Constrained Monte Carlo algorithm for the HERWIG evolution

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Evolution in Cracow, QCD

Monte Carlo modeling of $\bar{M}S$ DGLAP evolution:

- Markovian (forward) precision ($\sim 10^{-3}$) solutions of the full LL DGLAP equations (massless quarks). Acta.Phys.Pol. B35 (2004).
- Markovian precision solutions of the full NLL DGLAP equations (massless quarks). IFJPAN-V-04-08.
- Markovian study of the CCFM one-loop evolution IFJPAN-V-05-03.

Constrained Monte Carlo algorithms for DGLAP evolution:

- Constrained MC (non-Markovian) class II. Proc. Loops$\&$Legs 2004, Nucl. Phys. Proc. Suppl. 135 (2004) and IFJPAN-V-04-06.
- Constrained MC (non-Markovian) class I. October 2004 talk at HERA-LHC wshop and IFJPAN-V-04-07.

People involved:

- K.Golec-Biernat, S.Jadach, W.Płaczek, M.Skrzypek, Z.Wąs

Towards the parton shower (this talk):

- Constrained MC algorithm (class I) for HERWIG-style evolution.

Motivation and background

Known facts:

- Markovian MC implementing the QCD/QED evolution equations is the underlying ingredient in all parton shower type MCs
- Unconstrained forward Markovian MC, with evolution kernels from perturbative QCD/QED, inefficient for ISR.
- Backward evolution MC algorithm of Sjöstrand (Phys.Lett. 157B, 1985) is a widely adopted workaround.
- Backward Markovian MC does not solve the QCD evolution eqs. It merely exploits their solutions coming from the external non-MC methods

The old-standing problem:

- Is it possible to invent an efficient MC algorithm, solving internally the evolution eqs. by its own? No use of external PDFs.
- THE ANSWER IS YES! As shown in works listed on the previous page.

Motivation:

- Better modeling the ISR parton shower, possibly more friendly for inclusion of NLL and NNLL into parton shower MCs.
- Possibly easier MC modeling of the unintegrated parton distributions $D_k(p_T, x)$ and CCFM class of the QCD calculations/models.

Vocabulary

- **Markovian MC algorithm**
  The algorithm in which the number of emission (determining the dimension of the dimension of the integral, phase space), is generated as the last variable
- **non-Markovian MC algorithm**
  The algorithm in which the number of emission (the dimension of the integral), is generated as one of the first variables.
- **Constrained MC algorithm = CMC**
  The distributions are the same as in normal Markovian evolution, but the final energy $x = \prod z_i$ and the parton type $k = G, q_j, \bar{q}_j$ are predefined i.e. constrained.
- **HERWIG Evolution (terminology by P. Nason)**:
  Two ingredients:
  $\alpha_S(Q(1-z))$ (Amati+Basetto+Ciafaloni+Marchesini+Veneziano, NPB173, 1980)
  and $\varepsilon_{IR} = Q_0/Q$ where $Q_0 \sim 1$GeV (Webber+Marchesini, NPB310, 1988).
  For simplicity Q_0 coincides with the starting point of the QCD evolution.
- **MS-bar DGLAP evolution $\neq$ HERWIG evolution**
  At the LL they differ by large NLL and Q_0/Q terms.
  The difference going away at the NLL (Amati at.al.)

Discussion

- We have got efficient CMC algorithm (October talk) for the MS-bar DGLAP evolution.
- Is it much more difficult to extend it to HERWIG Evolution?
- In principle not. However, the CMC algorithm is quite complicated and we don’t really know, until it is actually done.
- The key points to check are MC efficiency and numerical stability.
- Pure bremsstrahlung is the critical part of the CMC algorithm.
- We are going to check its efficiency for the HERWIG Evolution. We shall show that it works well, for pure bremsstrahlung.
- The rest is modeling of (up to four) Quark<–>Gluon transitions.
- In the CMC class I Quark<–>Gluon transitions are modeled using general purpose MC tool FOAM, hence it should work almost automatically. Still to be checked.

