On the deterministic and stochastic solutions of Space Charge models and their impact on high resolution timing

C.C. Bueno *(IPEN São Paulo)* P. Fonte *(LIP Coimbra)* A. Gobbi *(GSI Darmstadt)* D. González-Díaz *(LabCaf Santiago de Compostela)* L. Lopes *(LIP Coimbra)* A. Mangiarotti *(PI Heidelberg)*
Talk given by A.M.

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Study of Strongly Interacting Matter

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LIP

Outline

The effect of the space charge

- analytical models of the space charge effect
- comparison with data

Deterministic and stochastic solutions

- time back extrapolation
- comparison with numerical solutions
- comparison with data

Timing

- single avalanche timing
- comparison with numerical solutions

Conclusions

THE EFFECT OF THE SPACE CHARGE

The space charge effect is a crucial ingredient of RPCs operation (M. Abbrescia et al., NPB (Proc. Suppl.) 78 (1999) 459):

- to have an efficency around 80% for MIPs even in small gaps (0.3 mm),
- to reach the very high electric fields strengths ($\approx$ 100 kV/cm) necessary to achieve a small time scale $1/((\alpha - \eta)v_d)$ (A.Blanco et al. NIMA 535 (2004) 272) still with a modest fraction of streamers and reasonable rate capabilities (R. Cardarelli et al., NIMA 333 (1993) 399, I. Crotty et al., NIMA 337 (1994) 379).

First experimental evidences were reported, for trigger RPCs, in R. Cardarelli et al., NIMA 382 (1996) 470 and were later extensively investigated, for timing RPCs, in P. Fonte and V. Peskov, NIMA 477 (2002) 17.

ANALYTICAL MODELS OF THE SPACE CHARGE

ABRUPT TRANSITION MODELS

- M. Abbrescia: sharp cut-off (M. Abbrescia et al., NPB (Proc. Suppl.) 78 (1999) 459). One free parameter: $n_{\mathrm{cut-off}}$.

$$\alpha(n) = \begin{cases} \alpha_0 & \text{if } n \leq n_{\mathrm{cut-off}} \\ 0 & \text{if } n > n_{\mathrm{cut-off}} \end{cases}$$

$$n(x) = \begin{cases} \exp(\alpha_0\, x) & \text{if } \alpha_0 x \leq \ln(n_{\mathrm{cut-off}}) \\ n_{\mathrm{cut-off}} & \text{if } \alpha_0 x > \ln(n_{\mathrm{cut-off}}) \end{cases}$$

- H. Raether: linear decrease of α in $\ln n$ (H. Raether, Electron Avalanches and Breakdown in Gases, Butterworths 1964). Two free parameters: n_0 and b.

$$\alpha(n) = \begin{cases} \alpha_0 & \text{if } n \leq n_0 \\ \alpha_0 \left(1 - b \ln(n/n_0)\right) & \text{if } n > n_0 \end{cases}$$

$$n(x) = \begin{cases} \exp(\alpha_0\, x) & \text{if } \alpha_0 x \leq \ln(n_0) \\ n_0 \exp\left[\frac{1}{b}\left(1 - e^{-b\left(\alpha_0 x - \ln(n_0)\right)}\right)\right] & \text{if } \alpha_0 x > \ln(n_0) \end{cases}$$

ANALYTICAL MODELS OF THE SPACE CHARGE
SMOOTH TRANSITION MODELS

- P. Fonte: sigmoidal behavior of α with n, (P. Fonte, NIMA 456 (2000) 6 – RPC 1999). One free parameter: n_{sat}.

$$\alpha(n) = \alpha_0 \, \frac{n_{\mathrm{sat}}}{n + n_{\mathrm{sat}}}$$

$$n(x) = n_{\mathrm{sat}} \, W\left(\frac{e^{\alpha_0 \, x + \frac{1}{n_{\mathrm{sat}}}}}{n_{\mathrm{sat}}}\right)$$

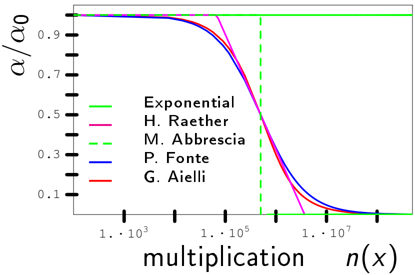
  W is the Lambert function, defined as $W(x)\, e^{W(x)} = x$ (R.M. Corless et al., Adv. in Comp. Math., 5 (1996) 329).

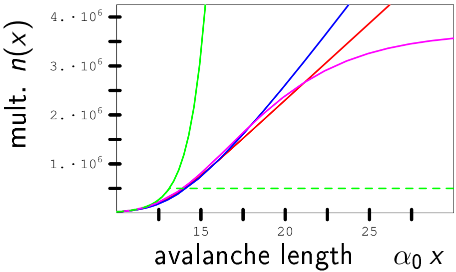
- G. Aielli: Logistic model (G. Aielli et al., NIMA 508 (2003) 6 – RPC 2001). One free parameter: x_0 ($n_{\mathrm{act}}(\infty)/n_{\mathrm{act}}(x_0) = 2$).

$$n_{\mathrm{act}}(x) = \frac{1 + e^{\alpha_0 \, x_0}}{1 + e^{-\alpha_0 (x - x_0)}}$$

$$n(x) = 1 + (1 + e^{\alpha_0 \, x_0}) \ln \frac{1 + e^{\alpha_0 (x - x_0)}}{1 + e^{-\alpha \, x_0}}$$

