From $e\,p$ to $p\,p$: high energy evolution with Pomeron loops

Michael Lublinsky

University of Connecticut

In collaborations with Alex Kovner and Eugene Levin

High Energy Evolution

DIS: small projectile vs. large target ($e\,p$, $e\,A$)

Projectile's wave function $|P\rangle$ (rapidity dependent)

Target wave function $|T\rangle$ (rapidity dependent)

S-matrix:

$$S(Y) \;=\; \langle T\,\langle\, P|\;\hat{s}(\rho^t,\,\rho^p)\;|P\rangle\,T\rangle$$

Projectile averaged s-matrix:

$$\Sigma^p_{Y\,-\,Y_0}(\rho^t) \;=\; \langle\, P|\;\hat{s}(\rho^t,\,\rho^p)\;|P\rangle$$

$$S(Y) \;=\; \int D\rho^t\;\Sigma^p_{Y\,-\,Y_0}[\rho^t]\;W^t_{Y_0}[\rho^t]$$

High energy evolution = boosting classical (Coulomb) field

High energy limit = soft gluon emission approximation

Projectile evolution: $|P\rangle_Y \ \rightarrow \ |P\rangle_{Y+\delta Y}$

$$\frac{d\Sigma^p}{dY} \ = \ \chi^{HE}_{\Sigma} \ \Sigma^p$$

Target evolution: $|T\rangle_Y \ \rightarrow \ |T\rangle_{Y+\delta Y}$

$$\frac{dW^t}{dY} \ = \ \chi^{HE}_{W} \ W^t$$

Lorentz invariance:

$$\frac{dS}{dY_0} \ = \ 0 \quad \rightarrow \quad \chi^{HE}_{\Sigma} \ = \ \chi^{HE}_{W}$$

Saturation in DIS

JIMWLK equation is an evolution of a dense target:

$$\chi^{JIMWLK} = \chi^{HE}(\rho^t \to \infty)$$

JIMWLK is equivalent to the evolution of Σ^p of a small projectile ($\rho^p \to 0$). Approach due to Balitsky.

Mueller's dipole Limit (Large N_c):

$$\chi^{dipole}[s] = \alpha_s \int_{x,y,z} K^{BFKL}_{x\,y\,z} \, [s(x,\, z)\, s(y,z) \, - \, s(x,y)] \, \frac{\delta}{\delta s(x,y)}$$

E. Levin and M.L. **NPA730** (2004) 191

Classical branching process. Violates t-channel unitarity. No Ploops!

$$\boxed{\chi^{JIMWLK} = \chi^{dipole}[\,s\,] + \frac{1}{N_c^2}\,\chi^{cc}}$$

$$\chi^{cc} \propto \frac{\delta^2}{\delta\, s\, \delta\, s}$$

A. Kovner and M.L., hep-ph/0502071

Kovchegov equation

DIS of a single dipole: $\Sigma^p \;=\; s(x,\,y)$

$$\frac{d\Sigma^p[\,s\,]}{d\,Y} \;=\; \chi^{dipole}[s]\;\Sigma^p[\,s\,] \;\;\rightarrow$$

$$\frac{d\,s(x,y)}{d\,Y} \;=\; \alpha_s\,\int_z K^{BFKL}_{x\,y\,z}\;[s(x,\,z)\,s(y,z)\;-\;s(x,y)]$$

Total S-matrix:

$$S_Y(x,\,y)\;=\;\int Ds\;s(Y\;-\;Y_0;\,x,\,y)\;W^t_{Y_0}[s]$$

No target correlations (large nucleus): $W^t[s]\;\sim\;\delta(s\;-\;s_0)$

$$S_Y(x,\,y)\;=\;s(Y\;-\;Y_0;\,x,\,y)\,;\qquad\qquad S_{Y_0}\;=\;s_0$$

Kovchegov equation $\rightarrow$ HERA phenomenology $\rightarrow$ LHC

Balitsky's hierarchy

Total S-matrix:

$$S_Y(x,\, y) \;=\; \int Ds\; s(Y \;-\; Y_0;\; x,\; y)\; W^t_{Y_0}[s]$$

Target correlations:

$$S(Y) \;=\; \langle\, s \,\rangle$$

$$d\,\langle\, s \,\rangle \,/\, dY \;=\; \alpha_s \int_z K^{BFKL}_{x\,y\,z}\; [\langle\, s\, s \,\rangle \;-\; \langle\, s \,\rangle]$$

$$d\,\langle\, s\, s \,\rangle \,/\, dY \;\;\propto\;\; \langle\, s\, s\, s \,\rangle$$