Pure bremsstrahlung from the "emitter" $k = G, q, \bar{q}$ line

Iterative solution of the QCD evolution equations,
for evolution $t_0 \to t$, where $t = \ln Q$ is the evolution time:

$$x\mathcal{D}_{kk}(t, t_0; x) = e^{-\Phi_k(t,t_0)} \Big\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int_{t_0}^{t} dt_i \int_0^1 dz_i \; \mathcal{P}^{\Theta}_{kk}(t_i, z_i) \; \delta_{x=\prod_{i=1}^{n} z_i} \Big\},$$

Notation:

- $\theta_{x>0} = 1$ for $x > y$ and $= 0$ otherwise.
- $\delta_{x=y} \equiv \delta(x-y)$.
- $\mathcal{P}_{kk}(t, z) \equiv \frac{\alpha(t,z)}{\pi} z P_{kk}(t) = -\mathcal{P}^{\delta}_{kk}(t)\delta_{z=1} + \mathcal{P}^{\Theta}_{kk}(t, z)$.
- $\mathcal{P}^{\Theta}_{kk}(t, z) = \mathcal{P}_{kk}(t, z)\theta_{1-z>\varepsilon(t)}$, the same as in LL DGLAP.
- $\mathcal{P}^{\delta}_{kk}(t) = \int\limits_0^{1-\varepsilon(t)} dz \mathcal{P}^{\Theta}_{kk}(z, t)$, from energy sum rule, valid up to NLL.
- Sudakov formfactor: $\Phi_k(t, t_0) = \int\limits_{t_0}^{t} dt' \; \mathcal{P}^{\delta}_{kk}(t')$.
- IR cut $\varepsilon(t) = Q_0/Q$; it is not anymore $<< 1$, as in the standard DGLAP.

Variable mapping more complicated than for normal DGLAP

$$\int_x^{1-\varepsilon(t)} dz_i \int_{t_0}^{t} dt_i \; \mathcal{P}^{\Theta}_{kk}(t_i, z_i) = h_k \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \int_0^1 ds_i, i = 1, 2, ..., n,$$

$$z_i(y_i) = 1 - \exp(\rho^{-1}(y_i)),$$

$$\hat{t}_i(s_i) = \hat{t}_0 \left(\frac{\hat{t} + \ln(1 - z_i)}{\hat{t}_0} \right)^{s_i} - \ln(1 - z_i).$$

where

$$\rho(v) \equiv (\hat{t} + v) \ln(\hat{t} + v) - v - v \ln \hat{t}_0 - \hat{t} \ln \hat{t}, \quad \hat{t} \equiv t - t_\Lambda = \ln Q - \ln \Lambda_0.$$

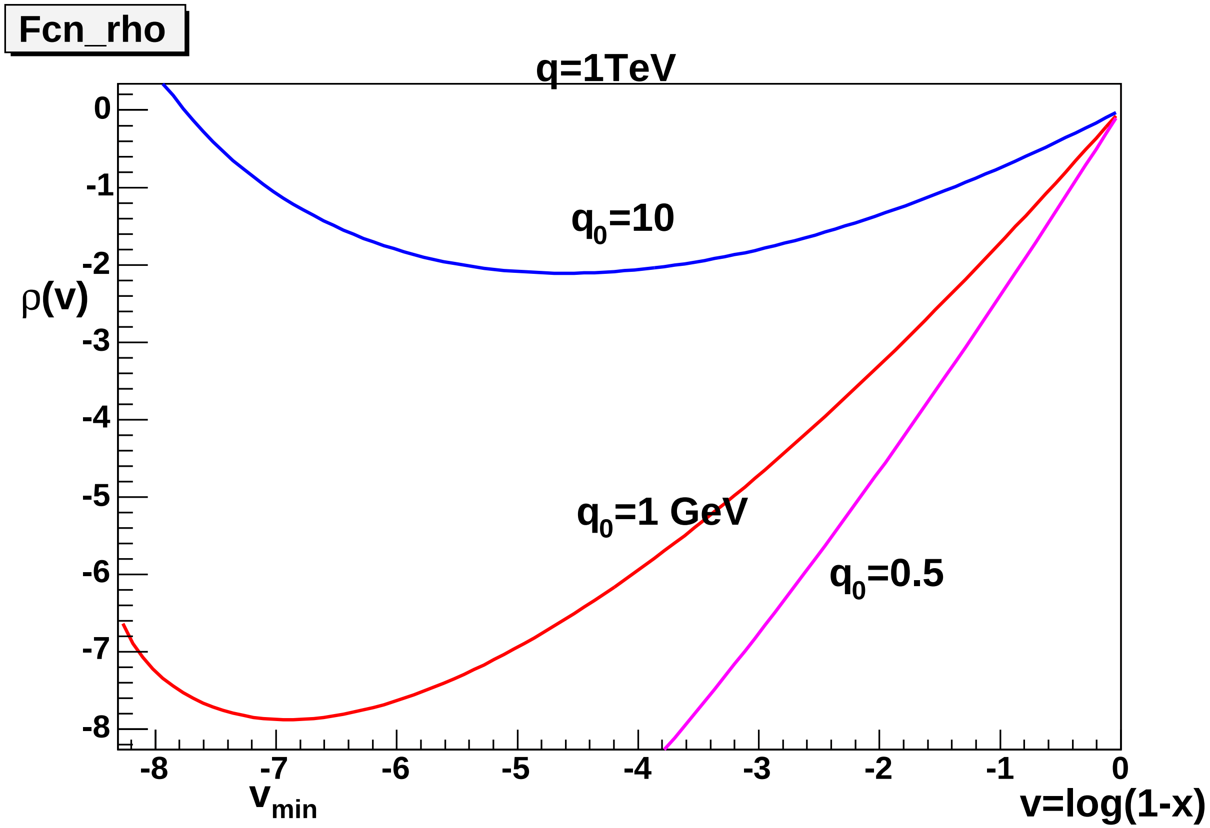
IMPORTANT: ρ^{-1} is not analytical!
Inversion has to be done numerically. ρ^{-1} will enter the constraint function $\prod z_i$!

