Recent Progress in Parton Distributions

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In determining partons need to consider that not only are there 6 different combinations of partons, but also wide distribution of x from 0.75 to 0.00003. Need many different types of experiment for full determination. All below are important (nearly).

H1 $F_2^{e^+p}(x,Q^2)$ 1996-97 moderate Q^2 and 1996-97 high Q^2, and $F_2^{e^-p}(x,Q^2)$ 1998-99 high Q^2 small x. ZEUS $F_2^{e^+p}(x,Q^2)$ 1996-97 small x wide range of Q^2. 1999-2000 high Q^2.

NMC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2), (F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2))$, E665 $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$ medium x.

BCDMS $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$, SLAC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2)$ large x.

CCFR (NuTeV) $F_2^{\nu(\bar{\nu})p}(x,Q^2), F_3^{\nu(\bar{\nu})p}(x,Q^2)$ large x , singlet, valence.

E605 (E866) $pN \rightarrow \mu\bar{\mu} + X$ large x sea.

E866 Drell-Yan asymmetry $\bar{u}, \bar{d}$ $\bar{d} - \bar{u}$.

CDF W-asymmetry u/d ratio at high x.

CDF D0 Inclusive jet data high x gluon.

CCFR (NuTev) Dimuon data constrains strange sea.

Parton Uncertainties - Experiment – recently a lot of work. Number of approaches.

Hessian (Error Matrix) approach.

$$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j} H_{ij}(a_i - a_i^{(0)})(a_j - a_j^{(0)})$$

Simple method problematic due to extreme variations in $\Delta\chi^2$ in different directions in parameter space - particularly with more parameters (more data). $\rightarrow$ numerical instability. Improved by CTEQ. Now used in slightly weaker form by MRST and ZEUS.

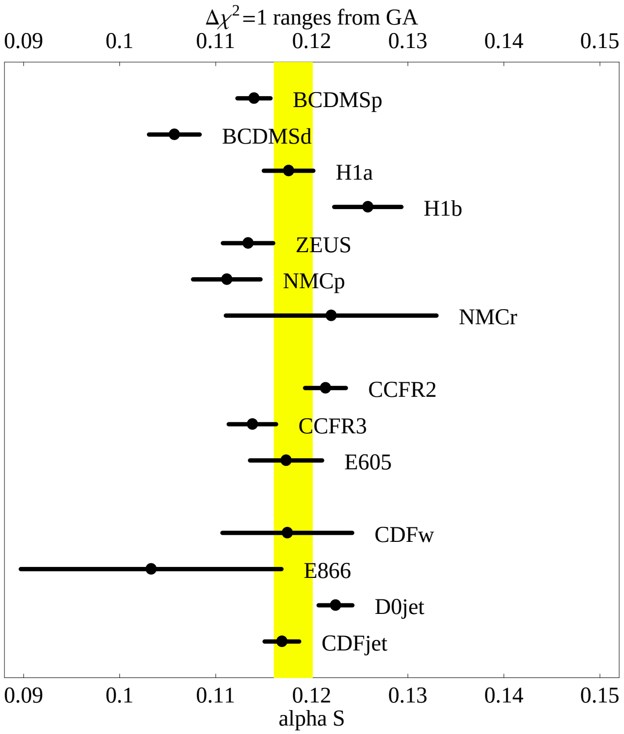
Can also look at uncertainty on a given physical quantity using Lagrange Multiplier method, first suggested by CTEQ and concentrated on by MRST. Minimize

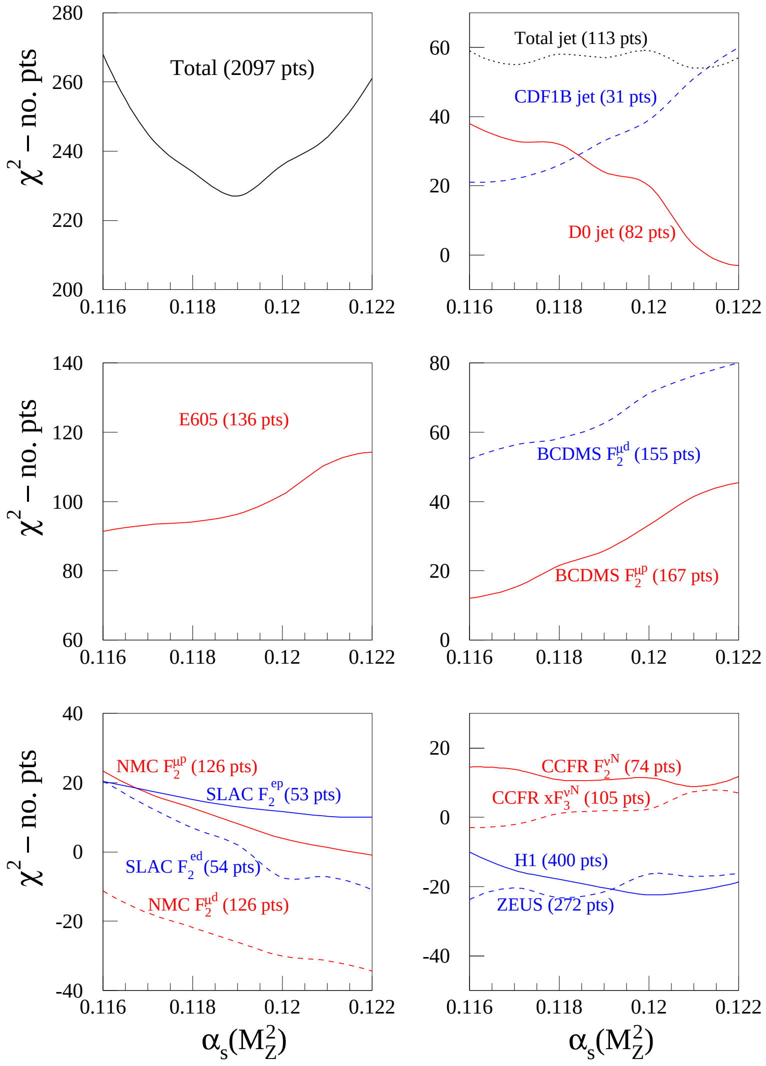
$$\Psi(\lambda, a) = \chi^2_{global}(a) + \lambda F(a).$$

Gives best fits for particular values of quantity $F(a)$ without relying on Gaussian approx for χ^2.

Also neural networks (Piccione).

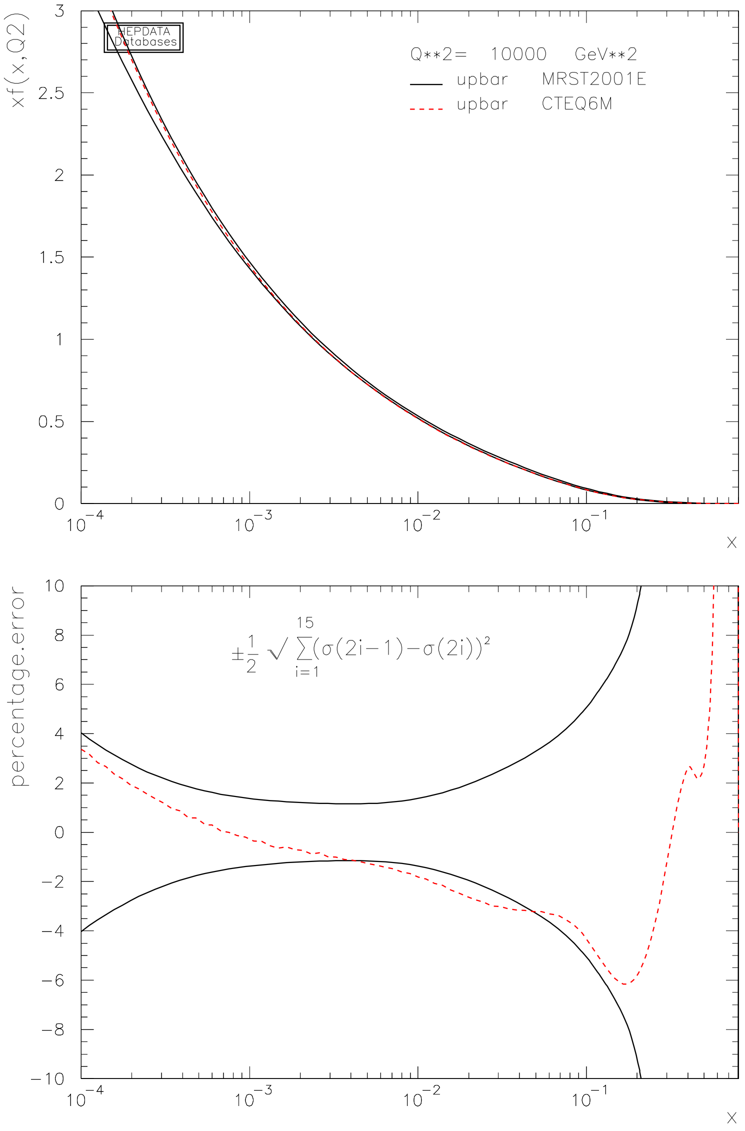
In full **global** fit art in choosing "correct" $\Delta\chi^2$ given complication of errors. Ideally $\Delta\chi^2 = 1$, but unrealistic.

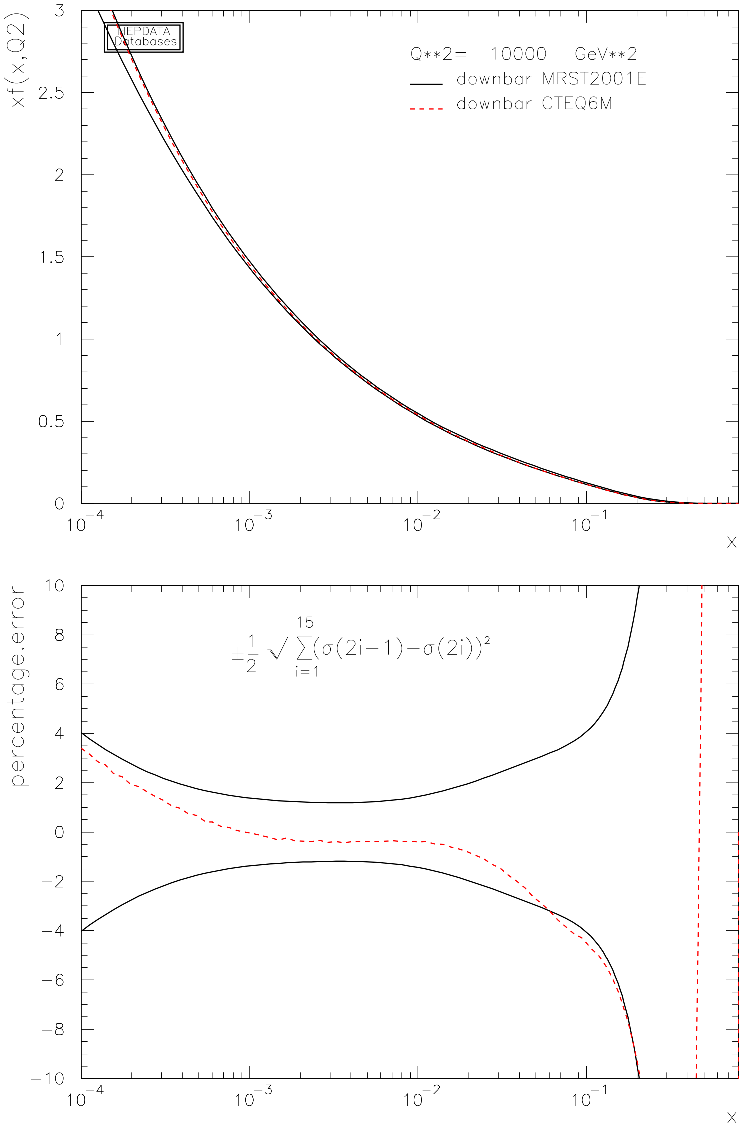




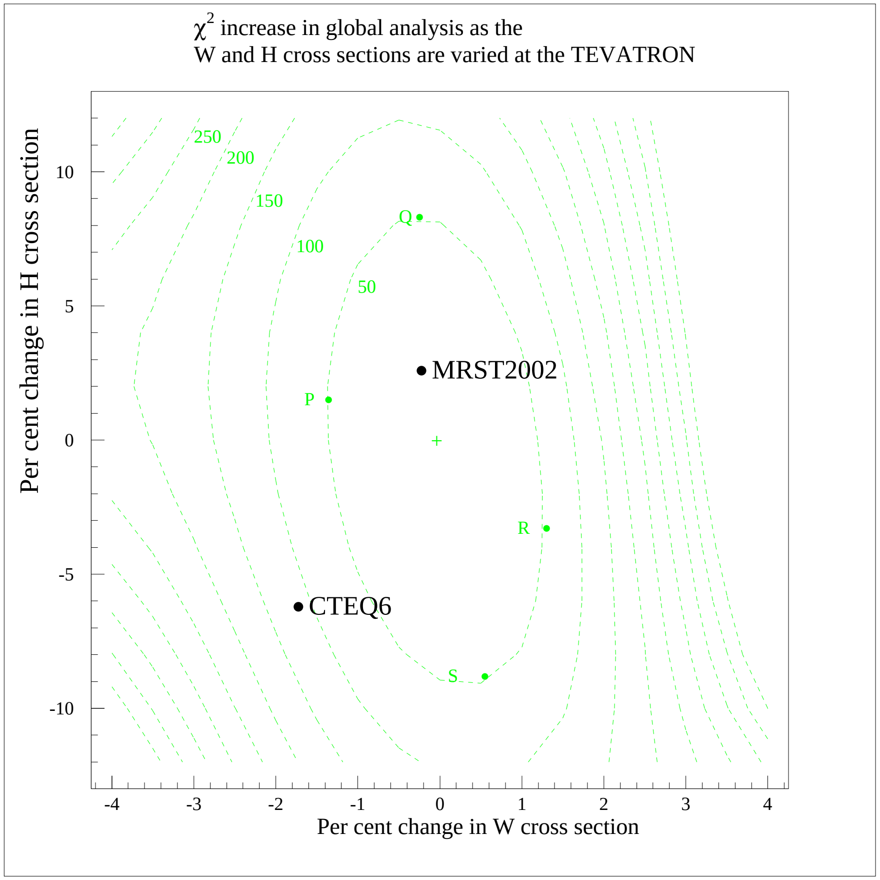
Many approaches use $\Delta\chi^2 \sim 1$. CTEQ choose $\Delta\chi^2 \sim 100$ for 90% confidence limit, i.e. ~ 40 for $1-\sigma$ error. MRST choose $\Delta\chi^2 \sim 20$ for $1-\sigma$ error.

Uncertainty on MRST $\bar{u}$ and $\bar{d}$ distributions, along with CTEQ6. Central rapidity $x = 0.006$ is ideal for MRST uncertainty in W, Z (Higgs?) at the LHC.

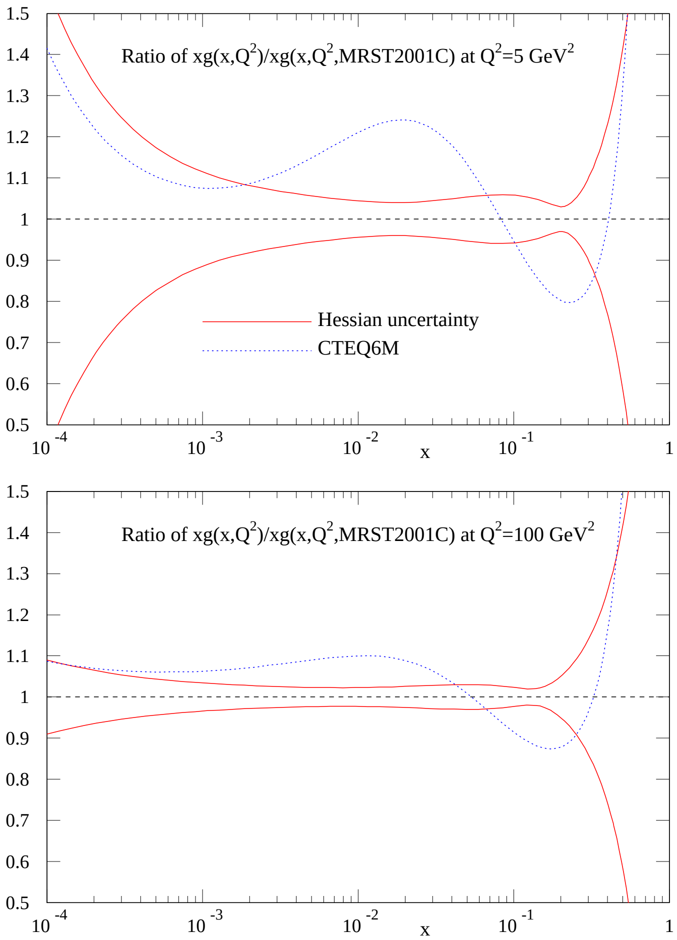




Different approaches lead to similar accuracy of measured quantities, but can lead to different central values. Must consider effect of assumptions made during fit.



Uncertainty of gluon from Hessian method



Cuts made on data, data sets fit, correctness of NLO QCD, parameterization for input sets, heavy flavour prescription, no isospin violation, strong coupling ......

Many can be as important as experimental errors on data used (or more so).

Results from LHC/LP Study Working Group (Bourilkov).

Table 1: Cross sections for Drell-Yan pairs (e^+e^-) with PYTHIA 6.206, rapidity < 2.5. The errors shown are the PDF uncertainties.

PDF set	Comment	xsec [pb]	PDF uncertainty %
$81 < M < 101$ GeV			
CTEQ6	LHAPDF	1065 ± 46	4.4
MRST2002	LHAPDF	1091 ± ...	3
Fermi2002	LHAPDF	853 ± 18	2.2

Comparison of $\sigma_W \cdot B_{l\nu}$ for MRST2002 and Alekhin partons.

PDF set	Comment	xsec [nb]	PDF uncertainty
Alekhin	Tevatron	2.73	± 0.05 (tot)
MRST2002	Tevatron	2.59	± 0.03 (expt)
CTEQ6	Tevatron	2.54	± 0.10 (expt)
Alekhin	LHC	215	± 6 (tot)
MRST2002	LHC	204	± 4 (expt)
CTEQ6	LHC	205	± 8 (expt)

In both cases differences (mainly) due to detailed constraint (by data) on quark decomposition.

Table 2: Cross sections for Drell-Yan pairs (e^+e^-) with `PYTHIA` 6.206. The errors shown are the statistical errors of the Monte-Carlo generation.

PDF set	Comment	xsec
$81 < M < 101$ GeV		
CTEQ5L	PYTHIA internal	1516 ± 5 pb
CTEQ5L	PDFLIB	1536 ± 5 pb
CTEQ6	LHAPDF	1564 ± 5 pb
MRST2001	LHAPDF	1591 ± 5 pb
Fermi2002	LHAPDF	1299 ± 4 pb
$M > 1000$ GeV		
CTEQ5L	PYTHIA internal	6.58 ± 0.02 fb
CTEQ5L	PDFLIB	6.68 ± 0.02 fb
CTEQ6	LHAPDF	6.76 ± 0.02 fb
MRST2001	LHAPDF	7.09 ± 0.02 fb
Fermi2002	LHAPDF	7.94 ± 0.03 fb

Note anti-correlation between deviations at high and low mass, i.e. high and low x. Typical result from sum rules and evolution.

Problems in the fit.

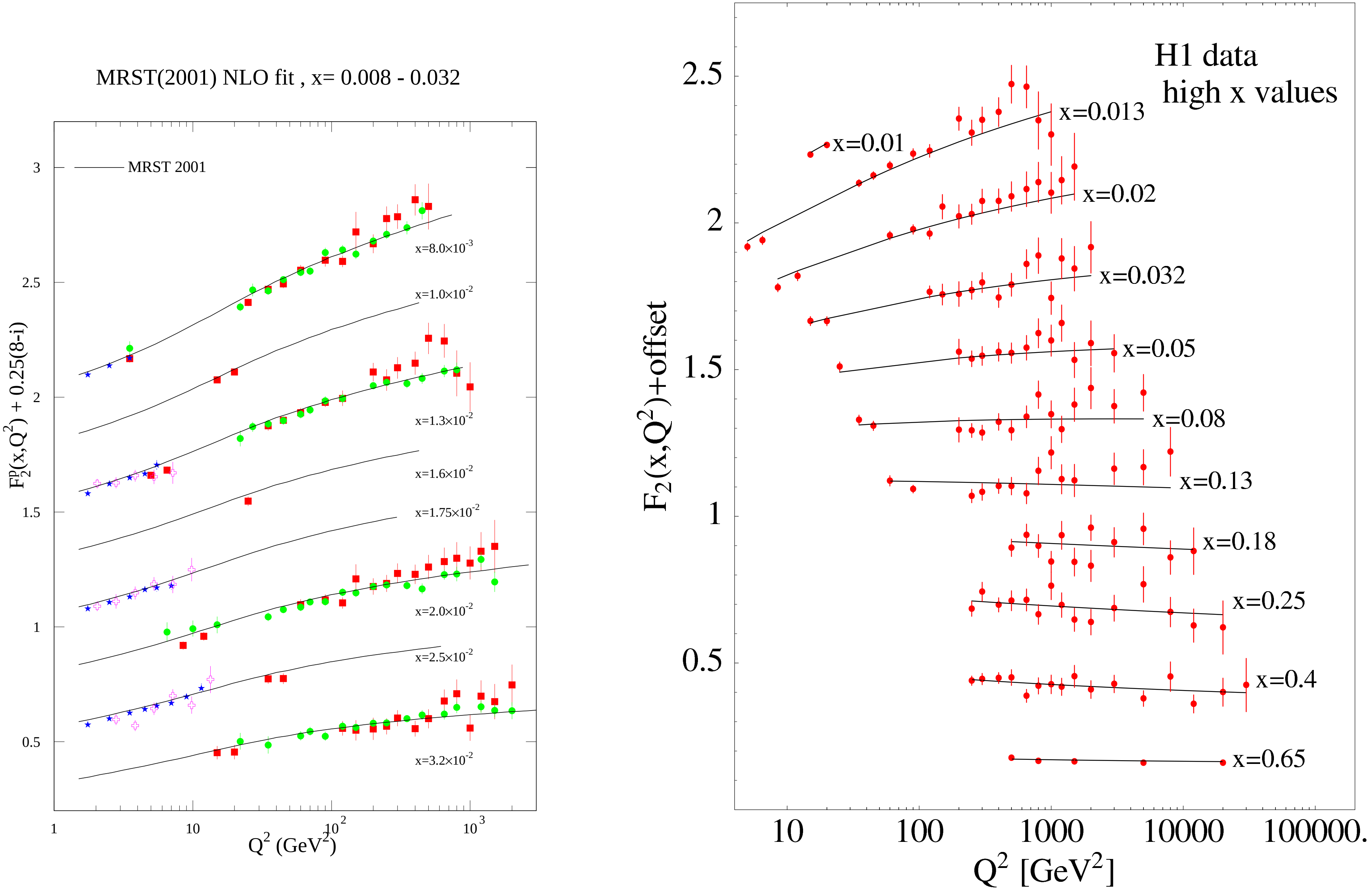
Variations from different approaches partially due to inadequacy of theory .

