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Hadroproduction of Heavy Mesons in a massive VFNS

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Outline

- Overview and Motivation
- Theoretical approaches to 1-particle inclusive heavy quark production
  - Fixed Order
  - Conventional (massless) Parton Model
  - Massive Parton Model
- Hard scattering coefficients with heavy quark masses
  - Massless limit of Fixed Order calculation
  - Mass factorization (with massive regularization)
- Numerical Results
- Conclusions and Outlook

Motivation

Heavy Quarks: $h = c, b, t$

$$m_h \gg \Lambda_{\rm QCD} \Rightarrow \alpha_s(m_h^2) \propto \ln^{-1}(\frac{m_h^2}{\Lambda_{\rm QCD}^2}) \ll 1$$

$$\alpha_s(m_h^2) \ll 1$$

$\Rightarrow$ Perturbation Theory (pQCD) applicable!

$\Rightarrow$ Test pQCD! Heavy Quark Production/Decay

Application of pQCD

Two Aspects:

(i) Factorization: $\sigma = \text{PDF} \otimes \hat{\sigma} \otimes \text{F}F$ (irresp. of heavy quarks)

$\longrightarrow$ $\hat{\sigma}$ computable within pQCD

(ii) m_h long distance cut-off (heavy quarks)

$\longrightarrow$ No collinear subtraction necessary; No non-pert. part associated with heavy line

$\longrightarrow$ No heavy quark PDF or perturbative boundary condition

Overview

Subject of this talk:

- 1-particle inclusive hadroproduction of D mesons: $p\bar{p} \to (D^0, D^{*+}, D^+, D_s^+)X$
- Massive Variable Flavour Number Scheme (Massive VFNS): [1]
  - Collinear logarithms of the heavy quark mass $\ln \mu/m_h$ are subtracted and resummed
  - finite non-logarithmic m_h/Q terms are kept in the hard part/taken into account
  - Scheme based on the factorization theorem of Collins with heavy quarks [2]

Further applications:

- 1-particle inclusive hadroproduction of B mesons: $p\bar{p} \to BX$
- Completes earlier work on D meson production in $\gamma\gamma$ and γp collisions:
  - $\gamma\gamma \to D^\star X$: direct process [3]
  - $\gamma\gamma \to D^\star X$: single-resolved process [4]
  - $\gamma p \to D^\star X$: direct process [5]

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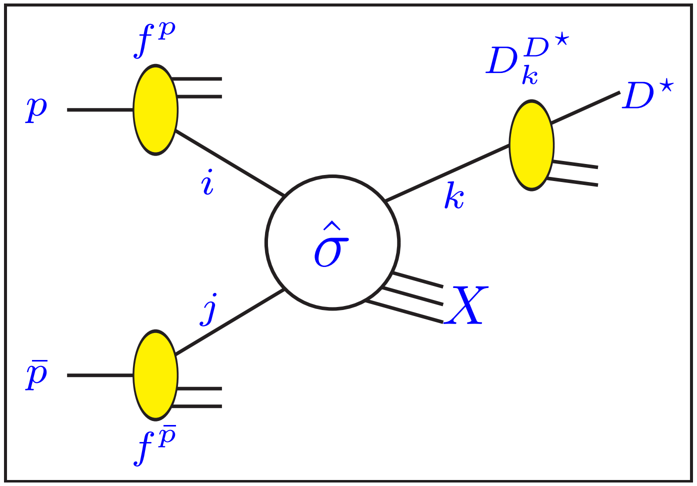
[1] B.A. Kniehl, G. Kramer, I.S.,H. Spiesberger, PRD71(2005)014018
[2] J. Collins, PRD58(1998)094002
[3] G. Kramer, H. Spiesberger, EPJC22(2001)289; [4] EPJC28(2003)495; [5] EPJC38(2004)309

Factorization for $p\bar{p} \longrightarrow D^\star X$

$$d\sigma = \sum_{i,j,k} f_i^p(x_1) \otimes f_j^{\bar{p}}(x_2) \otimes d\hat{\sigma}(ij \longrightarrow kX) \otimes D_k^{D^\star}(z)$$

Sum over all possible subprocesses $ij \longrightarrow kX$



- $d\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), [\frac{m_h}{p_T}])$: hard scattering cross sections, perturbatively computable, free of long-distance physics $\longrightarrow$ m_h can be kept
- Parton densities of the proton $f_i^p(x_1, \mu_F)$: non-perturbative input
  $i = g, q, \ldots \quad [q = u, d, s]$
- Parton densities of the anti-proton $f_j^{\bar{p}}(x_2, \mu_F)$: non-perturbative input
  $j = g, q, \ldots$
- Fragmentation functions $D_k^{D^\star}(z, [\mu'_F])$: non-perturbative input
  $k = c, \ldots$

Details (which subprocesses, PDFs, FFs; mass terms) depend on the Heavy Flavour Scheme

Heavy Flavour Schemes

Two basic approaches:

- Fixed Order Perturbation Theory (FFNS)
- Parton Model (ZM-VFNS)

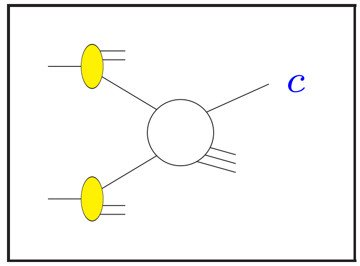
Interpolating schemes combining the good features:

- Parton Model with quark masses (GM-VFNS, ACOT)
- FONLL

Glossary:

- ZM: Zero Mass
- GM: General Mass
- VFNS: Variable Flavour Number Scheme
- FFNS: Fixed Flavour Number Scheme

Fixed Order (FFNS)



- $m_c \neq 0, \quad n_f = 3$ fixed
- Partons: g, u, d, s
  (NO charm parton: $f_c = 0$)
  Charm (only) in final state
- collinear logarithms $\ln \frac{s}{m_c^2}$ finite
  $\longrightarrow$ No factorization; no conceptual necessity for FFs
  $\longrightarrow$ fixed order perturbation theory; no resummation
- Usually c treated in on-shell scheme ($\overline{\mathrm{MS}}_m$)

