STOCHASTIC MODELING REGGEON FIELD THEORY

(REGGEONS and soft PARTONS)

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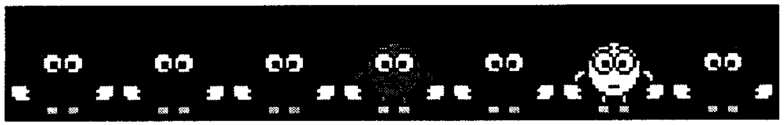


P. Grassberger, K. Sundermeyer, Phys.Lett. **B77** (1978) 220
K. Boreskov, hep-ph/0112325

- only soft processes will be discussed
- the nature of soft partons is not specified

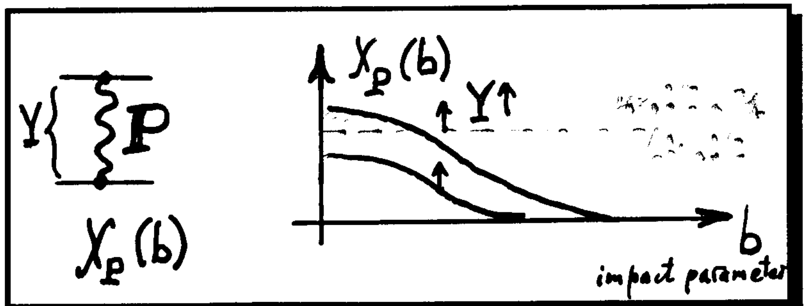
Contents

- ✦ Reggeon Field Theory (RFT) with supercritical pomeron ($\alpha_P > 1$)
- ✦ Multiparticle content of RFT
- ✦ Stochastic parton dynamics
  - without diffusion — toy model
  - with diffusion
- ✦ Interaction of hadrons as parton sets
  - Lorentz invariance
  - Operator of interaction
- ✦ Examples
  - Eikonal model
  - Schwimmer model

  } inconsistencies
- ✦ Concluding remarks — advantages, problems, first steps

RFT diagrams

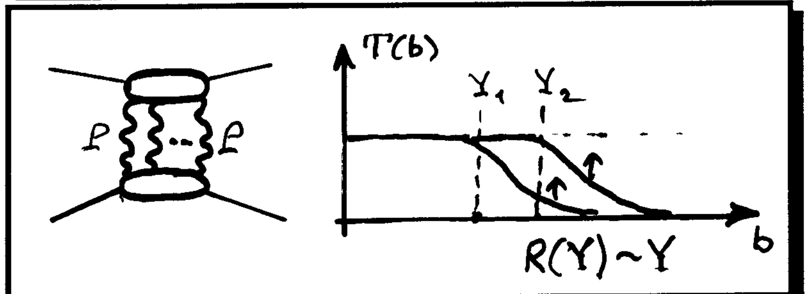
$Y \sim \ln s$



$\sigma_P(Y) \sim e^{\Delta \cdot Y}, \quad \Delta = \alpha_P(0) - 1$

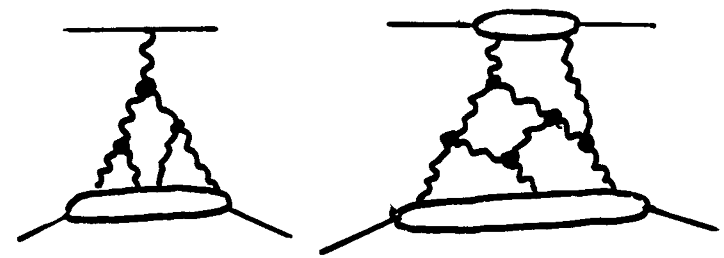
$\Delta > 0$ – the *supercritical* pomeron

violates s-channel unitarity



multipomeron exchanges
(non-enhanced diagrams)

Average number of exchanges
$\langle n \rangle \sim \exp(\Delta Y)$



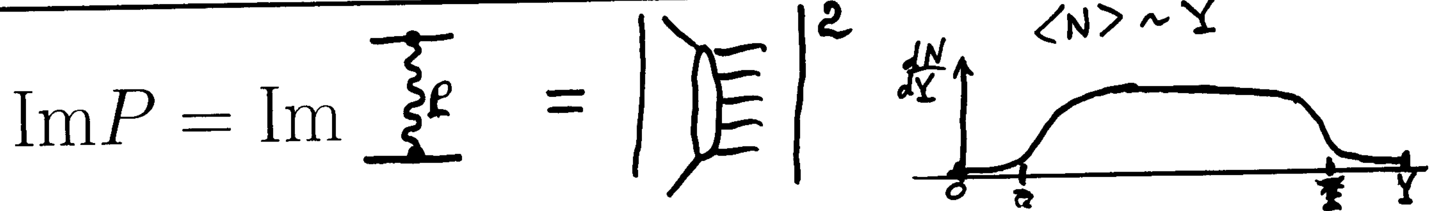
enhanced diagrams

Relative contributions
increase with energy !