Failings of NLO QCD indicated by some areas where fit quality could be improved.

Good fit to HERA data, but some problems at highest Q^2 at moderate x, i.e. in $dF_2/d\ln Q^2$. $\rightarrow$ possible underestimate of quarks in this region.

Want more gluon in the $x \sim 0.01$ range, and/or larger $\alpha_S(M_Z^2)$.

Possible sign of required $\ln(1/x)$ corrections.



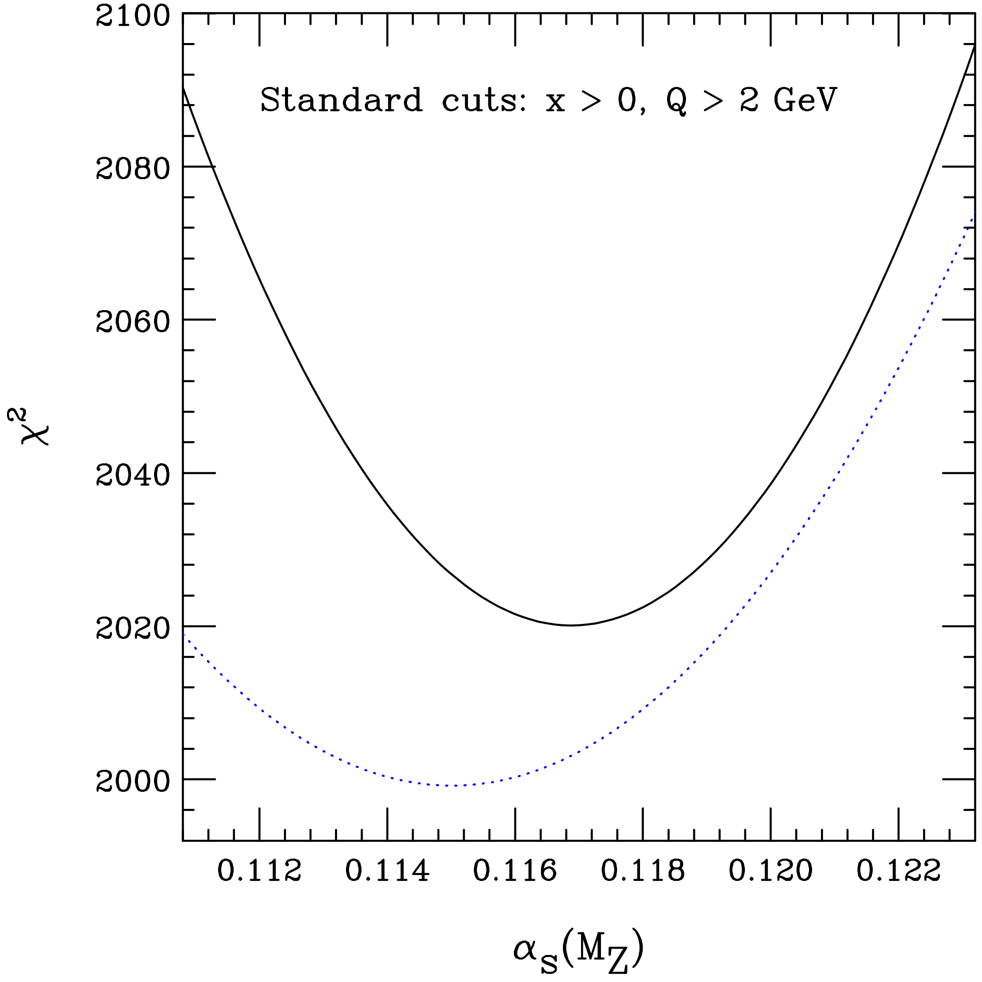
Comparison of MRST(2001) $F_2(x,Q^2)$ with HERA, NMC and E665 data (left) and of CTEQ6 $F_2(x,Q^2)$ and H1 data.

Data require gluon to be negative at low Q^2, e.g. MRST $Q_0^2 = 1\mathrm{GeV}^2$. Needed by all data (e.g Tevatron jets) not just low Q^2 low x data.

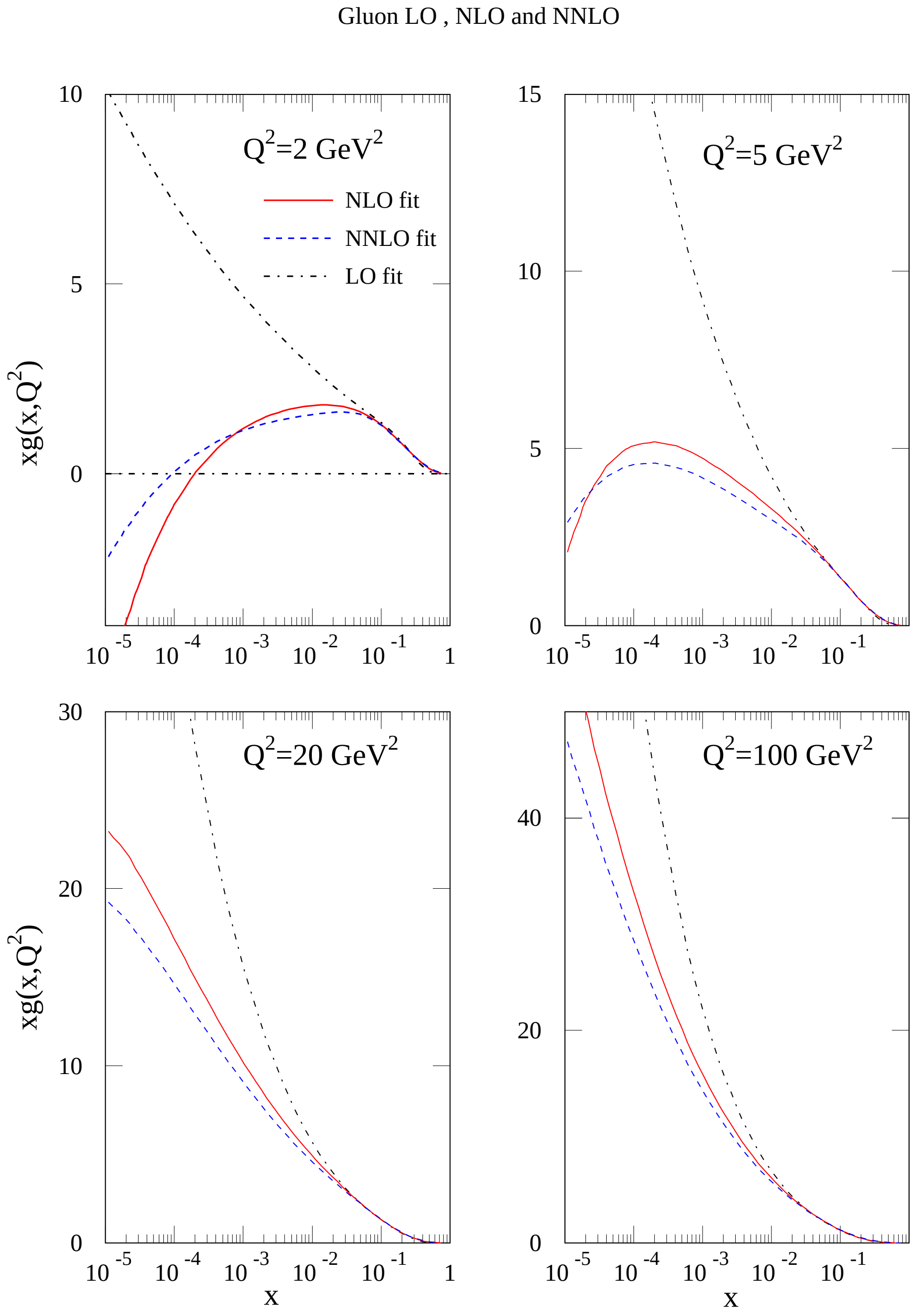
$\rightarrow F_L(x,Q^2)$ dangerously small at smallest x, Q^2.

Other groups find similar problems with gluon at low x.

CTEQ have valence-like input gluon at $Q_0^2 = 1.69\mathrm{GeV}^2$ which would like (at least a little) to be negative. (blue line - negative gluon allowed, **black** line - positive definite gluon.)

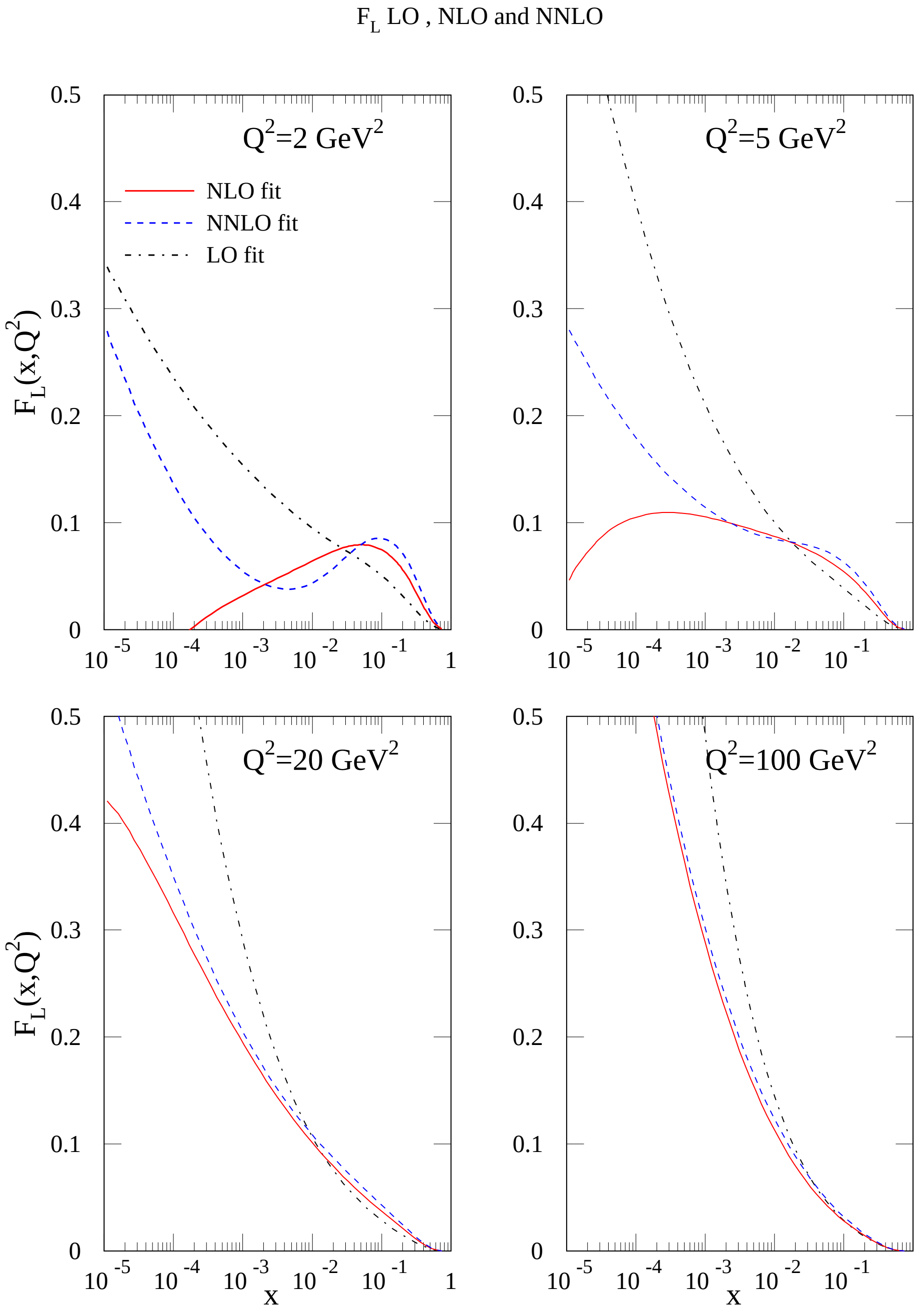


$xg(x,Q^2)$ going from LO → NLO → NNLO.



Also instability in physical, gluon dominated, quantity $F_L(x,Q^2)$ going from LO → NLO → NNLO.

Note very large effect of exact NNLO coefficient function.

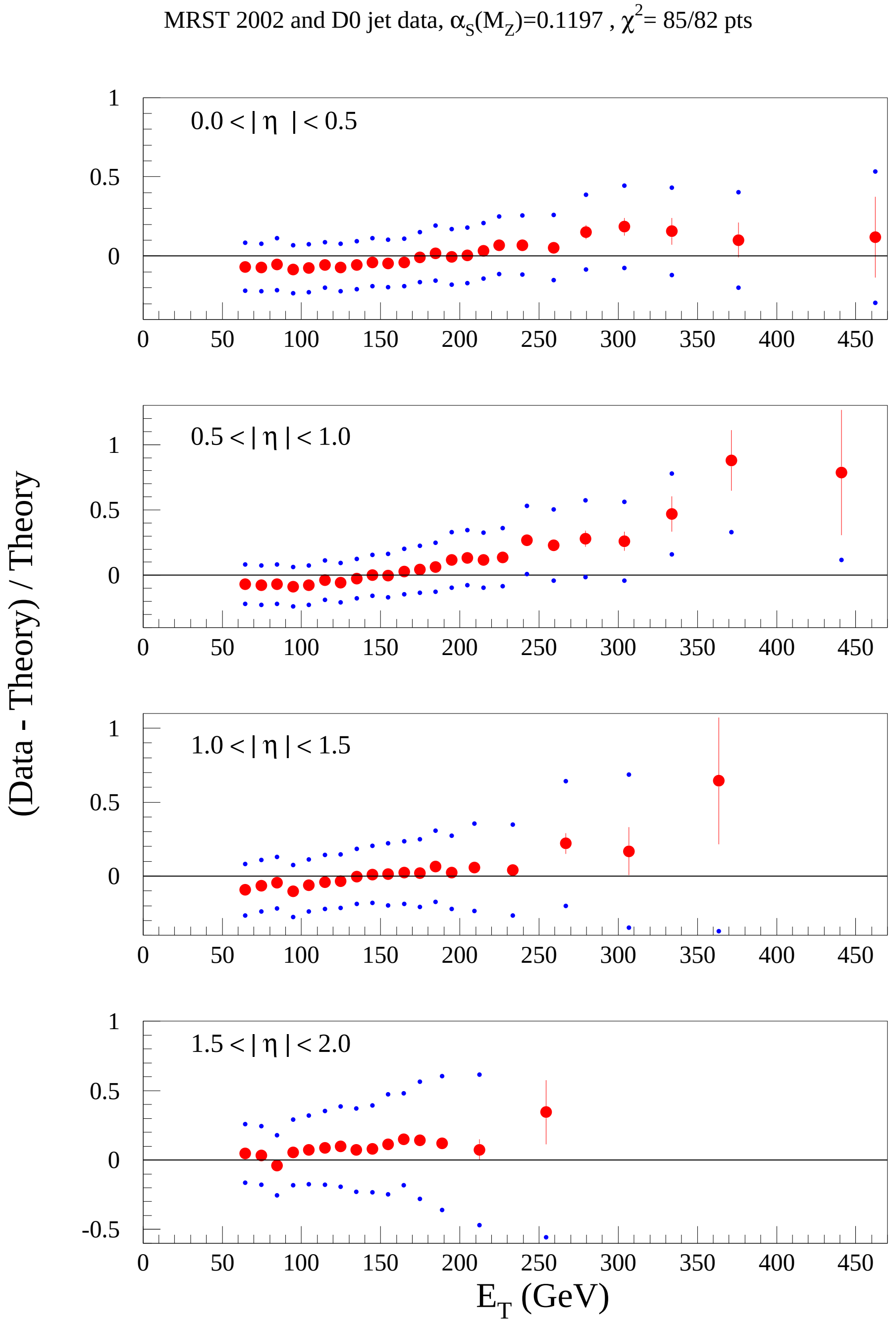


Difficult to reconcile fit to jets and rest of data.

MRST find a reasonable fit to jet data, but need to use the large systematic errors.

Better for CTEQ6 due to different cuts on other data, and different type of high-x parameterization. Usually worse for other partons (jets not in fits). General tension between HERA and NMC data and jets.

In general different data compete over the gluon and $\alpha_S(M_Z^2)$.



Comparison to CDF1B jet data.

Can explicitly see data move relative to theory using correlated systematic errors.

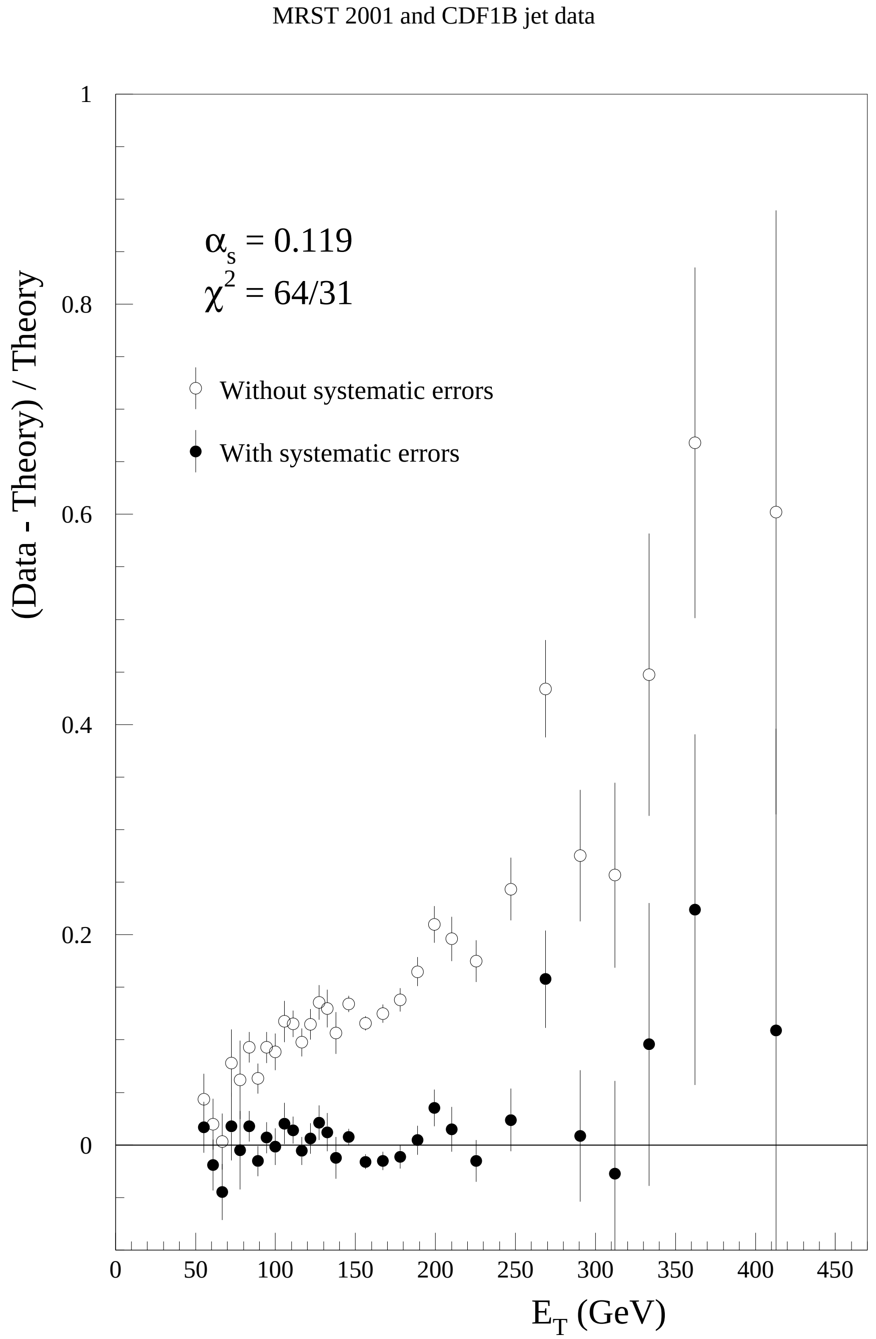


Illustration of problem with jets.

