be gained by looking at various approximations of the basic dynamical system.

Let us first discuss the temperature fluctuations. What is observed is the temperature autocorrelation:

$$C(\vartheta) := \left\langle \frac{\Delta T(\mathbf{n})}{T} \cdot \frac{\Delta T(\mathbf{n}')}{T} \right\rangle = \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} C_l P_l(\cos\vartheta), \tag{18}$$

where ϑ is the angle between the two directions of observation $\mathbf{n}, \mathbf{n}'$, and the average is taken ideally over all sky. The *angular power spectrum* is by definition $\frac{l(l+1)}{2\pi} C_l$ *versus* l ($\vartheta \simeq \pi/l$).

A characteristic scale, which is reflected in the observed CMB anisotropies, is the sound horizon at last scattering, i.e., the distance over which a pressure wave can propagate until decoupling. This can be computed within the unperturbed model and subtends about half a degree on the sky for typical cosmological parameters. For scales larger than this sound horizon the fluctuations have been laid down in the very early Universe. These have been detected by the COBE satellite. The (gauge invariant brightness) temperature perturbation $\Theta = \Delta T/T$ is dominated by the combination of the intrinsic temperature fluctuations and gravitational redshift or blueshift effects. For example, photons that have to climb out of potential wells for high-density regions are redshifted. One can show that these effects combine for adiabatic initial conditions to $\frac{1}{3}\Psi$, where Ψ is one of the two gravitational Bardeen potentials. The latter, in turn, is directly related to the density perturbations. For scale-free initial perturbations and almost vanishing spatial curvature the corresponding angular power spectrum of the temperature fluctuations turns out to be nearly flat (Sachs-Wolfe plateau in Fig. 1).

On the other hand, inside the sound horizon before decoupling, acoustic, Doppler, gravitational redshift, and photon diffusion effects combine to the spectrum of small angle anisotropies shown in Fig.1. These result from gravitationally driven synchronized acoustic oscillations of the photon-baryon fluid, which are damped by photon diffusion.

A particular realization of $\Theta(\mathbf{n})$, such as the one accessible to us (all sky map from our location), cannot be predicted. Theoretically, Θ is a random field $\Theta(\mathbf{x}, \eta, \mathbf{n})$, depending on the conformal time η, the spatial coordinates, and the observing direction $\mathbf{n}$. Its correlation functions should be rotationally invariant in $\mathbf{n}$, and respect the symmetries of the background time slices. If we expand Θ in terms of spherical harmonics,

$$\Theta(\mathbf{n}) = \sum_{lm} a_{lm} Y_{lm}(\mathbf{n}), \tag{19}$$

the random variables a_{lm} have to satisfy

$$\langle a_{lm} \rangle = 0, \quad \langle a^\star_{lm} a_{l'm'} \rangle = \delta_{ll'}\delta_{mm'} C_l(\eta), \tag{20}$$

where the $C_l(\eta)$ depend only on η. Hence the correlation function at the present time η_0 is given by (1), where $C_l = C_l(\eta_0)$, and the bracket now denotes the statistical average. Thus,

$$C_l = \frac{1}{2l+1} \left\langle \sum_{m=-l}^{l} a^\star_{lm} a_{lm} \right\rangle. \tag{21}$$

The standard deviations $\sigma(C_l)$ measure a fundamental uncertainty in the knowledge we can get about the C_l's. These are called *cosmic variances*, and are most pronounced for low l. In simple inflationary models the a_{lm} are Gaussian distributed, hence

$$\frac{\sigma(C_l)}{C_l} = \sqrt{\frac{2}{2l+1}}. \tag{22}$$

Therefore, the limitation imposed on us (only one sky in one universe) is small for large l.

6.2 Boltzmann hierarchy

The brightness temperature fluctuation can be obtained from the perturbation of the photon distribution function by integrating over the magnitude of the photon momenta. The linearized Botzmann equation can then be translated into an equation for Θ, which we now regard as a function of η, x^i, and γ^j, where the γ^j are the directional cosines of the momentum vector relative to an orthonormal triad field of the unperturbed spatial metric with curvature K. Next one performs a harmonic decomposition of Θ, which reads for the spatially flat case ($K = 0$)

$$\Theta(\eta, \boldsymbol{x}, \boldsymbol{\gamma}) = (2\pi)^{-3/2} \int d^3k \sum_l \theta_l(\eta, k) G_l(\boldsymbol{x}, \boldsymbol{\gamma}; \boldsymbol{k}), \tag{23}$$

where

$$G_l(\boldsymbol{x}, \boldsymbol{\gamma}; \boldsymbol{k}) = (-i)^l P_l(\hat{\boldsymbol{k}} \cdot \boldsymbol{\gamma}) \exp(i\boldsymbol{k} \cdot \boldsymbol{x}). \tag{24}$$

