HERA-LHC workshop 21/3/2005

Effect of HQ fragmentation in hadro-/photo- production

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Heavy hadron p_T distributions

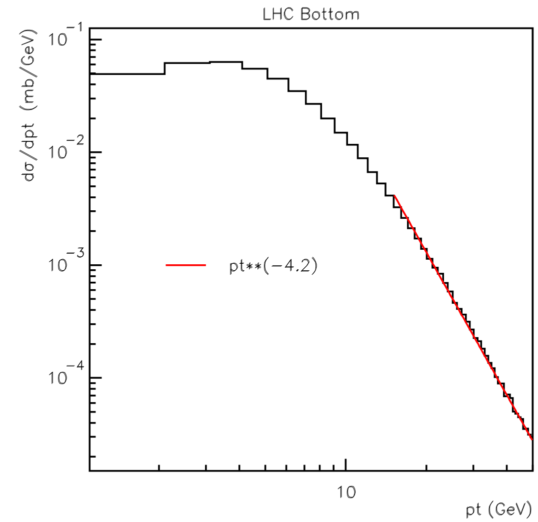
$$\frac{d\sigma}{dp_T^H}(p_T^H) = \int \frac{dx}{x}\, D^{\text{np}}(x)\, \frac{d\sigma^{\text{pert}}}{dp_T^Q}\left(\frac{p_T^H}{x}\right)$$

- $\frac{d\sigma^{\text{pert}}}{dp_T^Q}$ = perturbative quark diff. cross section

- $D^{\text{np}}(x)$ = non-perturbative Fragmentation Function (FF)

How well can we evaluate the effect of $D^{\text{np}}(x)$ on heavy-quark cross sections in ep, pp ?

Moments of $D^{\text{np}}(x)$



For bottom at LHC $N \le 4.2$

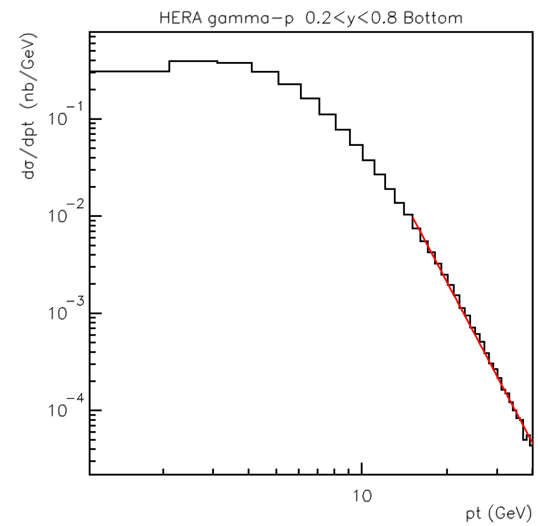
if $\quad \frac{d\sigma^{\text{pert}}}{dp_T^Q}(p_T) = Cp_T^{-N} \quad$ then

$$\frac{d\sigma}{dp_T^H}(p_T^H) = \int dx\, x^{(N-1)}\, Cp_T^{-N} = \frac{d\sigma^{\text{pert}}}{dp_T^Q}\widehat{D}_N^{\text{np}}$$

where $\quad \widehat{D}_N^{\text{np}} = \int dx\, x^{N-1}\, D^{\text{np}}(x)$

is the N^{th} Mellin moment of the non-pert. FF

the approximation of $\frac{d\sigma^{\text{pert}}}{dp_T^Q}(p_T)$ with an inverse power law is quite good at large p_T



For bottom at HERA $N \le 5.5$

Connection with central moments

We are used to describe distributions in terms of the mean value $\langle x \rangle$ and of the central moments:

central moments: $\mu_n = \int dx\, (x - \langle x \rangle)^n$, for $n \geq 2$

lowest μ_n have names:

$(\mu_2)^{1/2} = \sigma$, root mean square
$(\mu_3)^{1/3} = \mathcal{S}$, skewness
$(\mu_4)^{1/4} = \mathcal{K}$, kurtosis

Mellin moments can be written in terms of $\langle x \rangle$ and μ_n:

$$
\begin{aligned}
\widehat{D}_1 &= 1 \\
\widehat{D}_2 &= \langle x \rangle \\
\widehat{D}_3 &= \langle x \rangle^2 + \mu_2 \\
\widehat{D}_4 &= \langle x \rangle^3 + 3\mu_2 \langle x \rangle + \mu_3 \\
\widehat{D}_5 &= \langle x \rangle^4 + 6\mu_2 \langle x \rangle^2 + 4\mu_3 \langle x \rangle + \mu_4 \\
\widehat{D}_6 &= \langle x \rangle^5 + 10\mu_2 \langle x \rangle^3 + 10\mu_3 \langle x \rangle^2 + 5\mu_4 \langle x \rangle + \mu_5
\end{aligned}
$$