The above mapping leads to:

$$x\mathcal{D}_{kk}(t, t_0, x) = e^{-\Phi_k(t,t_0)} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \frac{1}{n!} h_k^n \prod_{i=1}^{n} \int_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i \; \delta_{x=\prod_{i=1}^n z_i(y_i)} \int_0^1 ds_i \right\}.$$

Variable mapping

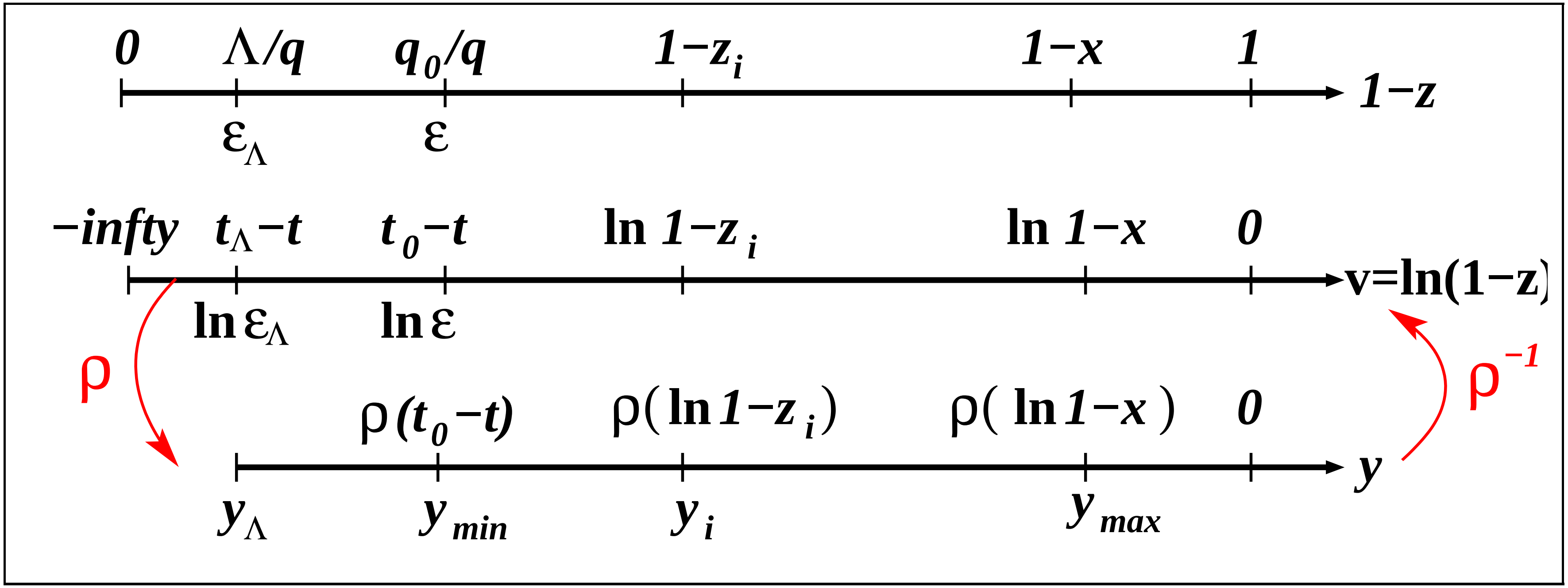
The middle (red) curve in following figure illustrates the shape of function $\rho(v)$ for $Q = 1$TeV and $Q_0 = 1$GeV:



The minimum position is at $v_{\min} = \ln(\varepsilon)$.
We only define the mapping $\rho(v)$ only for $v > v_{\min}$.