COMPARISON BETWEEN THE MODELS





H. Raether	$n_0 = n_{\rm sat}\, e^{-2} \quad b = 1/4$
P. Fonte	$n_{\rm sat} = 5.0 \cdot 10^5$
G. Aielli	$\alpha_0^{-1}\, x_0 \approx \ln(n_{\rm sat}) - \ln\left(2(1 + W(-2e^{-2}))\right)$

The models are different in the asymptotic behavior for $x \to \infty$:

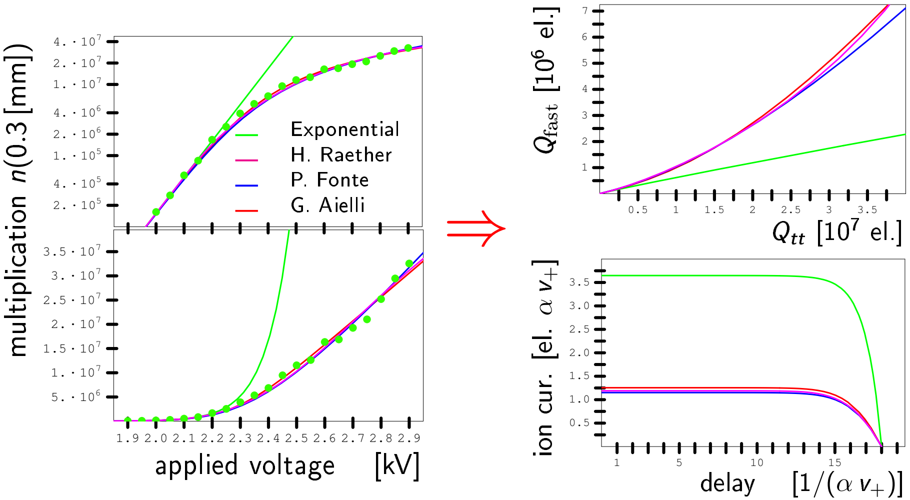
H. Raether	$n(x) \to n_0\, e^{1/b}$
P. Fonte	$n(x) \to n_{\rm sat}(\alpha_0\, x - \ln(\alpha_0\, x))$
G. Aielli	$n(x) \to e^{\alpha_0\, x_0} \alpha_0\, x$

A NEW SETUP

New measurements were performed with a special setup in the lab. of P. Fonte (Coimbra), an evolution of the one described in P. Fonte et al., NIMA 433 (1999) 513.

- A quartz fiber has been used to inject U.V. light directly in the gap and extract a single photoelectron form the cathode $\Rightarrow$ single avalanche regime.
- The gap was accurately made $\Rightarrow$ fixed avalanche length.
- Care was taken to operate the test RPC at a very low rate to avoid any charge up effect of the glass.
- A high bandwidth preamplifier and a new very fast discriminator were employed.
- An 0.3 mm gap operated in the standard electronegative mixture was used.

COMPARISON OF THE MODELS WITH DATA

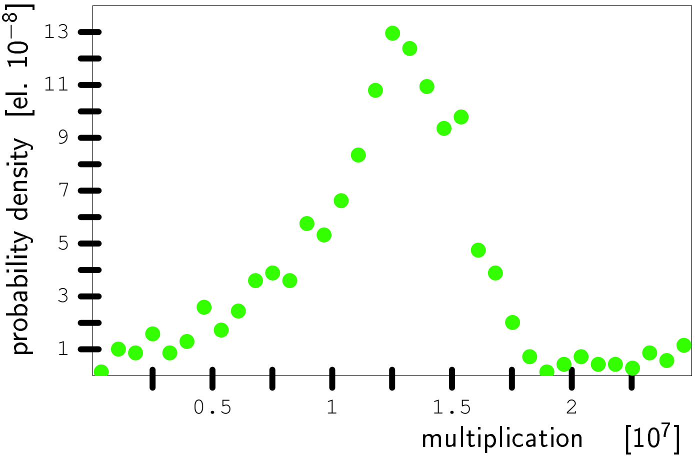


H. Raether $n_0 = 0.57 \cdot 10^6 \quad b = 0.215$

P. Fonte $n_{\mathrm{sat}} = 6.1 \cdot 10^6 \quad \ln(n_{\mathrm{sat}}) = 15.6\ [\alpha_0^{-1}]$

G. Aielli $x_0 = 15.3\ [\alpha_0^{-1}]$

PULSE HEIGHT SPECTRUM



- What information on the **avalanche dynamics** is contained in the shape of the pulse height spectrum?
- Can it be calculated analytically?

DETERMINISTIC AND STOCHASTIC SOLUTIONS

For sake of illustration the model of P. Fonte will be used.

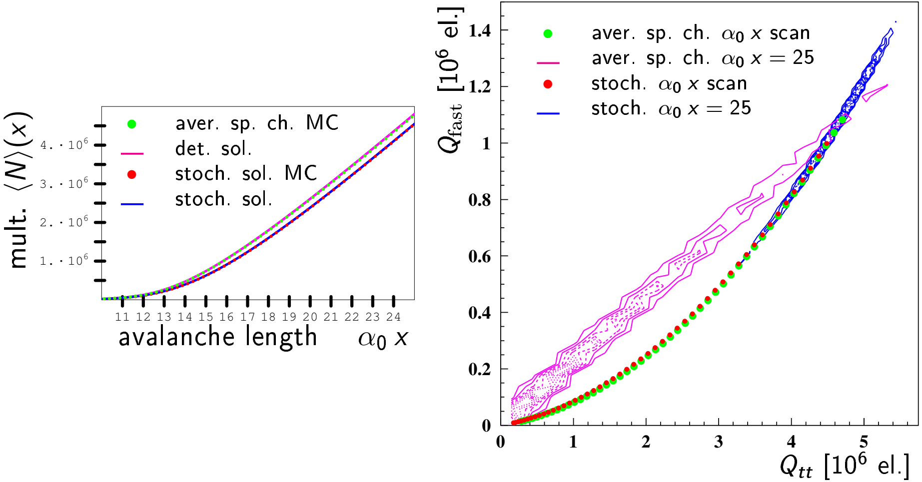
Deterministic
$$\begin{cases} \dfrac{dn}{dx} = n\ \alpha_0\ \dfrac{n_{\mathrm{sat}}}{n + n_{\mathrm{sat}}} \\ n(0) = 1 \end{cases}$$