Case of Heavy Quark fragmentation

Even if not calculable, we know something about $D^{\text{np}}(x)$:

$$\langle x \rangle = 1 - O(\epsilon) \text{ where } \epsilon = \frac{\Lambda_{\text{QCD}}}{m_Q} \ll 1$$

then for any well-behaved distribution peaked around $\langle x \rangle$ and going to zero at 0, 1:

$$\mu_n = O(\epsilon^n) \qquad n \geq 2$$

At order ϵ any Mellin moment depends only on $\langle x \rangle$:

$$\widehat{D}_N = \langle x \rangle^{N-1} + O(\epsilon^2)$$

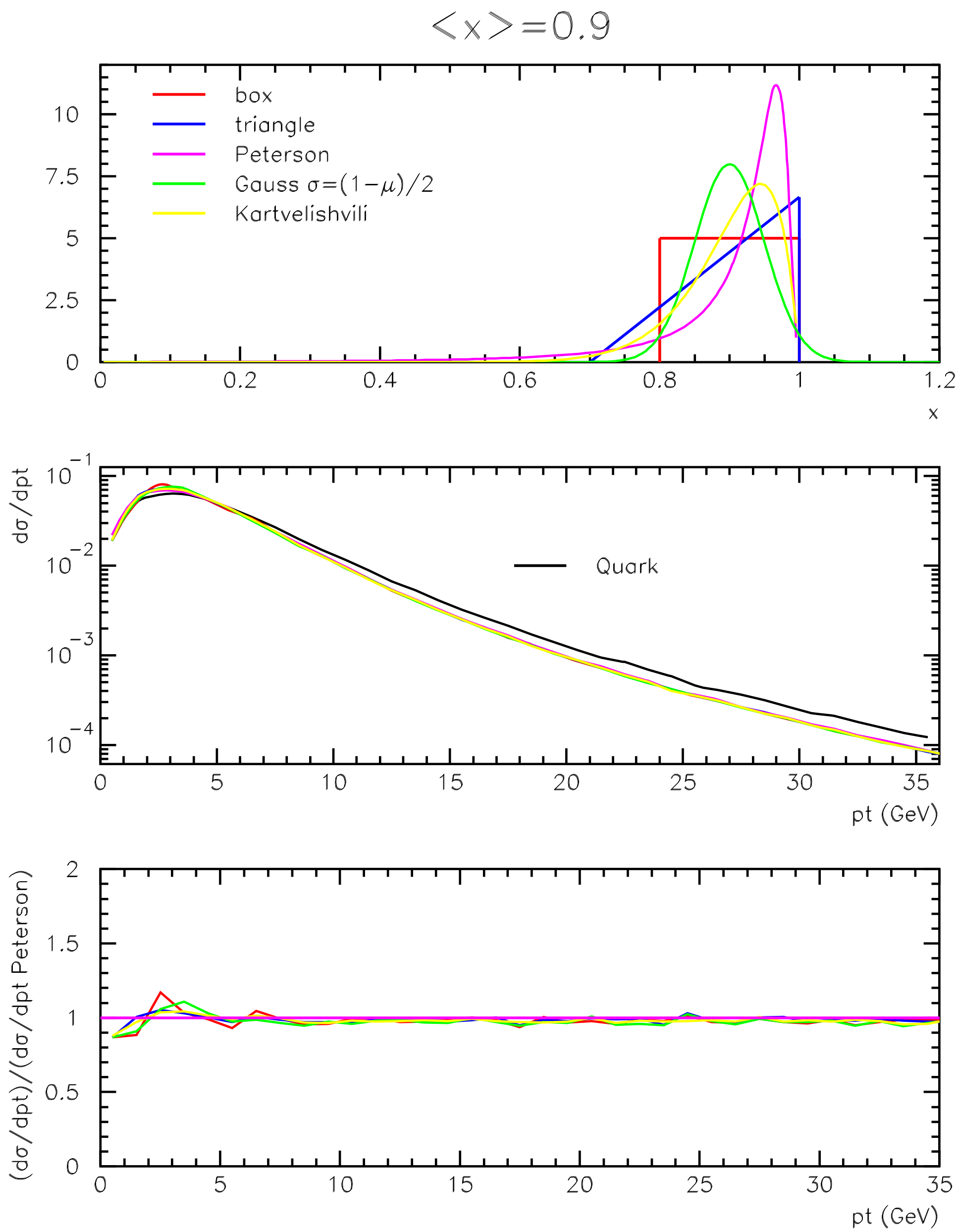
the expansion to ϵ^2 involves the RMS:

$$\widehat{D}_N = \langle x \rangle^{N-1} + \frac{(N-1)!}{2(N-3)!}\sigma^2 \langle x \rangle^{N-3} + O(\epsilon^3)$$

$$\frac{d\sigma}{dp_T^H}(p_T) = \frac{d\sigma^{\text{pert}}}{dp_T^Q}(p_T) \; (\langle x \rangle^{\text{np}})^{N-1} + O(\epsilon^2)$$

what is important is the mean of $D(x)$ not the shape !