Using simple spectator counting rules, at high x

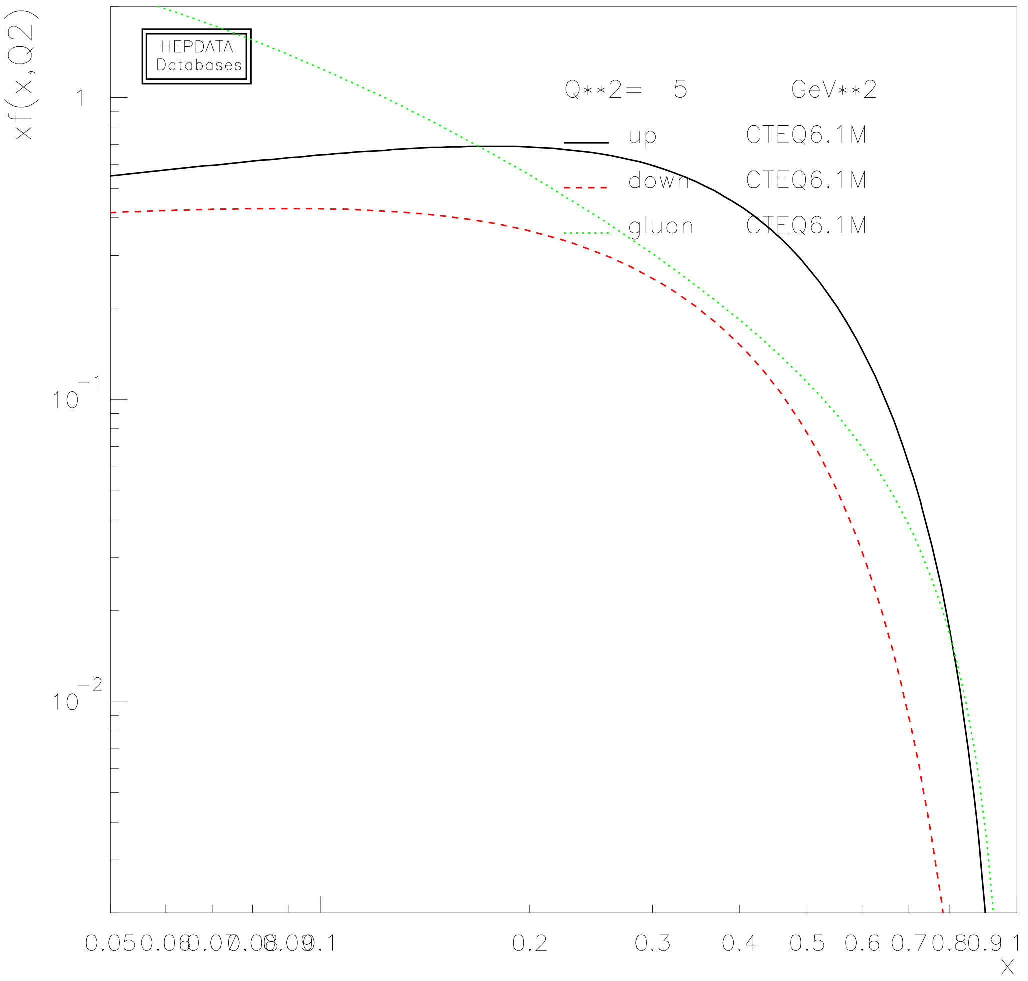
$q_V(x) \sim (1-x)^3, \; g(x) \sim (1-x)^5$

Clearly not true for CTEQ6.1M partons which give good jet fit.

Gluon is hardest as $x \rightarrow 1$.

MRST parameterizations don't allow such a hard gluon. Fits not as good as one would like ideally.

Worse at NNLO since high-x quarks smaller → even bigger gluon.



New approach to high-x gluon.

In DIS scheme $F_2(x,Q^2) \equiv \sum_i^{N_f} e_i^2\, x q_i(x,Q^2)$.

Under change of scheme from $\overline{MS}$ to DIS schemes we have.

$$q^{DIS}(x) = q^{\overline{MS}} + C_{2,q}^{\overline{MS}} \otimes q^{\overline{MS}} + C_{2,g}^{\overline{MS}} \otimes g^{\overline{MS}},$$

$$g^{DIS} = g^{\overline{MS}} - C_{2,q}^{\overline{MS}} \otimes q^{\overline{MS}} - C_{2,g}^{\overline{MS}} \otimes g^{\overline{MS}}.$$

Designed to maintain 100% momentum.

Scheme transformation should dominate high-x gluon if valence quarks naturally biggest at high x.

If $g^{\overline{MS}} \sim (1-x)^5$ then becomes negative in DIS scheme. Or if $g^{DIS} \sim (1-x)^5$ then transformation determines very high-x limit.

DIS scheme is certainly more physical for quarks. $\overline{MS}$ scheme not really physical at all.

Assume high-x gluon is smaller than high-x quarks in DIS scheme. Therefore in $\overline{MS}$ scheme

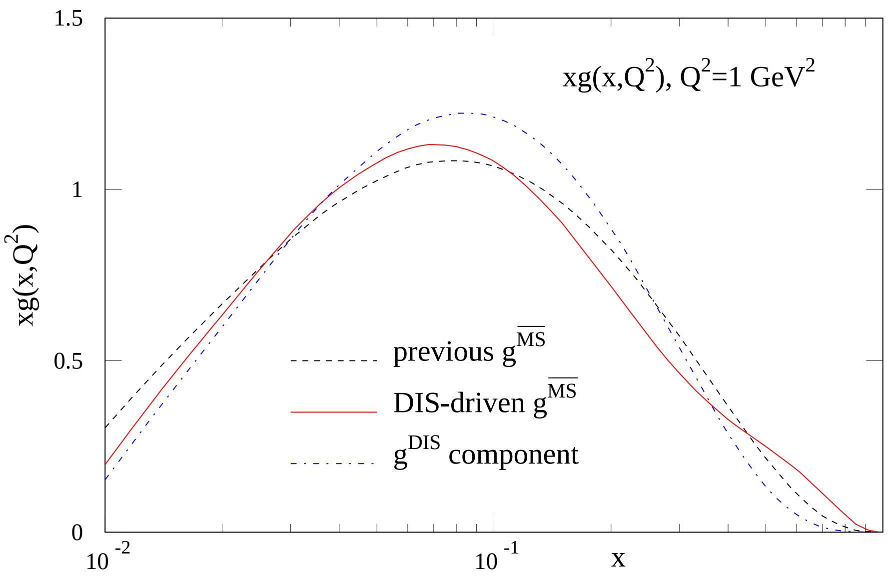
$$g^{\overline{MS}} = g^{DIS} + C_{2,q}^{\overline{MS}} \otimes q^{\overline{MS}},$$

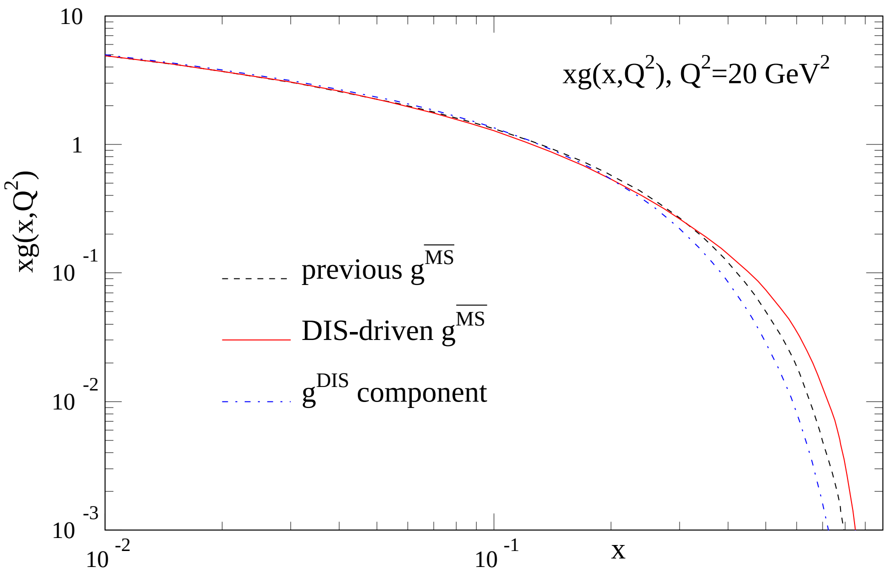
so high-x gluon determined from quarks.

Works extremely well. χ^2 for jets $154 \rightarrow 116$.

Total $\Delta\chi^2 = -26$.

DIS scheme gluon more natural at high x.





Works even better at NNLO.

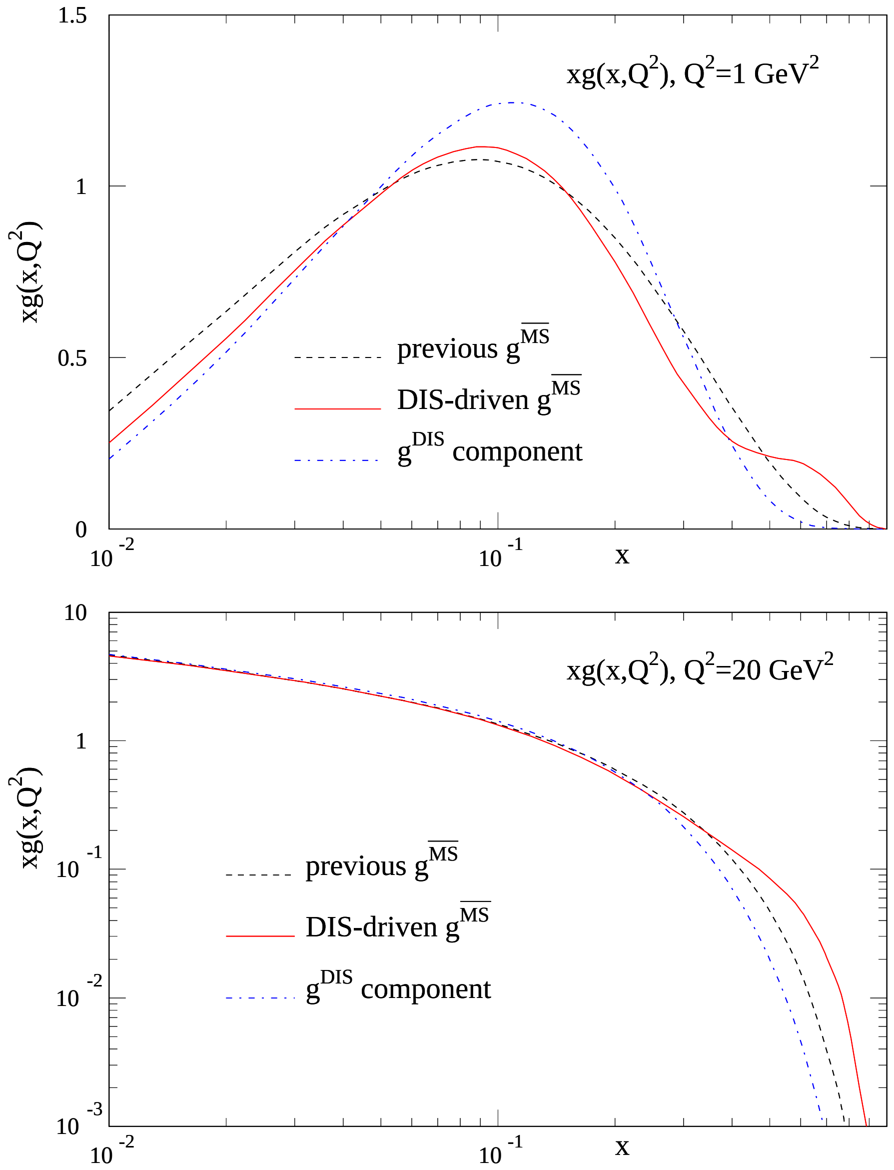
$C^{\overline{MS},(2)}_{2,q} \otimes q^{\overline{MS}}$ positive and significant at very high $x \longrightarrow$ high-x gluon even more determined from quarks.

Now χ^2 for jets $164 \longrightarrow 117$.

Total $\Delta\chi^2 = -79$.

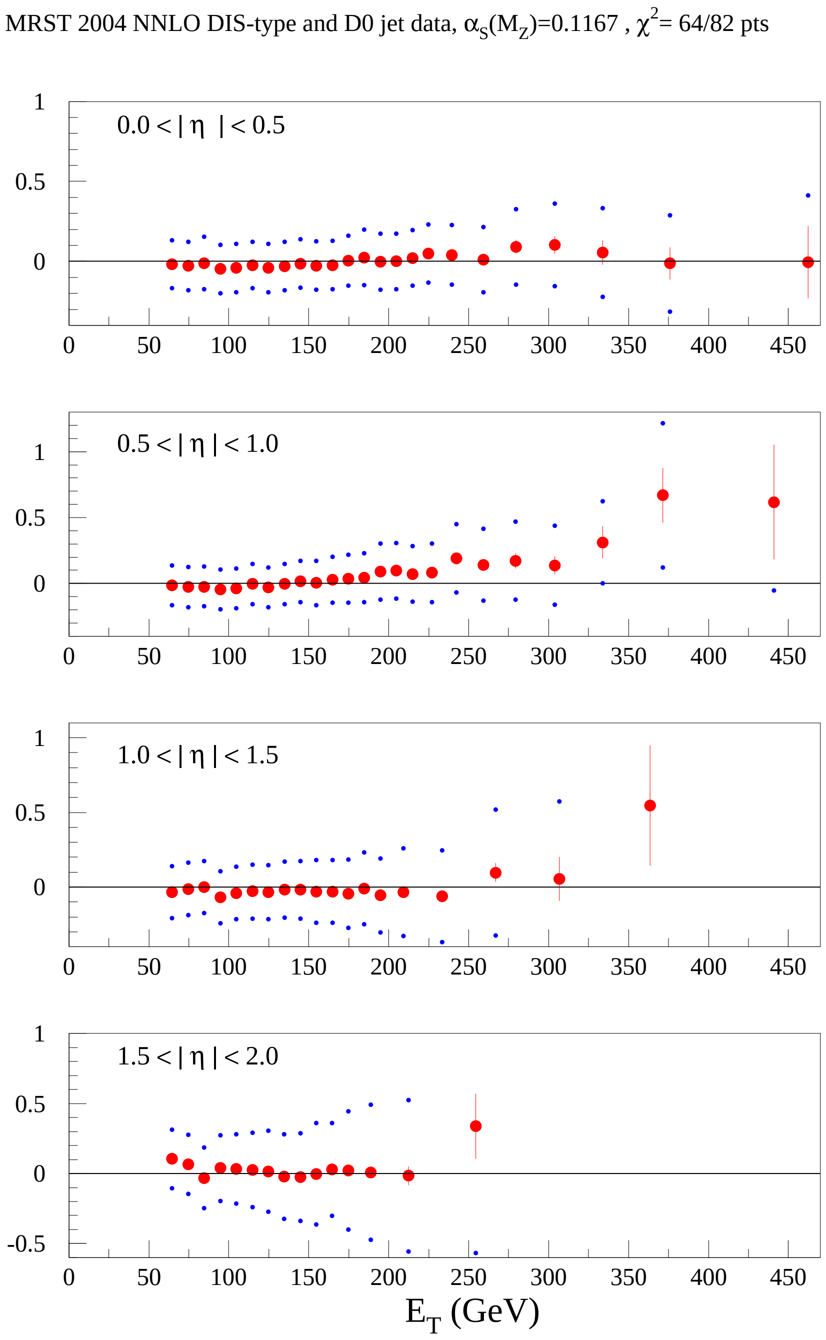
DIS scheme gluon again more natural at high x.

In $\overline{MS}$ scheme high-x gluon unphysical and determined entirely by quarks?



Comparison to D0 jet data for scheme change-inspired partons.

Shape much better.

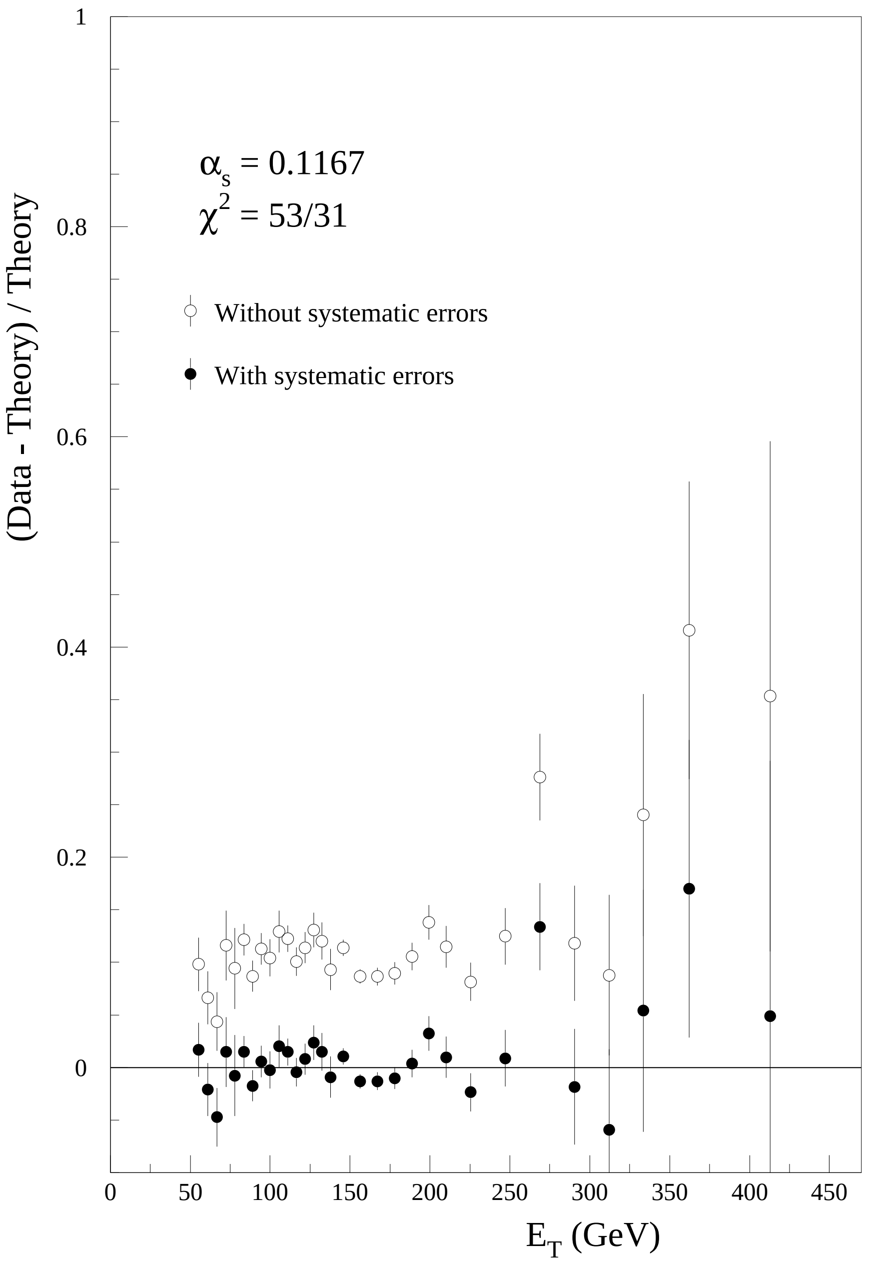


Comparison to CDF1B jet data for scheme change-inspired partons.

Shape now correct. Normalization shift of theory relative to data.