Pro and Contra:

+ $(\frac{m_c}{p_T})^n$ terms included; correct threshold suppression
  $\Rightarrow$ valid for $0 \leq p_T^2 \lesssim m_c^2$ $\Rightarrow$ σ_{tot} calculable

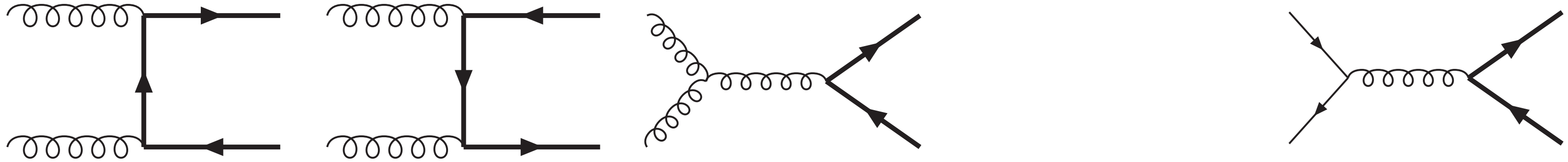
– fixed order logarithms $\ln \frac{p_T^2}{m_c^2}$ large for $p_T^2 \gg m_c^2$;
  resummation of these large logarithms necessary
  $\Rightarrow$ breaks down for $p_T^2 \gg m_c^2$

– non-perturbative function $D_c^{D^\star}(z)$, describing the transition $c \longrightarrow D^\star$ needs to be included to match data;
  $\longrightarrow$ not based on factorization theorem (no AP evolution)
  $\longrightarrow$ universal?

3-FFS: Partonic Subprocesses

• Leading Order (LO):

1. $gg \longrightarrow c\bar{c}$
2. $q\bar{q} \longrightarrow c\bar{c} \quad (q = u, d, s)$



• Next-To-Leading Order (NLO):

1. $gg \longrightarrow c\bar{c}g$
2. $q\bar{q} \longrightarrow c\bar{c}g$
3. $gq \longrightarrow c\bar{c}q \quad , \; g\bar{q} \longrightarrow c\bar{c}\bar{q}$ NEW@NLO

$\longrightarrow$ Feynman diagrams

Conventional Parton Model (ZM-VFNS)

- $m_c = 0 \rightarrow$ 'Zero Mass'
- variable number of partons $\rightarrow$ 'VFNS'

$$\text{partons} = \begin{cases} g, u, d, s & : \mu_F < Q_0 \\ g, u, d, s + c & : Q_0 < \mu_F \end{cases}$$

$n_f = 3 \rightarrow n_f = 4$ in VFNS: finite transformation

$$\left. \begin{array}{l} \alpha_s^{(3)} \rightarrow \alpha_s^{(4)} = \alpha_s^{(3)} + \mathcal{O}(\alpha_s^3) \\ f_i^{(3)} \rightarrow f_i^{(4)} = f_i^{(3)} + \mathcal{O}(\alpha_s^2) \end{array} \right\} @\ Q_0 = m_c$$

$\boxed{f_c^{(4)}(x, Q_0^2 = m_c^2) = 0}$ pert. boundary condition

- Collinear divergences related to c lines factorized into non-perturbative PDFs and FFs

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Pro and Contra:

+ large collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ resummed in evolved $f_c(x, \mu^2)$ and $D_c^{D^\star}(x, \mu^2)$ to LL and NLL accuracy
  $\Rightarrow$ $\boxed{\text{good for large } \mu^2 \simeq p_T^2 \gg m_c^2}$
+ Universality of PDFs and FFs guaranteed by factorization theorem $\rightarrow$ predictive power, global data analysis

– $(\frac{m_c}{p_T})^n$ terms neglected in the hard part
  $\Rightarrow$ $\boxed{\text{breaks down for } p_T^2 \lesssim m_c^2}$ $\Rightarrow$ No σ_{tot}

Massive VFNS

- VFNS with $m_c \neq 0$
- Partons: g, u, d, s, c ($\exists$ charm parton: $f_c \neq 0$)
- collinear $\ln \frac{\mu^2}{m_c^2}$ terms:
  subtracted from hard part (avoid double counting!) and
  resummed by AP evolution equations ($\rightarrow f_c \neq 0$)
- $D_c^{D^\star}(z, {\mu'_F}^2)$ evolved

Pro and Contra:

- technically more involved:
  - calculation with $m_c \neq 0$
  - subtraction of collinear parts $\leftrightarrow$ 'IR-safe' hard parts
    Mass factorization with massive regularization
  - kinematics: factorization with massive partons $\rightarrow$
    'ACOT-χ' in DIS

+ large collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ resummed in evolved
  $f_c(x, \mu^2)$ and $D_c^{D^\star}(x, \mu^2)$
+ $(\frac{m_c}{p_T})^n$ included

$\Rightarrow$ **good for all p_T: $0 \leq p_T^2 \lesssim m_c^2$ and $p_T^2 \gg m_c^2$**

FONLL = FO+NLL [1]

$$\mathrm{FONLL} = \mathrm{FO} + (\mathrm{RS} - \mathrm{FOM0})\mathrm{G}(\mathrm{m}, \mathrm{p_T})$$

FO: Fixed Order; $\mathrm{FOM0}$: Massless limit of FO; RS: Resummed

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2}$$

$$\Rightarrow \mathrm{FONLL} = \begin{cases} \mathrm{FO} & : \quad \mathrm{p_T} \lesssim 5\mathrm{m} \\ \mathrm{RS} & : \quad \mathrm{p_T} \gtrsim 5\mathrm{m} \end{cases}$$

[1] Cacciari, Greco, Nason, JHEP05(1998)007

Hard scattering coefficients with heavy quark masses

- Massless limit of fixed order calculation
- Mass factorization with massive regularization
- Implementation freedom