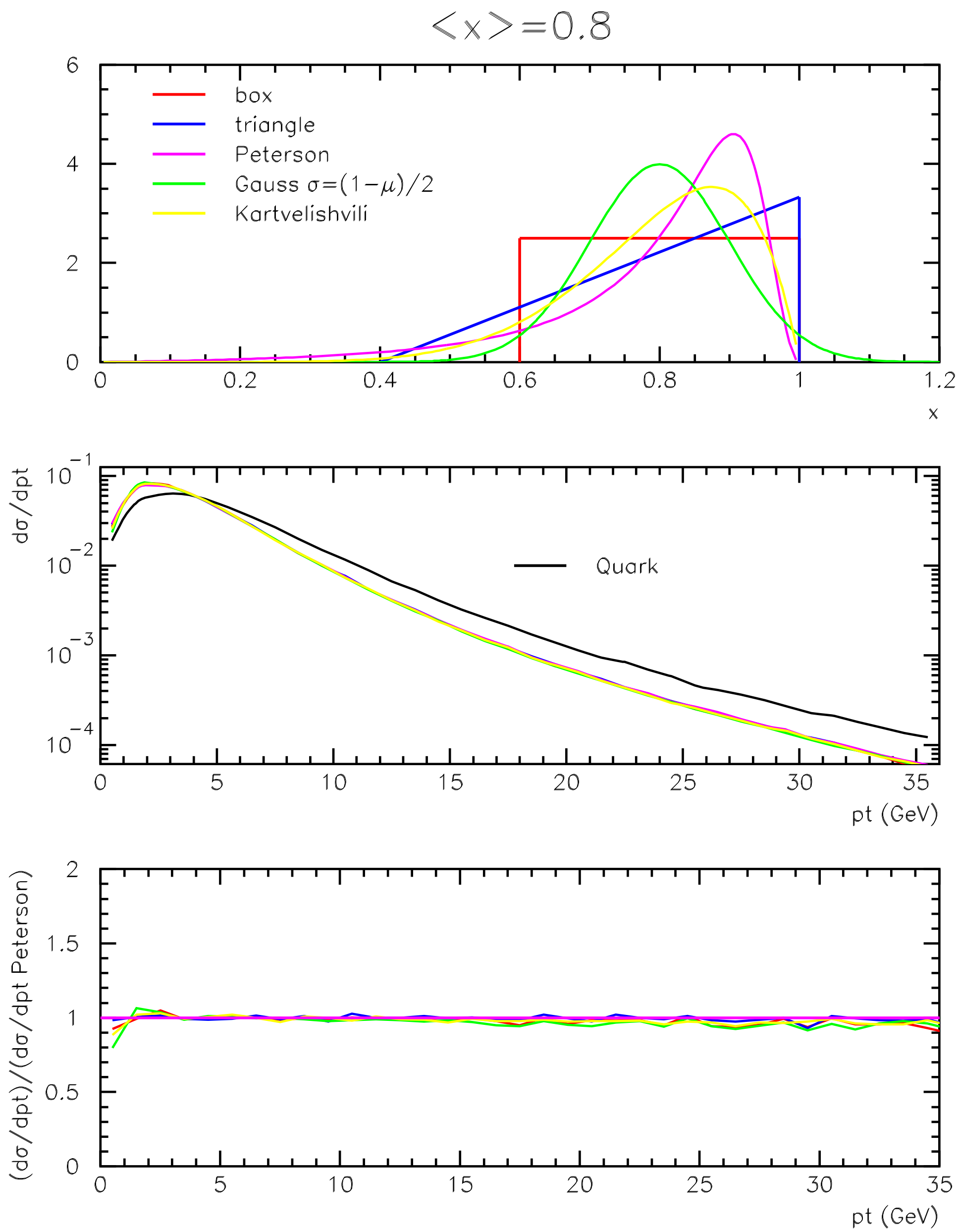
Does it work in practice ?



- fix $\langle x \rangle^{\rm np} = 0.9$, typical for b fragmentation
- try different shapes:
  - Kartvelishvili
  - Peterson
  - Gaussian with $\sigma = (1 - \langle x \rangle)/2$
  - flat probability between $1 - 2(1 - \langle x \rangle)$ and 1
  - triangular: slope between $1 - 3(1 - \langle x \rangle)$ and 1

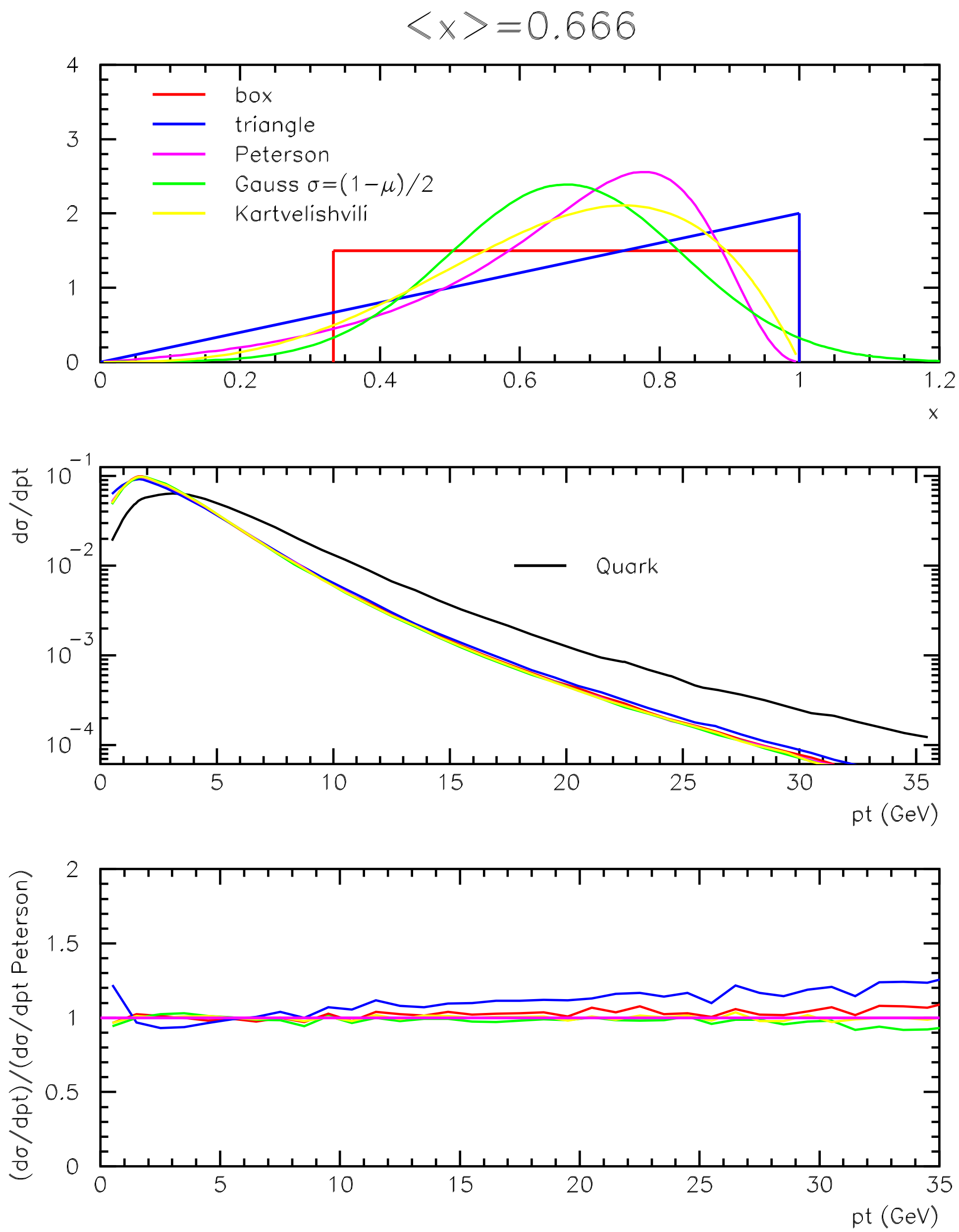
  .
- Smear p_T^Q distribution at LHC from NLO (thanks to A. Dainese)
- all the functions give the same result, within numerica accuracy!