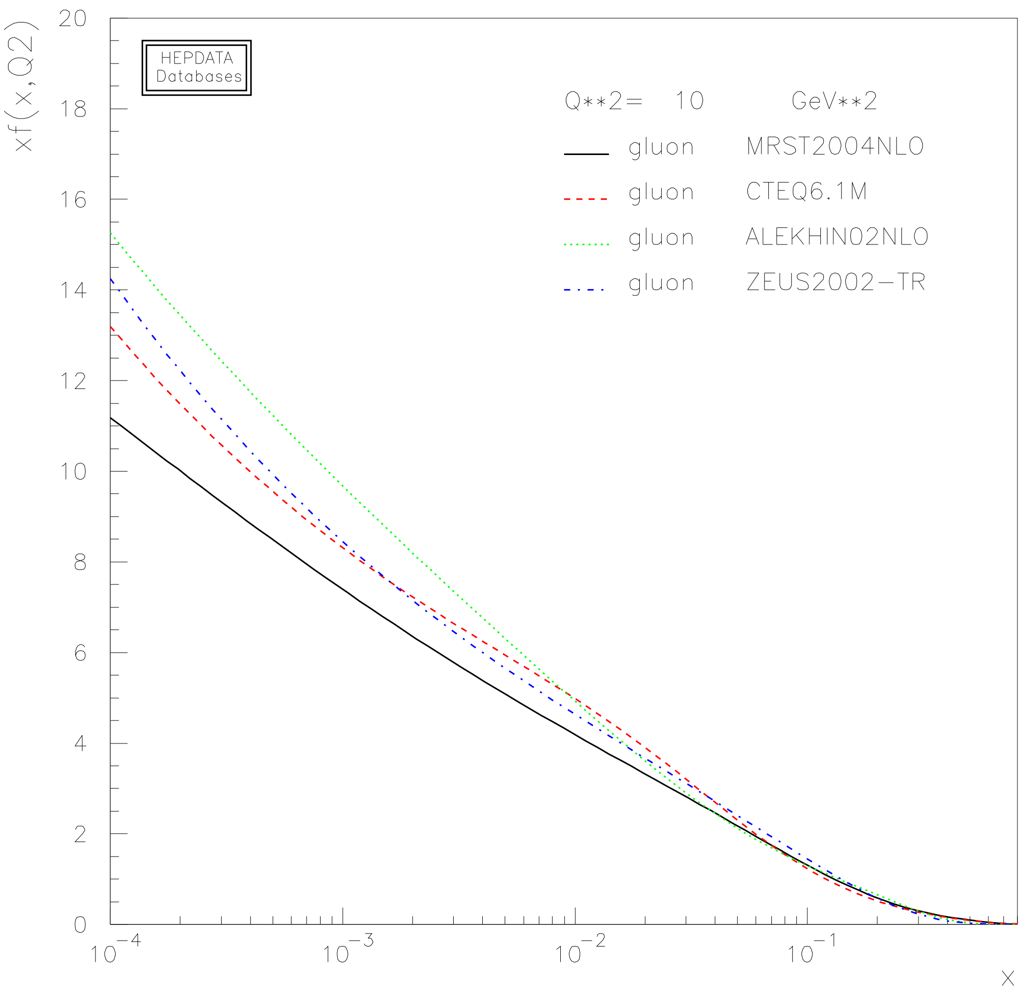
6% normalization difference between CDF and $D0$.

MRST NNLO 2004 DIS-type and CDF1B jet data



Current comparison of gluons over full x range.

MRST smallest at small x - probably due to allowed negative input plus largest amount of data.

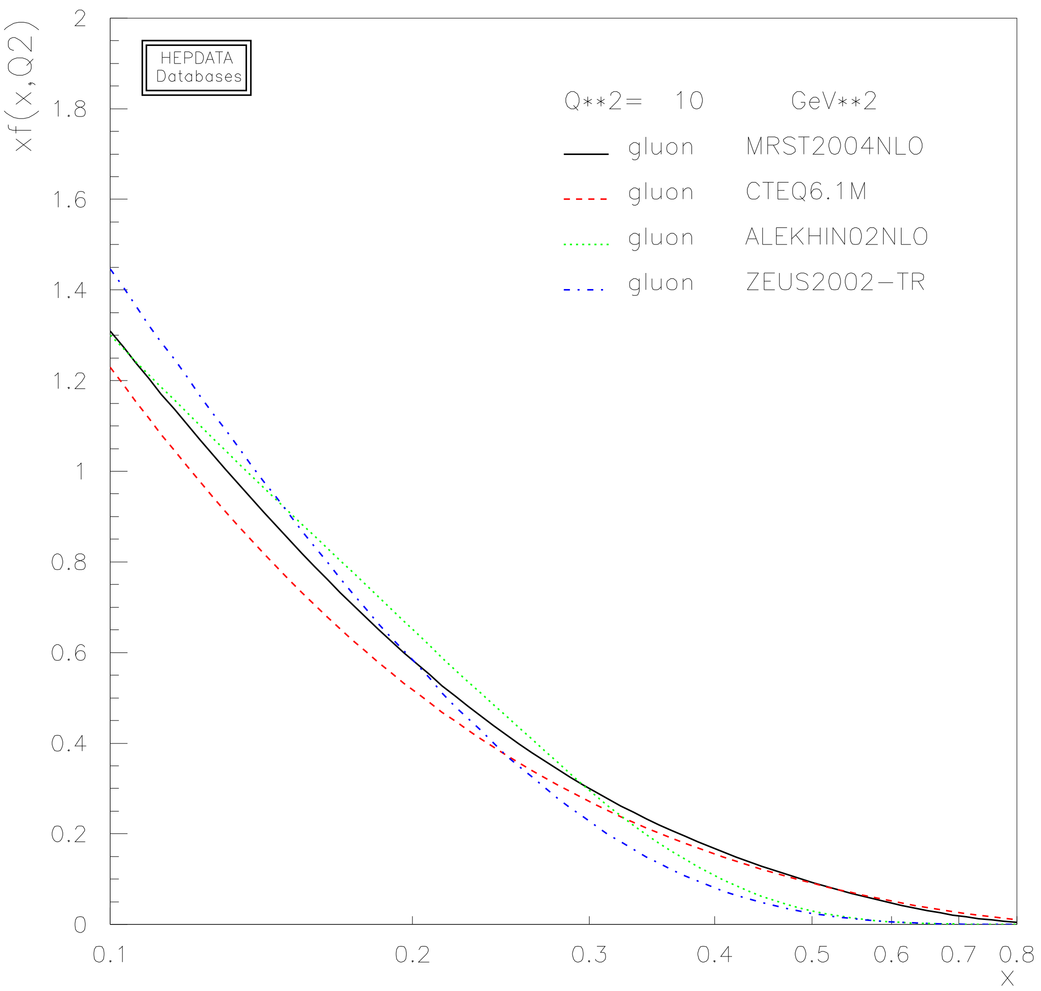


Current comparison of gluons focusing on high x.

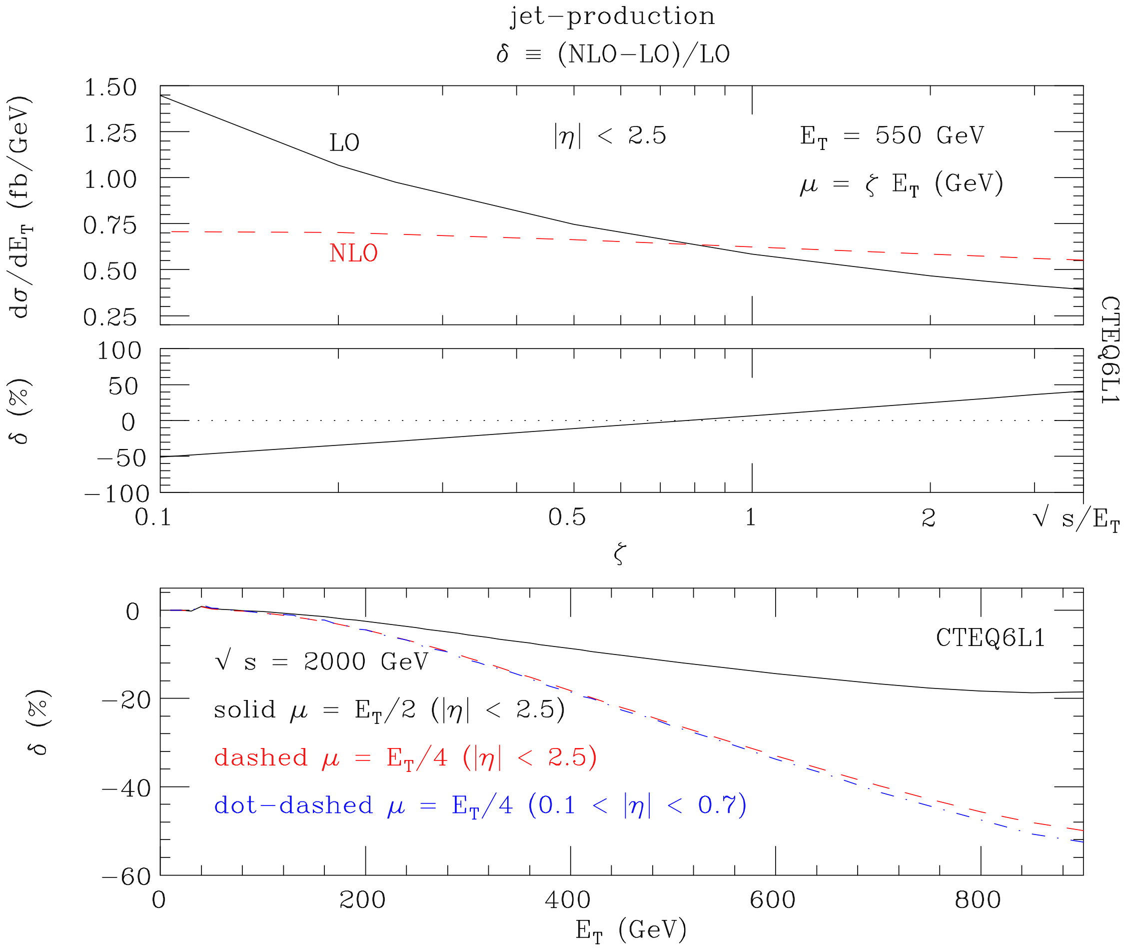
MRST and CTEQ bigger at high x.

Better fit to jet data than Alekhin and ZEUS. These quite possibly perfectly acceptable though, depends on shape as well as size.

Won't be as good though, and will require full utilisation of systematic errors.



Weak corrections



Calculation by Moretti, Nolten, Ross, goes like $(1 - \frac{2}{3} C_F \frac{\alpha_W}{\pi} \log^2(E_T^2/M_W^2))$.

Dominated by quark-(anti)quark processes.

They suggest $\approx 12\%$ correction at $E_T = 450\mathrm{GeV}$. Question validity of recent partons.

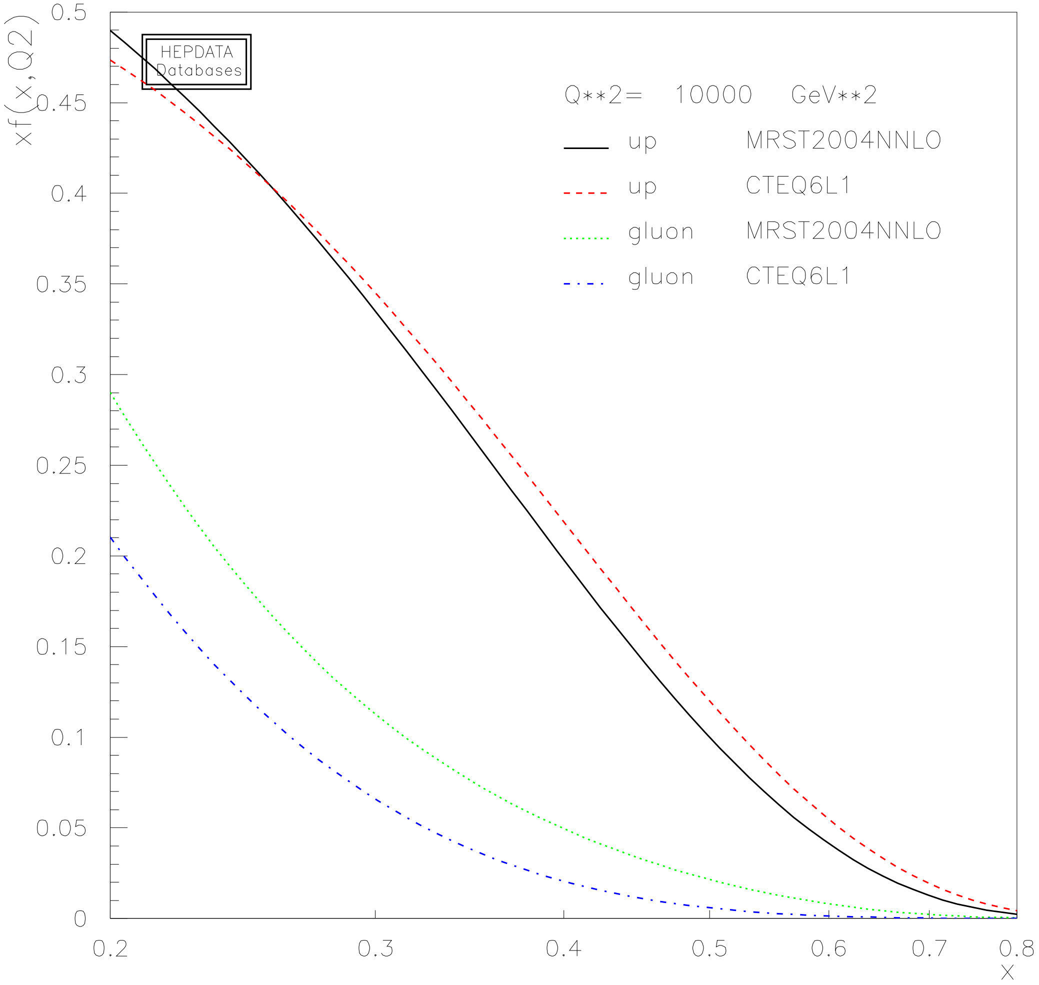
Not quite as big in reality. Use LO partons with big high-x quarks, very small gluon → high-E_T cross-section almost all quarks.

Not the case with most recent partons (look at $x = 0.5$).

qq qg gg matrix elements in ratio 5 6 30.

Even at highest E_T gluon contributes $\sim 30\%$.

Estimate max suppression reduced to $\approx 7 - 8\%$.



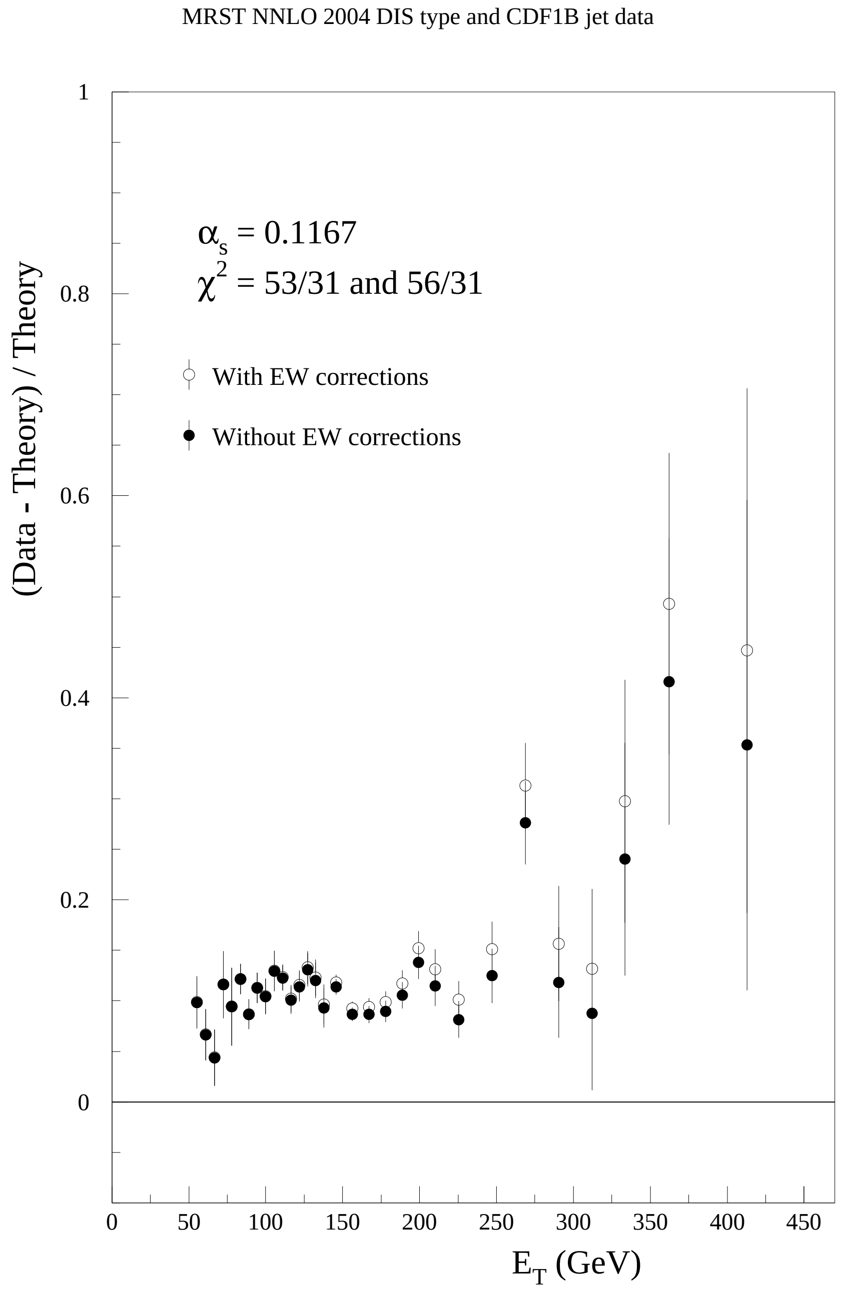
Phenomenological impact not huge.

Movement of both CDF and D0 data relatively small.

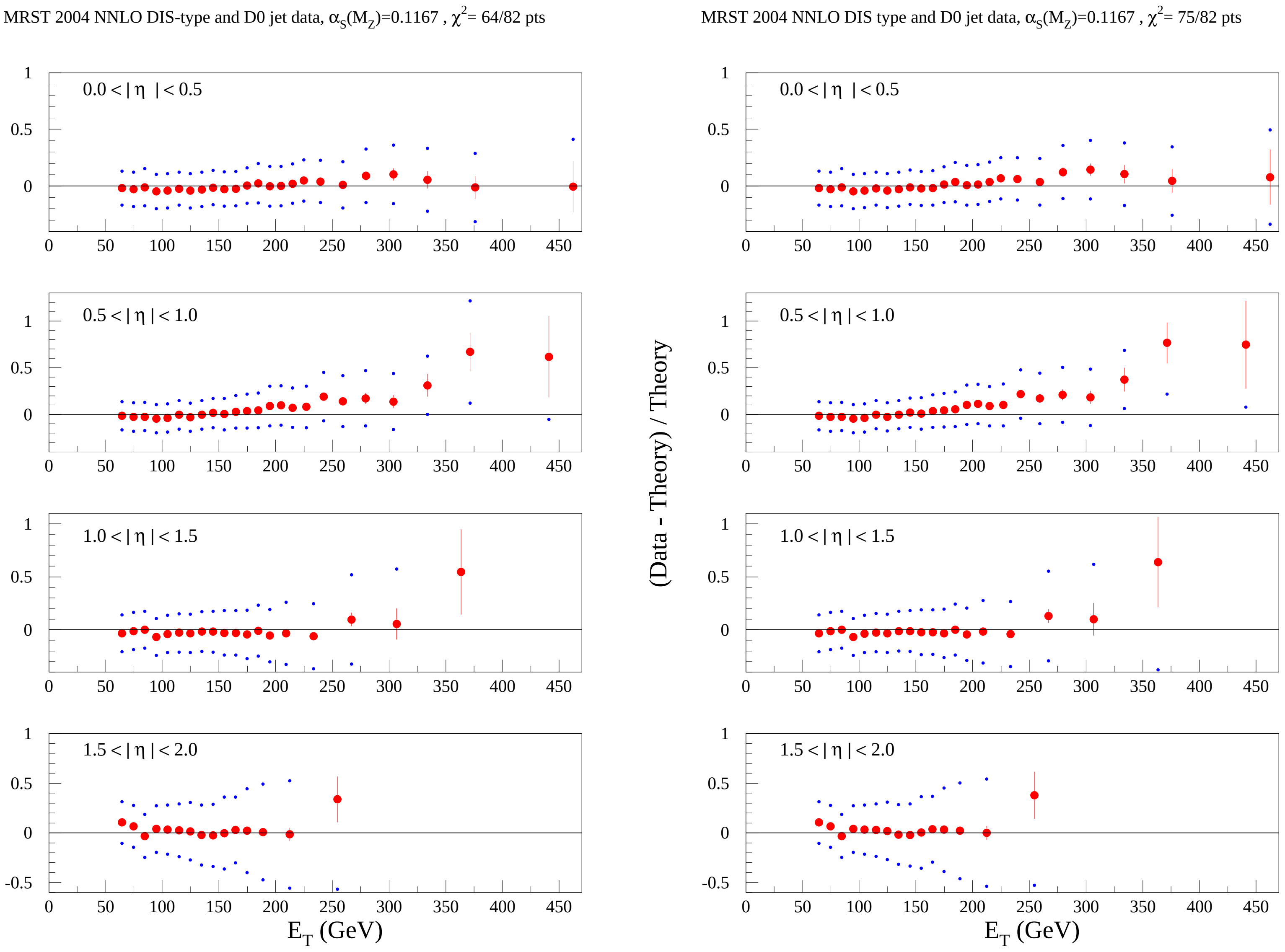
Total χ^2 goes from $117/113$ to $131/113$ (without refitting).

Significant but not a disaster by any means.

More important at higher E_T.



Change in fit to D0 data.


MRST 2004 NNLO DIS-type and D0 jet data, $\alpha_S(M_Z)$=0.1167 , χ^2= 64/82 pts

MRST 2004 NNLO DIS type and D0 jet data, $\alpha_S(M_Z)$=0.1167 , χ^2= 75/82 pts

Theoretical Errors and Fine Details

It is vital to consider theoretical corrections, and to look at data which determines the small differences in parton distributions. These include ....

- Data determining quark decomposition, e.g. W-asymmetry, dimuon data and Drell-Yan asymmetry.

- possibility of isospin violation, $s(x) \neq \bar{s}(x)$, etc.