especially for nuclei (RHIC $\Rightarrow$ LHC)

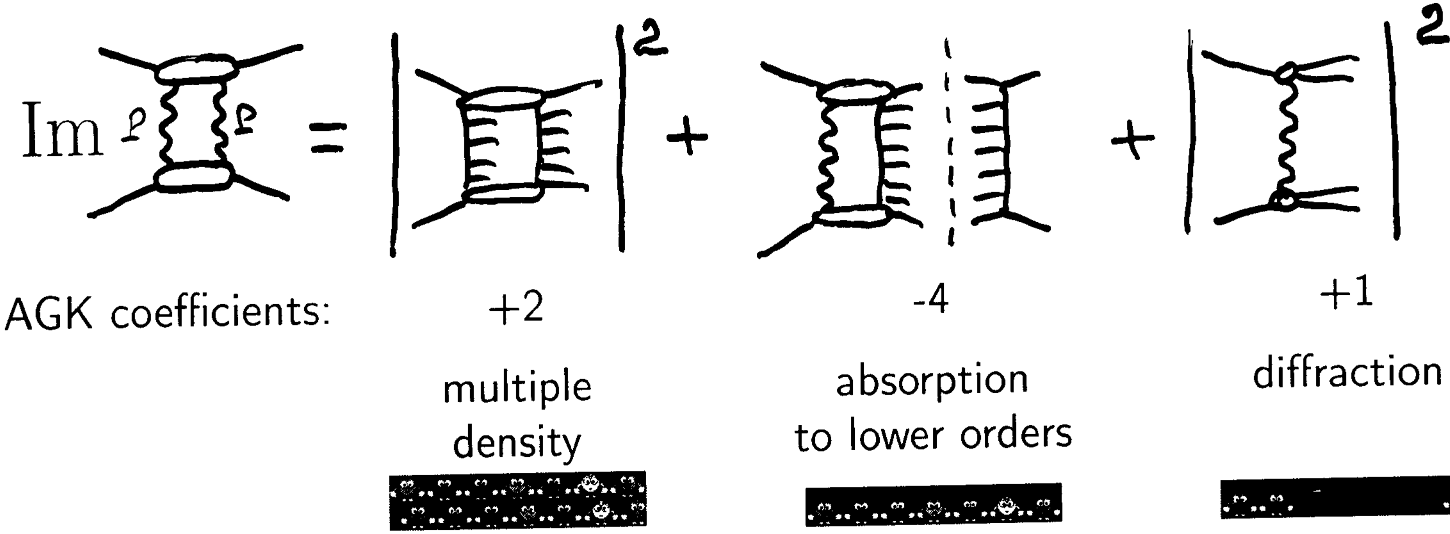
Very complicated problem of diagram summation

Multiparticle contents of RFT



multiperipheral process (ordering in rapidity)

Cutting many-pomeron graphs includes also *absorption* effects



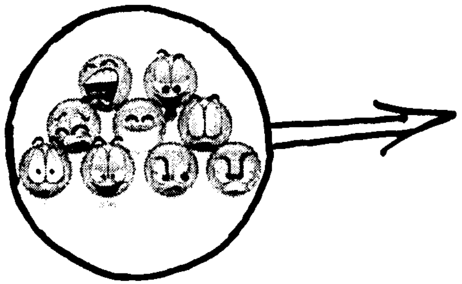
AGK coefficients: +2 -4 +1

multiple density

absorption to lower orders

diffraction

Inelastic and absorption contributions are strongly correlated!