The dynamical variables $\theta_l(\eta)$ are the *brightness moments*, and should be regarded as random variables. Boltzmann's equation implies the following hierarchy of ordinary differential equations for the brightness moments[5] $\theta_l(\eta)$ (if polarization effects are neglected):

$$\theta_0' = -\frac{1}{3}k\theta_1 - \Phi', \tag{25}$$

$$\theta_1' = k\Big(\theta_0 + \Psi - \frac{2}{5}\theta_2\Big) - \dot{\tau}(\theta_1 - V_b), \tag{26}$$

$$\theta_2' = k\Big(\frac{2}{3}\theta_1 - \frac{3}{7}\theta_3\Big) - \dot{\tau}\frac{9}{10}\theta_2, \tag{27}$$

$$\theta_l' = k\Big(\frac{l}{2l-1}\theta_{l-1} - \frac{l+1}{2l+3}\theta_{l+1}\Big), \quad l > 2. \tag{28}$$

Here, V_b is the gauge invariant scalar velocity perturbation of the baryons, $\dot{\tau} = x_e n_e \sigma_T a/a_0$, where a is the scale factor, $x_e n_e$ the unperturbed free electron density (x_e = ionization fraction), and σ_T the Thomson cross section.

[5]In the literature the normalization of the θ_l is sometimes chosen differently: $\theta_l \to (2l+1)\theta_l$.

Moreover, Φ and Ψ denote the Bardeen potentials. (For further details, see Sect. 6 of [1] or [39].)

The C_l are determined by an integral over k, involving a primordial power spectrum (of curvature perturbations) and the $|\theta_l(\eta)|^2$, for the corresponding initial conditions (their transfer functions).

This system of equations is completed by the linearized fluid and Einstein equations. Various approximations for the Boltzmann hierarchy provide already a lot of insight. In particular, one can very nicely understand how damped acoustic oscillations are generated, and in which way they are influenced by the baryon fraction (again, see [2] or [39]). A typical theoretical CMB spectrum is shown in Fig. 1. (Beside the scalar contribution in the sense of cosmological perturbation theory, considered so far, the tensor contribution due to gravity waves is also shown there.)

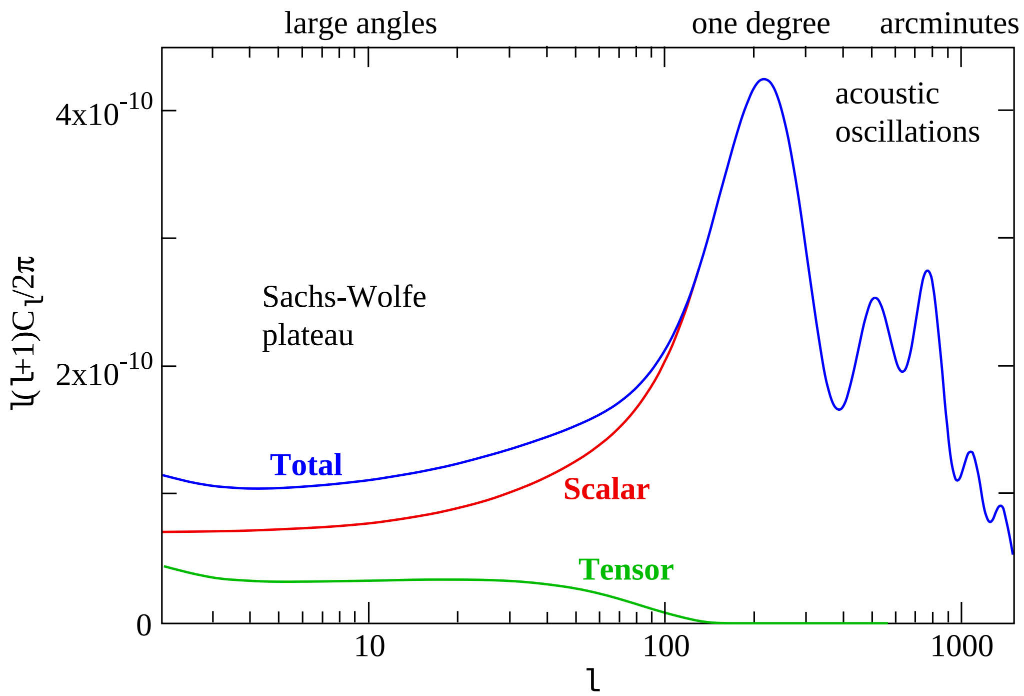

Figure 1: Theoretical angular power spectrum for adiabatic initial perturbations and typical cosmological parameters. The scalar and tensor contributions to the anisotropies are also shown.

7 Polarization

A polarization map of the CMB radiation provides important additional information to that obtainable from the temperature anisotropies. For example, we can get constraints about the epoch of reionization. Most importantly, future polarization observations may reveal a stochastic background of gravity waves, generated in the very early Universe. In this section we give a brief introduction to the study of CMB polarization.