Does it work in practice ?



- more difference expected for smaller $\langle x \rangle$ (larger ϵ)
- $\langle x \rangle^{\text{np}} = 0.8$
- differences below few %

Does it work in practice ?



- $\langle x \rangle^{\rm np} = 0.666$, lower than typical charm values
- large difference (20%) for unrealistic triangular function
- difference of less than 10% for Gauss and box
- no difference between Peterson and Kartvelishvili
- For reasonable shapes of FF, $\langle x \rangle^{\rm np}$ is the only relevant parameter for heavy hadron spectra in pp (ep)

$\langle x \rangle^{\mathrm{np}}$ from e^+e^-

Let's evaluate $\langle x \rangle^{\mathrm{np}}$ for beauty from e^+e^- beauty.

Obsevable at e^+e^-:
scaled energy distribution of the B hadron: $f(x_B)$, $x_B = \frac{2E_B}{Q}$

$$f(x_B) = \int \frac{dx}{x} D^{\mathrm{np}}(x)\, f^{\mathrm{pert}}(\frac{x_B}{x})$$

therefore

$$\langle x_B \rangle = \langle x \rangle^{\mathrm{np}} \langle x \rangle^{\mathrm{pert}}$$

Two ingredients are needed:
$\langle x_B \rangle$ from direct measurements
$\langle x \rangle^{\mathrm{pert}}$ from perturbative theory

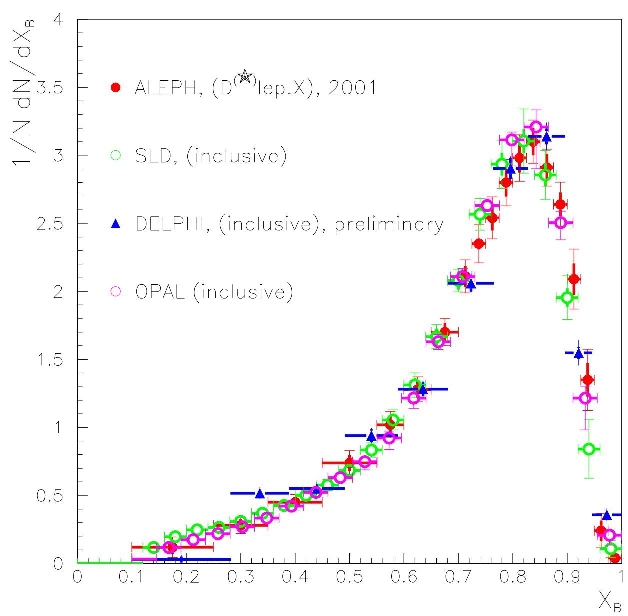
$$\langle x \rangle^{\mathrm{np}} = \frac{\langle x_B \rangle \Longleftarrow \text{experiment}}{\langle x \rangle^{\mathrm{pert}} \Longleftarrow \text{theory}}$$

$\langle x_B \rangle$: data

$\langle x_B \rangle$ measured at the Z^0 peak by single experiments to better than 1%

Use the results for the weakly-decaying B hadron: $x_B^w = 2E_{B^w}/Q$

Experiment		$\langle x_B^w \rangle$	
SLD	0.709	$\pm$0.003(stat.)	$\pm$0.003(syst.) $\pm$ 0.002(model)
ALEPH	0.716	$\pm$0.006(stat.)	$\pm$0.006(syst.)
OPAL	0.7193	$\pm$0.0016(stat.)	$^{+0.0038}_{-0.0033}$(syst.)
DELPHI (prel.)	0.7153	$\pm$0.0007(stat.)	$^{+0.0049}_{-0.0052}$(syst.)
Crude average	0.715	$\pm$0.03	

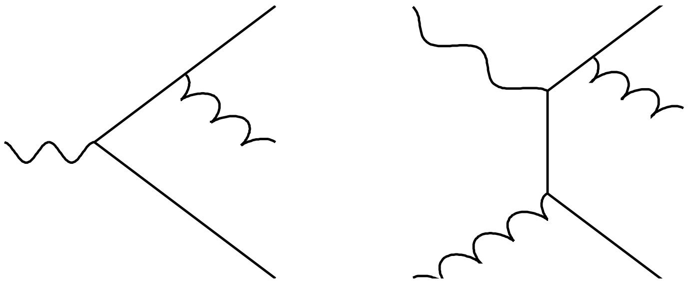


$\langle x_b \rangle^{\text{pert}}$: Theory

Pert. theory for e^+e^- should correspond to that used for ep or pp.