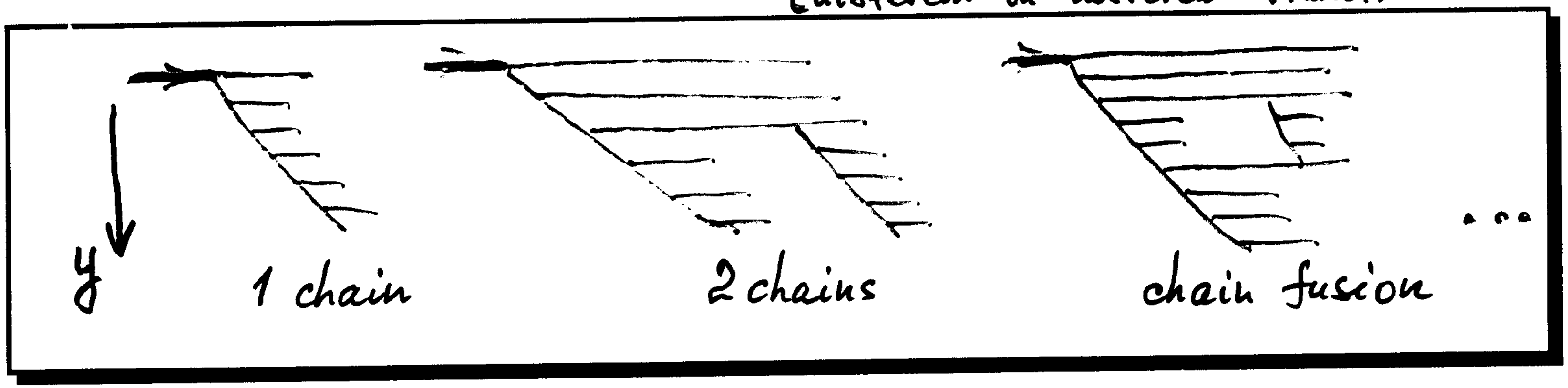


What is a fast hadron?

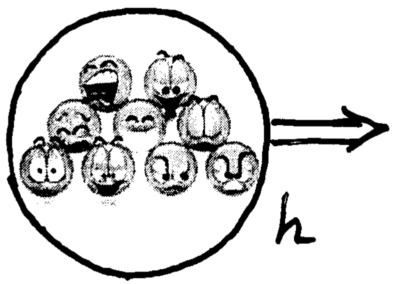
At what moment does the high density of particles arise ?

It **already presents** in the Fock wave function
of the fast hadron !

Due to quantum mechanical fluctuations
a fast hadron is a complicated multiparticle state
(different in different frames)

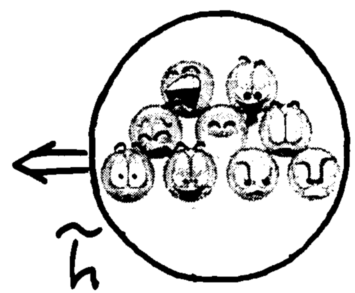


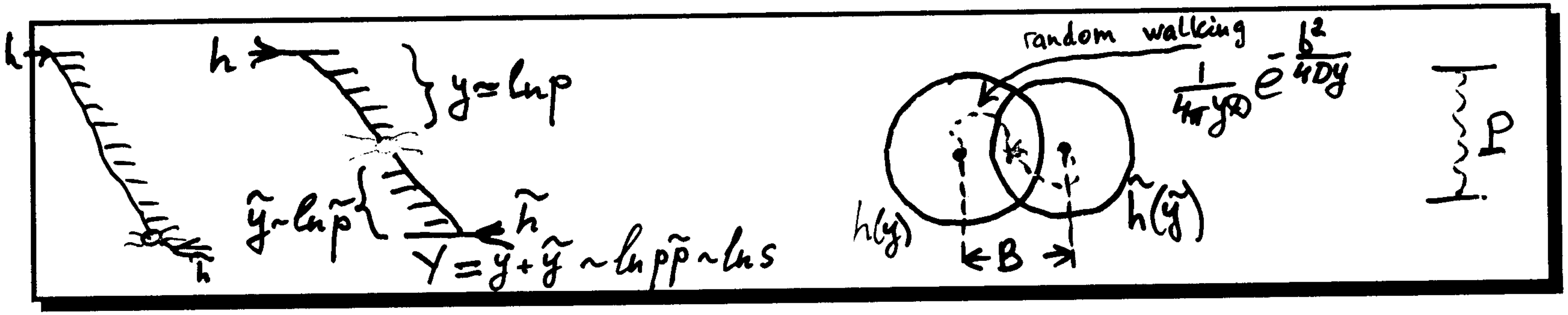
Interactions of fast hadrons



Fast hadron interacts due to its slow components

(short-range interaction in rapidity space)





Probability of interaction:

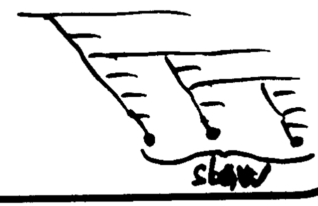
$$T(Y,b) = \int d^2b \left(\frac{1}{4\pi y D} e^{-b^2/4Dy}\right) \left(\frac{1}{4\pi \tilde{y} D} e^{-\tilde{b}^2/4D\tilde{y}}\right) \delta(\vec{b} + \vec{\tilde{b}} - \vec{B}) = \frac{1}{4\pi Y D} e^{-B^2/4DY}$$

$$e^{\Delta y} \cdot e^{\Delta \tilde{y}} = e^{\Delta Y}$$

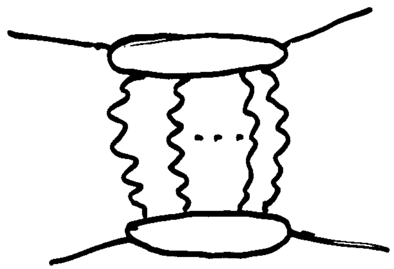
Supercritical pomeron: $\sigma \sim e^{\Delta Y}$