- higher orders (NNLO)

- QED (comparable to NNLO ? ($\alpha_s^3 \sim \alpha$)

- large x $(\alpha_s^n \ln^{2n-1}(1-x))$

- low Q^2 (higher twist)

- small x $(\alpha_s^n \ln^{n-1}(1/x))$

W- or lepton asymmetry at the Tevatron.

$$\begin{aligned} A_W(y) &= \frac{d\sigma(W^+)/dy - d\sigma(W^-)/dy}{d\sigma(W^+)/dy + d\sigma(W^-)/dy} \\ &\approx \frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)}, \end{aligned}$$

Since $u(x) > d(x)$ at large x, whereas they become roughly equal at smaller x, $A_W(y)$ is positive for $x_1 > x_0 = 0.05$ $(y > 1)$ where measurements are taken.

This helps pin down the u and d quarks in the region $x \sim 0.1$ as well as giving compatible information to NMC and CCFR at higher x.

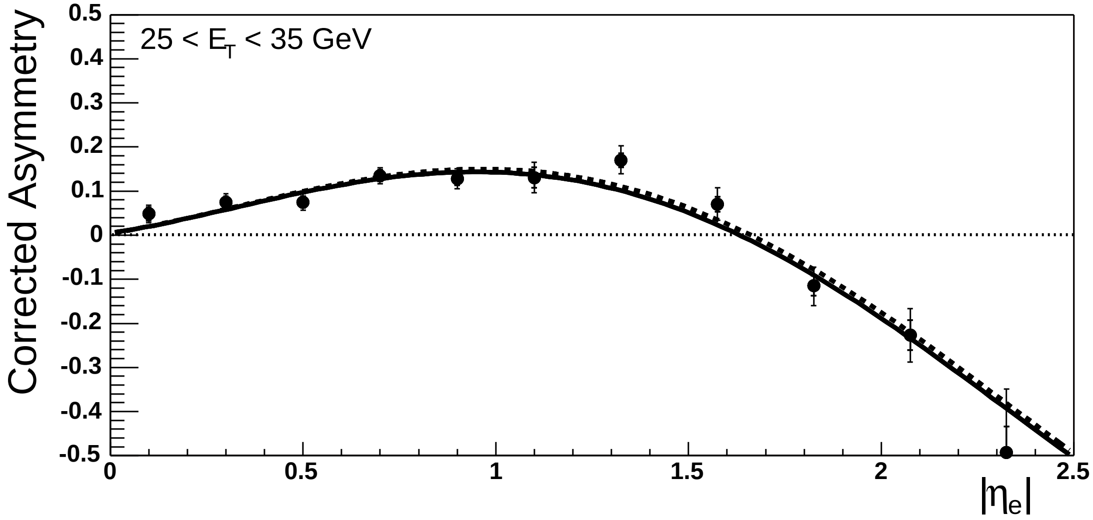
In practice it is the final state leptons that are detected, so it is really the lepton asymmetry

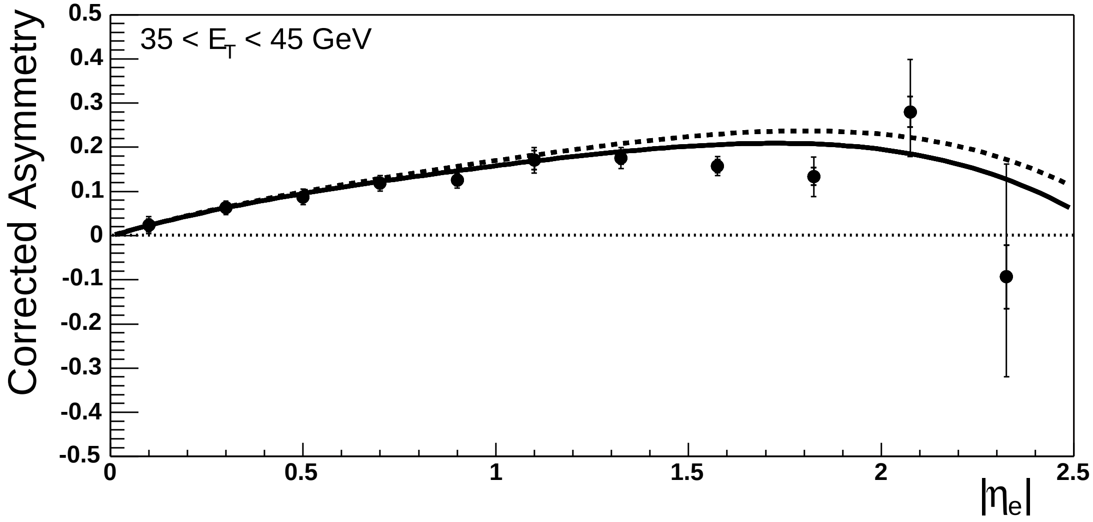
$$A(y_l) = \frac{\sigma(l^+) - \sigma(l^-)}{\sigma(l^+) + \sigma(l^-)}$$

which is measured.

New data from CDF very recently. Now in bins of different E_T, → more discriminating power.

Matches very well to current MRST (dashed) and CTEQ (solid) partons.





Strange quark distribution related to the $\bar{u}$ and $\bar{d}$ quarks using unlike sign dimuon production at CCFR and NuTeV.

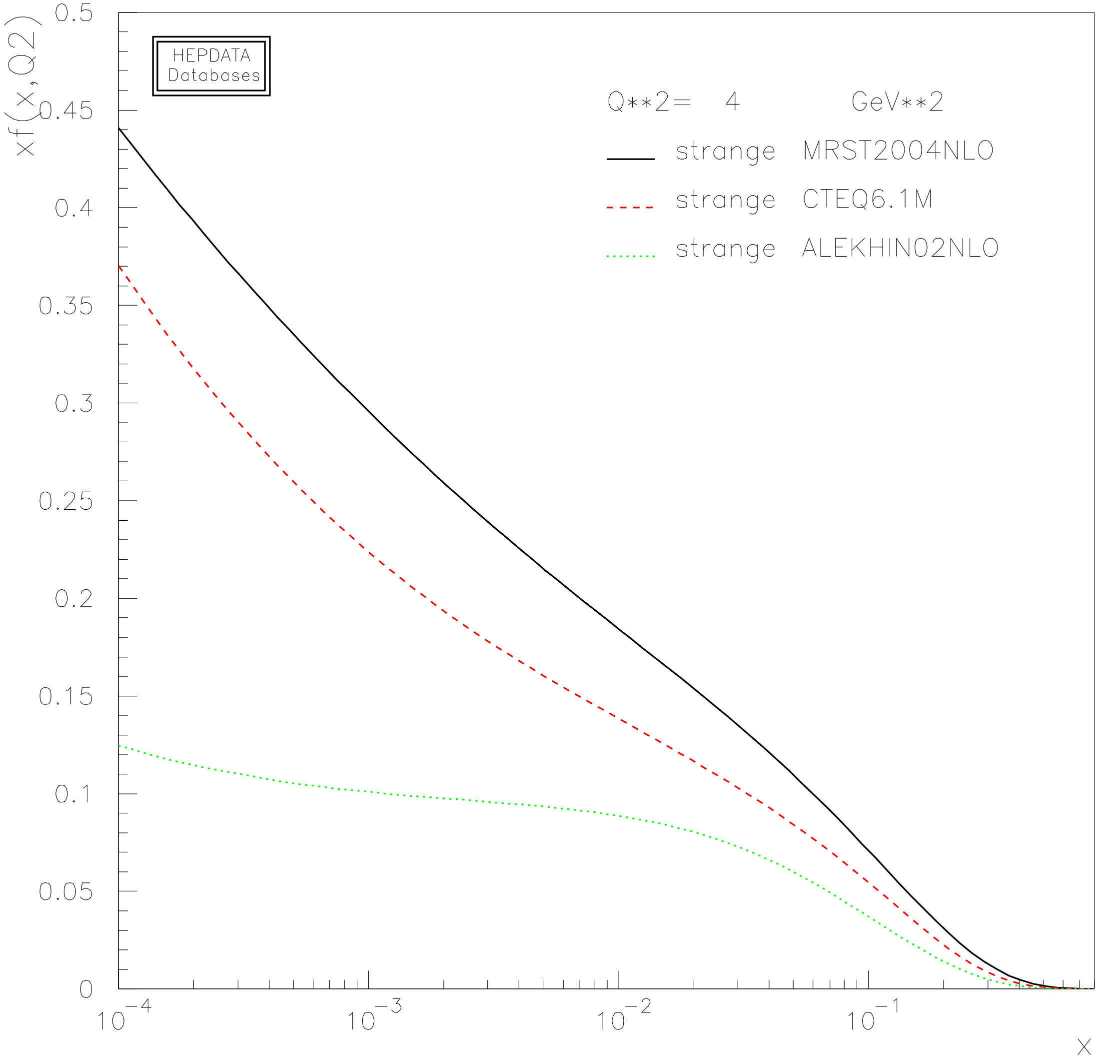
$$\nu_\mu \rightarrow \mu^- + W^+$$

followed by

$$W^+ + s \rightarrow c \rightarrow D^+ \rightarrow \mu^+.$$

At $Q^2 \sim 1\mathrm{GeV}^2$ $s(x) \approx 0.4 \times 0.5(\bar{u}+\bar{d})$, i.e. strange is 18% of the input sea. Fraction increases as Q^2 increases.

Important constraint on partons.



Information more directly available from Drell-Yan asymmetry

$$A_{DY} = \frac{\sigma_{pp} - \sigma_{pn}}{\sigma_{pp} + \sigma_{pn}} = \frac{1-r}{1+r},$$

where

$$r \approx \frac{4u_1\bar{d}_2 + d_1\bar{u}_2 + 4\bar{u}_1 d_2 + \bar{d}_1 u_2}{4u_1\bar{u}_2 + d_1\bar{d}_2 + 4\bar{u}_1 u_2 + \bar{d}_1 d_2},$$

and 1 labels the proton and 2 the neutron.

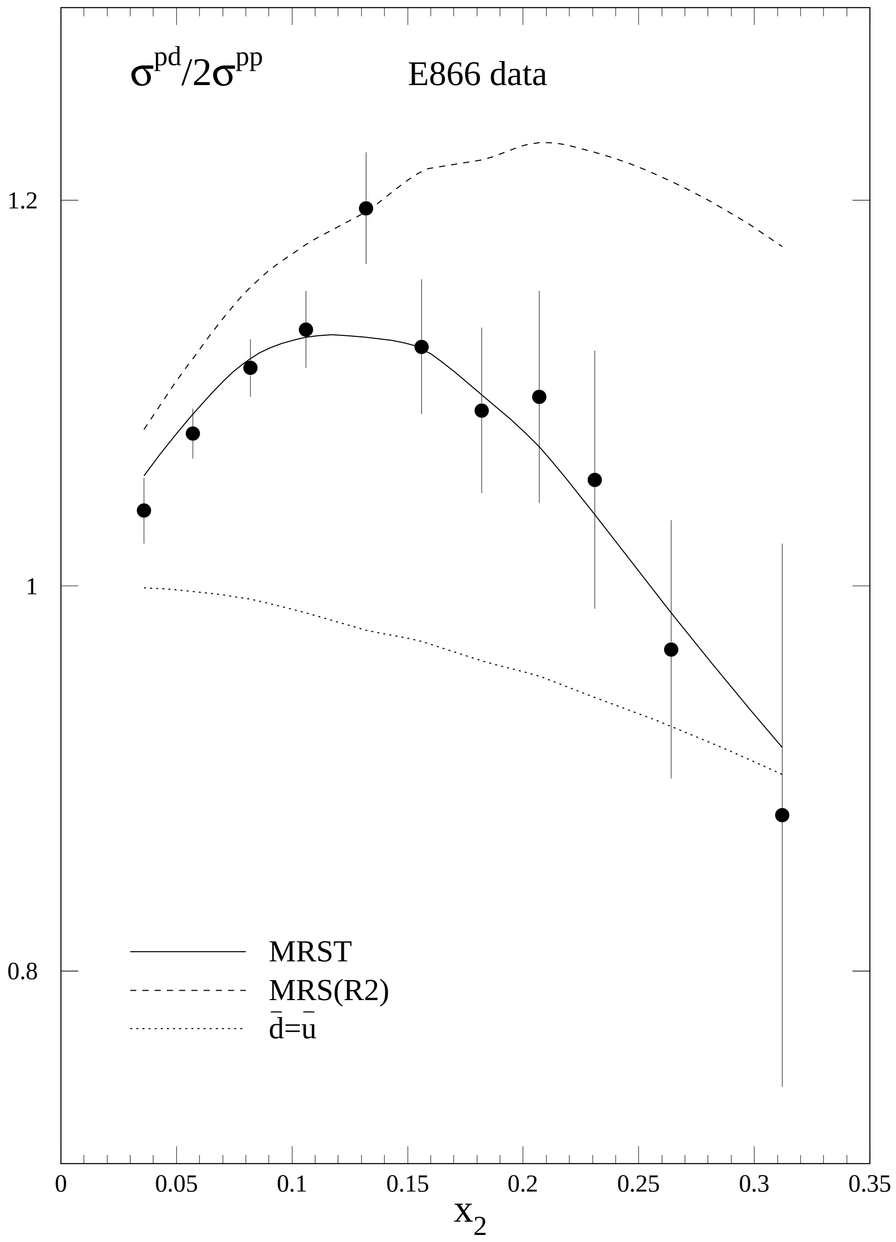
In fact measure the quantity

$$R_{dp} = \frac{\sigma_{pd}}{2\sigma_{pp}} = \frac{1}{2}(1+r),$$

E866 measured very accurately from $0.04 < x < 0.3$. Gives clear evidence of $\bar{u} - \bar{d}$ asymmetry, but not as much as suggested previously.

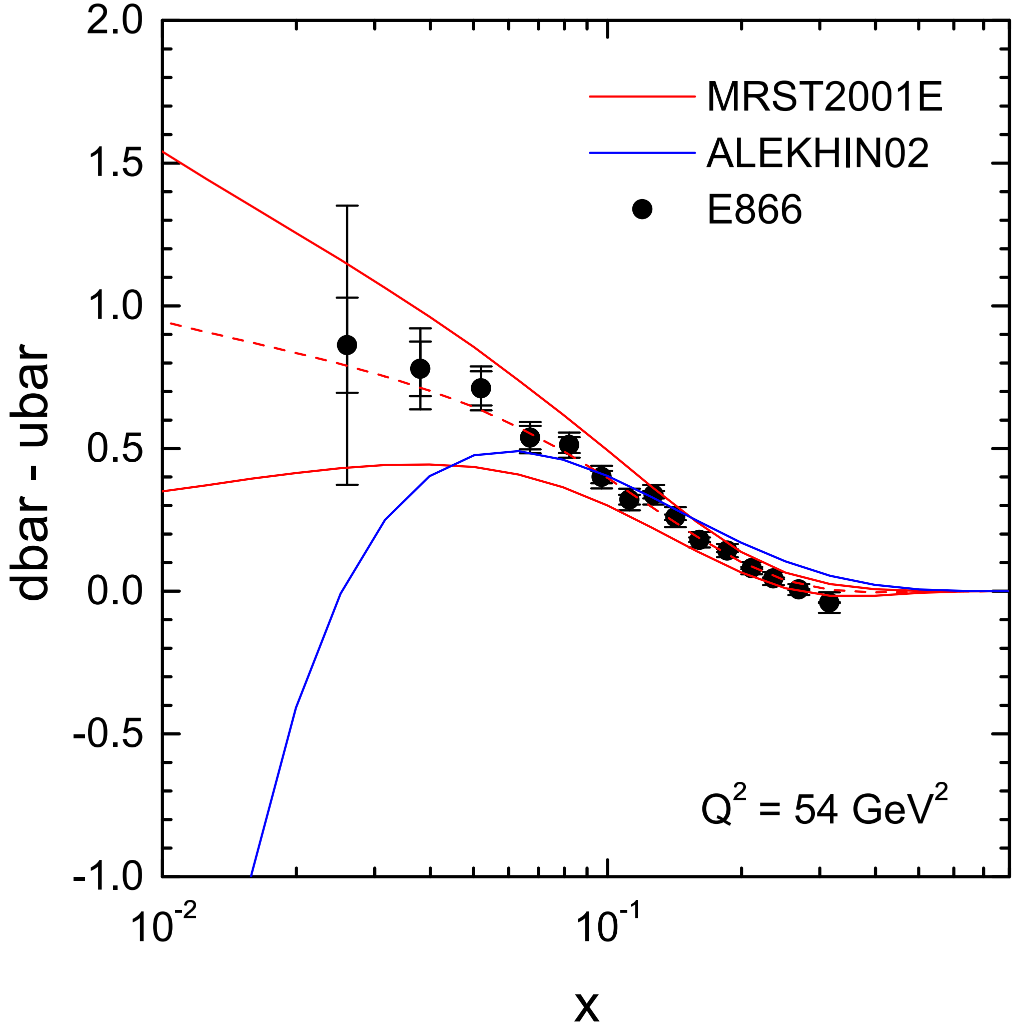
Seems to reach maximum at $x \approx 0.2$. Not clear what happens as $x \to 1$. Consistent with $\to 1$ as $x \to 0$.

Drell-Yan asymmetry compared to E866 data.



$\bar{d} - \bar{u}$ data and partons

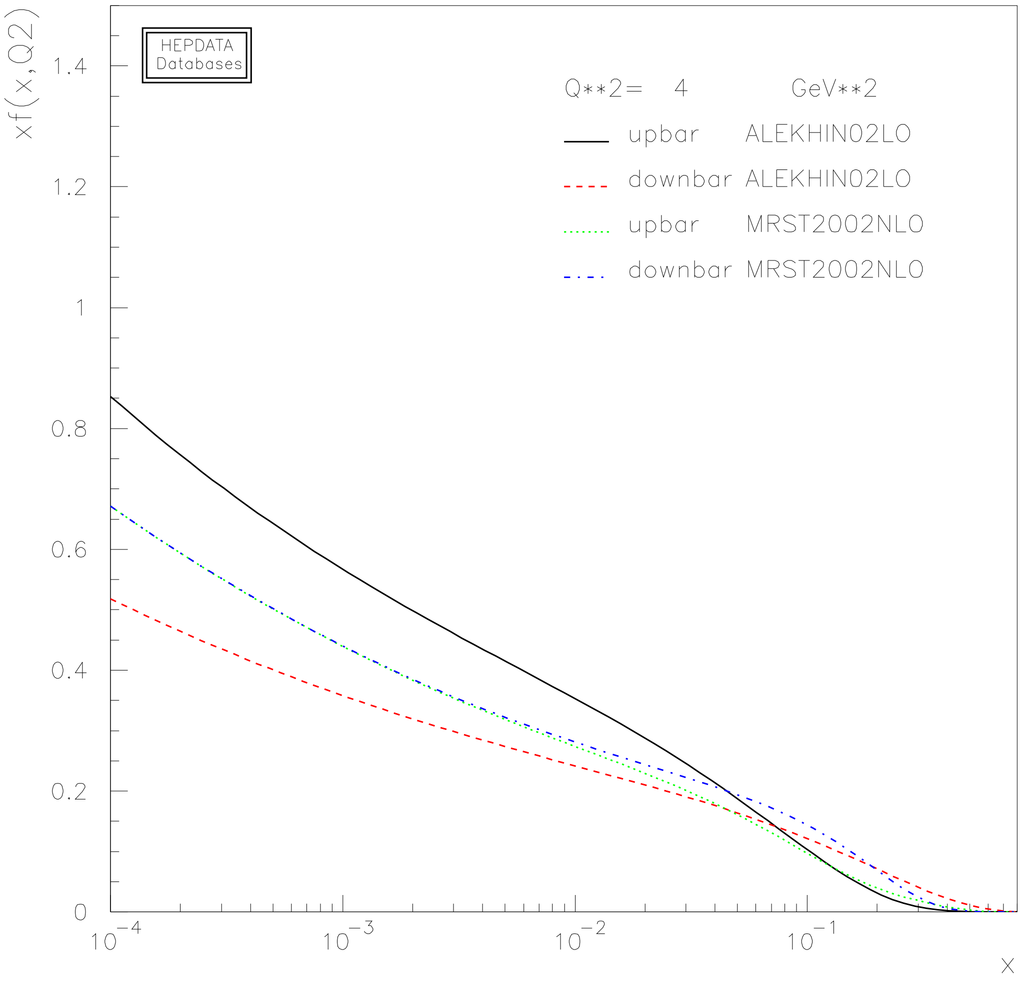
Comparison of the $\bar{d}$ and $\bar{u}$ difference for MRST and Alekhin compared to the E866 data.



$\bar{d}$ and $\bar{u}$ partons

Both the strange and $\bar{d}$-$\bar{u}$ asymmetry feed into the determination of the u and d quarks at low x.

Comparison of the $\bar{d}$ and $\bar{u}$ partons for MRST and Alekhin.