The mechanism which partially polarizes the CMB radiation is similar to that for the scattered light from the sky. Consider first scattering at a single electron of unpolarized radiation coming in from all directions . Due to the familiar polarization dependence of the differential Thomson cross section, the scattered radiation is, in general, polarized. It is easy to compute the corresponding Stokes parameters. Not surprisingly, they are not all equal to zero if and only if the intensity distribution of the incoming radiation has a non-vanishing quadrupole moment. The Stokes parameters Q and U are proportional to the overlap integral with the combinations $Y_{2,2} \pm Y_{2,-2}$ of the spherical harmonics, while V vanishes.) This is basically the reason why a CMB polarization map traces (in the tight coupling limit) the quadrupole temperature distribution on the last scattering surface.

The polarization tensor of an all sky map of the CMB radiation can be parametrized in temperature fluctuation units, relative to the orthonormal basis $\{d\vartheta, \sin\vartheta\ d\varphi\}$ of the two sphere, in terms of the Pauli matrices as $\Theta \cdot 1 + Q\sigma_3 + U\sigma_1 + V\sigma_2$. The Stokes parameter V vanishes (no circular polarization). Therefore, the polarization properties can be described by the following symmetric trace-free tensor on S^2:

$$(\mathcal{P}_{ab}) = \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}. \tag{29}$$

As for gravity waves, the components Q and U transform under a rotation of the 2-bein by an angle α as

$$Q \pm iU \to e^{\pm 2i\alpha}(Q \pm iU), \tag{30}$$

and are thus of spin-weight 2. $\mathcal{P}_{ab}$ can be decomposed uniquely into *'electric'* and *'magnetic'* parts:

$$\mathcal{P}_{ab} = E_{;ab} - \frac{1}{2} g_{ab}\Delta E + \frac{1}{2}(\varepsilon_a{}^c B_{;bc} + \varepsilon_b{}^c B_{;ac}). \tag{31}$$

Expanding here the scalar functions E and B in terms of spherical harmonics, we obtain an expansion of the form

$$\mathcal{P}_{ab} = \sum_{l=2}^{\infty} \sum_m \left[a^E_{(lm)} Y^E_{(lm)ab} + a^B_{(lm)} Y^B_{(lm)ab} \right] \tag{32}$$

in terms of the tensor harmonics:

$$Y^E_{(lm)ab} := N_l(Y_{(lm);ab} - \frac{1}{2}g_{ab}Y_{(lm);c}{}^{c}), \quad Y^B_{(lm)ab} := \frac{1}{2}N_l(Y_{(lm);ac}\varepsilon^c{}_b + a \leftrightarrow b), \tag{33}$$

where $l \geq 2$ and

$$N_l \equiv \left(\frac{2(l-2)!}{(l+2)!}\right)^{1/2}.$$

Equivalently, one can write this as

$$Q + iU = \sqrt{2}\sum_{l=2}^{\infty}\sum_m \left[a^E_{(lm)} + ia^B_{(lm)}\right] {}_2Y^m_l, \tag{34}$$

where ${}_sY^m_l$ are the spin-s harmonics.

As in Eq.(19) the multipole moments $a^E_{(lm)}$ and $a^B_{(lm)}$ are random variables, and we have equations analogous to (21):

$$C^{TE}_l = \frac{1}{2l+1}\sum_m \langle a^{\Theta\star}_{lm}a^E_{lm}\rangle, \quad etc. \tag{35}$$

(We have now put the superscript Θ on the a_{lm} of the temperature fluctuations.) The C_l's determine the various angular correlation functions. For example, one easily finds

$$\langle\Theta(\boldsymbol{n})Q(\boldsymbol{n'})\rangle = \sum_l C^{TE}_l\frac{2l+1}{4\pi}N_lP^2_l(\cos\vartheta). \tag{36}$$

For the space-time dependent Stokes parameters Q and U of the radiation field we can perform a normal mode decomposition analogous to (23). If, for simplicity, we again consider only scalar perturbations this reads

$$Q \pm iU = (2\pi)^{-3/2}\int d^3k\sum_l(E_l \pm iB_l)\,{}_{\pm2}G^0_l, \tag{37}$$

where

$${}_sG^m_l(\boldsymbol{x},\boldsymbol{\gamma};\boldsymbol{k}) = (-i)^l\left(\frac{2l+1}{4\pi}\right)^{1/2}\,{}_sY^m_l(\boldsymbol{\gamma})\exp(i\boldsymbol{k}\cdot\boldsymbol{x}), \tag{38}$$

if the mode vector $\mathbf{k}$ is chosen as the polar axis. (Note that G_l in (24) is equal to ${}_0G^0_l$.)