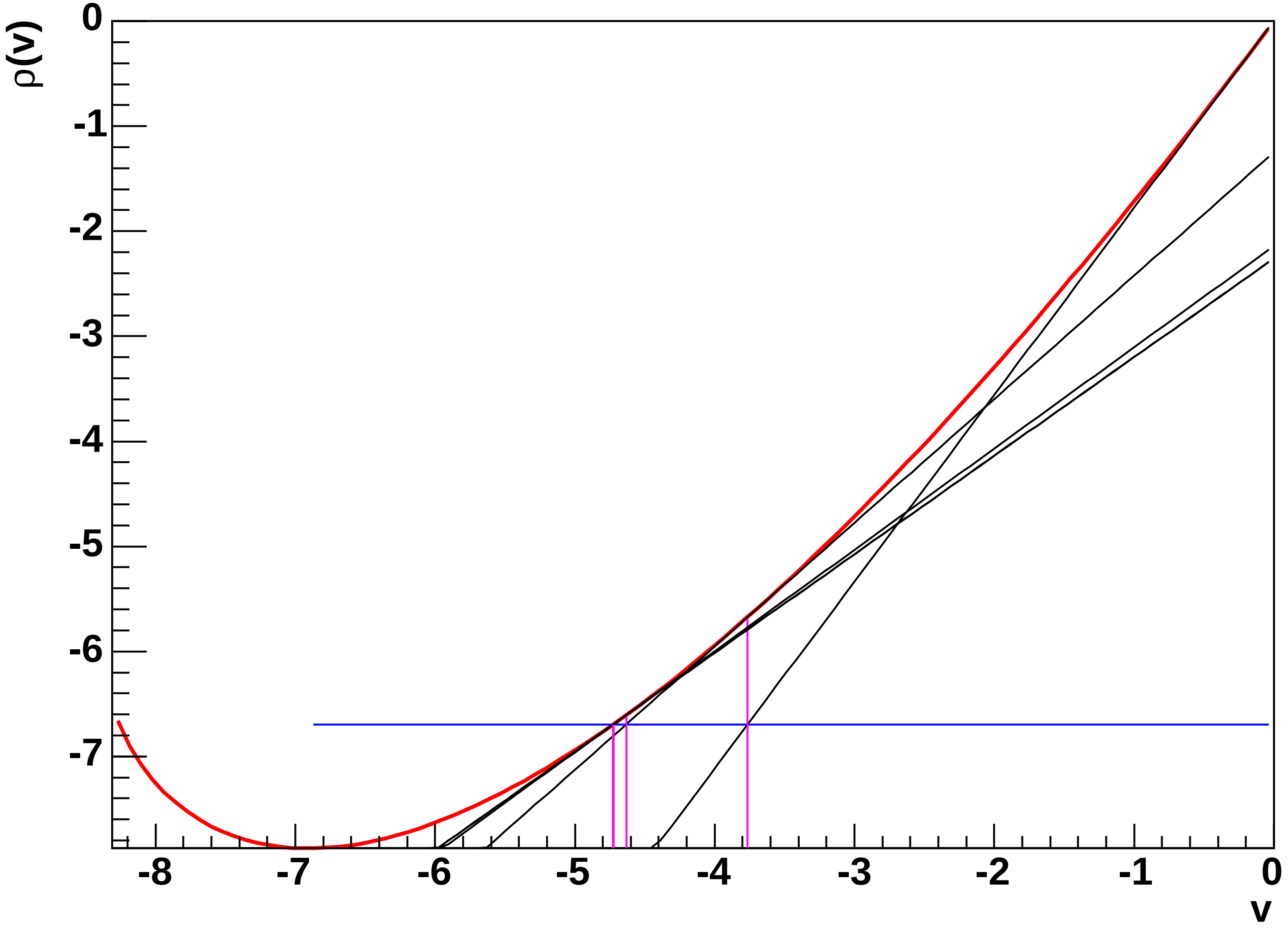
Variable mapping

Closer look into mapping of z into $v = v(z) = \ln(1-z)$ and further into another variable $\rho(v(z))$:



Solving ρ^{-1} numerically

Red curve is the function $\rho(v)$ for $Q_0 = 1$GeV, $Q = 100$GeV, together with the illustration of the iterative method of finding $v = \rho^{-1}(y)$ using method of tangential (starting at $v = 0$):



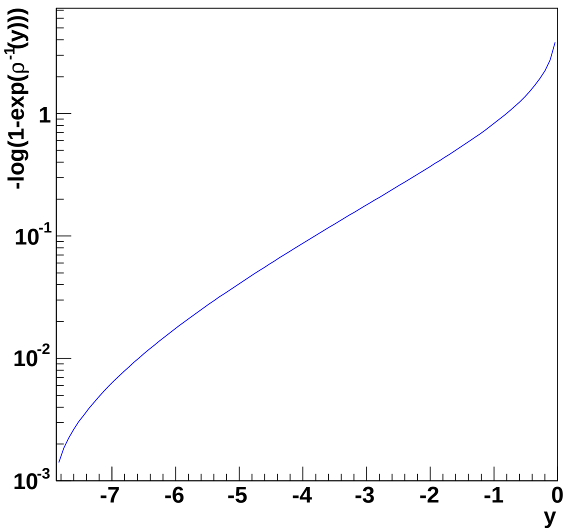
The energy constraint

Using symmetry of the integrand we finally trade the ordering in evolution time variables t_i into ordering in the energy variables y_i ($y_0 \equiv 0$):

$$x\mathcal{D}_{kk}(t,t_0,x) = e^{-\Phi_k(t,t_0)} \Big\{ \delta_{x=1} +$$

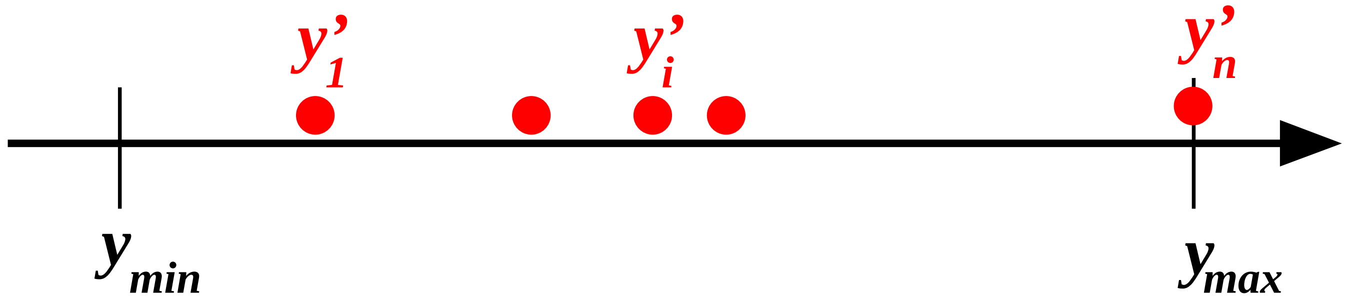
$$+\, x^{-1} \sum_{n=1}^{\infty} h_k^n \prod_{i=1}^{n} \int\limits_{\rho(t_0-t)}^{\rho(\ln(1-x))} dy_i\; \theta_{y_i>y_{i-1}}\, \delta\left(\ln\frac{1}{x} - \sum_j f(y_j) \right) \int\limits_0^1 ds_i \Big\}.$$

Here, $f(y_i)$ is very steeply (exponentially) rising,
see plot below for $q_0 = 1$GeV and $q = 1000$GeV