From the parton viewpoint $\Rightarrow$ multiplication of partons with rapidity (chain splitting)

number of slow partons $\sim e^{\Delta Y}$



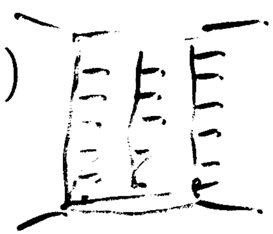
High Energies → High densities



Average number of multiperipheral chains

$$\langle N \rangle_{chain} \sim e^{\Delta Y} \rightarrow \infty$$

High energies ⇒ large density of multiperipheral chains
(or pomerons, or strings, or whatever you like)

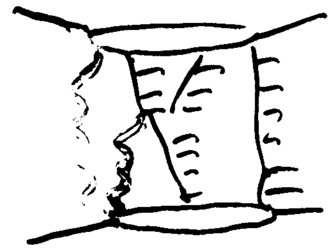


⇒ strong interactions between chains

⇒ large absorption effects

One has to account for several mechanisms:

- chain splitting
- chain fusion
- correct absorption



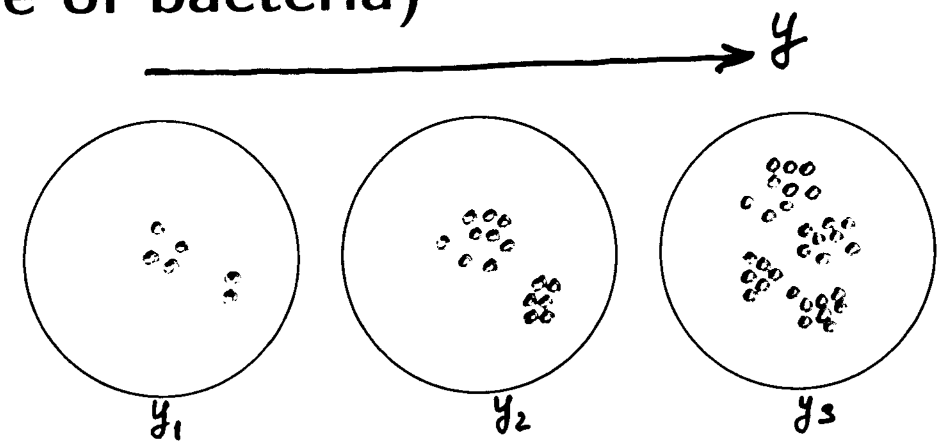
One way: to calculate and sum a huge set of reggeon diagrams

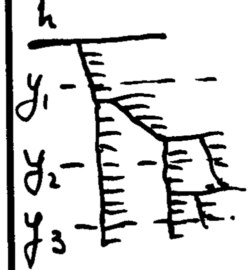
Another way: to solve (numerically) *the* equivalent classical problem