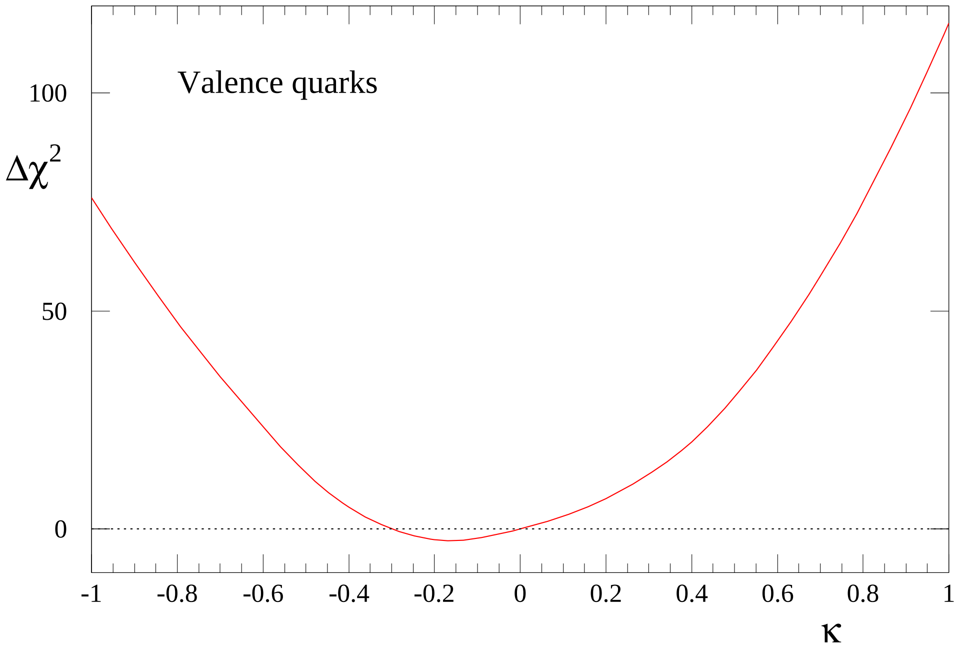
MRST look at effect of **isospin violation**. NuTeV measure (with a $3-\sigma$ discrepancy)

$$R^- = \frac{1}{2} - \sin^2\theta_W + (1 - \frac{7}{3}\sin^2\theta_W)\frac{[\delta U_{\rm v}] - [\delta D_{\rm v}]}{2[V^-]}.$$

$$[\delta U_{\rm v}] = [U^p_{\rm v}] - [D^n_{\rm v}], \qquad [\delta D_{\rm v}] = [D^p_{\rm v}] - [U^n_{\rm v}].$$

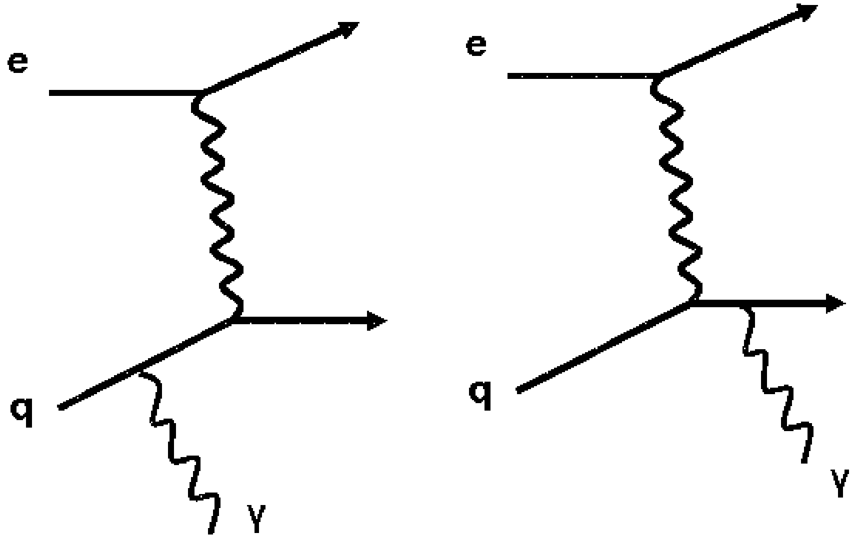
$$u^p_{\rm v}(x) = d^n_{\rm v}(x) + \kappa f(x), \qquad d^p_{\rm v}(x) = u^n_{\rm v}(x) - \kappa f(x).$$

$\kappa = -0.2 \rightarrow$ reduction of the NuTeV anomaly to $\sim 1.5 - \sigma$, i.e. $\Delta sin^2\theta_W \sim -0.002$. Larger (more negative) κ allowed.



QED **Effects**.

QED corrections to DIS include



⇒ mass singularity when $\gamma \parallel q$

QED collinear singularities are *universal* and can be absorbed into pdfs, exactly as for QCD collinear singularities, leaving finite (as $m_q \to 0$) $\mathcal{O}(\alpha)$ QED corrections in coefficient functions

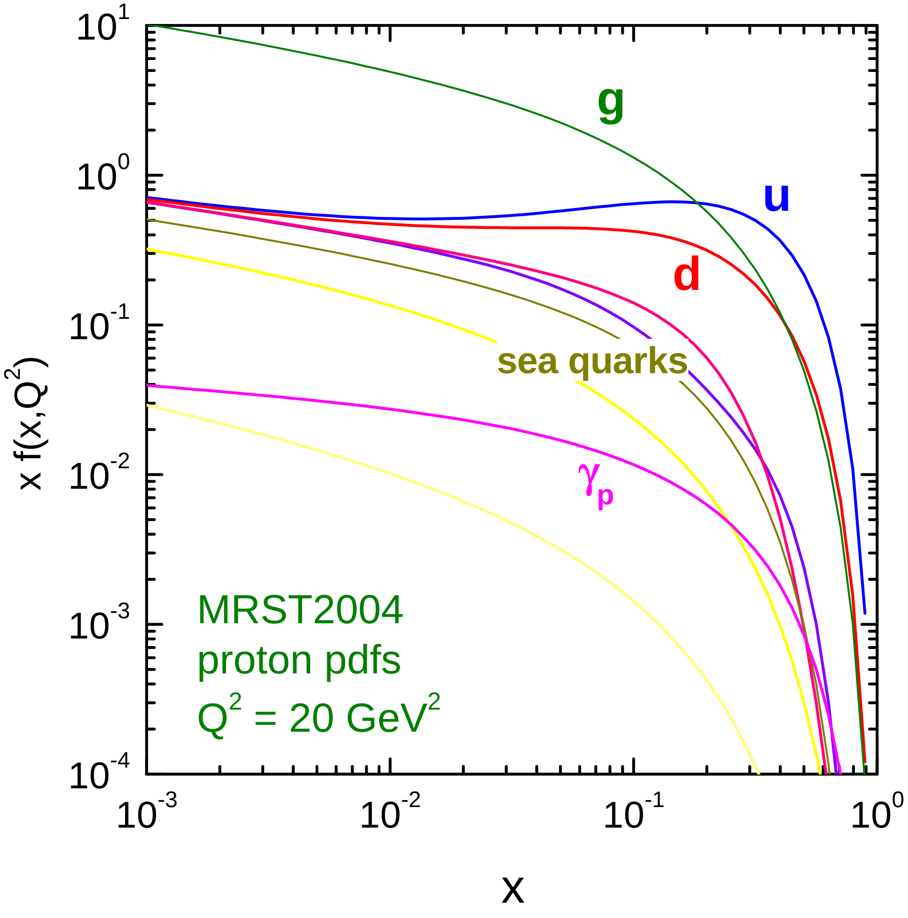
New study by MRST.

Effect on quark distributions is entirely negligible at small x where gluon contribution dominates DGLAP evolution. However, gluon loses a little momentum to photon. Small effect at small x. Not helpful. Lose a few χ^2 at NLO. No problem at NNLO.

At large x, photon radiation from quarks leads to faster evolution, roughly equivalent to a slight shift in α_S : $\Delta\alpha_S(M_Z^2) \simeq +0.0003$

Photon similar size to b-quark. Bigger at high x.

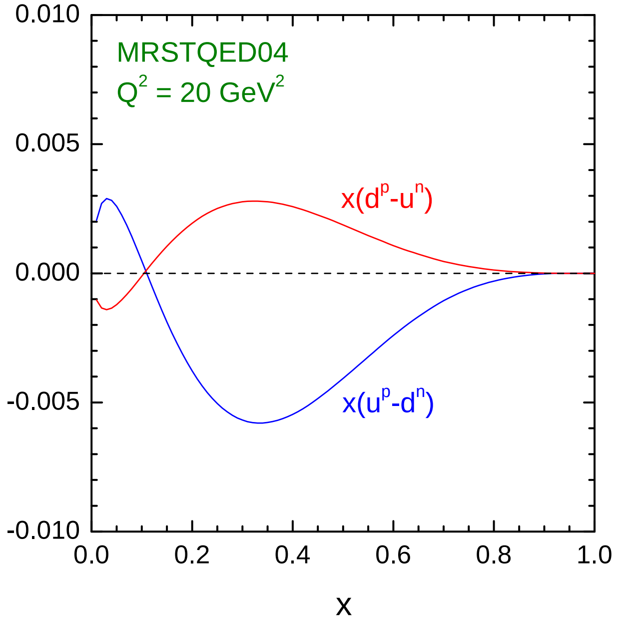
Overall QED effects much smaller than many sources of uncertainty.



However, QED effects to lead to small isospin violation.

$u_V^p(x)$ quarks radiate more photon than $d_V^n(x)$ quarks.

To rough approximation



$$\gamma(x,Q^2) = \sum_j e_j^2 \frac{\alpha}{2\pi} \ln(Q^2/m_q^2) \int_x^1 \frac{dy}{y} P_{\gamma q}(y) q_j(\frac{x}{y}, Q^2).$$

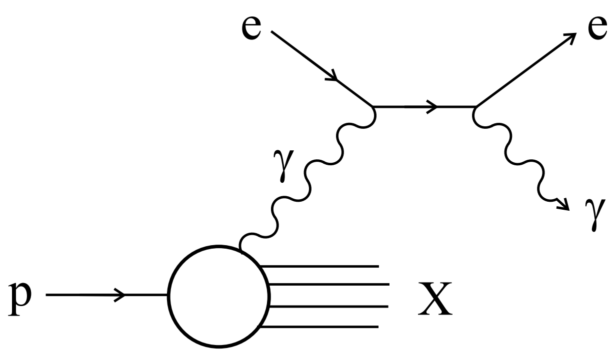
So more photon momentum in proton than neutron due to high-x up quarks radiating more than high-x down quarks.

Momentum conservation $\rightarrow u_V^p(x) < d_V^n(x)$ at high .

Hence, $[\delta U_\mathrm{v}] < 0$ as required by NuTeV anomaly.

Estimates for $m_u = 6$ MeV and $m_d = 10$ MeV imply similar to isospin violation observed by best fit! Reduces NuTeV anomaly to about $1/2$.

Model supported by wide angle photon scattering, i.e. $ep \to e\gamma X$ where final state electron and photon have equal and opposite large transverse momentum.



ZEUS has recently published a measurement of this cross section ($x_\gamma \approx 0.005$):

$$\sigma(ep \to e\gamma X) \;=\; 5.64 \;\pm\; 0.58\,\text{(stat.)}\;{}^{+0.47}_{-0.72}\,\text{(syst.) pb.}$$

Neither PYTHIA nor HERWIG can explain the observed rate - underestimating the cross-section by factors of 2 and 8 respectively.

Using the proton's photon parton distribution we find

$$\sigma(ep \to e\gamma X) \;=\; 6.2 \;\pm\; 1.2 \text{ pb.}$$

Using constituent quark masses our prediction nearly halves.

NNLO

Coefficient functions known at NNLO. Singular limits $x \to 1$, $x \to 0$ known for NNLO splitting functions as well as limited moments. Approximate NNLO splitting functions devised by van Neerven and Vogt. Improve quality of fit very slightly (MRST). Reduces $\alpha_S \to 0.1155$.

Now complete! (Moch, Vermaseren and Vogt). Extremely similar to average of best estimates → no real change in NNLO partons.

Full updated NNLO partons very soon.

Not only exact NNLO splitting functions, but rigorous heavy quark thresholds (partons **discontinuous** at NNLO), NNLO Drell-Yan cross-sections, and uncertainties.

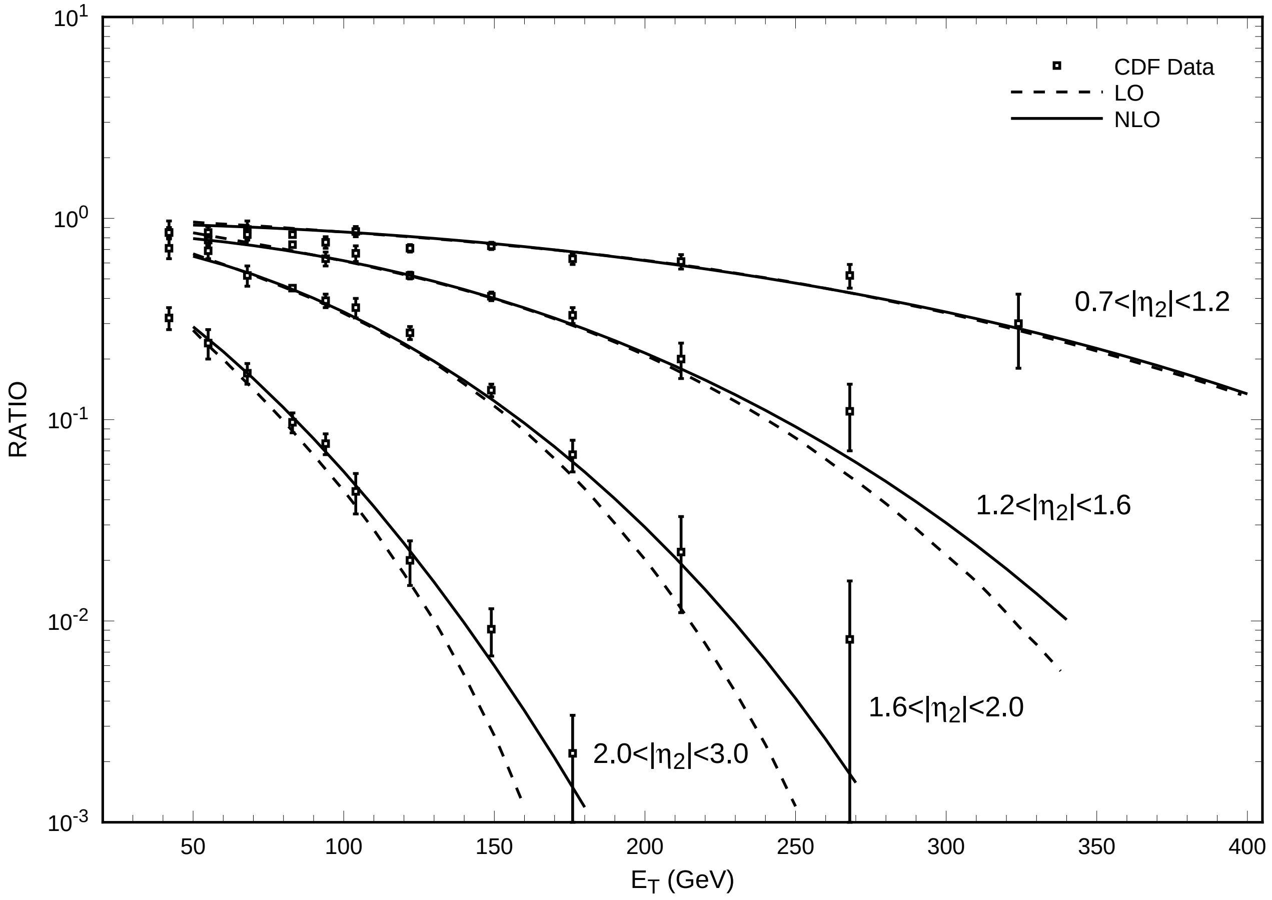
Essentially full NNLO determination of partons very soon.

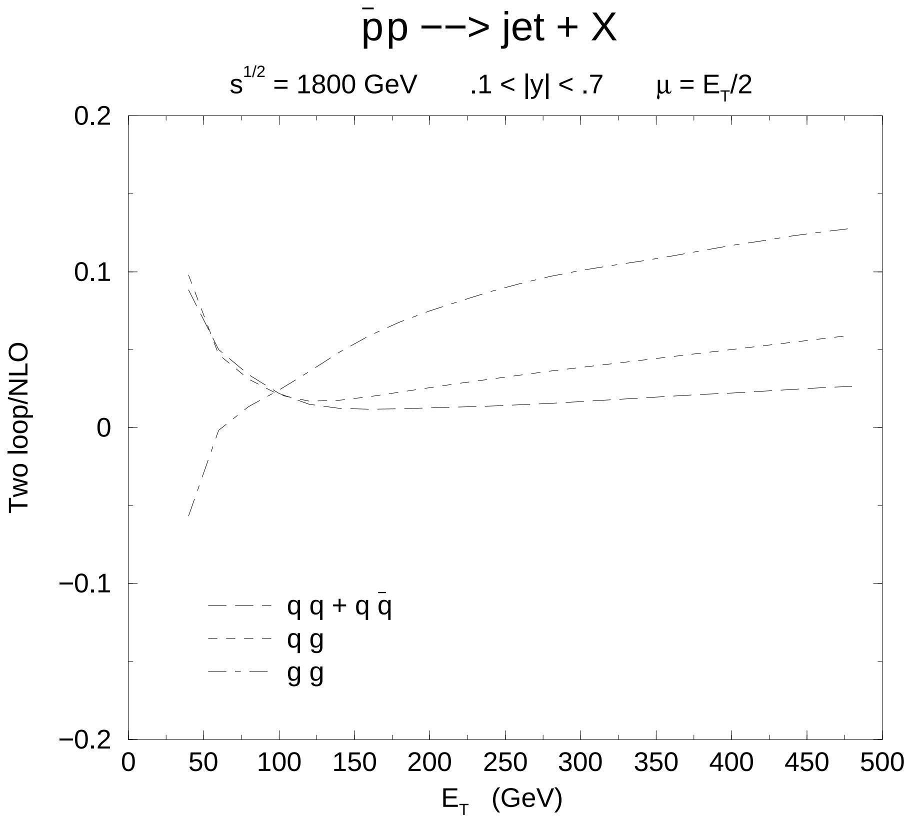
Only NNLO jet cross-sections missing. Is this important?

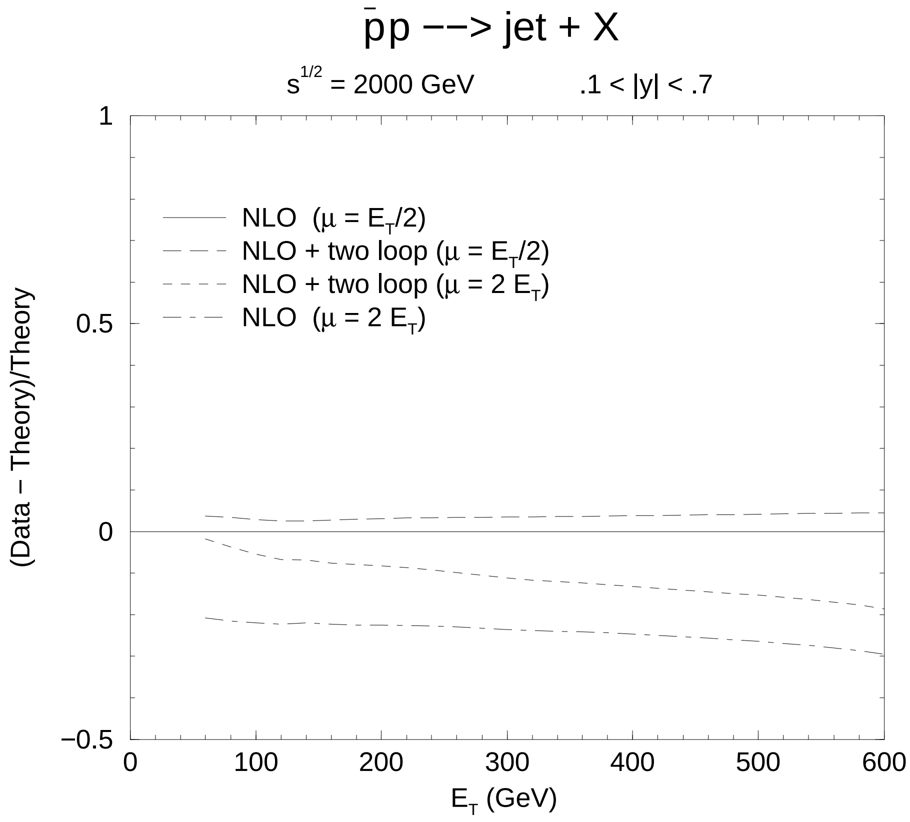
Probably not!

NLO corrections themselves not large, except at high rapidities.

At central rapidities $\leq 10\%$. Similar to correlated errors.