Fixed order NLO theory at ep, pp:
quarks radiate ≤ 1 gluon

Equivalent to $O(\alpha_S)$ in e^+e^-

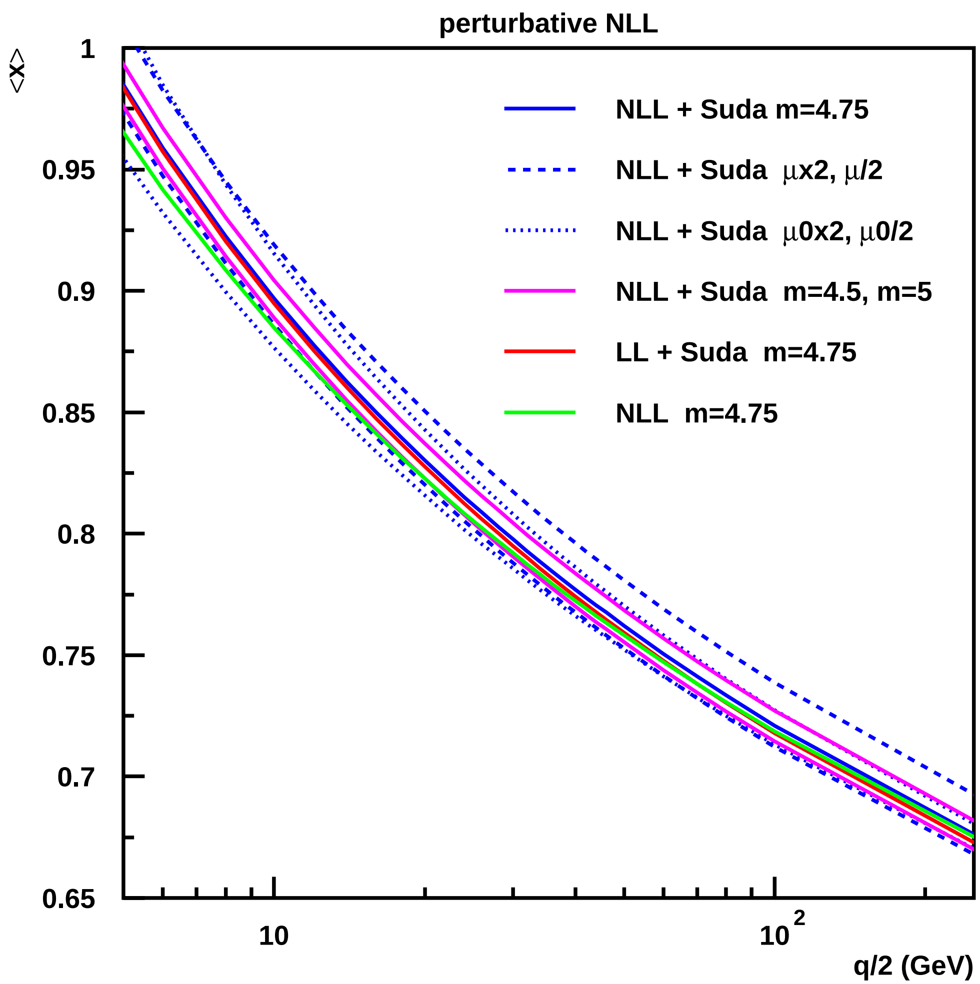


- for FONLL at ep, pp use NLL theory for e^+e^-: HVQF from Matteo
  $\hat{f}(x_b) = \hat{C}(\mu)\, \hat{E}(\mu, \mu^0)\, \hat{D}^{\text{pert}}(\mu^0)$ where $\mu = Q, \mu^0 = m_b$

- Fixed order NLO: HVQF with $\mu = \mu^0 = Q$ (no evolution)