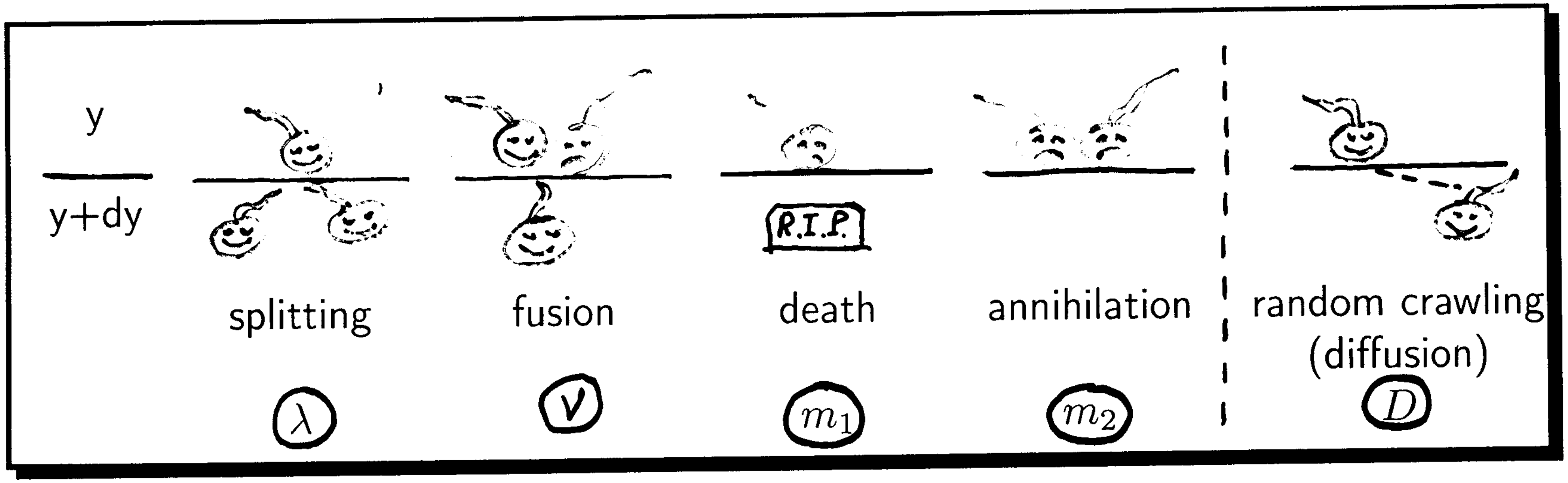
STOCHASTIC MODEL for RFT

(Life of bacteria)

Colony of bacteria
in the Petri plate
evolving with "time" y







Evolution equation

(toy model: no diffusion)

$p_N(y)$ – probability to have N partons at the "moment" y

$$\frac{dp_N}{dy} = -(\lambda + m_1)Np_N - (\nu + m_2)N(N-1)p_N$$

loss

$$+ \lambda N(N-1)p_{N-1} + m_1(N+1)p_{N+1} + \nu(N+1)Np_{N+1} + m_2(N+2)(N+1)p_{N+2}$$

income

$$\frac{dp_0}{dy} = m_1 p_1 + 2m_2 p_2, \quad \frac{dp_1}{dy} = -(\lambda + m_1)p_1 + 2(\nu + m_1)p_2 + 6m_2 p_3 \, .$$

Kinetic equations

Probability conservation

$$p(y) = \sum_0^\infty p_N(y) = 1 \, ; \qquad \frac{dp(y)}{dy} = 0 \, .$$

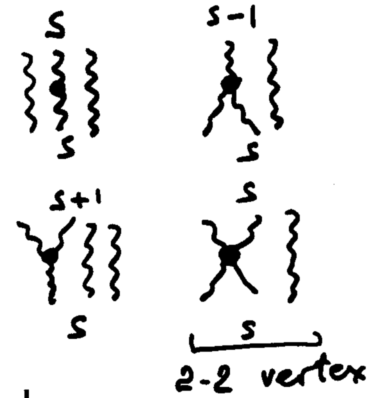
Moments

$$\mu_s(y) = \sum_{N=0}^{\infty} \frac{N!}{(N-s)!} p_N(y)$$

$$\mu_0 = p = 1, \quad \mu_1 = \langle N \rangle,$$

$$\mu_2 = \langle N(N-1) \rangle, \quad \ldots$$

$$\frac{d\mu_s}{dy} = \overbrace{(\lambda - m_1)}^{\Delta} s\mu_s + \lambda s(s-1)\mu_{s-1} - (\nu + m_2)s(s-1)\mu_s - (\nu + 2m_2)s\mu_{s+1}$$



Equivalent to 0-dim RFT !

Generating function:

$$G(w;y) = \sum_{N=0}^{\infty} p_N(y) w^N = \sum_{s=0}^{\infty} \mu_s(y) \frac{(w-1)^s}{s!}$$

$$\frac{\partial G(w;y)}{\partial y} = (1-w)(m_1 - \lambda w)\frac{\partial G}{\partial w} + (1-w)[m_2 + (m_2 + \nu)w]\frac{\partial^2 G}{\partial w^2}$$

Two regimes

- **without fusion** $(\nu = 0,\ m_2 = 0),\ \Delta = \lambda - m_1$

$\Delta > 0$ (splitting larger than death-rate):	$\Delta < 0$ (splitting less than mortality):
$\langle N \rangle = N_0\, e^{\Delta Y}, \quad p_0(y) \to m_1/\lambda$	$\langle N \rangle \to 0, \quad p_0(y) \to 1$
(strong parton correlations in cascade)	

Partonless state $|N = 0\rangle$ is called the *absorbing* state: $dp_0/dy > 0$ (p_0 only increases)

- **with fusion** $(\nu \neq 0, m_1 = m_2 = 0)$

stationary Poisson-like state at $y \to \infty$

(saturation) (small correlations)

$$p_N = B \frac{a^N}{N!} e^{-a}\,, \quad a = \lambda'/\nu \qquad (y \to \infty)$$