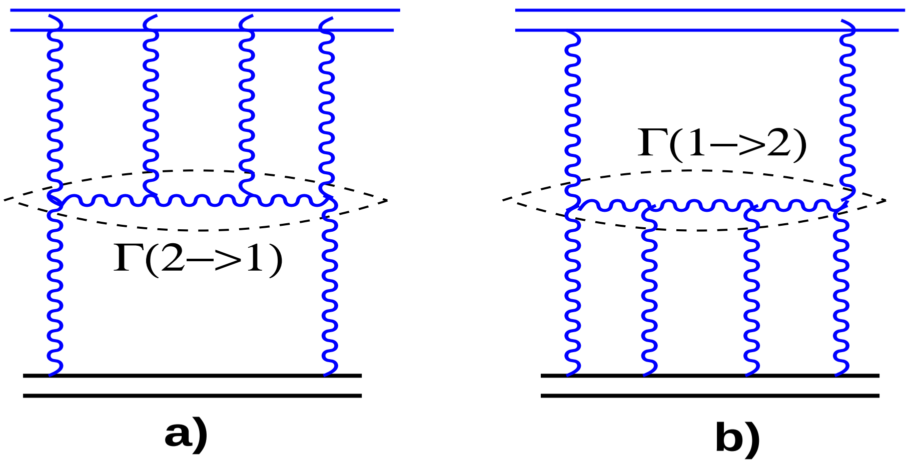
etc

Balitsky's hierarchy

M. Lublinsky

Dipole evolution with Pomeron loops

Dipole merging ($2 \; \rightarrow \; 1$) ?



E. Levin and M.L., hep-ph/0501173

$$\chi[s] \;=\; \chi^{dipole}[s] \;+\; \chi^{2\rightarrow 1}[s]$$

$$\chi^{2\rightarrow 1} \;=\; \int \Gamma^{\,2\rightarrow 1}_{x\,y\,u\,v\,r\,p}\;[s(x,y)\;-\;s(u,v)\,s(r,p)]\;\frac{\delta}{\delta\,s(u,v)}\,\frac{\delta}{\delta\,s(r,p)}$$

E. Iancu and D. Triantafyllopoulos, hep-ph/0411405; hep-ph/0501193

A. Mueller, A. Shoshi and S. Wong, hep-ph/0501088

KLWMIJ equation

JIMWLK equation is an evolution of a dense target:

$$\chi^{JIMWLK} = \chi^{HE}(\rho^t \to \infty)$$

JIMWLK: $1 \to n$ splittings (if viewed from P-side)

A. Kovner and M.L., hep-ph/0501198

KLWMIJ equation is an evolution of a dilute target:

$$\chi^{KLWMIJ} = \chi^{HE}(\rho^t \to 0)$$

KLWMIJ: $n \to 1$ mergings (if viewed from P-side)

$$\chi^{KLWMIJ}(N_c \to \infty) \to s \sum_n \left(\frac{\delta}{\delta s}\right)^n$$

$n = 2$ case includes corrections to $\Gamma^{2 \to 1}$

A. Kovner and M.L., hep-ph/0503155

Model with Pomeron Loops:

$$\chi^{HE} = \chi^{JIMWLK} \text{ "}+\text{" } \chi^{KLWMIJ}$$

DDD - Dense Dilute Duality

Classical field generated by a color charge ρ

$$\Delta\,\alpha(x)\;=\;\rho(x)\qquad\qquad (YM)$$

$$\chi^{JIMWLK}\;=\;\chi^{JIMWLK}\,(\alpha,\,\frac{\delta}{\delta\,\alpha})$$

$$\chi^{KLWMIJ}\;=\;\chi^{KLWMIJ}\,(\frac{\delta}{\delta\,\rho},\,\rho)$$

DDD transformation:

$$\alpha\;\rightarrow\;\frac{\delta}{\delta\,\rho}\,;\qquad\qquad \frac{\delta}{\delta\alpha}\;\rightarrow\;\rho$$

$$\boxed{\chi^{JIMWLK}\;\rightarrow\;\chi^{KLWMIJ}}$$

Self-Duality of High Energy Evolution

From $e\,p$ to $p\,p$:

A. Kovner and M.L., hep-ph/0502119

- Projectile - Target democracy
- Lorentz invariance
- Eikonal approximation

$$\chi^{HE}(\alpha\ ,\delta/\delta\,\alpha) \;=\; \chi^{HE}(\delta/\delta\,\rho,\ \rho)$$

Outlook

We are in a quest for a complete selfdual evolution kernel for high energy QCD.