Average Space Charge
$$\begin{cases} \dfrac{dN}{dx} = N\ \alpha_0\ \dfrac{n_{\mathrm{sat}}}{\langle N \rangle + n_{\mathrm{sat}}} \\ N(0) = 1 \end{cases}$$

Stochastic
$$\begin{cases} \dfrac{dN}{dx} = N\ \alpha_0\ \dfrac{n_{\mathrm{sat}}}{N + n_{\mathrm{sat}}} \\ N(0) = 1 \end{cases}$$

For a non-linear stochastic differential equation, the average of the stochastic solution $\langle N \rangle$ does not coincide with the deterministic solution n (W. Feller, Acta Biotheoretica 5 (1939) 1).

AVERAGE SPACE CHARGE vs STOCHASTIC SOLUTIONS

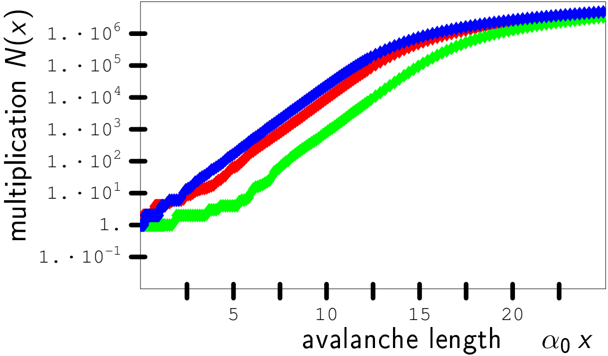


- The average space charge and the deterministic solutions coincide.
- If the stochastic solution is fitted to the data, the variation in $n_{\rm sat}$ is 15%.
- The biggest difference between the average space charge and the stochastic solutions is in the spectral shape of the fluctuations.

BACK EXTRAPOLATION IN TIME

(P. Fonte, NIMA 456 (2000) 6)

Fluctuations are produced only in the first steps of the avalanche, when the number of electrons is small.



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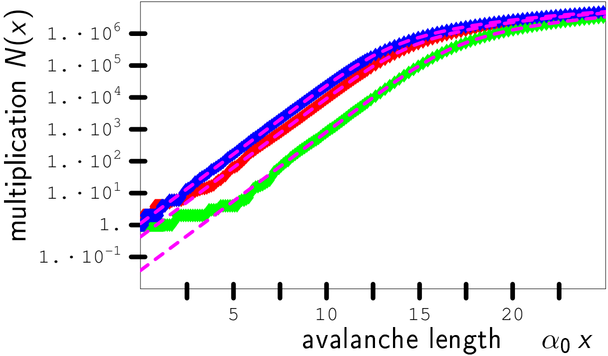


The deterministic multiplication can then be used to extrapolate back to an initial avalanche magnitude n_0. Because n_0 represents the fluctuations generated in the initial phase of the growth, it will follow the Furry law (P. Fonte, NIMA 456 (2000) 6).

ANALYTICAL SOLUTIONS

Within the back extrapolation scheme, the distribution in multiplication is always a Furry law in the initial n_0!

$$\rho_M(m) = e^{-n_0} \left| \frac{dn_0}{dm} \right|$$

No Space Charge

$$n_0 = \frac{m}{\overline{m}(x)} \qquad \Rightarrow \qquad \rho_M(m) = e^{-n_0} \frac{1}{\overline{m}(x)}$$

with $\overline{m}(x) = e^{\alpha_0 x}$

Average Space Charge

$$n_0 = \frac{m}{\overline{m}(x)} \qquad \Rightarrow \qquad \rho_M(m) = e^{-n_0} \frac{1}{\overline{m}(x)}$$

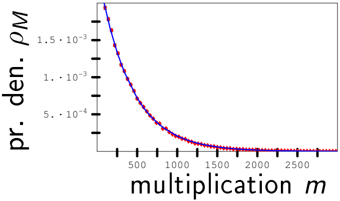
with $\overline{m}(x) = n_{\mathrm{sat}}\, W\left(\frac{1}{n_{\mathrm{sat}}} e^{1/n_{\mathrm{sat}} + \alpha_0 x}\right)$

Stochastic

$$n_0 = n_{\mathrm{sat}}\, W\left(\frac{m}{n_{\mathrm{sat}}} e^{\frac{m}{n_{\mathrm{sat}}} - \alpha_0 x}\right) \qquad \Rightarrow \qquad \rho_M(m) = e^{-n_0} \frac{1 + \frac{n_{\mathrm{sat}}}{m}}{1 + \frac{n_{\mathrm{sat}}}{n_0}}$$

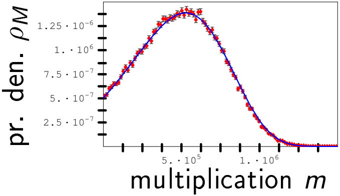
W is Lambert function, defined as $W(x)\, e^{W(x)} = x$ (R.M. Corless et al., Adv. in Comp. Math., 5 (1996) 329).