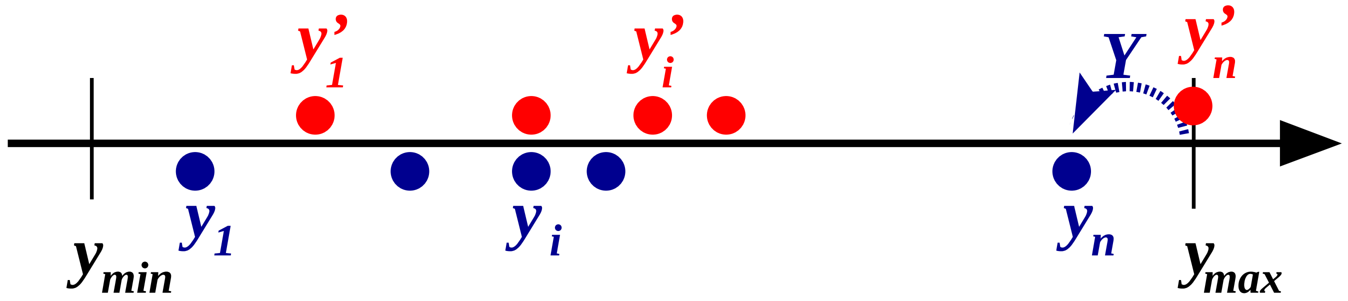
hence the constraint $x = \prod_{i=1}^{n} z_i(y_i)$ is “saturated” by a single z.

Linear shift: $y'_i \to y_i = y'_i - Y$



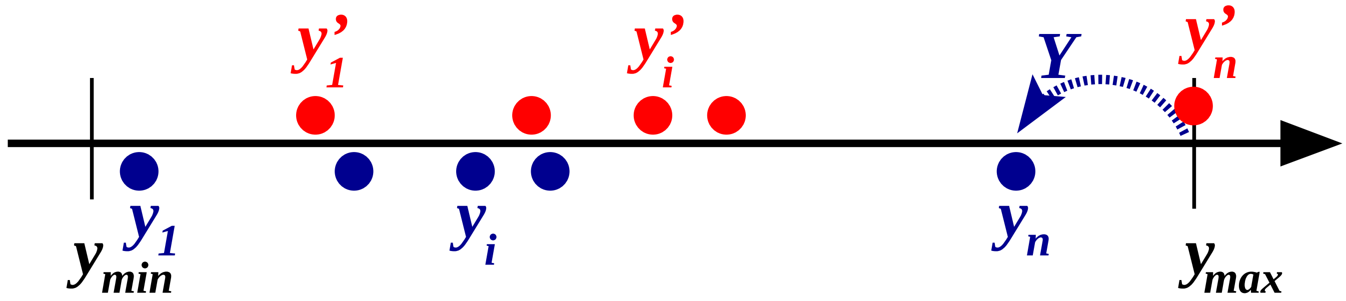
- Begin with y'_i such that one of them $y_n \equiv y_{\max}$

Linear shift: $y'_i \to y_i = y'_i - Y$



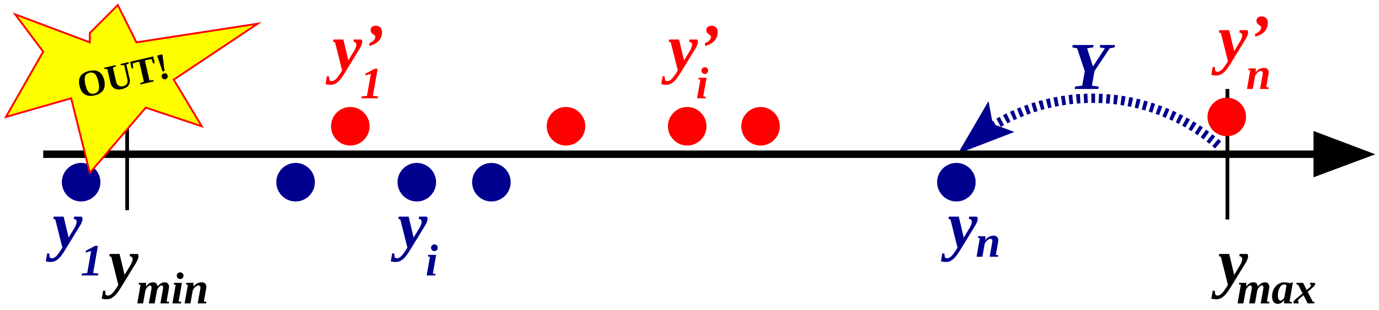
- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$

Linear shift: $y_i' \to y_i = y_i' - Y(y_1', y_2', ..., y_n')$



- Begin with y_i' such that one of them $y_n \equiv y_{\max}$
- Shift $y_i' \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y_i'