Our theoretical basis for $p\bar{p} \to D^\star X$

Factorization Formula: [1]

$$d\sigma(p\bar{p} \to D^\star X) = \sum_{i,j,k} \int dx_1\, dx_2\, dz\, f_i^p(x_1)\, f_j^{\bar{p}}(x_2) \times$$
$$\mathrm{d}\hat{\sigma}(ij \to kX)\, D_k^{D^\star}(z) + \mathcal{O}(\alpha_s^{n+1}, (\tfrac{\Lambda}{Q})^p)$$

Q: hard scale, $p = 1, 2$

- $\mathrm{d}\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), \frac{m_h}{p_T})$: hard scattering cross sections free of long-distance physics $\longrightarrow$ m_h kept
- PDFs $f_i^p(x_1, \mu_F)$, $f_j^{\bar{p}}(x_2, \mu_F)$: $i, j = g, q, c$ $\quad [q = u, d, s]$
- FFs $D_k^{D^\star}(z, \mu'_F)$: $k = g, q, c$

$\Rightarrow$ need short distance coefficients including heavy quark masses

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[1] J. Collins, 'Hard-scattering factorization with heavy quarks: A general treatment', PRD58(1998)094002

Adopted Procedure

- Calculate $m \to 0$ limit of massive 3-FFNS calculation of heavy quark production (at the partonic level) [1]
  Only keep m as regulator in $\ln \frac{m^2}{s}$

  Partonic subprocesses in 3-FFNS:
  - Leading Order (LO):
    1. $gg \to c\bar{c}$
    2. $q\bar{q} \to c\bar{c} \quad (q = u, d, s)$
  - Next-To-Leading Order (NLO):
    1. $gg \to c\bar{c}g$
    2. $q\bar{q} \to c\bar{c}g$
    3. $gq \to c\bar{c}q$ New@NLO

  Limiting procedure non-trivial:
  - Map from PS-slicing to Subtraction method
  - care needed to recover $\delta(1-w)$, $(\frac{1}{1-w})_+$, $(\frac{\ln(1-w)}{1-w})_+$

  Checks:
  - Compare Abelian parts with results in [2]
  - Numerical tests

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[1] Bojak, Stratmann, PRD67(2003)034010; FORTRAN code provided by I. Bojak

[2] Kramer, Spiesberger, EPJC22(2001)289; hep-ph/0302081

Adopted Procedure –continued–

- Compare $m \to 0$ limit of massive calculation with massless $\overline{\mathrm{MS}}$ calculation [1]

$$\lim_{m\to 0} \mathrm{d}\sigma(m) = \mathrm{d}\hat{\sigma}(\overline{\mathrm{MS}}) + \Delta\mathrm{d}\sigma$$

$\Rightarrow$ Subtraction terms

$$\mathrm{d}\sigma_{\mathrm{SUB}} \equiv \Delta\mathrm{d}\sigma = \lim_{m\to 0} \mathrm{d}\sigma(m) - \mathrm{d}\hat{\sigma}(\overline{\mathrm{MS}})$$

- Subtract $\mathrm{d}\sigma_{\mathrm{SUB}}$ from massive partonic cross section while keeping mass terms

$$\mathrm{d}\hat{\sigma}(m) = \mathrm{d}\sigma(m) - \mathrm{d}\sigma_{\mathrm{SUB}}$$

$\to$ $\mathrm{d}\hat{\sigma}(m)$ short distance coefficient including m

This is equivalent to $\overline{\mathrm{MS}}$ mass factorization in a scheme where collinear divergences are massively regularized with help of a quark mass m

$\to$ allows to use PDFs and FFs with $\overline{\mathrm{MS}}$ factorization
$\oplus$ massive short distance cross sections

- Treat contributions with charm in the initial state with $m_c = 0$; $\rightsquigarrow$ scheme choice of practical importance; tiny effect in DIS [2]

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[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105

[2] Kretzer, I.S., PRD58(1998)094035

List of Subprocesses

Calculation: $\overline{\mathrm{MS}}$-scheme, heavy quark: $m_Q = 0$ [1]

- Red: Heavy quark mass effects included
- Green: Heavy quark initiated: $m_Q = 0$
- Blue: only light lines involved

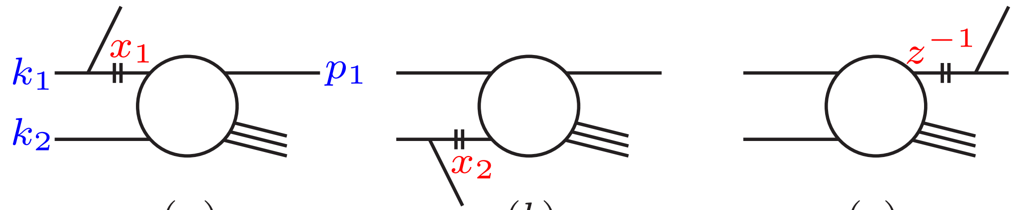
$gg \to qX$	$gg \to QX$	
$gg \to gX$		
$qg \to gX$	$Qg \to gX$	
$qg \to qX$	$Qg \to QX$	
$q\bar{q} \to gX$	$Q\bar{Q} \to gX$	
$q\bar{q} \to qX$	$Q\bar{Q} \to QX$	
$qg \to \bar{q}X$	$Qg \to \bar{Q}X$	
$qg \to \bar{q}'X$	$Qg \to \bar{q}X$	$qg \to \bar{Q}X$
$qg \to q'X$	$Qg \to qX$	$qg \to QX$
$qq \to gX$	$QQ \to gX$	
$qq \to qX$	$QQ \to QX$	
$q\bar{q} \to q'X$	$Q\bar{Q} \to qX$	$q\bar{q} \to QX$
$q\bar{q}' \to gX$	$Q\bar{q} \to gX$	$q\bar{Q} \to gX$
$q\bar{q}' \to qX$	$Q\bar{q} \to QX$	$q\bar{Q} \to qX$
$qq' \to gX$	$Qq \to gX$	$qQ \to gX$
$qq' \to qX$	$Qq \to QX$	$qQ \to qX$