COMPARISON WITH NUMERICAL SOLUTIONS



Below Space Charge threshold

$$n_{\mathrm{sat}} = 5.0 \cdot 10^5$$
$$\alpha_0\, x = 6$$
$$\langle N \rangle = 4.0 \cdot 10^2$$

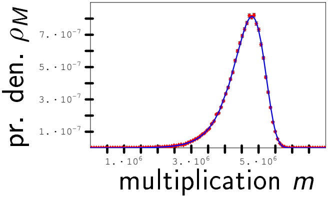


At Space Charge threshold

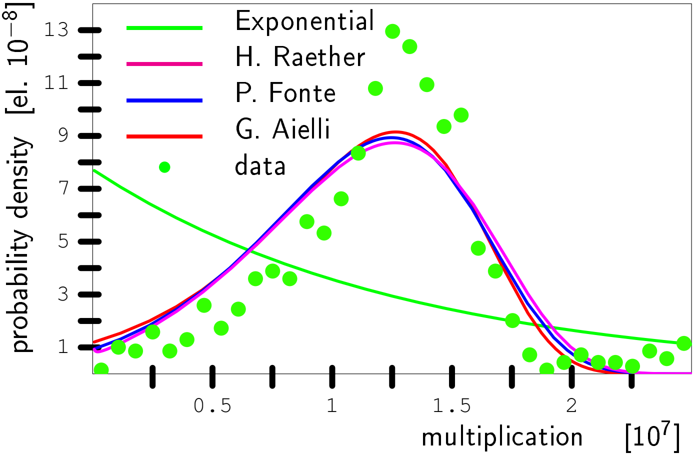
$$\alpha_0\, x = 14.5$$
$$\langle N \rangle = 5.0 \cdot 10^5$$

Above Space Charge threshold

$$\alpha_0\, x = 25$$
$$\langle N \rangle = 4.5 \cdot 10^6$$



COMPARISON WITH DATA



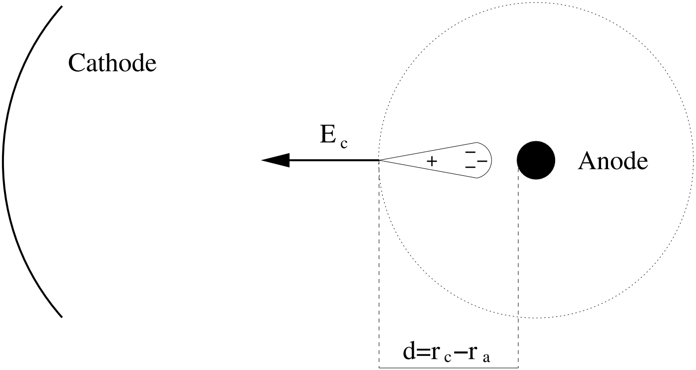
$\alpha_0 x = 18$ $V = 2.6$ kV in data

Raether $n_0 = 0.57 \cdot 10^6$ $b = 0.215$

Fonte $n_{\rm sat} = 6.1 \cdot 10^6$ $\ln(n_{\rm sat}) = 15.6$ $[\alpha_0^{-1}]$

Aielli $x_0 = 15.3$ $[\alpha_0^{-1}]$

FROM FLUCTUATIONS IN MULTIPLICATION....



Fluctuations in avalanche multiplication are a very well known phenomena in proportional counters, where the avalanche length is constrained to be constant.

$$\rho_M(m, d) = \frac{1}{\overline{m}(d)} e^{-\frac{m}{\overline{m}(d)}} \qquad \text{(Furry Law)}$$

....TO FLUCTUATIONS IN DELAY

(A. Mangiarotti and A. Gobbi, NIMA 482 (2002) 192)

The probability that the avalanche has to grow past a length l, to reach a fixed threshold in multiplication m_t, is equal to the probability that, over the same length l, it has not yet reached m_t:

$$p(L > l) = p(M < m_t, l)$$

$$\begin{aligned} p(M < m_t, l) &= \frac{1}{\overline{m}(l)} \int_0^{m_t} e^{-\frac{m'}{\overline{m}(l)}} \, dm' \\ &= 1 - e^{-\frac{m_t}{\overline{m}(l)}} \end{aligned}$$

$$p(L > l) = 1 - e^{-\frac{m_t}{\overline{m}(l)}}$$

$$\rho_L(l) = -\frac{dp}{dl} = e^{-\frac{m_t}{\overline{m}(l)}} \frac{m_t}{\overline{m}^2(l)} \frac{d\overline{m}(l)}{dl}$$

SINGLE AVALANCHE TIMING

A saturated electron drift velocity v_d is assumed in the Space Charge region so that the non dimensional time $\boxed{\tau = \alpha_0 \, v_d \, t}$ can be introduced. The results can be re-interpreted as a back extrapolation from an ideal discriminator threshold m_t to a Furry law in the initial n_0!

$$\boxed{\rho_{\mathcal{T}}(\tau) = e^{-n_0} \left| \frac{dn_0}{d\tau} \right|}$$

No Space Charge

$n_0 = \frac{m_t}{\overline{m}(\tau)}$ $\quad\Rightarrow\quad$ $\rho_{\mathcal{T}}(\tau) = e^{-n_0} \, n_0$

with $\overline{m}(\tau) = e^{\tau}$

Average Space Charge

$n_0 = \frac{m_t}{\overline{m}(\tau)}$ $\quad\Rightarrow\quad$ $\rho_{\mathcal{T}}(\tau) = e^{-n_0} \frac{n_0}{1+\frac{m_t}{n_{\mathrm{sat}}} n_0}$