Model with diffusion

$$p_N(y) \Rightarrow \rho_N(y;\mathcal{B}_N)\ , \quad \frac{1}{N!}\int d\mathcal{B}_N \rho_N(y;\mathcal{B}_N) = p_N(y)\ , \quad \mathcal{B}_N = \{b_1,\dots,b_N\}$$

Set of equations (local interactions in b-space)
of range ϵ

$$\frac{d\rho_N(y;\mathcal{B}_N)}{dy} = \underbrace{D\,\nabla_N^2\,\rho_N(y;\mathcal{B}_N)}_{\text{diffusion}} - (m_1+\lambda)N\rho_N(y;\mathcal{B}_N)$$
$$+\, m_1\int db_{N+1}\rho_{N+1}(y;\mathcal{B}_N,\mathbf{b}_{N+1}) + \lambda\sum_{k,l=1}^{N\geqslant 2}\rho_{N-1}(y;\mathcal{B}_N^{(l)})\delta(\mathbf{b}_k-\mathbf{b}_l)$$
$$-\,(m_2+\nu)\sum_{k,l=1}^{N\geqslant 2}\rho_N(y;\mathcal{B}_N)\delta(\mathbf{b}_k-\mathbf{b}_l) + m_2\int d\mathbf{b}_{N+1}\rho_{N+2}(y;\mathcal{B}_N,\mathbf{b}_{N+1},\mathbf{b}_{N+1})$$
$$+\,\nu\int d\mathbf{b}_{N+1}\rho_{N+1}(y;\mathcal{B}_N,\mathbf{b}_{N+1})\sum_{k=1}^{N}\underbrace{\delta(\mathbf{b}_k-\mathbf{b}_{N+1})}_{\text{local interaction}}\ ,$$

Multiparton distributions

(analogues of moments μ_s)

$$f_s^{(N)}(y;\{\mathbf{z}_1,\ldots,\mathbf{z}_s\}) = \frac{1}{N!}\int d\mathcal{B}_N\ \rho_N(y;\mathcal{B}_N) \times \sum_{perm} \delta(\mathbf{z}_1-\mathbf{b}_{i_1})\delta(\mathbf{z}_2-\mathbf{b}_{i_2})\ldots\delta(\mathbf{z}_s-\mathbf{b}_{i_s})\ ,$$

$s = 1, 2, \ldots$

inclusive distributions

$$f_s(y;\mathcal{Z}_s) = \sum_{N=0}^{\infty} f_s^{(N)}(y;\mathcal{Z}_s)$$

$$\mathcal{Z}_s = \{\mathbf{z}_1,\ldots,\mathbf{z}_s\}$$

coordinates of s partons of N are fixed;
$f_1(y,\mathbf{z})$ – parton density

Momentum representation

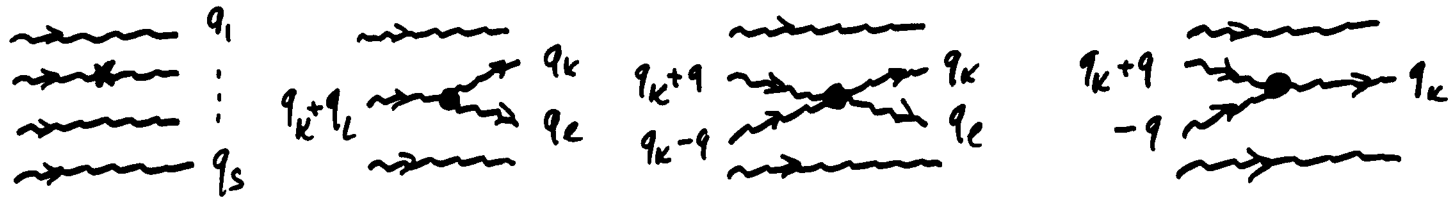
$$g_s^{(N)}(y;Q_s) = \int d\mathcal{Z}_s e^{i\mathbf{q}_1\mathbf{z}_1} \dots e^{i\mathbf{q}_s\mathbf{z}_s} f_s^{(N)}(y;\mathcal{Z}_s) \ ,$$

$$g_s(y;Q_s) = \sum_N g_s^{(N)}(y;Q_s) \ , \quad Q_s = \{\mathbf{q}_1, \dots, \mathbf{q}_s\}$$

transverse momenta

Closed set of eqs for g_s (much simpler than for $g_s^{(N)}$)