$\oplus$ charge conjugated processes

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105

Subtraction terms via $\overline{\rm MS}$ mass factorization: $a(k_1)b(k_2) \to Q(p_1)X$ [1]

Sketch of kinematics:



(a) (b) (c)

Fig. (a):

$$\mathrm{d}\sigma^{\rm sub}(ab \to QX) = \int_0^1 \mathrm{d}x_1\ f^{(1)}_{a\to i}(x_1, \mu_F^2)\ \mathrm{d}\hat\sigma^{(0)}(ib \to QX)[x_1 k_1, k_2, p_1]$$
$$\equiv f^{(1)}_{a\to i}(x_1) \otimes \mathrm{d}\hat\sigma^{(0)}(ib \to QX)$$

Fig. (b):

$$\mathrm{d}\sigma^{\rm sub}(ab \to QX) = \int_0^1 \mathrm{d}x_2\ f^{(1)}_{b\to j}(x_2, \mu_F^2)\ \mathrm{d}\hat\sigma^{(0)}(aj \to QX)[k_1, x_2 k_2, p_1]$$
$$\equiv f^{(1)}_{b\to j}(x_2) \otimes \mathrm{d}\hat\sigma^{(0)}(aj \to QX)$$

Fig. (c):

$$\mathrm{d}\sigma^{\rm sub}(ab \to QX) = \int_0^1 \mathrm{d}z\ \mathrm{d}\hat\sigma^{(0)}(ab \to kX)[k_1, k_2, z^{-1} p_1]\ d^{(1)}_{k\to Q}(z, {\mu'_F}^2)$$
$$\equiv \mathrm{d}\hat\sigma^{(0)}(ab \to kX) \otimes d^{(1)}_{k\to Q}(z)$$

[1] Kniehl, Kramer, I.S., Spiesberger, hep-ph/0502194

Subtraction terms via $\overline{\rm MS}$ mass factorization: Partonic PDFs and FFs

1. initial state:

$$f^{(1)}_{g\to Q}(x,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} P^{(0)}_{g\to q}(x) \ln\frac{\mu^2}{m^2}$$

$$f^{(1)}_{Q\to Q}(x,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} C_F \left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu^2}{m^2} - 2\ln(1-z) - 1\right)\right]_+ \qquad [2]$$

$$f^{(1)}_{g\to g}(x,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi}\frac{1}{3}\ln\frac{\mu^2}{m^2}\delta(1-x)$$

2. final state:

$$d^{(1)}_{g\to Q}(z,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} P^{(0)}_{g\to q}(z) \ln\frac{\mu^2}{m^2}$$

$$d^{(1)}_{Q\to Q}(z,\mu^2) = C_F \frac{\alpha_s(\mu)}{2\pi} \left[\frac{1+z^2}{1-z}\left(\ln\frac{\mu^2}{m^2} - 2\ln(1-z) - 1\right)\right]_+ \qquad [1,2,3]$$

- Other distributions are zero to this order in α_s
- Analogous for photon splitting: $g \to \gamma$, $\alpha_s \to \alpha$, color factors

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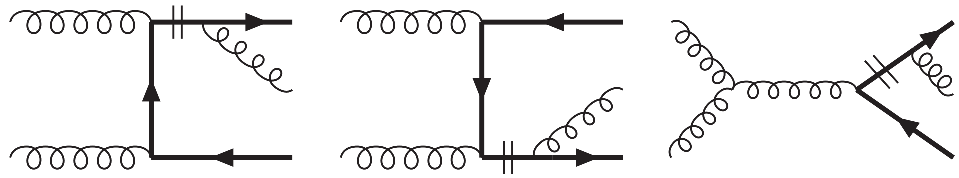
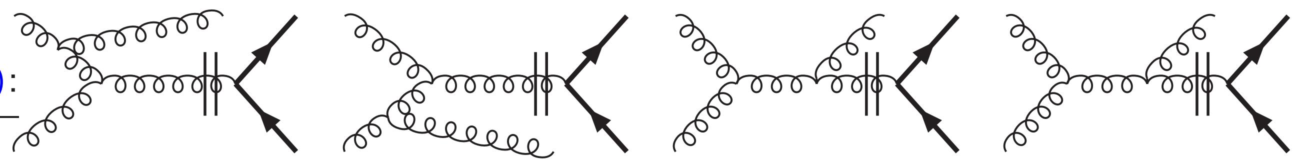
[1] Mele, Nason, NPB361(91)626 [$\gamma^\star \to c\bar{c}g$]
[2] Kretzer, I.S., PRD58(1998)094035;D59(1999)054004 [$c\gamma^\star \to cg$]
[3] Melnikov, Mitov, PRD70(2004)034027; Mitov, hep-ph/0410205 [$\mathcal{O}(\alpha_s^2)$]

Graphical representation of Subtraction terms for $gg \to Q\bar{Q}g$

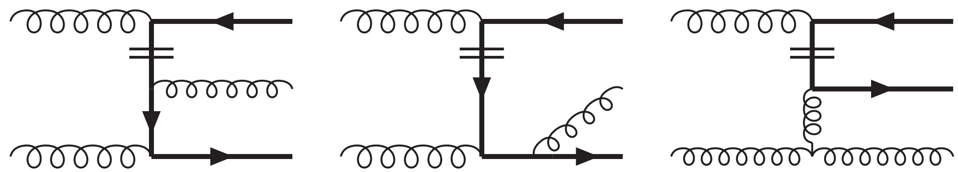
$\mathrm{d}\hat{\sigma}^{(0)}(gg \to Q\bar{Q}) \otimes d^{(1)}_{Q\to Q}(z)$:



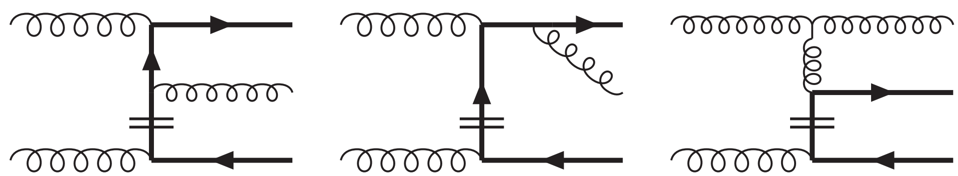
$\mathrm{d}\hat{\sigma}^{(0)}(gg \to gg) \otimes d^{(1)}_{g\to Q}(z)$:



$f^{(1)}_{g\to Q}(x_1) \otimes \mathrm{d}\hat{\sigma}^{(0)}(Qg \to Qg)$:

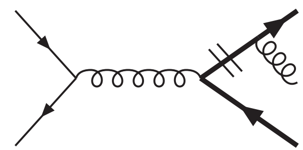


$f^{(1)}_{g\to Q}(x_2) \otimes \mathrm{d}\hat{\sigma}^{(0)}(gQ \to Qg)$:

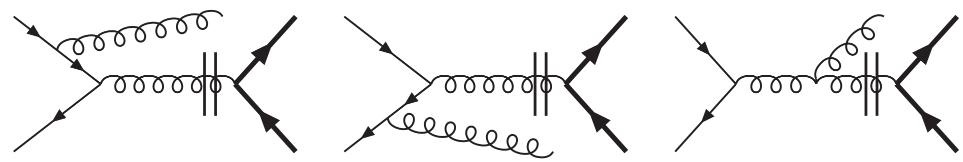


Graphical representation of subtraction terms for $q\bar{q} \to Q\bar{Q}g$ and $gq \to Q\bar{Q}q$

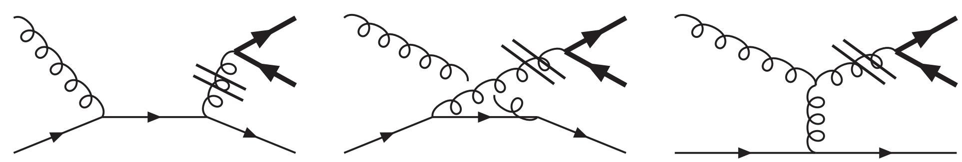
$\mathrm{d}\hat{\sigma}^{(0)}(q\bar{q} \to Q\bar{Q}) \otimes d^{(1)}_{Q\to Q}(z)$:



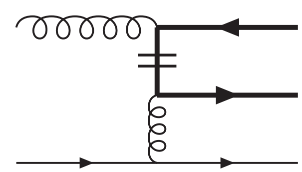
$\mathrm{d}\hat{\sigma}^{(0)}(q\bar{q} \to gg) \otimes d^{(1)}_{g\to Q}(z)$:



$\mathrm{d}\hat{\sigma}^{(0)}(gq \to gq) \otimes d^{(1)}_{g\to Q}(z)$:



$f^{(1)}_{g\to Q}(x_1) \otimes \mathrm{d}\hat{\sigma}^{(0)}(Qq \to Qq)$:



Comparison with FONLL

$\mathrm{FONLL} = \mathrm{FO} + (\mathrm{RS} - \mathrm{FOM0})\mathrm{G}(\mathrm{m}, \mathrm{p_T})$ with

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2}$$

GM-VFNS $= \mathrm{FO} + (\mathrm{RS} - \mathrm{FOM0})\tilde{\mathrm{G}}(\mathrm{m}, \mathrm{p_T})$ with $\tilde{G}(m, p_T) = 1$ (very similar to S-ACOT scheme)

FO: Fixed Order; $\mathrm{FOM0}$: Massless limit of FO; $\mathrm{RS} \equiv$ ZM-VFNS: Resummed

- Both approaches interpolate between FO and ZM-VFNS
  - FONLL: obvious;
  - GM-VFNS: matching with FO at quark level (see Olness,Scalise,Tung,PRD59(1998)014506)
- Different point-of-view: GM-VFNS finally needs PDFs and FFs in this scheme.
- Numerical comparisons interesting and should be done!

Numerical Results

$$\int_{-1}^{1} \mathrm{d}y \; \frac{\mathrm{d}\sigma}{\mathrm{d}p_T \mathrm{d}y}$$

for

$$p + \bar{p} \longrightarrow (D^0, D^{\star +}, D^+, D_s^+) + X$$

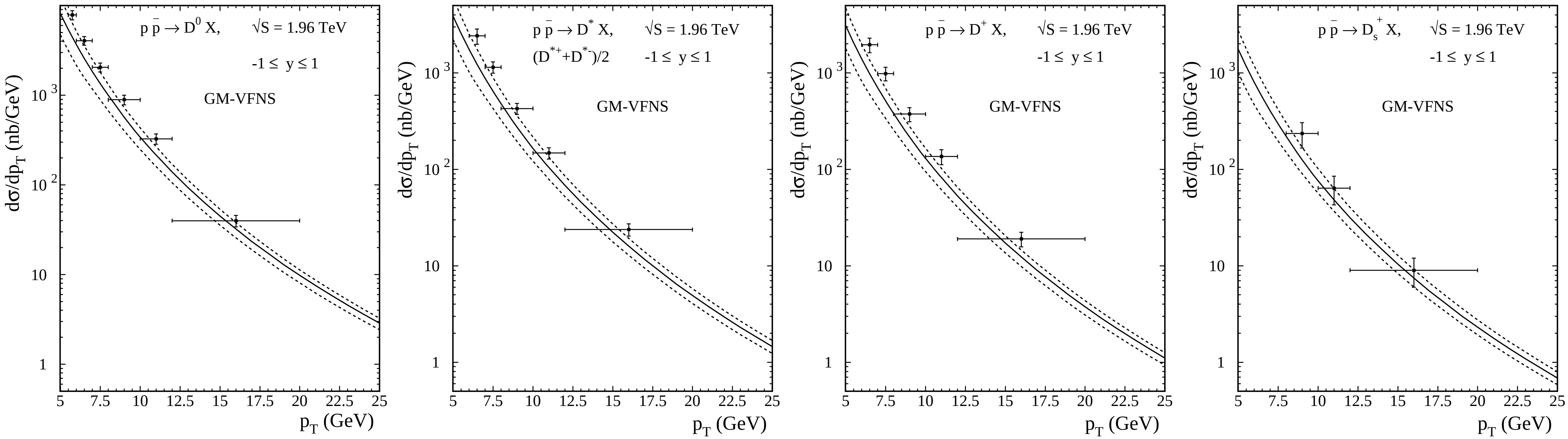
at the Tevatron

Input parameters:

- PDFs: CTEQ6M (NLO)
- FFs: BKK NLO OPAL
- $\alpha_s(M_Z) = 0.118$
- $m_c = 1.5 \text{ GeV}, m_b = 5 \text{ GeV}$

Comparison with CDF II data for $p\bar{p} \to (D^0, D^{\star +}, D^+, D_s^+)X$ [1]

- $\mathrm{d}\sigma/\mathrm{d}p_T$ (nb/GeV), $|y| \leq 1$, massive VFNS (GM-VFNS)
- Uncertainty band: independent variation of $\mu_R, \mu_F, \mu'_F = \xi m_T$, $\xi \in [1/2, 2]$



- Prompt charm (no secondary charm from B decay)
- Data and Theory compatible within errors
- Central values: $\text{Data/Theory} \simeq 1.5 - 1.8$

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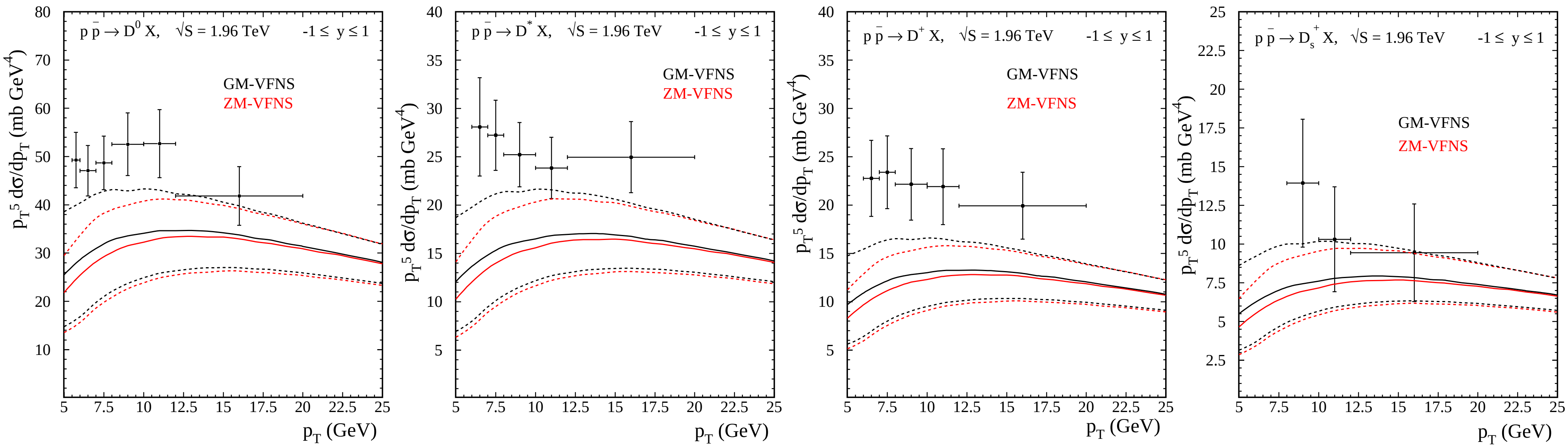
[1] Acosta et al, PRL91(2003)241804

Comparison with CDF II data for $p\bar{p} \to (D^0, D^{\star +}, D^+, D_s^+)X$ [1]

- $p_T^5\ \mathrm{d}\sigma/\mathrm{d}p_T\ (\mathrm{mb\ GeV}^4)$, $|y| \leq 1$: almost flat
- Uncertainty band: independent variation of $\mu_R, \mu_F, \mu'_F = \xi m_T$, $\xi \in [1/2, 2]$
  Large scale uncertainty dominant!