Linear shift: $y'_i \to y_i = y'_i - Y(y'_1, y'_2, ..., y'_n)$



- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i
- Sometimes the smallest y'_i is shifted OUT of the phase space, below IR the limit $y_{\min}$. Such an event gets MC weight $w = 0$

Master formula for the bremsstrahlung Monte Carlo

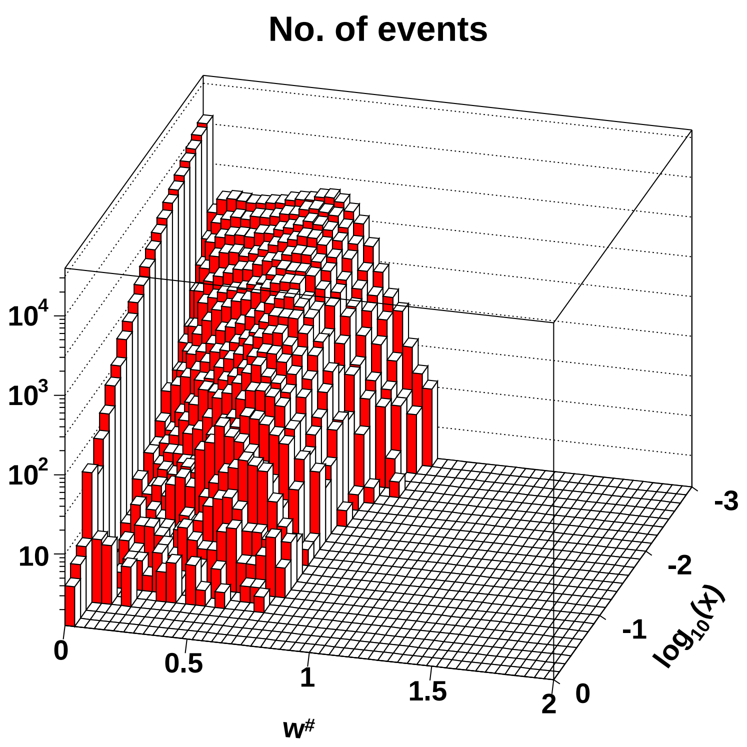
$$x\mathcal{D}_{kk}(\tau,\tau_0;x) = e^{(\tau-\tau_0)a_k}\sum_{n=0}^{\infty}\Big\{ e^{b_k\mathcal{R}(\varepsilon)}\delta_{n=0}\delta_{x=1} + \delta_{n>0}\theta_{1-x>\varepsilon}e^{b_k\mathcal{R}(1-x)}\frac{b_k x^{\omega_k-1}}{xg(x)}$$
$$\times P_n\big(b_k[\mathcal{R}(1-x)-\mathcal{R}(\varepsilon)]\big)\prod_{i=1}^{n}\int_0^1 dr_i\,\frac{\delta(1-\max r_j)}{n}\int_0^1 ds_i\; w^{\#}\Big\}$$

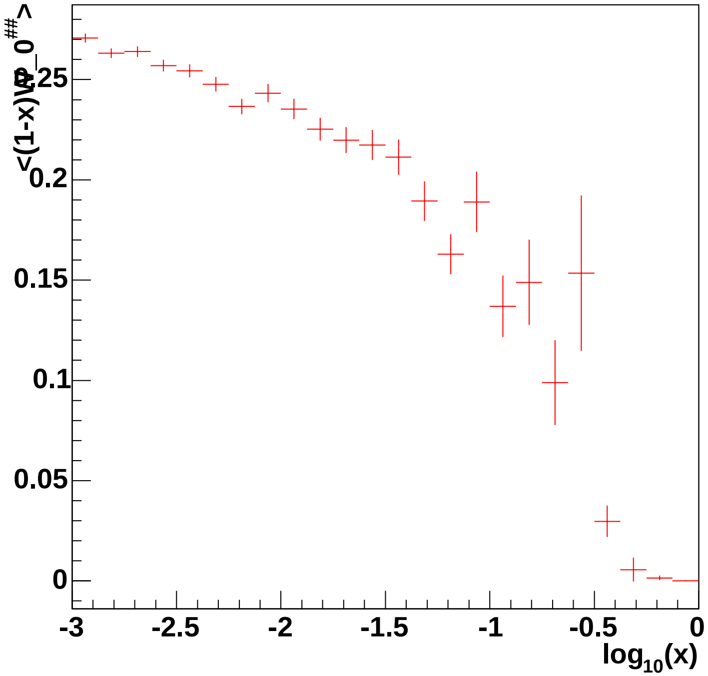
NOTATION:

- Mapping $z_i(y_i) = 1 - \exp(\rho^{-1}(y_i))$.
- Mapping $\hat{t}_i(s_i) = \hat{t}_0\left(\frac{\hat{t}+\ln(1-z_i)}{\hat{t}_0}\right)^{s_i} - \ln(1-z_i)$.
- Poisson distribution: $P_n(\lambda) = e^{-\lambda}\lambda^n/n!,\quad \lambda =< n >$.
- $\mathcal{R}(1-z) \equiv \rho(\ln(1-z))$, (implicitly depends on t and t_0).
- MC weight: $\boxed{w^{\#} = w_P\,\frac{xg(x)}{|\partial_Y \ln F(Y_0)|}\,\theta_{y_1'-Y_0>y_{\min}}}$,
- where $g(x) = |\partial_y \ln z(y)|_{z=x} = \frac{1-x}{x}$ is to stabilize the MC weight.
- Ordering of y_i' is here relaxed (to get explicit $1/(n-1)!$ of Poisson).

Prototype Monte Carlo I.d

The starting parton distribution is that of gluon in the proton.

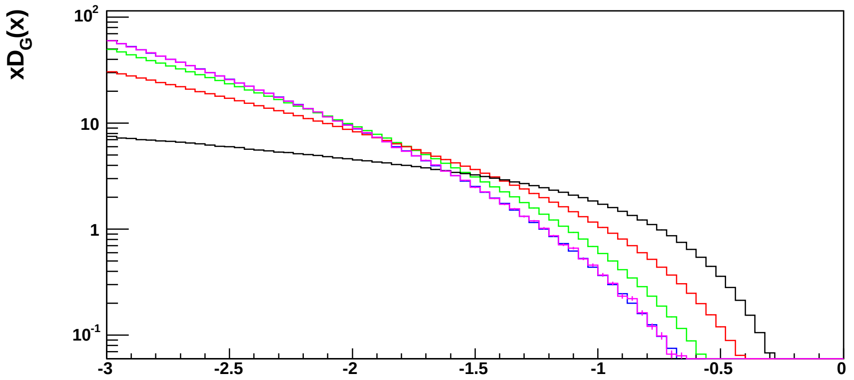


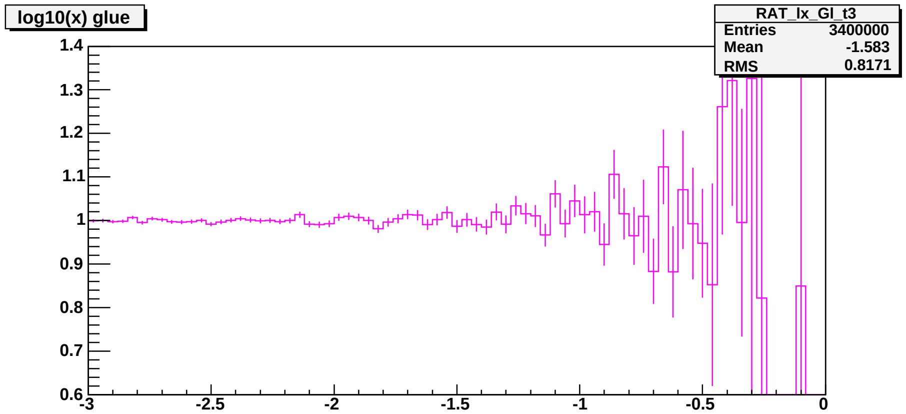


Plotted are weight distribution and the average weight as a function of x.
The maximum weight is below one and the acceptance rate 0.25 is surprisingly good!
About 1/3 of events has zero weight.

The comparison with the simpler Markovian MC

Below is the comparison of CMC I.d with the unconstrained Markovian MC for the HERWIG evolution, gluonstrahlung:





There is a reasonable agreement, within the statistical error, for small statistics, so far.

Summary

- It is demonstrated using prototype program (bremsstrahlung) that the Constrained MC works in practice for the HERWIG evolution.
- Still to be done: implementing Quark—Gluon transitions for the HERWIG evolution, as it was already done for the MS-bar DGLAP
- Including the rest of NLL corrections into CMC,
- and more...
- However, most difficult technical problems in constructing Constrained MCs for DGLAP-like evolutions are solved!