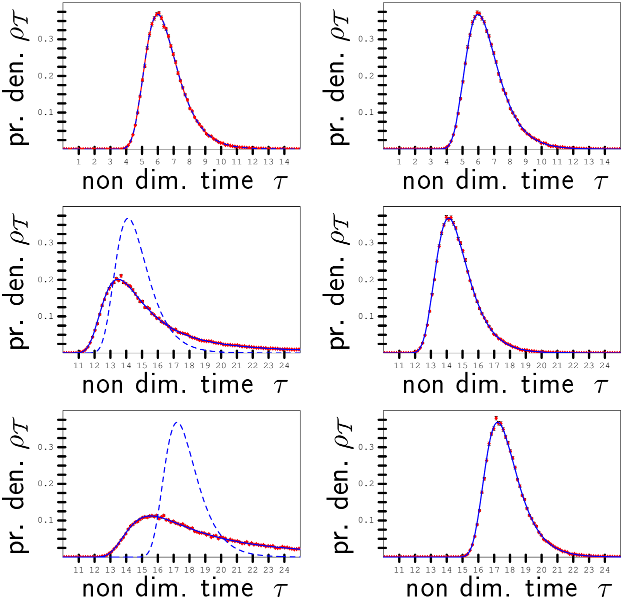
with $\overline{m}(\tau) = n_{\mathrm{sat}} \, W\left(\frac{1}{n_{\mathrm{sat}}} e^{1/n_{\mathrm{sat}}+\tau}\right)$

Stochastic

$n_0 = n_{\mathrm{sat}} \, W\left(\frac{m_t}{n_{\mathrm{sat}}} e^{\frac{m_t}{n_{\mathrm{sat}}}-\tau}\right)$ $\quad\Rightarrow\quad$ $\rho_{\mathcal{T}}(\tau) = e^{-n_0} \frac{n_0}{1+\frac{n_0}{n_{\mathrm{sat}}}}$