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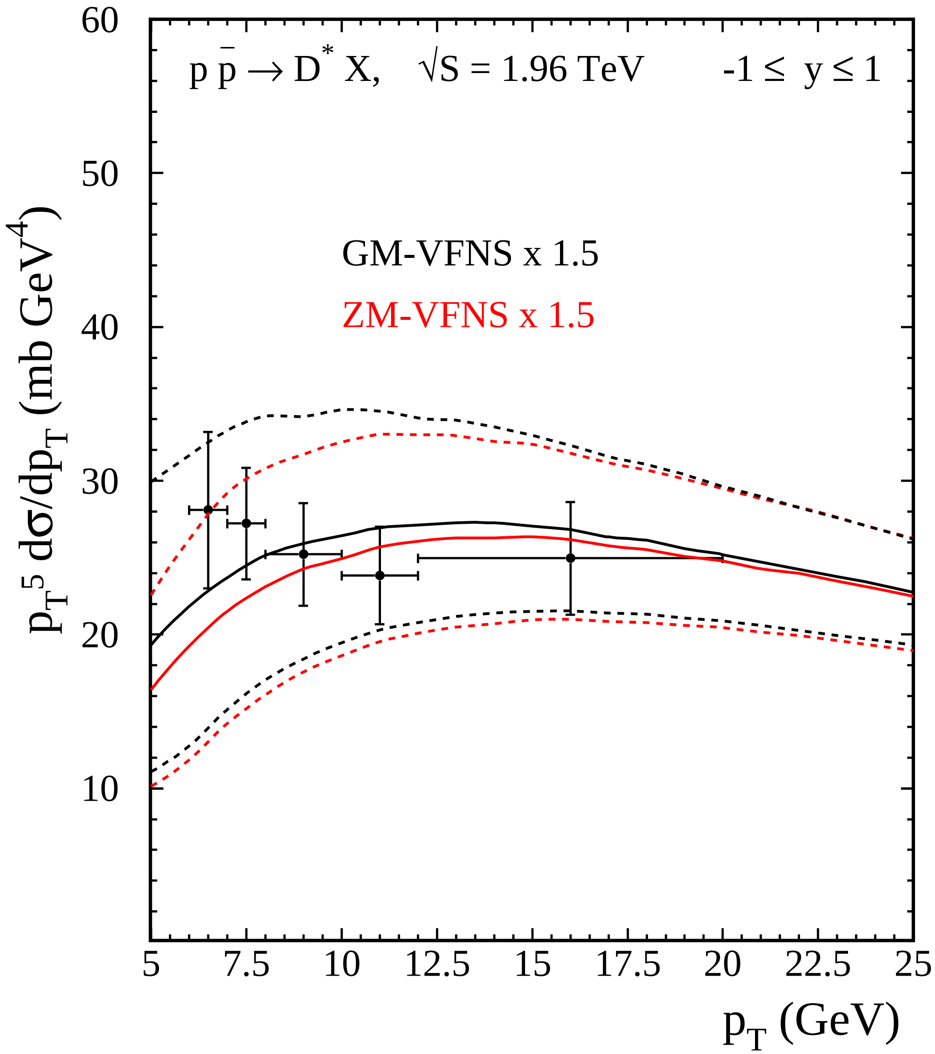
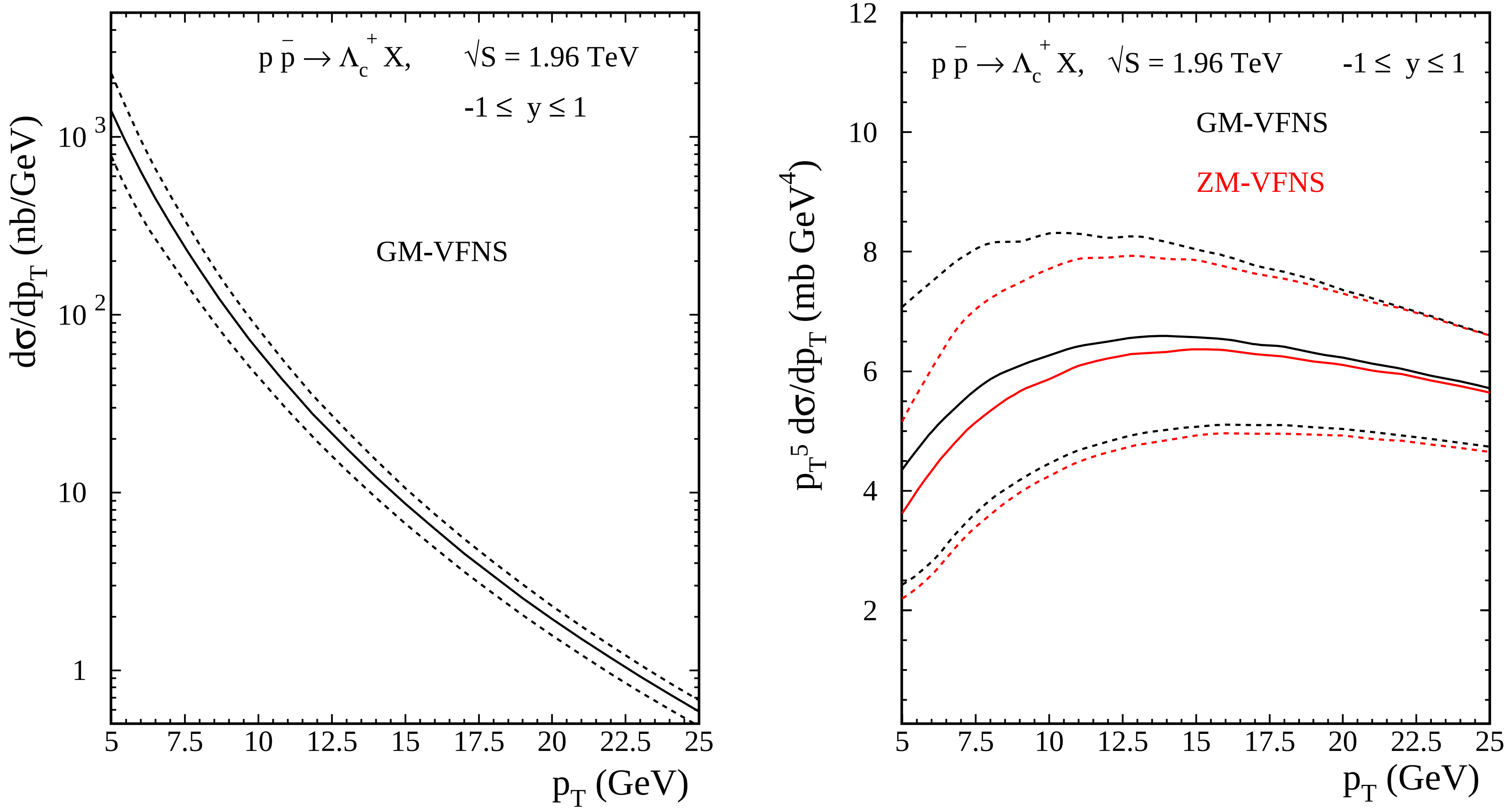
[1] Acosta et al, PRL91(2003)241804