The Boltzmann equation implies a coupled hierarchy for the moments θ_l, E_l, and B_l [40], [41]. It turns out that the B_l vanish for scalar perturbations. Non-vanishing magnetic multipoles would be a unique signature for a spectrum of gravity waves. In a sudden decoupling approximation, the present electric multipole moments can be expressed in terms of the brightness quadrupole moment on the last scattering surface and spherical Bessel functions as

$$\frac{E_l(\eta_0, k)}{2l+1} \simeq \frac{3}{8}\theta_2(\eta_{dec}, k)\frac{l^2 j_l(k\eta_0)}{(k\eta_o)^2}. \tag{39}$$

Here one sees how the observable E_l's trace the quadrupole temperature anisotropy on the last scattering surface. In the tight coupling approximation the latter is proportional to the dipole moment θ_1.

8 Observational results

In recent years several experiments gave clear evidence for multiple peaks in the angular temperature power spectrum at positions expected on the basis of the simplest inflationary models and big bang nucleosynthesis [42]. These results have been confirmed and substantially improved by WMAP [43] (see Fig. 2).

In spite of the high accuracy of the data, it is not possible to extract unambiguously cosmological parameters, because there are intrinsic degeneracies, especially when tensor modes are included. These can only be lifted if other cosmological information is used. Beside the supernova results, use has been made for instance of the available information for the galaxy power spectrum (in particular from the 2-degree-Field Galaxy Redshift Survey (2dFGRS)), and limits for the Hubble parameter. For example, if one adds to the CMB data the well-founded constraint $H_0 \geq 50\ km/s/Mpc$, then the total density parameter Ω_{tot} has to be in the range $0.98 < \Omega_{tot} < 1.08$ (95 %). The Universe is thus *spatially almost flat.* (For further evidence, see Figs. 3,4) In what follows we therefore always assume $K = 0$.

Table 1 is extracted from the extended analysis [44] of the WMAP data and other cosmological information. It shows the 68% confidence ranges for some of the cosmological parameters for two types of fits, assuming a ΛCDM model. In the first only the CMB data are used (but tensor modes are included), while in the second these data are combined with the 2dFGRS power spectrum (assuming adiabatic, Gaussian initial conditions described by power laws).