$$\frac{d}{dy} g_s(y;Q_s) = -D \left(\sum_{a=1}^{s} \mathbf{q}_a^2 \right) g_s(y;Q_s) + (\lambda - m_1) s g_s(y;Q_s)$$
$$+ \lambda \sum_{k,l=1}^{s\geqslant 2} g_{s-1}(y; Q_s^{(k,l)}, \mathbf{q}_k + \mathbf{q}_l) - \nu \int \frac{d\mathbf{q}}{(2\pi)^2} \sum_{k,l=1}^{s\geqslant 2} g_s(y; Q_s^{(k,l)}, \mathbf{q}_k + \mathbf{q}, \mathbf{q}_l - \mathbf{q})$$
$$- (2m_2 + \nu) \int \frac{d\mathbf{q}}{(2\pi)^2} \sum_{k=1}^{s} g_{s+1}(y; Q_s^{(k)}, \mathbf{q}_k + \mathbf{q}, -\mathbf{q}) \ , \quad (s = 1, 2, \dots) \ .$$



Equivalence to RFT

ω-representation:

$$G_s(\omega;Q_s) = \int_0^\infty dy e^{-\omega y} g_s(y;Q_s) \ ,$$
$$g_s(y,Q_s) = \frac{1}{2\pi i}\int_\uparrow d\omega e^{\omega y} G_s(\omega;Q_s) \ ,$$

$$D_s^{(0)}(\omega;Q_s) = \left(\omega - D\sum_{a=1}^{s} q_a^2\right)^{-1}$$

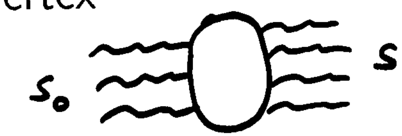
bare propagator of s pomerons

$$D_s(\omega;Q_s) = \left(\omega - s(\lambda - m_1) - D\sum_{a=1}^{s} q_a^2\right)^{-1}$$

renormalized propagator of s pomerons

$$\Gamma_s^{s_0}(\omega;Q_s) = \left[D_s(\omega;Q_s)\right]^{-1} G_s(\omega;Q_s)$$

reduced vertex

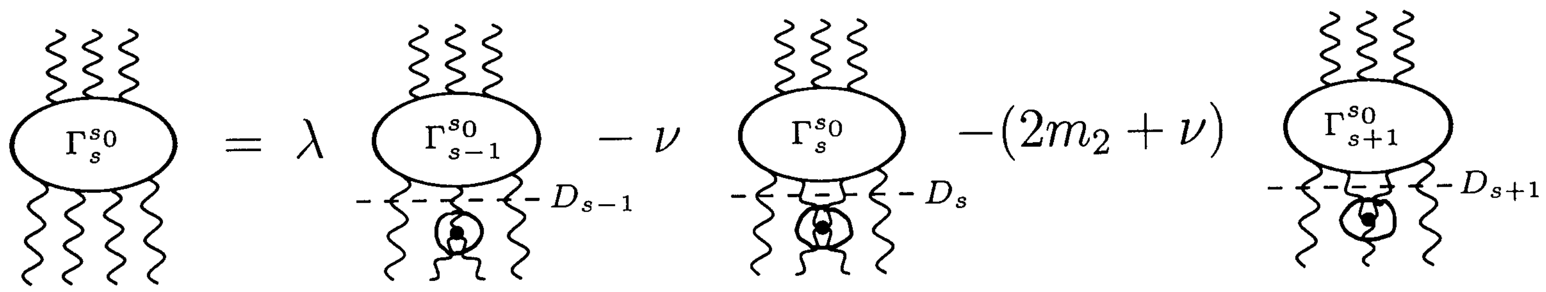


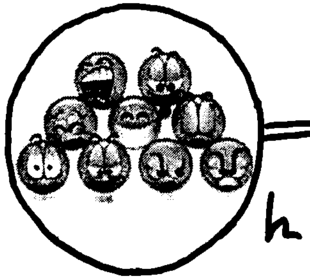
Equations for pomeron vertices

$$\Gamma_s^{so}(\omega; Q_s) = \lambda \sum_{k,l=1}^{s} \Gamma_{s-1}^{so}(\omega; Q_s^{(kl)}, \mathbf{q}_k + \mathbf{q}_l) D_{s-1}(\omega; Q_s) -$$

$$-\nu \int \frac{d\mathbf{q}}{(2\pi)^2} \sum_{k,l=1}^{s} \Gamma_s^{so}(\omega; Q_s^{(kl)}, \mathbf{q}_k + \mathbf{q}, \mathbf{q}_l - \mathbf{q}) D_s(\omega; Q_s) -$$