Central values: Data/Theory $\simeq 1.5$



Predictions for Λ_c^+



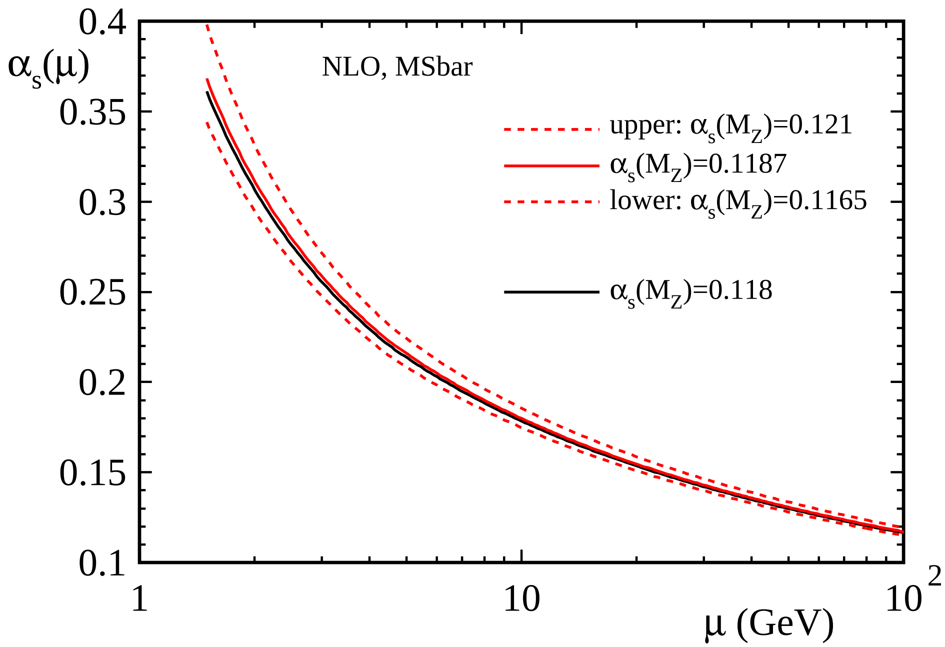
Status

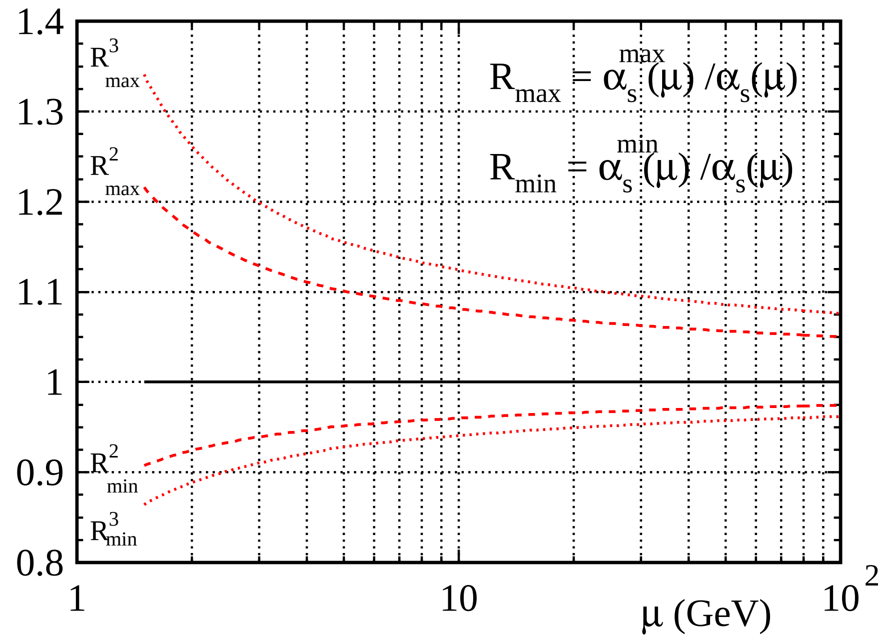
Theory compatible with exp. results within errors. However, for central curves:
Data/Theory $\simeq 1.5 \ldots 2$

Explanations? [Assuming: pQCD/Factorization valid; Data correct (within errors)]

- Non-pert. Input: PDFs, FFs
  Note: Gluon fragmentation contributes about 30% to the cross section at Tevatron.
- α_s uncertainty: increase of $10\% - 20\%$ possible
  (of course increase of α_s feeds back on PDFs) [→ Figure]
- NNLO corrections
- New physics
- . . . ?

- PDG'04: $\alpha_s(M_Z) = 0.1187 \pm 0.0020$
- CTEQ6M PDFs: $\alpha_s(M_Z) = 0.118$; MRST03 $\alpha_s(M_Z) = 0.1165$;
- $\mathrm{d}\sigma(p\bar{p} \to DX) \propto \alpha_s^2(1 + \alpha_s K)$





Conclusions and Outlook

We have established a massive VFNS for heavy quark production in hadron hadron collisions. Same approach as in previous work by G. Kramer and H. Spiesberger for $\gamma\gamma, \gamma p \to D^* X$:

- Massless limit of massive calculation compared with massless $\overline{\mathrm{MS}}$ calculation
  Subtraction terms identified:

$$\mathrm{d}\sigma_{\mathrm{SUB}} \equiv \Delta \mathrm{d}\sigma = \lim_{m \to 0} \mathrm{d}\sigma(m) - \mathrm{d}\hat{\sigma}(\overline{\mathrm{MS}})$$

- Subtraction terms in agreement with calculation based on mass factorization with massive regularization. Useful for future applications.

- Short distance coefficients including heavy quark mass m constructed:

$$\mathrm{d}\hat{\sigma}(m) = \mathrm{d}\sigma(m) - \mathrm{d}\sigma_{\mathrm{SUB}}$$

- Numerical checks of subtraction terms: $\mathrm{d}\hat{\sigma}(m) \to \mathrm{d}\hat{\sigma}(\overline{\mathrm{MS}})$ for $m \to 0$.

- allows to use PDFs and FFs with $\overline{\mathrm{MS}}$ factorization together with massive short distance cross sections

- Applications: $p\bar{p} \to DX$ and $p\bar{p} \to BX$ at the Tevatron. Resolved contribution to $\gamma p \to D^\star X$, $\gamma\gamma \to D^\star X$.