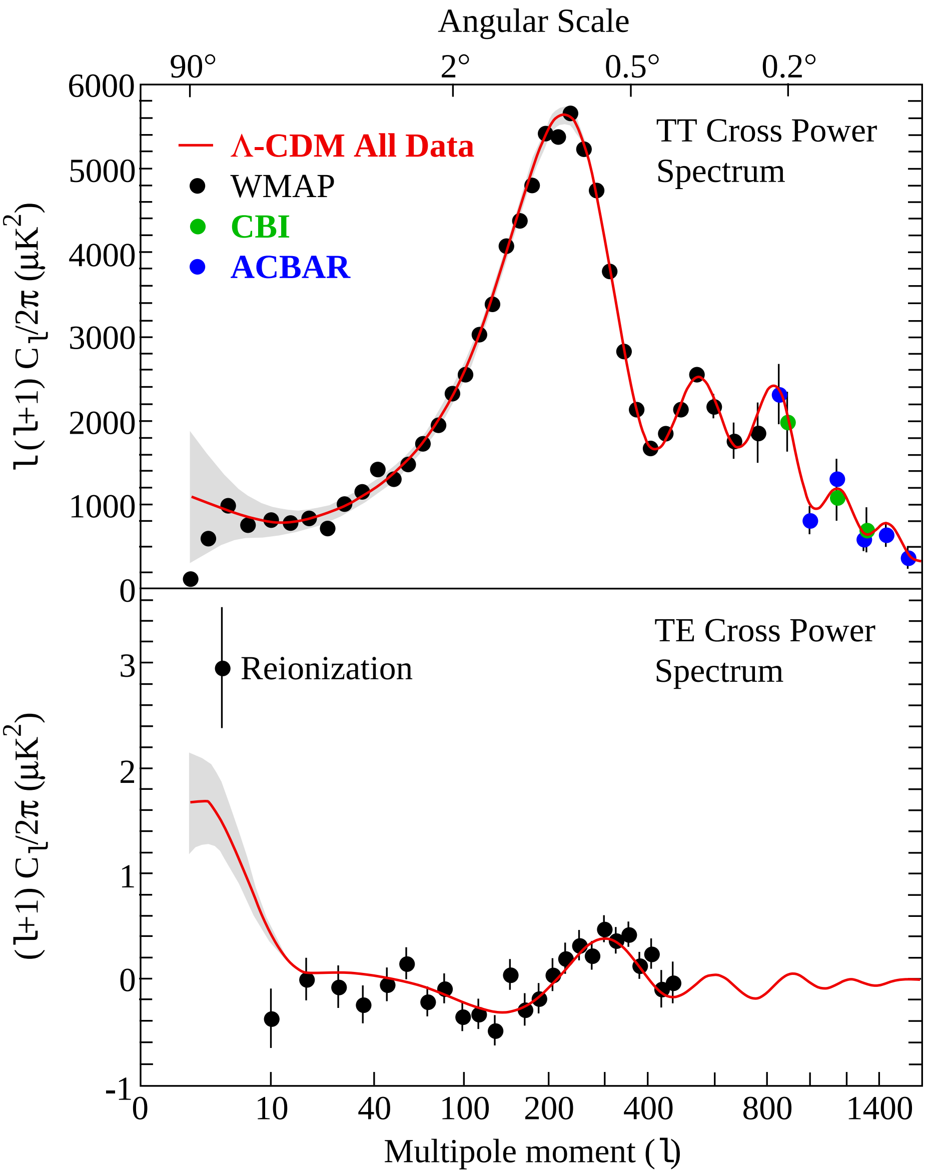

Figure 2: Temperature-temperature (TT) and temperature-polarization TE power spectra. The best fit ΛCDM model is also shown (Fig. 12 of Ref. [27]).

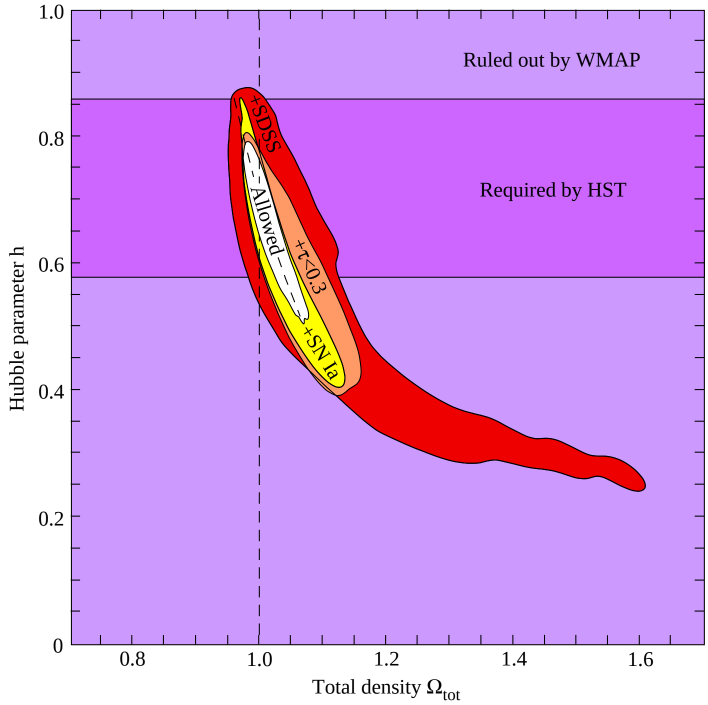

Figure 3: 95% constraints in the (Ω_{tot}, h_0) plane. The WMAP data alone, when analyzed with with a 7-parameter curved model, allows only the banana-shaped region. This becomes considerably smaller if $h_0 > 0.5$ is imposed. Additional information reduces the allowed region even further. (Adapted from [46], Fig. 7.)

Table 1.

Parameter	CMB alone	CMB and 2dFGRS
$\Omega_b h_0^2$	0.024 ± 0.001	0.023 ± 0.001
$\Omega_M h_0^2$	0.14 ± 0.02	0.134 ± 0.006
h_0	0.72 ± 0.05	0.71 ± 0.04
Ω_b	0.047 ± 0.006	≃ same
Ω_M	0.29 ±0.07	≃ same

Note that there is little difference between the two columns. The age of the Universe for these parameters is close to 14 Gyr. Another interesting result coming from the rise of the temperature-polarization correlation function at large scales (small l) in Fig. 2 is that reionization of the Universe has set in surprisingly early –, at a redshift of $z_r = 17 \pm 5$, with a corresponding

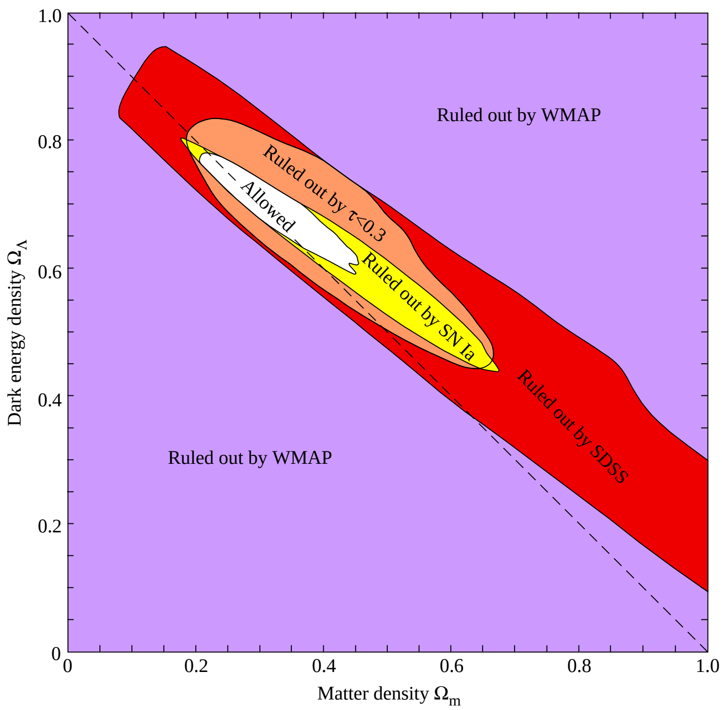

Figure 4: 95% constraints in the $(\Omega_M, \Omega_\Lambda)$ plane. These are based on the same models and data as in Fig. 8.3. (Adapted from [46], Fig. 9.)

optical depth $\tau = 0.17 \pm 0.06$.