COMPARISON WITH NUMERICAL SOLUTIONS

Average Space Charge | Stochastic



Below Space Charge threshold

$$n_{\rm sat} = 5.0 \cdot 10^5$$
$$m_t = 4.0 \cdot 10^2$$

At Space Charge threshold

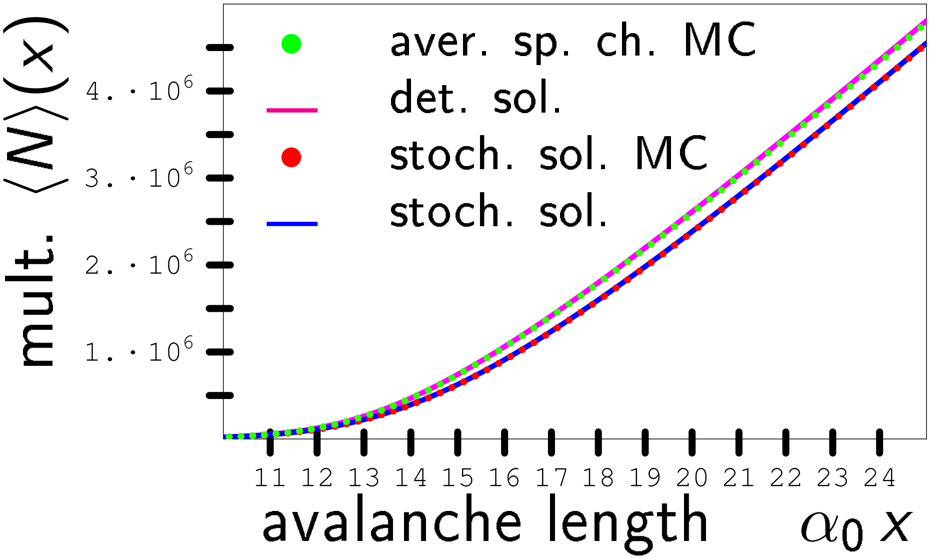
$$m_t = 5.0 \cdot 10^5$$

Above Space Charge threshold

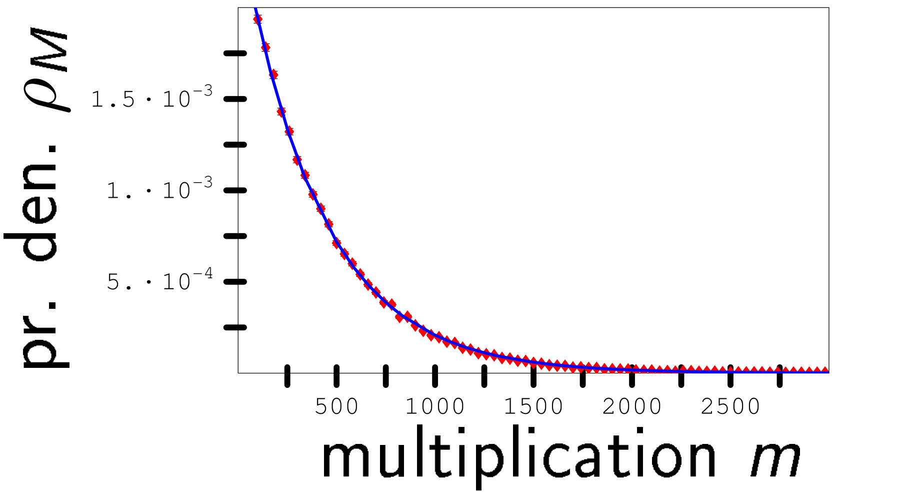
$$m_t = 1.5 \cdot 10^6$$

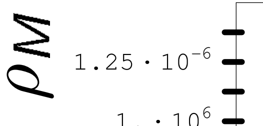
CONCLUSIONS

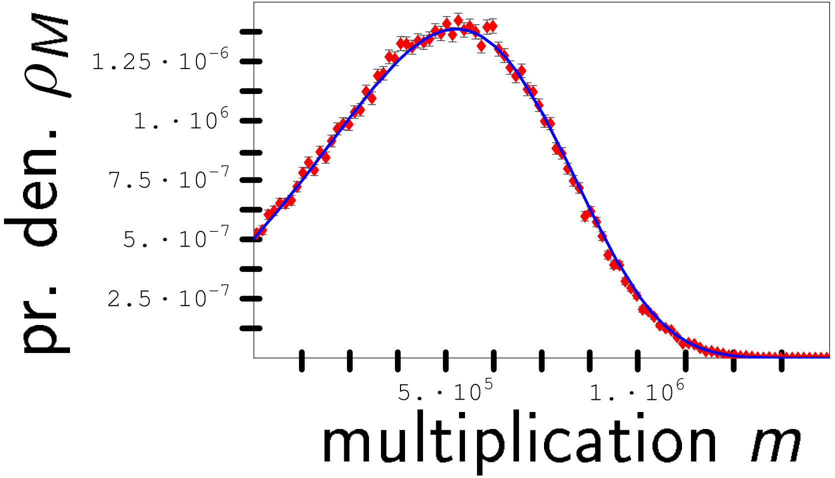
- The difference between the three analytical models of the Space Charge effect is, once they are fitted to the data, minor.
- The discrepancy in the extracted parameters can be as large as 15% if the deterministic or the stochastic version are used.
- The **pulse height spectrum** predicted by the deterministic / average space charge and the stochastic versions are very different. The stochastic one is closer to the data, indicating that the **negative feed back mechanism**, regulating the avalanche development under deep space charge induced saturation, is active even inside a single growth history.
- The same huge differences are expected for the single avalanche timing in presence of the space charge effect.
- Since data prefer the stochastic scenario, it can be argued that space charge induced saturation is expected to have a negligible impact on timing, hence validating its neglect in the the analytical approach developed so far.



aver. sp. ch. MC
det. sol.
stoch. sol. MC
stoch. sol.
$1.\cdot 10^6$
$2.\cdot 10^6$
$3.\cdot 10^6$
$4.\cdot 10^6$
mult. $\langle N\rangle(x)$
11 12 13 14 15 16 17 18 19 20 21 22 23 24
avalanche length $\alpha_0 x$

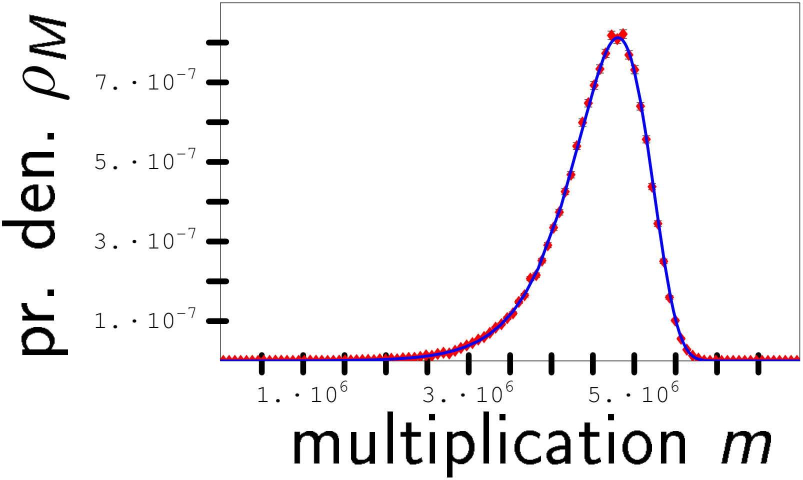

pr. den. ρ_M
$1.5 \cdot 10^{-3}$
$1. \cdot 10^{-3}$
$5. \cdot 10^{-4}$
500
1000
1500
2000
2500
multiplication m