- Pythia 6.2: same program used for ep, pp, ee

Theoretical Uncertainty on NLL Theory

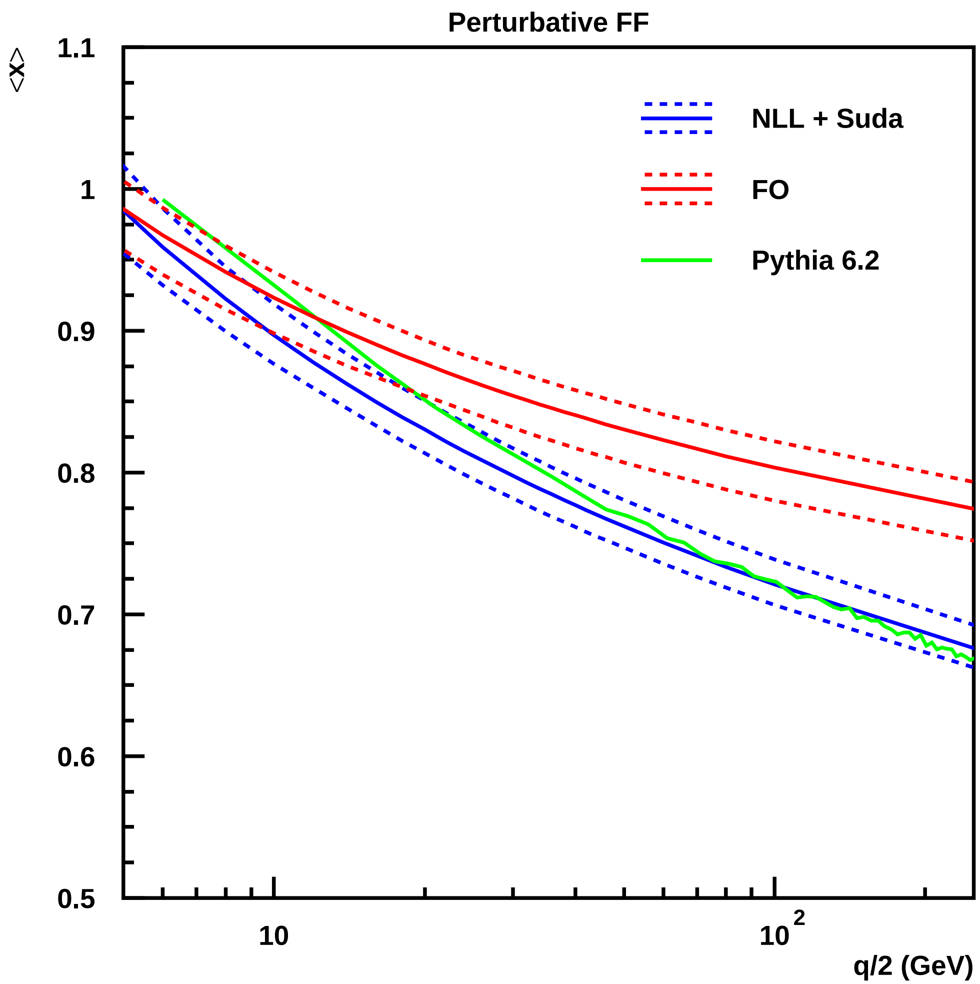


- HVQF nominal:
  Large-x Sudakov resummation ON,
  $\mu_F = \mu_R = Q$, $\mu_F^0 = \mu_R^0 = m_b$,
  $m_b = 4.75\text{GeV}$, $\Lambda^5 = 0.226\text{GeV}$
- vary scales by factors 2
- vary m_b 4.5-5.0 GeV
- result at $Q = 92 GeV$ from envelope of scale variations:

$$\langle x \rangle^{\text{pert,NLL}}(M_Z) = 0.768^{+0.019}_{-0.015}$$

  uncertainty $\sim 2\%$, larger than experim.
- Sudakov res. OFF: small effect
- LL evolution only: tiny effect

Perturbative results



- FO:
  from envelope of scale variations:

  $$\langle x\rangle^{\text{pert,FO}}(M_Z) = 0.834^{+0.018}_{-0.023}$$

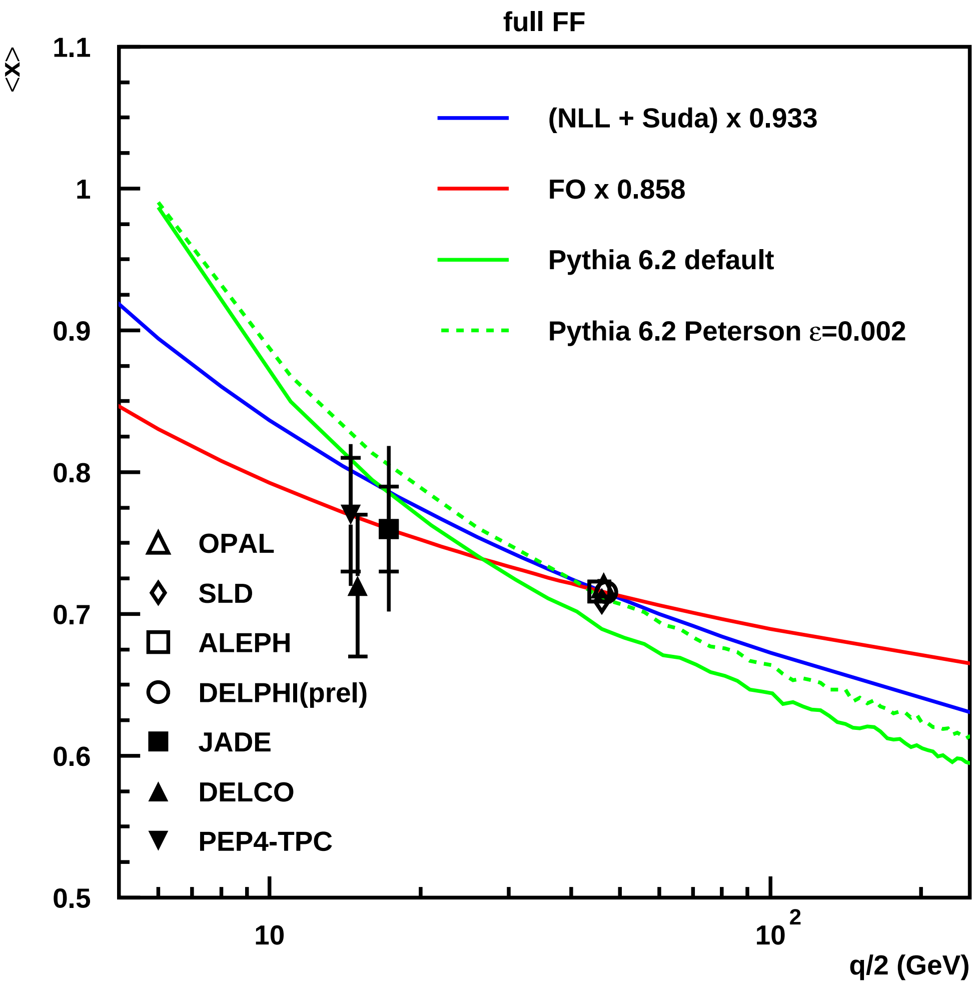
  but not compatible with NLL !