Before the new results possible admixtures of isocurvature modes were not strongly constraint. But now the measured temperature-polarization correlations imply that the primordial fluctuations were primarily *adiabatic*. Admixtures of isocurvature modes do not improve the fit.

One worry is that the quadrupole amplitude (C_2) measured by WMAP is lower than expected according to the best fit ΛCDM model [28]. This issue has led to lots of discussions. A recent reanalysis [45] of the effects of Galactic cuts indicates that this discrepancy is not particularly significant, being in the region of a few percent. This issue may look differently, once the second year WMAP data have been analyzed. (Late in 2005 we are still eagerly waiting for seeing this.)

9 Concluding remarks

A wide range of astronomical data support the following 'concordance' ΛCDM model: The Universe is spatially flat and dominated by vacuum energy density and weakly interacting cold dark matter. Furthermore, the primordial fluctuations are adiabatic and nearly scale invariant, as predicted in simple inflationary models.

A vacuum energy with density parameter $\Omega_\Lambda \simeq 0.7$ is so surprising that it should be examined whether this conclusion is really unavoidable. Since we do not have a tested theory predicting the spectrum of primordial fluctuations, it appears reasonable to consider a wider range of possibilities than simple power laws. An instructive attempt in this direction has been made in [47], by constructing an Einstein-de Sitter model with $\Omega_\Lambda = 0$, fitting the CMB data as well as the power spectrum of 2dFGRS. In this the Hubble constant is, however, required to be rather low: $H_0 \simeq 46\ km/s/Mpc$. The authors argue that this cannot definitely be excluded, because 'physical' methods lead mostly to relatively low values of H_0. In order to be consistent with matter fluctuations on cluster scales they add relic neutrinos with degenerate masses of order eV or a small contribution of quintessence with zero pressure ($w = 0$). In addition, they have to ignore the direct evidence for an accelerating Universe from the Hubble-diagram for distant Type Ia supernovas, on the basis of remaining systematic uncertainties. While these have not yet been fully eliminated, significant progress has been made in the meantime. On the basis of new data for redshifts larger than one, obtained in conjunction with the GOODS (Great Observatories Origins Deep Survey) Treasury program, conducted with the Advanced Camera for Surveys (ACS) aboard the Hubble Space Telescope (HST), the results presented in [48] are quite impressive. These are consistent with the "cosmic concordance" model ($\Omega_M = 0.3,\ \Omega_\Lambda = 0.7$), with $\chi^2_{dof} = 1.06$). For a flat universe with a cosmological constant, the fit gives $\Omega_M = 0.29\pm^{0.13}_{0.19}$ (equivalently, $\Omega_\Lambda = 0.71$).

It has recently been suggested [49], [50] that perturbations on scales larger than the Hubble length, likely generated in the context of inflation, could mimic dark energy and cause acceleration. This suggestion caused a lot of discussion, and several papers addressed the question whether this is really possible. We repeat below a simple general argument given in [51] that the proposed mechanism can not lead to acceleration, under the assumptions made in the cited papers. These include that the 4-velocity field u^μ of the CDM particles is geodesic and has zero vorticity $\omega_{\mu\nu}$. It is easy to see that these assumptions imply that the 1-form $\mathbf{u}$, belonging to the velocity field has a vanishing exterior derivative. Hence we have locally $\mathbf{u} = dt$, thus u^μ is perpendicular to the slices $\{t = const\}$. Moreover the metric and the velocity have the form

$$g = dt^2 - \bar{g}_t, \quad u = \partial_t,$$

where $\bar{g}_t$ is a t-dependent metric on slices of constant time t.