ρ_M
$1.25 \cdot 10^{-6}$

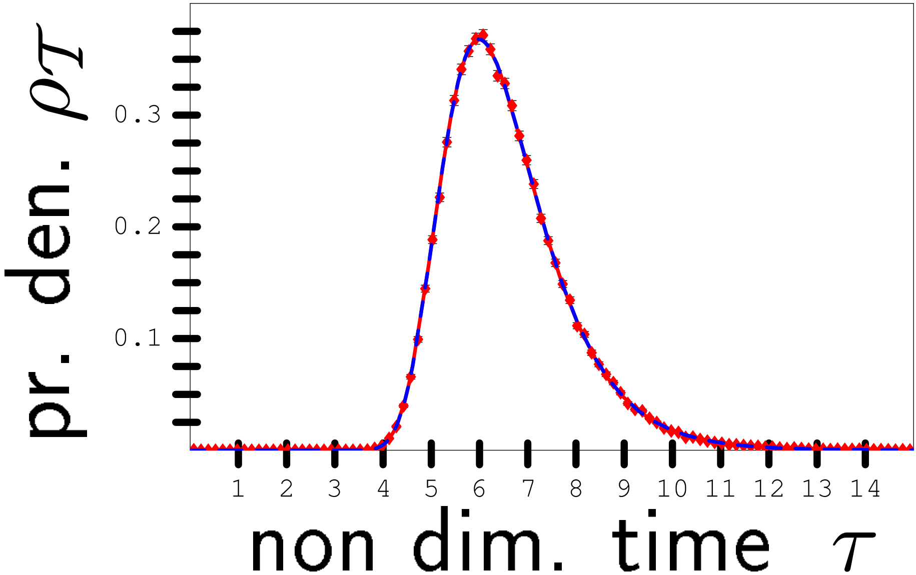


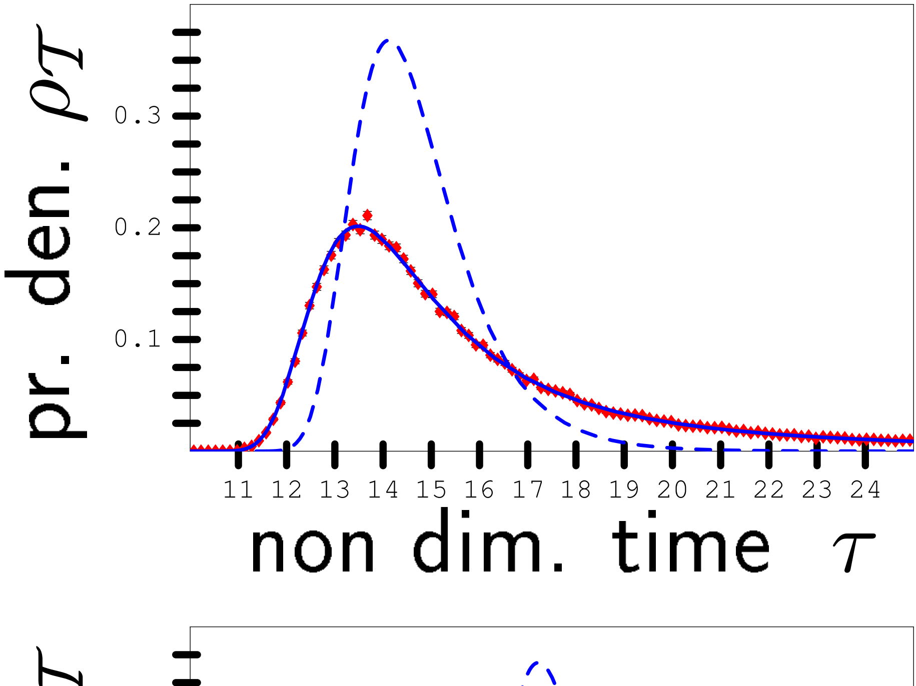
Above Space Charge

on m



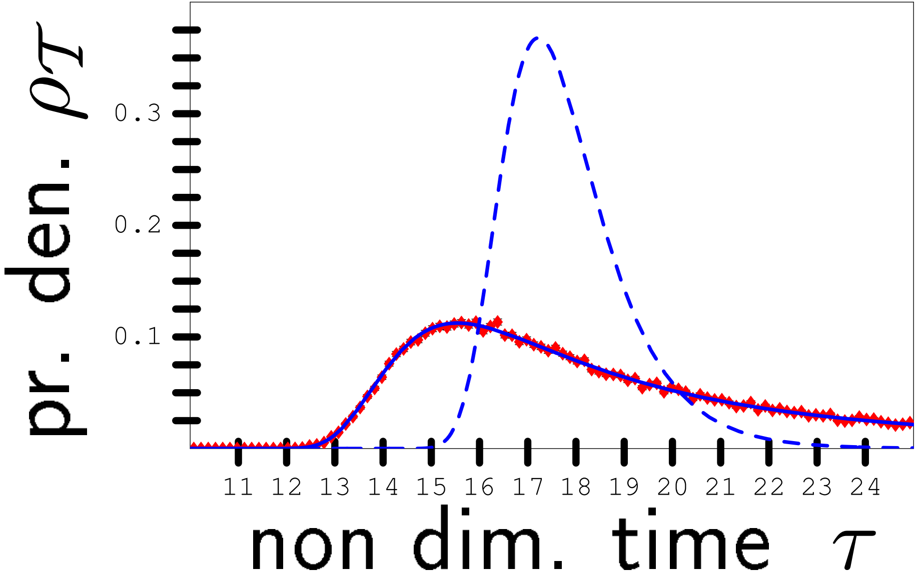
pr. den. ρ_M
$7. \cdot 10^{-7}$
$5. \cdot 10^{-7}$
$3. \cdot 10^{-7}$
$1. \cdot 10^{-7}$
$1. \cdot 10^{6}$
$3. \cdot 10^{6}$
$5. \cdot 10^{6}$
multiplication m