  Difference with NLL increases with Q, $\sim$ 10% difference at M_Z

- Pythia 6.2 (b quark after PS) compatible with NLL, a bit steeper

  $$\langle x\rangle^{\text{pert,Pythia}}(M_Z) = 0.774$$

Compare with data and extract $\langle x \rangle^{\text{np}}$



- NLL: $\langle x \rangle^{\text{np,NLL}} = 0.93 \pm 0.02$
  uncertainty dominated by theory

- FO: $\langle x \rangle^{\text{np,FO}} = 0.86 \pm 0.02$
  but uncertainty must be underestimated !

  Considering difference with NLL:
  $\langle x \rangle^{\text{np,FO}} \sim 0.93$ at low p_T
  $\langle x \rangle^{\text{np,FO}} \sim 0.86$ a $p_T \sim M_Z/2$

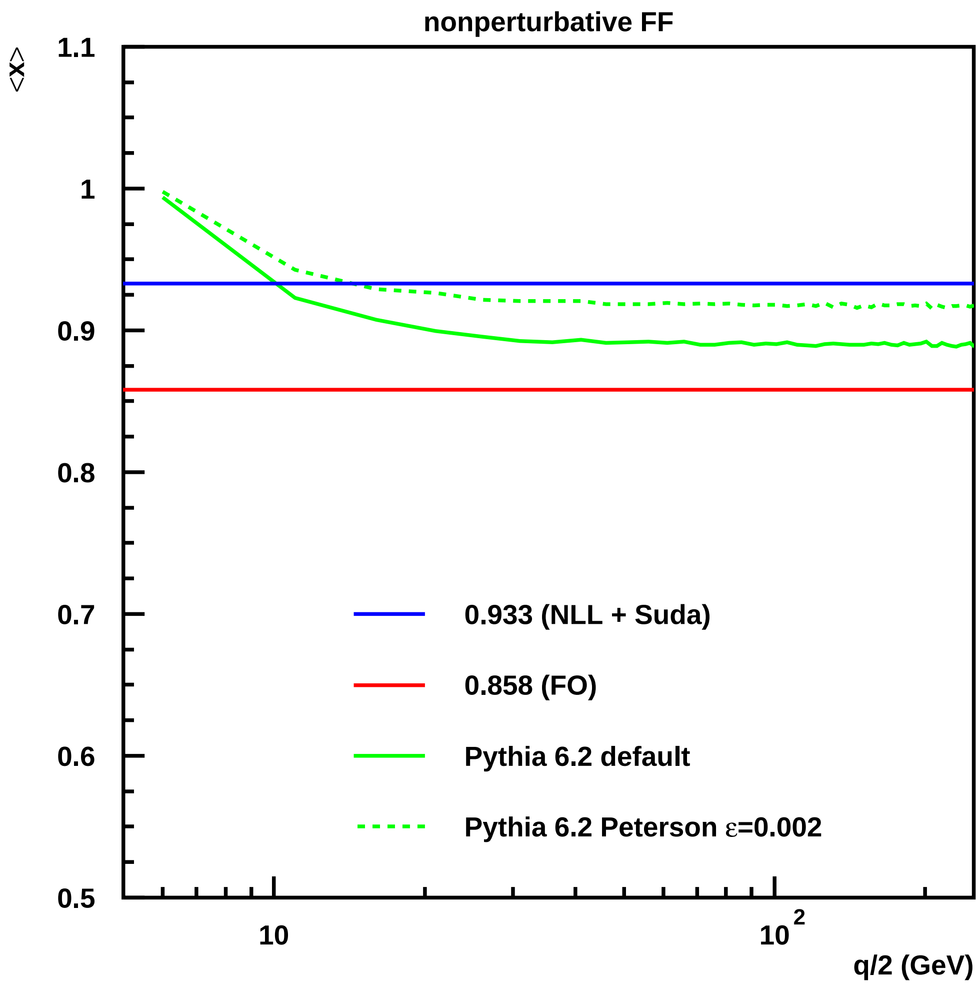
  my suggestion:
  $\langle x \rangle^{\text{np,FO}} = 0.90 \pm 0.05$
  use FO only for $p_T < M_Z/2$

- Pythia 6.2:

  Default (Lund-Bowler) too soft

  Reasonable agreement with data with Peterson with $\epsilon = 0.002$

is $\langle x \rangle^{\rm np}$ indepedent from Q ?



- factorization breaking terms $O(m_b/Q)$
- NLL, FO: factorization ansatz, $D_N^{\rm np}(Q)$ =constant
- Pythia 6.2: $\langle x \rangle^{\rm np} = \langle x_B \rangle / \langle x_b \rangle$
  asyntotic value
  $\langle x \rangle^{\rm np,pythia(Pet.)}(Q \to \infty) = 0.918$

  factorization breaks at low Q:
  $\langle x \rangle^{\rm np,pythia(Pet.)}(Q \to 2m_b) = 1$

  $\Delta \langle x \rangle^{np} / \langle x \rangle^{np} = 1\%$ at $Q/2 = 20$ GeV
  $\Delta \langle x \rangle^{np} / \langle x \rangle^{np} = 5\%$ at $Q/2 = 10$ GeV

  goes empirically like:
  $(\langle x \rangle^{np}(Q) - \langle x \rangle^{np}(\infty)) \sim 0.5 (m_b/Q)^2$

Translating into usual parameters

- Parameters for FO:

$\langle x\rangle^{\mathrm{np}}$	ϵ Poisson	α Kartvelishvili
0.90	0.0011	17.0
0.95	0.0002	37.0
0.85	0.0039	10.3