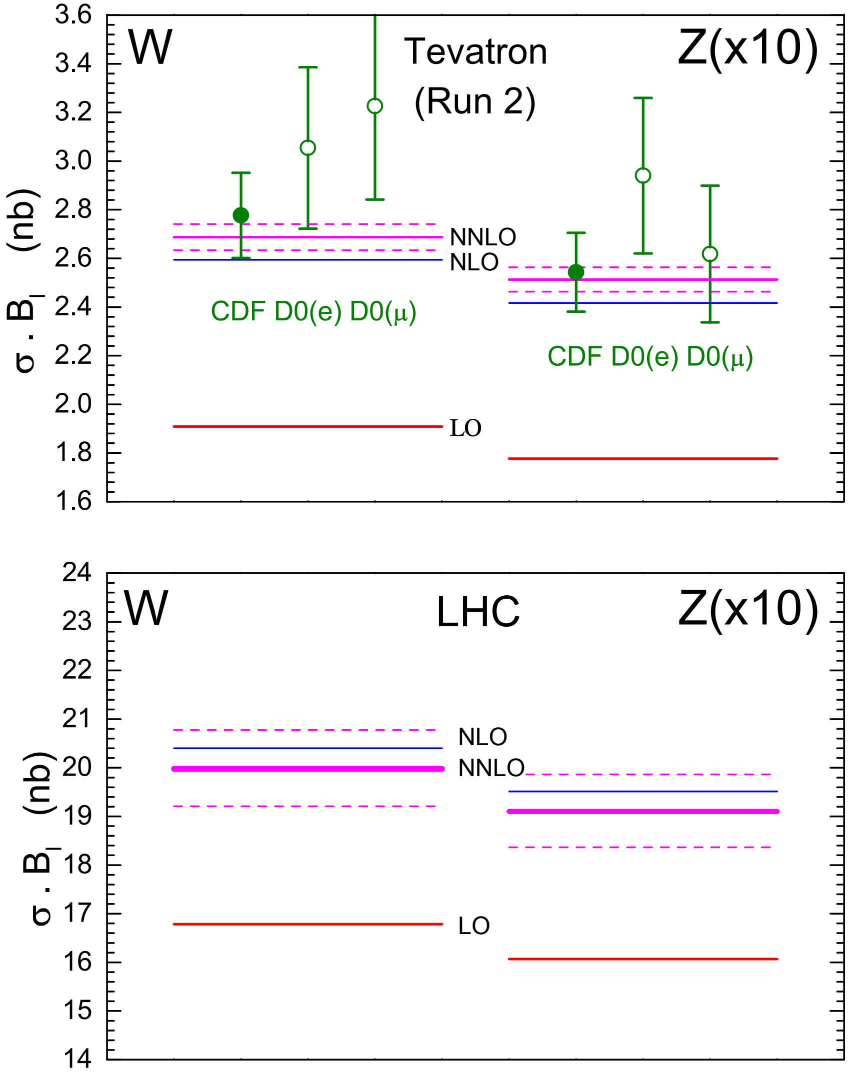
Also good NNLO estimates Kidonakis, Owens. Calculated threshold correction logarithms. Expected to be major component of total NNLO correction.

→ Flat $3 - 4\%$ correction. Consistent with what is known from NLO. Smaller than systematics on data.

Mistakes from ignoring jets in fits bigger than mistakes made at NNLO by not knowing exact hard cross-section.

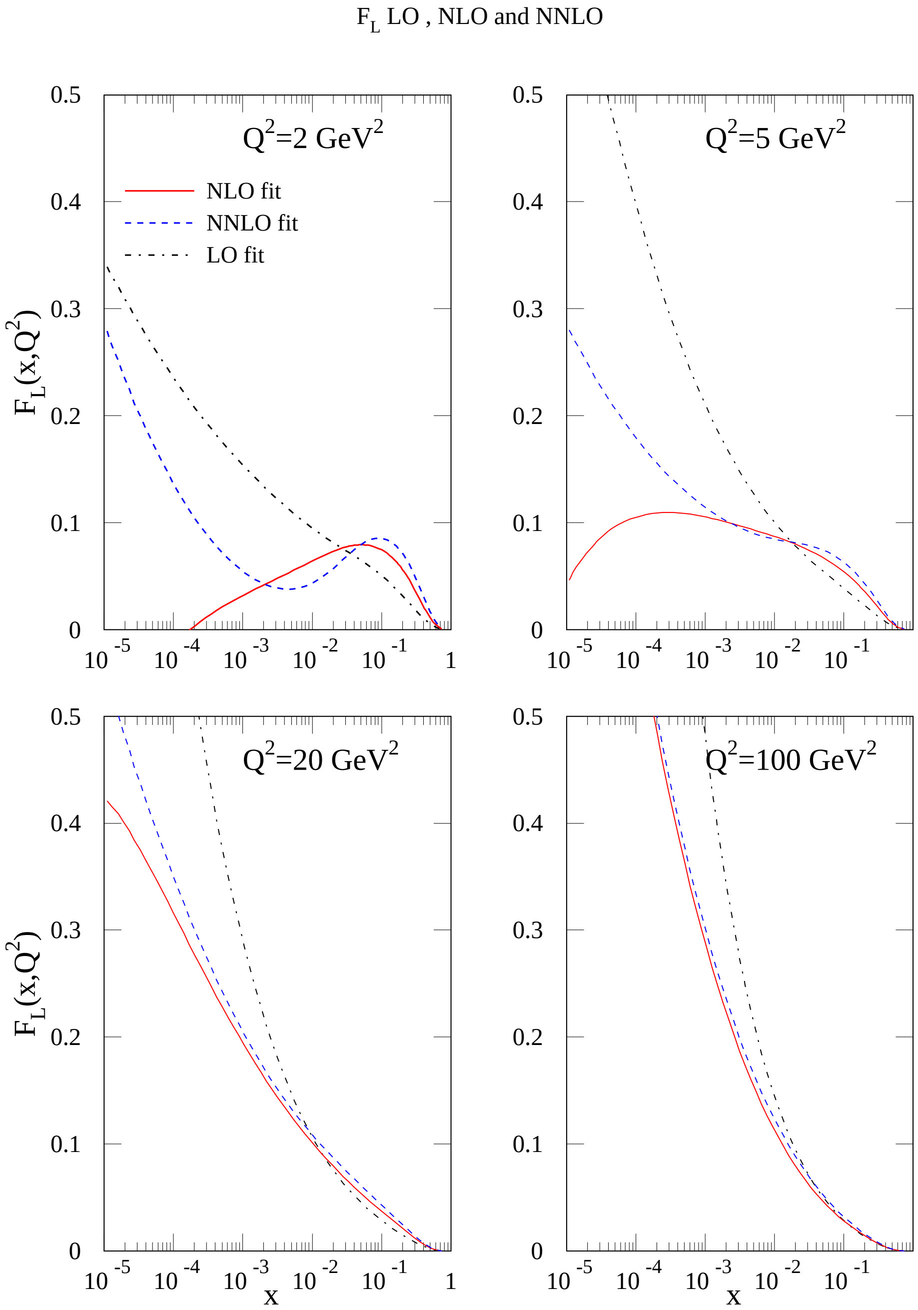
Reasonable stability order by order for (quark-dominated) W and Z cross-sections.

This fairly good convergence is largely guaranteed because the quarks are fit directly to data. Much worse for gluon dominated quantities e.g. $F_L(x,Q^2)$.



partons: MRST2002
NNLO evolution: van Neerven, Vogt approximation to Vermaseren et al. moments
NNLO W,Z corrections: van Neerven et al. with Harlander, Kilgore corrections

Instability in physical, gluon dominated, quantity $F_L(x, Q^2)$ going from LO → NLO → NNLO.



Alternative approach.

In order to investigate real quality of fit and regions with problems vary kinematic cuts on data.

Procedure – change W^2_{cut}, Q^2_{cut} and x_{cut}, re-fit and see if quality of fit to remaining data improves and/or input parameters change dramatically. Continue until quality of fit and partons stabilize.

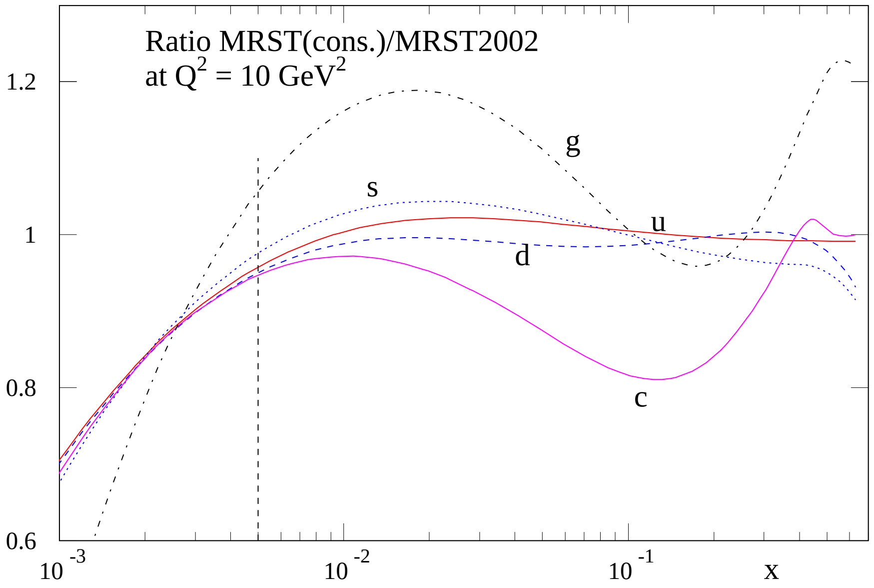
For W^2_{cut} raising from $12.5\mathrm{GeV}^2$ to $15\mathrm{GeV}^2$ sufficient.

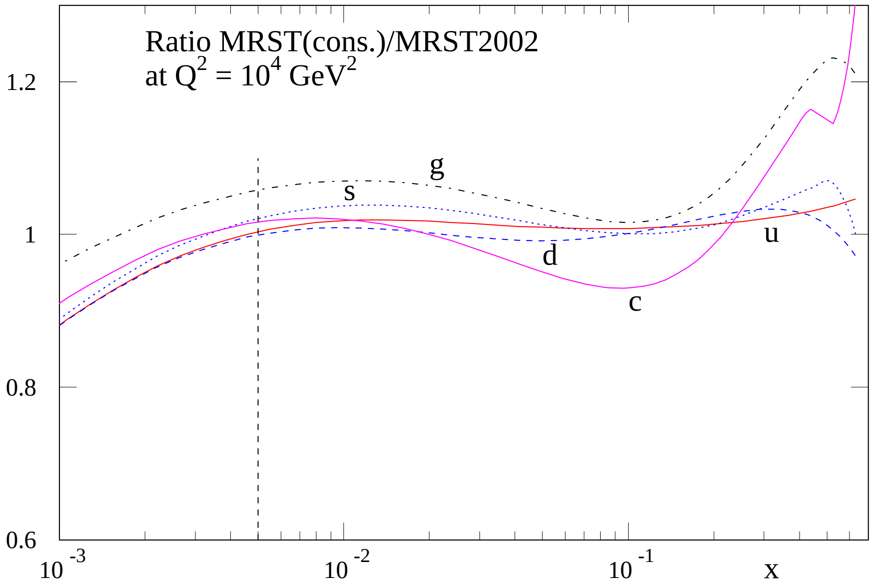
Raising Q^2_{cut} from $2\mathrm{GeV}^2$ in steps there is a slow continuous and significant improvement for higher Q^2 up to $> 10\mathrm{GeV}^2$ (cut 560 data points) – suggests any corrections mainly higher orders not higher twist.

Raising x_{cut} from 0 to 0.005 (cut 271 data points) continuous improvement. At each step moderate x gluon becomes more positive.

→ MRST2003 conservative partons. Should be most reliable method of parton determination ($\Delta\chi^2 = -70$ for remaining data), but only applicable for restricted range of x, Q^2. $\rightarrow \alpha_S(M_Z^2) = 0.1165 \pm 0.004$.

The ratio of the conservative partons to the default partons at NLO. One can see the dip of the conservative partons below $x_{cut} = 0.005$.

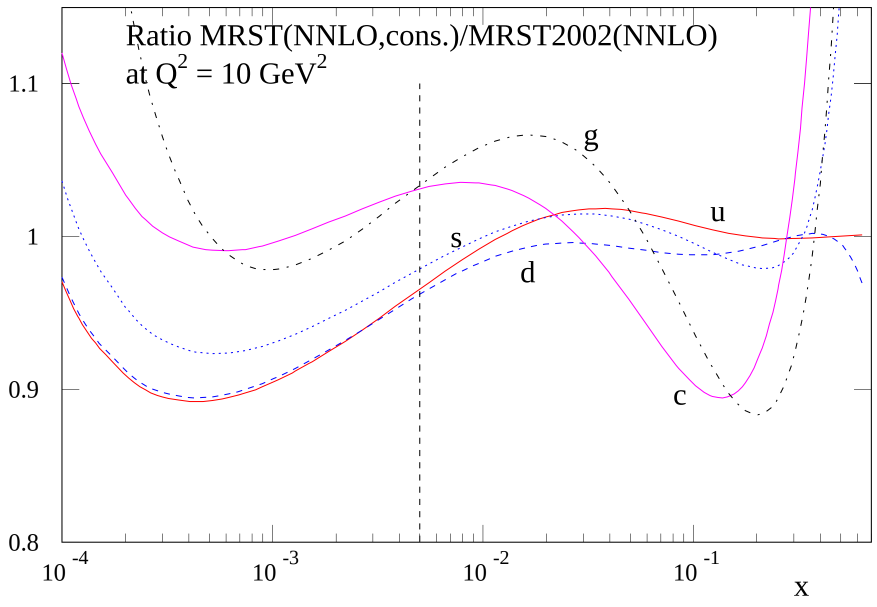


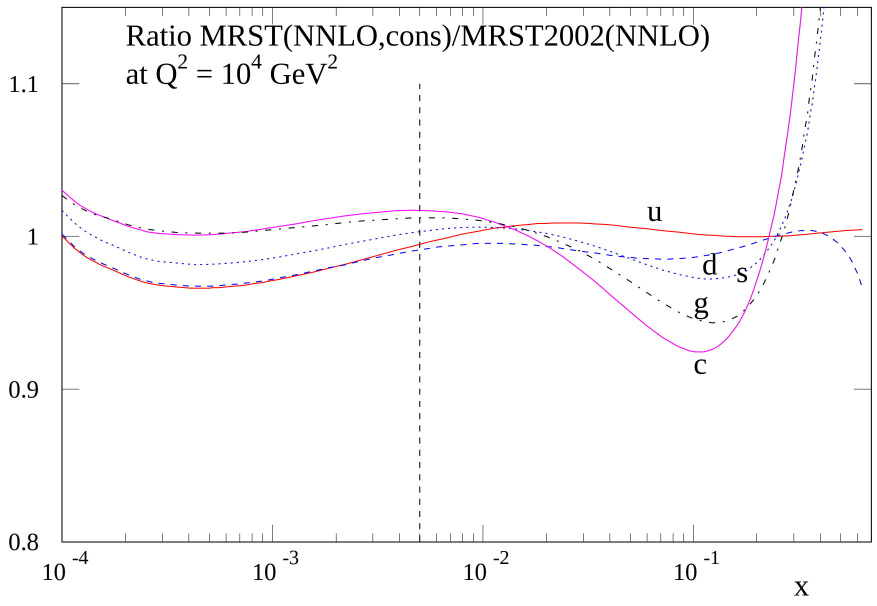


The ratio of the conservative partons to the default partons at NNLO. Now $x_{cut} = 0.005$ and $Q^2_{cut} = 7\mathrm{GeV}^2$. Slight improvement.

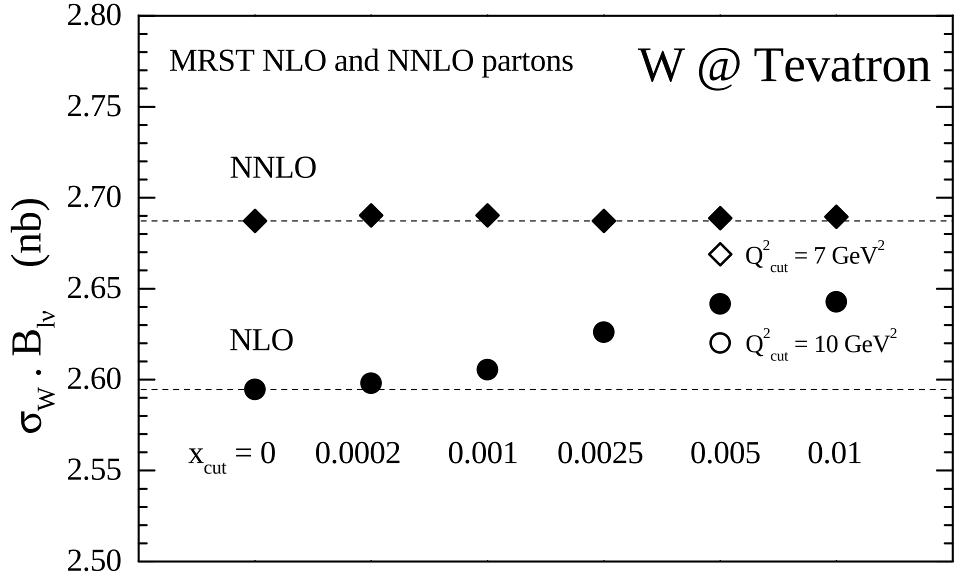
$\Delta\chi^2$ still large.

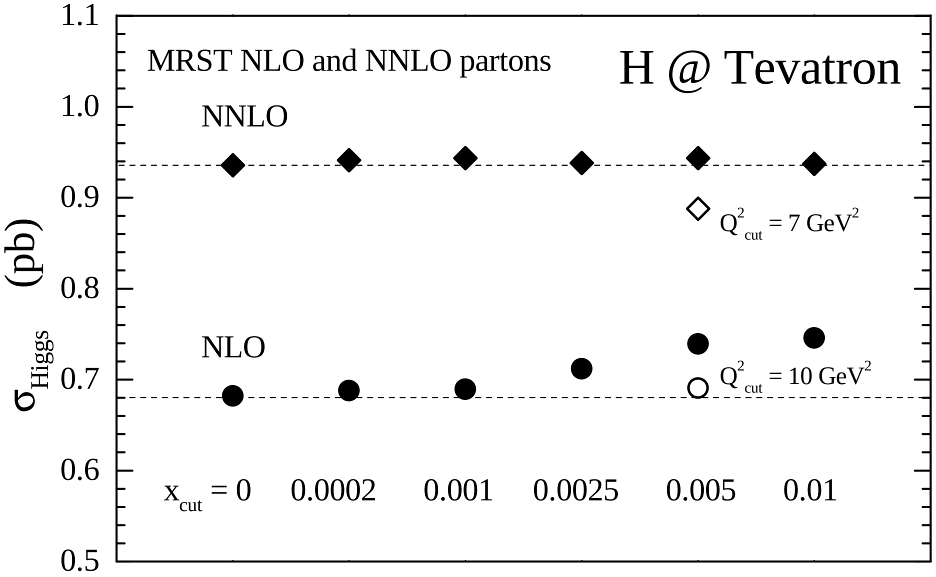
However, now the partons are similar below $x_{cut} = 0.005$. Significant or partially accidental?

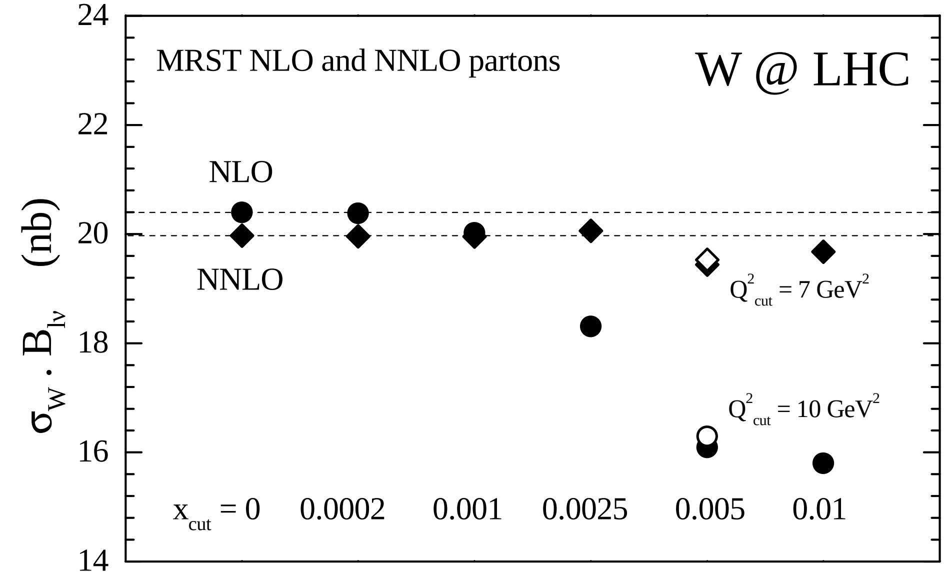


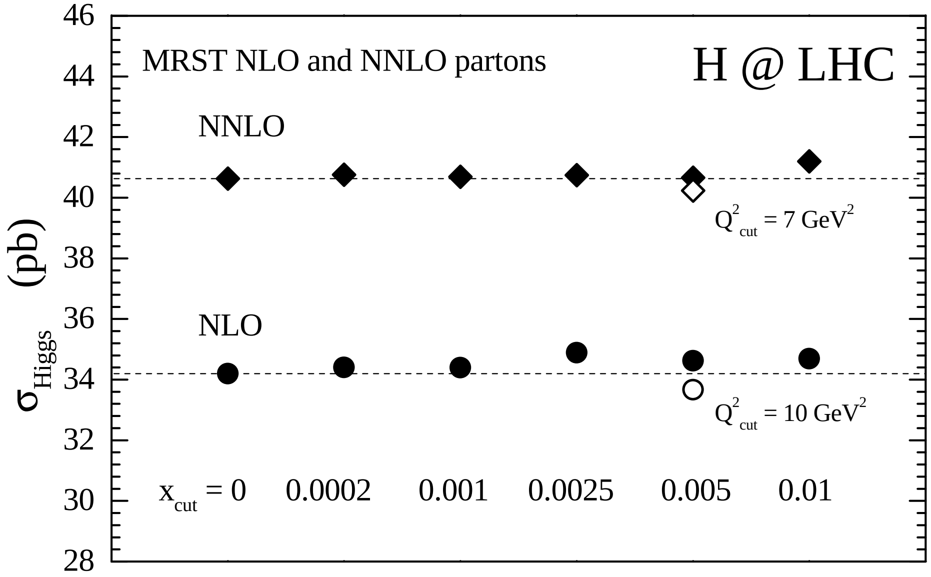


Variation in predictions with cuts. Follows patterns expected. Range of possible theoretical error.