For such an inhomogeneous cosmological model one can introduce various definitions of the deceleration parameter which reduce to the familiar one for Friedmann models. We adopt here the one used in [50]. To motivate this, consider for some initial time t_{in} a spatial domain D and let this evolve according to the flow of u. If ω_t denotes the volume form belonging to $\bar{g}_t$, then we have for the volume $|D_t|$ and its time derivatives

$$|D_t| = \int \omega_t, \quad |\dot{D}_t| = \int \theta\omega_t, \quad |\ddot{D}_t| = \int (\dot{\theta} + \theta^2)\omega_t,$$

where $\theta = \nabla \cdot u$ denotes the expansion. If $l := |D_t|^{1/3}$, a natural definition of the deceleration parameter is $q = -(l\ddot{l})/\dot{l}^2$. This can be expressed as follows

$$\frac{1}{3}\frac{(|\dot{D}_t|)^2}{|D_t|^2} q = -\left(\frac{|\ddot{D}_t|}{|D_t|} - \frac{2}{3}\frac{(|\dot{D}_t|)^2}{|D_t|^2} \right).$$

For an *infinitesimal* $|D_t|$ we obtain from the previous equations

$$\frac{1}{3}\theta^2 q = -(\dot{\theta} + \frac{1}{3}\theta^2).$$

For the right-hand side we can now use the Raychaudhuri equation

$$\dot{\theta} + \frac{1}{3}\theta^2 = -\sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu,$$

where $\sigma_{\mu\nu}$ is the shear. For a vanishing vorticity, and imposing the strong energy condition (assumed in [50]), we see that $q \geq= 0$. In this sense there is no acceleration.

A priori, a way out proposed by [49], is to argue that q as defined above is not what is measured in SN Ia observations. To analyse these one has to generalize the redshift-luminosity distance relation to inhomogeneous models. In doing this, two possible definitions for the deceleration parameter arise. One of them (q_4 in [51]) again has to be non-negative if the strong energy condition holds. The other (q_3 in [51]) may be negative, but in this case the supernova data would have to show acceleration in certain directions and deceleration in others. This is, however, not observed.

Kolb et al. have reacted to these considerations [52]. They admit that super-Hubble modes can not lead to an acceleration, but they maintain that sub-Hubble modes may cause a large backreaction that may imply an effective acceleration. The authors stress that for investigating the effective dynamics averaging over a volume of size comparable with the present-day Hubble volume is essential. Let me add a few remarks on this. Adopting the notation

$$\langle\theta\rangle = \frac{\int \theta\omega_t}{\int \omega_t}, \quad etc,$$

and using the Raychaudhuri equation, we can write

$$\frac{1}{3}\frac{(|D_t|^{\cdot})^2}{|D_t|^2}q = -\langle\dot{\theta} + \theta^2\rangle + \frac{2}{3}\langle\theta\rangle^2$$

$$= -\langle\dot{\theta} + \frac{1}{3}\theta^2\rangle - \frac{2}{3}(\langle\theta^2\rangle - \langle\theta\rangle^2) = \langle\sigma_{\mu\nu}\sigma^{\mu\nu} + R_{\mu\nu}u^\mu u^\nu\rangle - \frac{2}{3}(\langle\theta^2\rangle - \langle\theta\rangle^2).$$

The first term in the last eq. is non-negative if the strong energy condition holds, while the second term is non-positive.

The authors of [52] suggest that the second term may win and make q negative. To decide on the basis of detailed calculations whether this is indeed possible is a very difficult task and seems unlikely.

It is very likely that the present concordance model will survive, but for the time being it is healthy to remain sceptical until further evidence is accumulating. The Dark Energy problem will presumably stay with us for a long time.

References

[1] G.E. Lemaître, in *Albert Einstein: Philosopher-Scientist*, P.A. Schilpp, ed., Illinois: The Library of Living Philosophers (1949).

[2] N. Straumann, *On the Cosmological Constant Problems and the Astronomical Evidence for a Homogeneous Energy Density with Negative Pressure*, in *Poincarè Seminar 2002, Vacuum Energy – Renormalization*, B. Duplantier, and V. Rivasseau, eds.; Birkhäuser-Verlag 2003, p.7-51; astro-ph/0203330.

[3] N. Straumann, *The History of the Cosmological Constant Problem*, in *On the Nature of Dark Energy*, IAP Astrophysics Colloquium 2002, Frontier Group, 2003, p.17; gr-qc/0208027.

[4] A. Einstein, *Sitzungsber. Preuss. Akad. Wiss.* phys.-math. Klasse VI, 142 (1917). See also: [4], Vol. 6, p.540, Doc. 43.

[5] A. Einstein, *The Collected Papers of Albert Einstein*, Vols. 1-9, Princeton University Press, 1987–. See also: [http:www. einstein.caltech.edu/].

[6] A. Einstein, *On the Foundations of the General Theory of Relativity.* Ref.[4], Vol. 7, Doc. 4.

[7] A. Pais, *'Subtle is the Lord...': The Science and the Life of Albert Einstein.* Oxford University Press (1982). See especially Sect.15e.

[8] W. de Sitter, Proc. Acad. Sci., **19**, 1217 (1917); and **20**, 229 (1917).

[9] A.S. Eddington, *The Mathematical Theory of Relativity.*Chelsea Publishing Company (1924). Third (unaltered) Edition (1975). See especially Sect.70.

[10] Letter from Hermann Weyl to Felix Klein, 7 February 1919; see also Ref. [5], Vol. 8, Part B, Doc. 567.