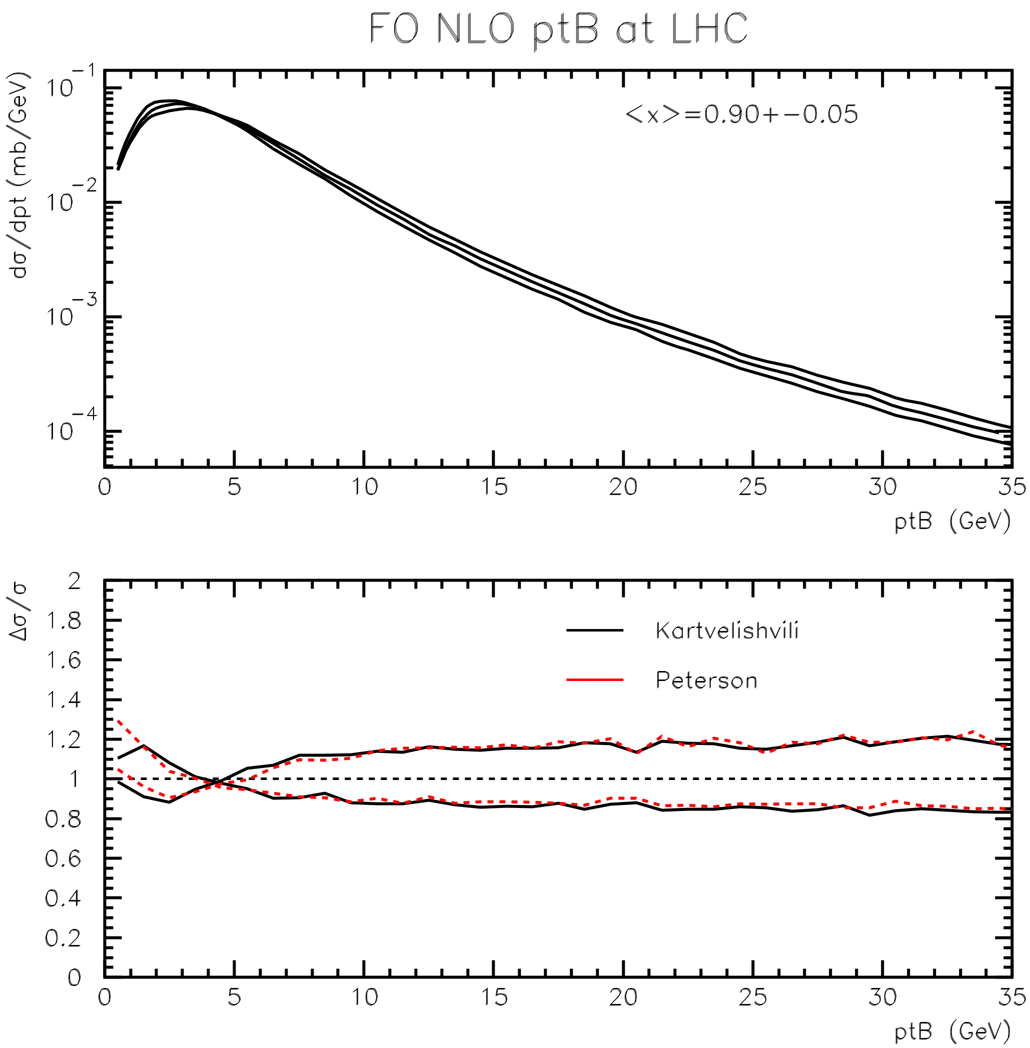
- Parameters for FONLL:

$\langle x\rangle^{\mathrm{np}}$	ϵ Poisson	α Kartvelishvili
0.93	0.0004	25.6
0.95	0.0002	37.0
0.91	0.0008	19.2

- Central values larger than usual results from fits...

- Uncertainty larger (mostly theoretical)

Effect on p_T^B spectrum at LHC



- Apply smearing to FO b spectrum for LHC $\langle x\rangle^{\mathrm{np}} = 0.90 \pm 0.05$
- 5.5% uncertainty on $\langle x\rangle^{\mathrm{np}}$ $\rightarrow\sim$ 20% uncertainty on $d\sigma/dp_T$
- as expected from $\frac{\Delta(\sigma)}{\sigma} = (N-1)\frac{\Delta\langle x\rangle^{\mathrm{np}}}{\langle x\rangle^{\mathrm{np}}}$
- for NLL, $\frac{\Delta\langle x\rangle^{\mathrm{np}}}{\langle x\rangle^{\mathrm{np}}} = 2\% \Longrightarrow \frac{\Delta(\sigma)}{\sigma} = 7\%$
- No difference between Peterson or Kartvelishvili

To understand/ to do

Things to investigate:

- why gap between FO and NLL not covered by scale variations ?
- why FF found harder than fits in literature

Things doable for the Writeup:

- extend uncertainty on p_T^B spectrum to HERA and to FONLL theory
- study factorization breaking with Pythia at HERA (and LHC?)
- extend to charm ?

Conclusions

- Effect of FF on B-hadron p_T spectra at (HERA)/LHC studied

- Details of $D^{\mathrm{np(x)}}$ not relevant, only $\langle x\rangle^{\mathrm{np}}$ matters

- $\langle x\rangle^{\mathrm{np}}$ extracted from e^+e^- data in different theoretical frameworks: FO NLO, NLL, Pythia6.2

- uncertainty of fragmentation on p_T^B spectrum at LHC evaluated for FO NLO (NLL) to be 20% (7%)

- few things to be studied in more detail...