CTEQ **results**

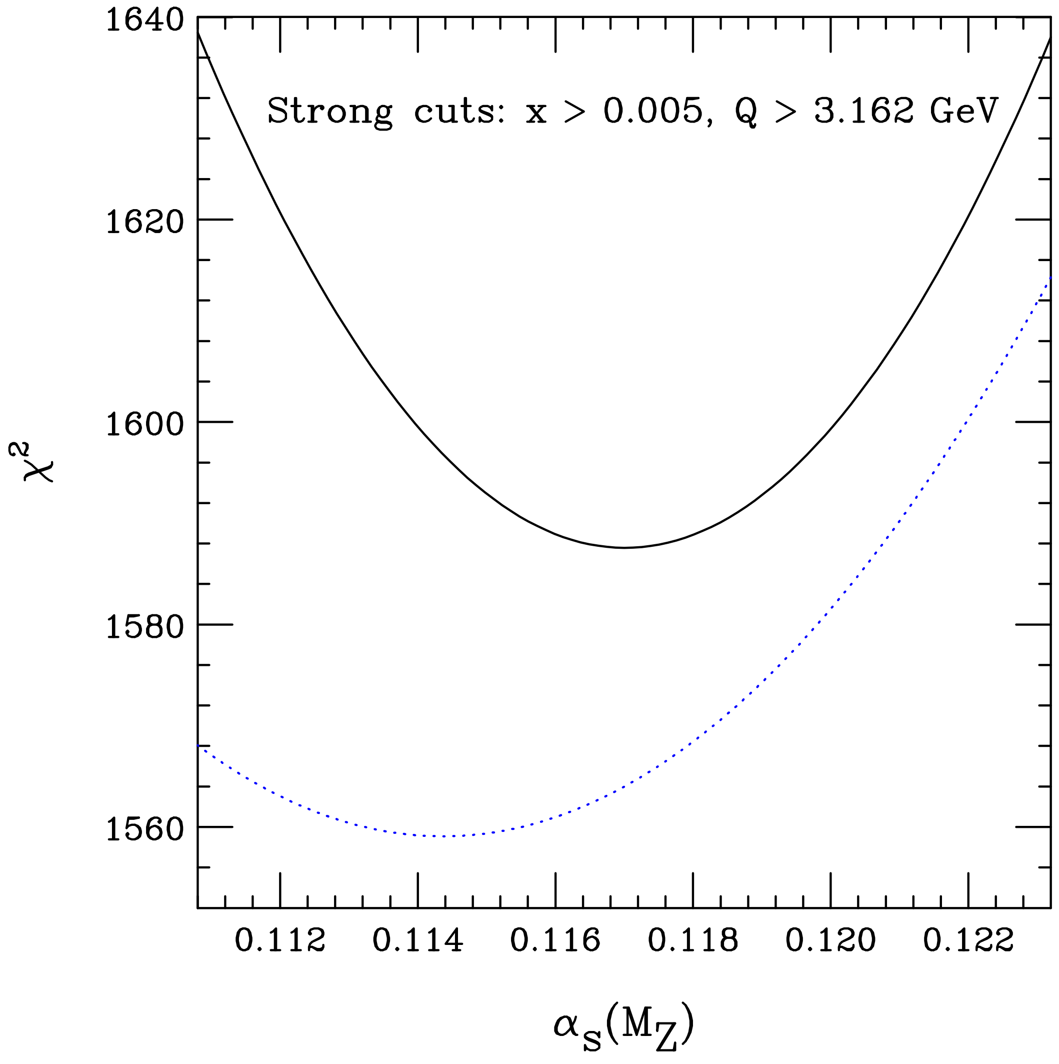
CTEQ see similar type of behaviour (despite protestations), though not as dramatic.

With conservative cuts on data their input gluon is as keen to have negative component (remember $Q_0^2 = 1.69\mathrm{GeV}^2$), and best value of $\alpha_S(M_Z^2)$ moves lower.

blue line - negative gluon allowed

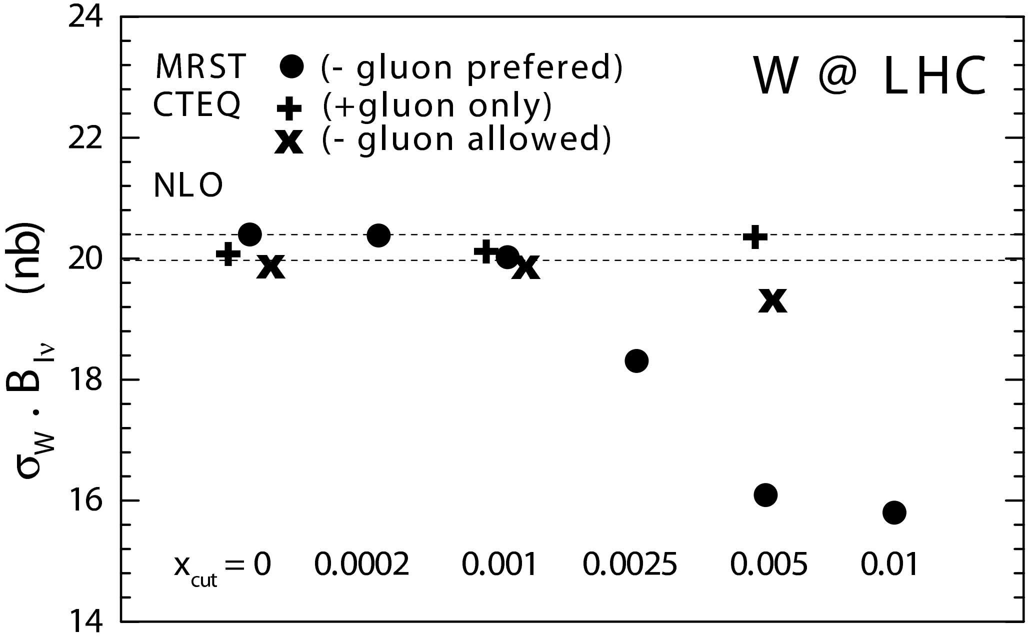
black line - positive definite gluon.

Verifies negative/small gluon at low x and Q^2 **not** due to data at low x and Q^2.



Prediction stability.

Also find prediction for σ_W at the LHC moves down a little as more cuts imposed. Not as significant as MRST by a long way it appears.



However, loss of data leads to larger errors, and χ^2 profile is very flat indeed in the downwards direction.

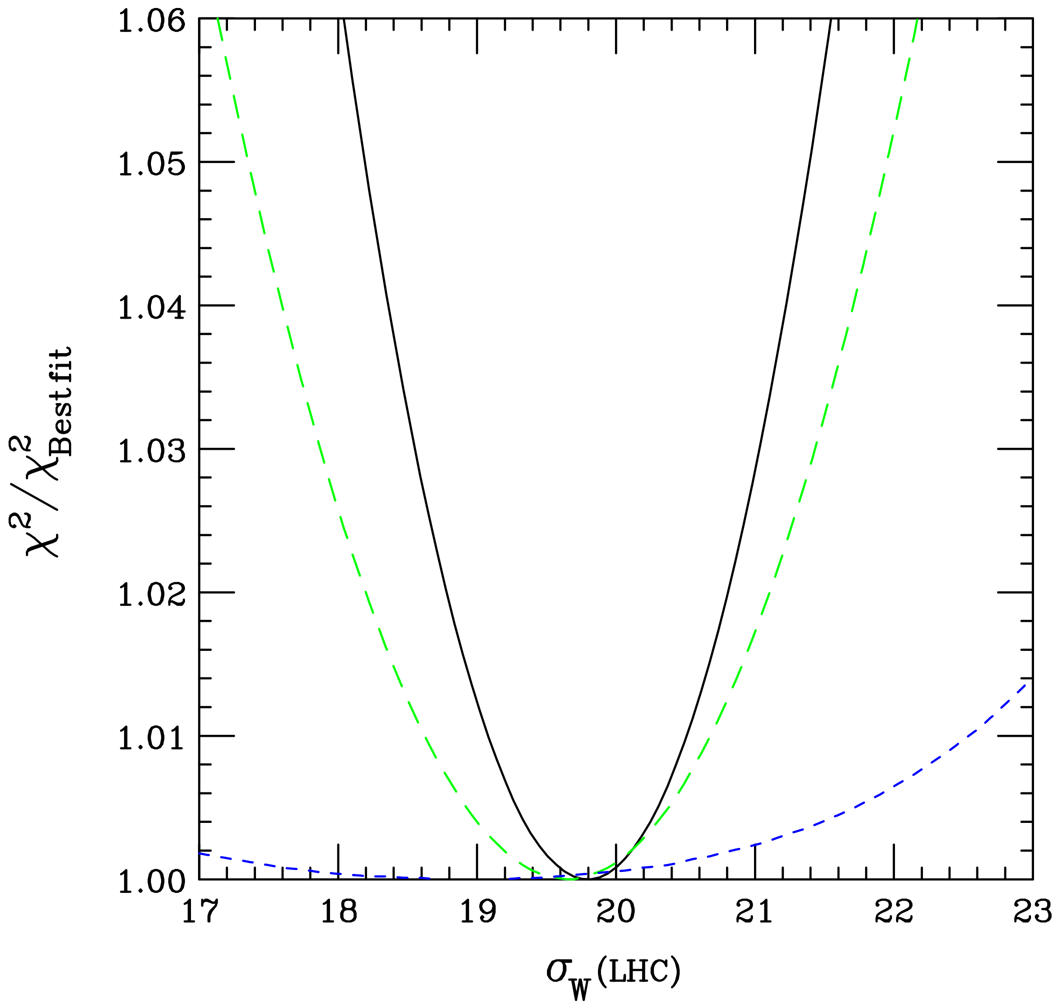
Not really any inconsistency with MRST.

If one is cautious about accuracy of theory at low x and Q^2, conclusion that uncertainty large on small x-sensitive quantities holds. CTEQ claim no reason to be cautious.

blue line - conservative cuts

green line - semi-conservative cuts

black line - normal cuts.



Large x and Low Q^2.

Perform fits with the known NNNLO large $\ln(1-x)$ terms included explicitly.

Also parameterize higher twist contributions by

$$F_i^{\mathrm{HT}}(x,Q^2) = F_i^{\mathrm{LT}}(x,Q^2)\left(1+\frac{D_i(x)}{Q^2}\right)$$

where i spans bins of x (and/or try saturation corrections at low x and Q^2).

In this type of expansion $\ln(1-x)$-corrections become indistinguishable from $1/W^2$ corrections at low W^2. Alternative $\ln(1-x)$ expansions possible.

No evidence for any higher twist except at low W^2.

x	LO	NLO	NNLO	NNNLO
0–0.0005	-0.07	-0.02	-0.02	-0.03
0.0005–0.005	-0.03	-0.01	0.03	0.03
0.005–0.01	-0.13	-0.09	-0.04	-0.03
0.01–0.06	-0.09	-0.08	-0.04	-0.03
0.06–0.1	-0.02	0.02	0.03	0.04
0.1–0.2	-0.07	-0.03	-0.00	0.01
0.2–0.3	-0.11	-0.09	-0.04	0.00
0.3–0.4	-0.06	-0.13	-0.06	-0.01
0.4–0.5	0.22	0.01	0.07	0.11
0.5–0.6	0.85	0.40	0.41	0.39
0.6–0.7	2.6	1.7	1.6	1.4
0.7–0.8	7.3	5.5	5.1	4.4
0.8–0.9	20.2	16.7	16.1	13.4

Table 3: The values of the higher-twist coefficients D_i, in the chosen bins of x, extracted from the LO, NLO, NNLO and NNNLO (NNLO with the approximate NNNLO non-singlet quark coefficient function) global fits.

Small x – gluon outside conservative range very negative, and $dF_2(x,Q^2)/d\ln Q^2$ incorrect, (NNLO much more stable than NLO). Theory corrections could cure this (quite plausible). Empirical resummation corrections improve global fit, e.g.

$$P_{gg} \to .... + \frac{1}{x}\left[A\bar{\alpha}_S^4\left(\frac{\ln^3(1/x)}{6} - \frac{\ln^2(1/x)}{2}\right) + B\bar{\alpha}_S^5\left(\frac{\ln^4(1/x)}{24} - \frac{\ln^3(1/x)}{6}\right)\right],$$

$$P_{qg} \to .... + \alpha_S\frac{N_f}{3\pi x}\left[C\bar{\alpha}_S^3\left(\frac{\ln^2(1/x)}{2} - \ln(1/x)\right) + D\bar{\alpha}_S^4\left(\frac{\ln^3(1/x)}{6} - \frac{\ln^2(1/x)}{2}\right)\right].$$

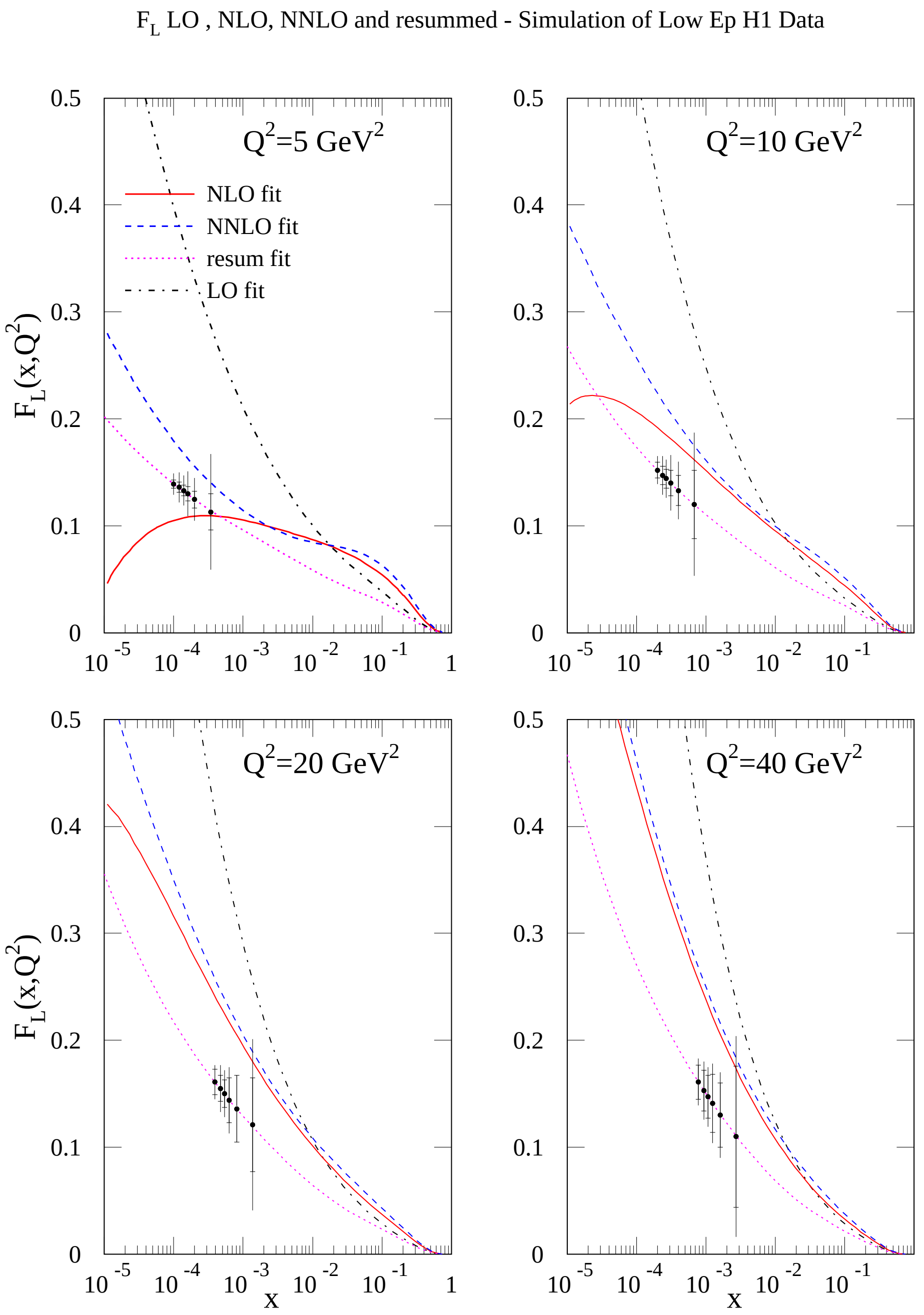
At NLO $A = -0.27, \quad B = 4.08, \quad C = 1.09, \quad D = 2.79$.

At NNLO $A = -0.35, \quad B = 5.49, \quad C = 2.81, \quad D = -2.00$.

Cuts suggestive of possible/probable theoretical errors for small x and/or small Q^2.

Much explicit work on $\ln(1/x)$-resummation in structure functions. Can suggest improvements to fit and changes in predictions.

Comparison of prediction for $F_L(x,Q^2)$ at LO, NLO and NNLO using MRST partons and also a $\ln(1/x)$-resummed prediction RT.



Accurate and direct measurements of $F_L(x,Q^2)$ and other quantities at low x and/or Q^2 (predicted range and accuracy of $F_L(x,Q^2)$ measurements possible at HERA II shown on picture) would be a great help in determining whether $NNLO$ is sufficient or whether resummed (or other) corrections are necessary, or helpful for maximum precision.

Conclusions

One can determine the parton distributions and predict cross-sections, by performing global fits over wide range of data. The fit quality using NLO or NNLO QCD is fairly good.

Various ways of looking at uncertainties due to errors on data. Uncertainties rather small – $\sim 1-5\%$ except in certain regions of parameter space.

QED and isospin violation corrections small, but important for NuTeV anomaly.

Uncertainty from input assumptions e.g. cuts on data, data used (particularly when it is sole constraint on a parton flavour), ..., comparable and potentially larger. Can shift central values of predictions significantly. Assumptions about input form can solve apparent high-E_T jet problem (even with weak corrections).

Errors from higher orders/resummation potentially large. Cutting out low x and/or Q^2 allows improved fit to remaining data, and altered partons. CTEQ see effects, but much smaller. NNLO more stable than NLO.

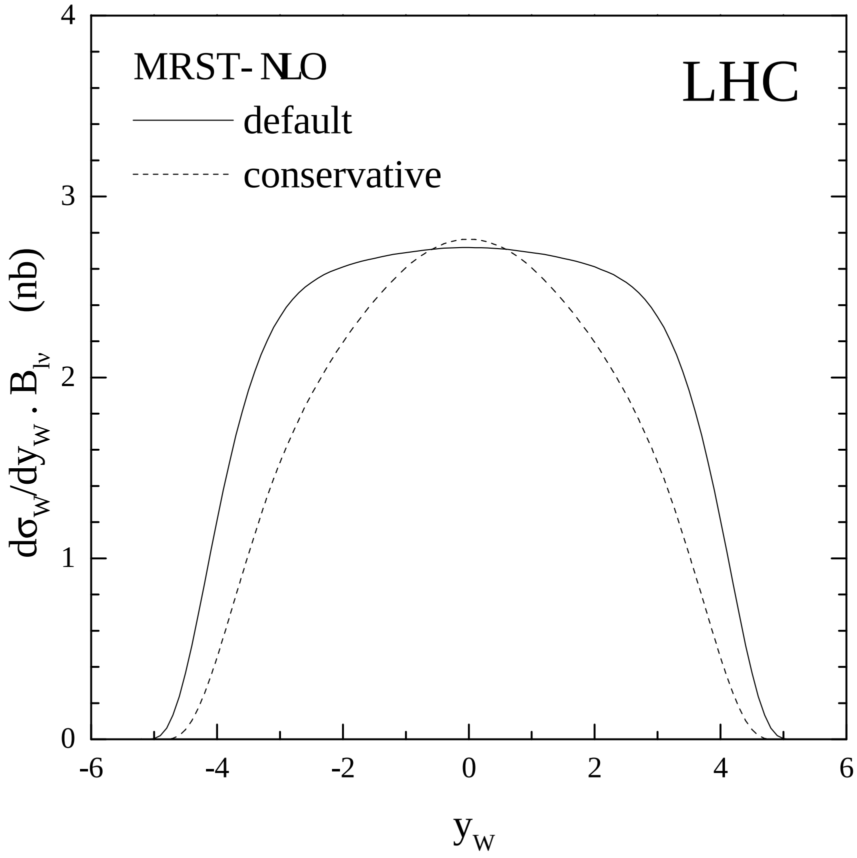
Theory (in general terms) often the dominant source of uncertainty. Much progress – NNLO, resummations ..., but much still to do. Both for theory and in obtaining useful new data.

Rapidity Distribution

Comparison of prediction for $(d\sigma_W/dy_W)$ for the standard MRST partons and the conservative set, where only central rapidity from direct fit to data.

The reduction in the total cross-section in the latter case is clearly due to the huge reduction at high y_W and represents the possible type of theoretical uncertainty in this region when working at NLO.

Note a slight increase in cross-section for $y_W = 0$ $(x = 0.006)$. Due to increased evolution of quarks here.



Saturation corrections do not help at NLO or NNLO.

MRST fit with slightly steep input gluon and fairly large shadowing corrections extrapolated to $Q^2 \leq 5\mathrm{GeV}^2$