[11] H. Weyl, Phys. Zeits. **24**, 230, (1923); Phil. Mag. **9**, 923 (1930).

[12] C. Lanczos, Phys. Zeits. **23**, 539 (1922).

[13] C. Lanczos, Zeits. f. Physik **17**, 168 (1923).

[14] A. Friedmann, Z.Phys. **10**, 377 (1922); **21**, 326 (1924).

[15] G.E. Lemaître, Ann. Soc. Sci. Brux. A **47**, 49 (1927).

[16] G.E. Lemaître, Monthly Not. Roy. Astron. Soc. **91**, 483 (1931).

[17] A. Einstein, S.B. Preuss. Akad. Wiss. (1931), 235.

[18] A. Einstein, Appendix to the 2nd edn. of *The Meaning of Relativity*, (1945); reprinted in all later editions.

[19] W. Pauli, *Theory of Relativity.* Pergamon Press (1958); Supplementary Note **19**.

[20] O. Heckmann, *Theorien der Kosmologie*, berichtigter Nachdruck, Springer-Verlag (1968).

[21] N. Straumann, *General Relativity, With Applications to Astrophysics*, Texts and Monographs in Physics, Springer Verlag, 2004.

[22] V. Petrosian, E.E. Salpeter, and P. Szekeres, Astrophys. J. **147**, 1222 (1967).

[23] W. Pauli, *Pauli Lectures on Physics*; Ed. C.P.Enz. MIT Press (1973); Vol.4, especially Sect.20.

[24] C.P. Enz, and A. Thellung, Helv. Phys. Acta **33**, 839 (1960).

[25] W. Pauli, *Die allgemeinen Prinzipien der Wellenmechanik*. Handbuch der Physik, Vol. XXIV (1933). New edition by N. Straumann, Springer-Verlag (1990); see Appendix III, p. 202.

[26] W. Heisenberg and H. Euler, Z. Phys. **38**, 714 (1936).

[27] V.S. Weisskopf, Kongelige Danske Videnskabernes Selskab, Mathematisk-fysiske Meddelelser XIV, No.6 (1936).

[28] M. Bordag, U. Mohideen, and V.M. Mostepanenko *New Developments in the Casimir Effect*, quant-ph/0106045.

[29] Y.B. Zel'dovich, JETP letters **6**, 316 (1967); Soviet Physics Uspekhi **11**, 381 (1968).

[30] C.G. Callan, S. Coleman, and R. Jackiw, Ann.Phys.**59**, 42 (1970).

[31] T. Schäfer and E.V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).

[32] R.D. Peccei and H. Quinn, Phys. Rev. Lett, **38**, 1440 (1977); Phys. Rev. **D16**, 1791 (1977).

[33] E. Witten, hep-ph/0002297.

[34] S.M. Caroll, *Living Reviews in Relativity*, astro-ph/0004075.

[35] N. Straumann, in *Dark Matter in Astro- and Particle Physics*, Edited by H.V.Klapdor-Kleingrothaus, Springer (2001), p.110.

[36] S.E. Rugh and H. Zinkernagel, hep-th/0012253.

[37] W. Hu and S. Dodelson, Annu. Rev. Astron. Astrophys. **40**, 171-216 (2002).

[38] U. Seljak, and M. Zaldarriaga, Astrophys. J.**469**, 437 (1996). (See also *http://www.sns.ias.edu/matiasz/CMBFAST/cmbfast.html*)

[39] N. Straumann, *From primordial quantum fluctuations to the anisotropies of the cosmic microwave background radiation*, to appear in Annalen der Physik (2006); hep-ph/0505249.

[40] W. Hu and M. White, Phys. Rev. D **56**, 596(1997).

[41] W. Hu, U. Seljak, M. White, and M. Zaldarriaga, Phys. Rev. D **57**, 3290 (1998).

[42] G. Steigman, astro-ph/0308511.

[43] C.L. Bennett, et al., ApJS **148**, 1 (2003); ApJS **148**, 97 (2003).

[44] D.N. Spergel, et al., ApJS **148** 175 (2003).

[45] G. Efstathiou, astro-ph/ 0310207.

[46] M. Tegmark et al., Phys. Rev. **D69**, 103501 (2004); astro-ph/0310723.

[47] A. Blanchard, M. Douspis, M. Rowan-Robinson, and S. Sarkar, astro-ph/0304237.

[48] A.G. Riess, et al., Astrophys. J. **607**, 665 (2004); astro-ph/0402512.

[49] E. Barausse, S. Matarrese, and A. Riotto, astro-ph/0501152.

[50] E. W. Kolb, S. Matarrese, A. Notari, and A. Riotto, hep-th/0503117.

[51] Ch. M. Hirata and U. Seljak, astro-ph/0503582.

[52] E. W. Kolb, S. Matarrese, and A. Riotto, astro-ph/0506534.