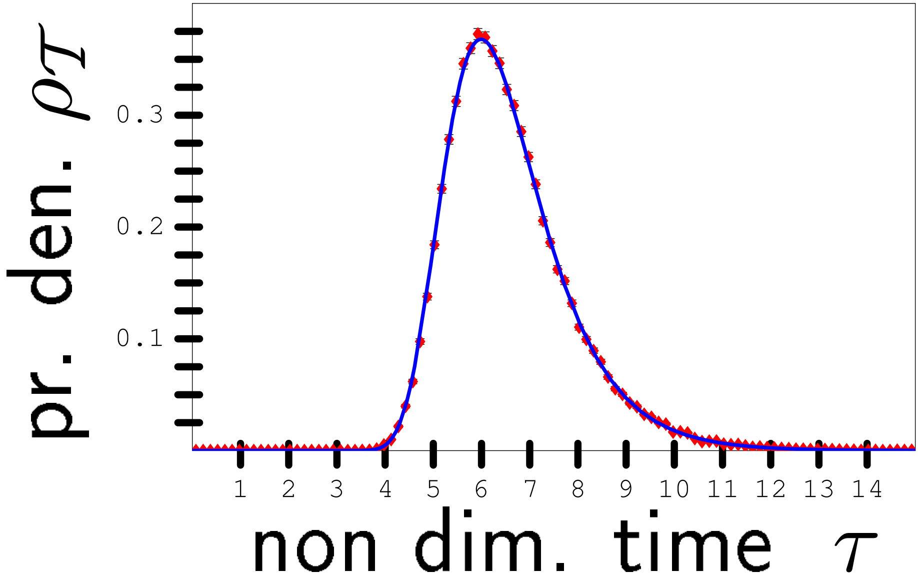

pr. den. $\rho_{\mathcal{T}}$
0.1
0.2
0.3
1
2
3
4
5
6
7
8
9
10
11
12
13
14
non dim. time τ

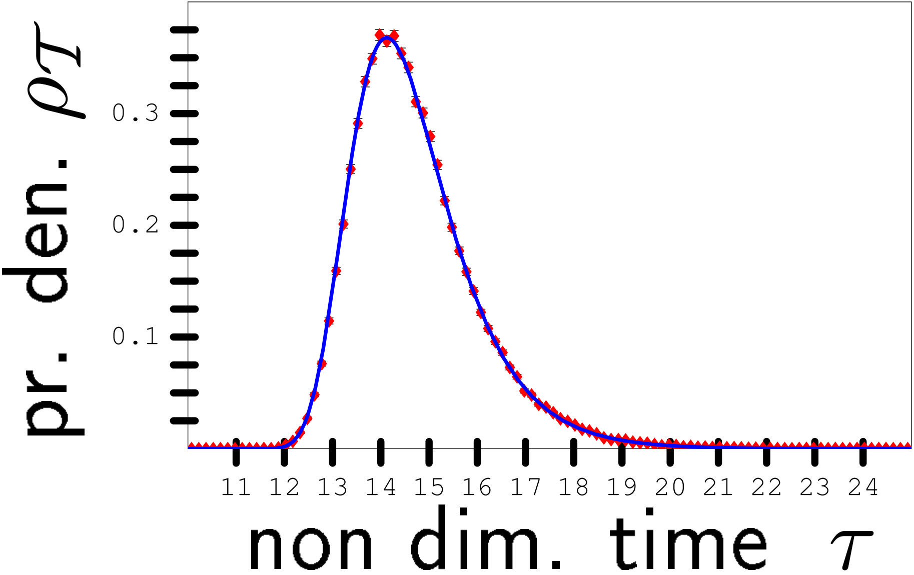


0.1
0.2
0.3
pr. den. $\rho_{\mathcal{T}}$
11
12
13
14
15
16
17
18
19
20
21
22
23
24
non dim. time τ

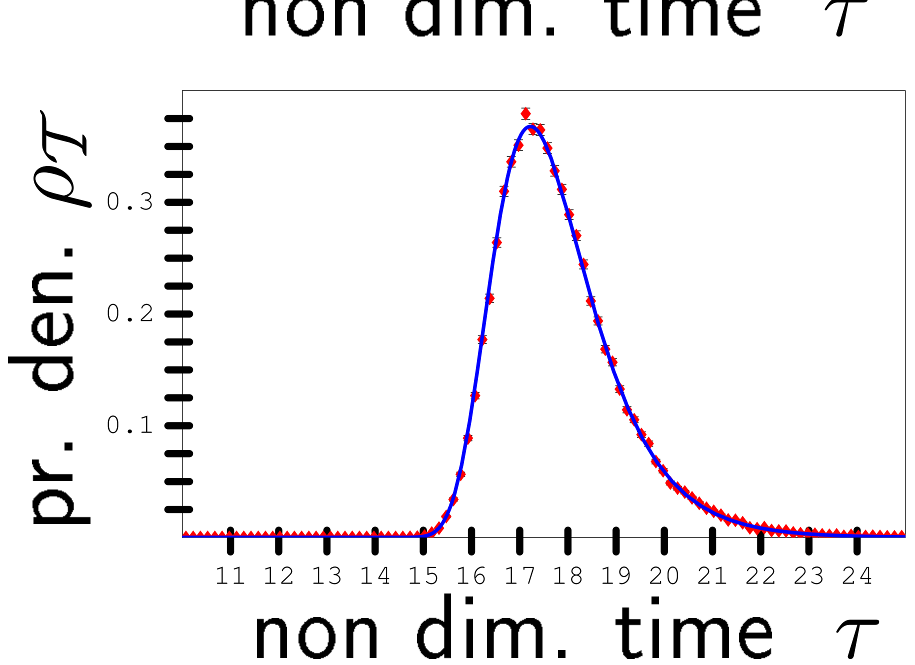
non dim. time τ




pr. den. $\rho_{\mathcal{T}}$
0.3
0.2
0.1
1 2 3 4 5 6 7 8 9 10 11 12 13 14
non dim. time τ



pr. den. $\rho_{\mathcal{T}}$
0.3
0.2
0.1
11 12 13 14 15 16 17 18 19 20 21 22 23 24
non dim. time τ


non dim. time τ
pr. den. $\rho_{\mathcal{T}}$
0.1
0.2
0.3
11 12 13 14 15 16 17 18 19 20 21 22 23 24
non dim. time τ