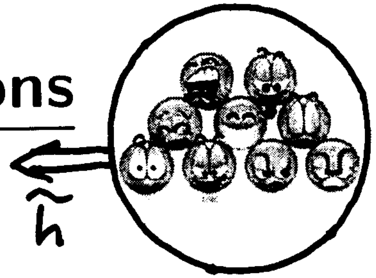
$$-(2m_2 + \nu) \int \frac{d\mathbf{q}}{(2\pi)^2} \sum_{k=1}^{s} \Gamma_{s+1}^{so}(\omega; Q_s^{(k)}, \mathbf{q}_k + \mathbf{q}, -\mathbf{q}) D_{s+1}(\omega; Q_s) \ .$$





Interactions of colliding hadrons

as sets of partons



Hadron state h can be described by a set of s-parton distributions

$$|\mathcal{F}(y;0)\rangle = \{f_s(y;\mathcal{Z}_s)\} \quad \text{or} \quad |\mathcal{G}(y)\rangle = \{g_s(y;Q_s)\}$$

and hadron state $\tilde{h}$ with rapidity $\tilde{y}$ at impact parameter $\mathbf{b}$ by

$$|\mathcal{F}(\tilde{y};\mathbf{b})\rangle = \{f_s(\tilde{y};\mathcal{Z}_s+\mathbf{b})\} \quad \text{or} \quad |\mathcal{G}(\tilde{y};\mathbf{b})\rangle = \{g_s(\tilde{y};Q_s)e^{i\mathbf{b}\sum_1^s \mathbf{q}_a}\}$$

Fock state

Evolution equation

$$\frac{d}{dy}|\mathcal{F}(y)\rangle = \hat{H}|\mathcal{F}(y)\rangle \quad \text{or} \quad \frac{d}{dy}|\mathcal{G}(y)\rangle = \hat{H}|\mathcal{G}(y)\rangle$$

Solution:

$$|\mathcal{F}(y)\rangle = \exp\left[\hat{H}y\right]|\mathcal{F}(0)\rangle \quad \text{or} \quad |\mathcal{G}(y)\rangle = \exp\left[\hat{H}y\right]|\mathcal{G}(0)\rangle$$

Amplitude of interaction:

$$T(y,\tilde{y};b) = \langle \mathcal{F}(\tilde{y})|\hat{T}|\mathcal{F}(y)\rangle = \langle \mathcal{G}(\tilde{y})|\hat{T}|\mathcal{G}(y)\rangle$$

Definition of interaction operator $\hat{T}$:

$$T(y,\tilde{y};b) = \sum_{s=1}^{\infty} \frac{(-1)^{s-1}\epsilon^s}{s!} \int d\tilde{\mathcal{Z}}_s d\mathcal{Z}_s \tilde{f}_s(\tilde{y};\tilde{\mathcal{Z}}_s) f_s(y;\mathcal{Z}_s)\delta^{(s)}(\mathcal{Z}_s - \tilde{\mathcal{Z}}_s - \mathbf{b})$$

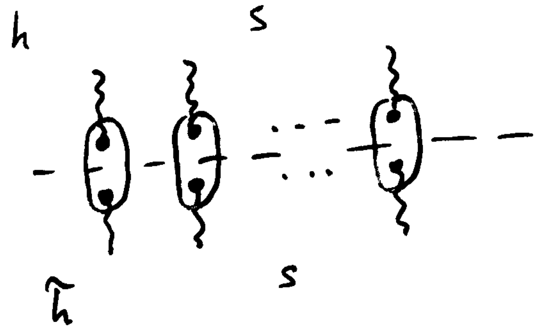
$$= \sum_{s=1}^{\infty} \frac{(-1)^{s-1}\epsilon^s}{s!} \int \frac{dQ_s}{(2\pi)^{2s}} e^{(i\mathbf{b}\sum_1^s \mathbf{q}_a)} \tilde{g}_s(\tilde{y};Q_s) g_s(y;Q_s)$$

(ϵ – parton size)

Important: all f_s should be involved
(in some approximations $f_s = \prod_1^s f_1$)

✓ corresponds to RFT

✓ provides boost invariance



Boost invariance

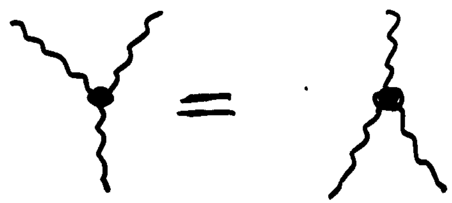
$$T(y,\tilde{y};b) = \langle \mathcal{G}(0;b)|e^{\hat{H}^t\tilde{y}}\hat{T}e^{\hat{H}y}|G(0;0)\rangle$$ must depend on $Y = y+\tilde{y}$

if $\boxed{\hat{H}^t\hat{T} = \hat{T}\hat{H}}$ then

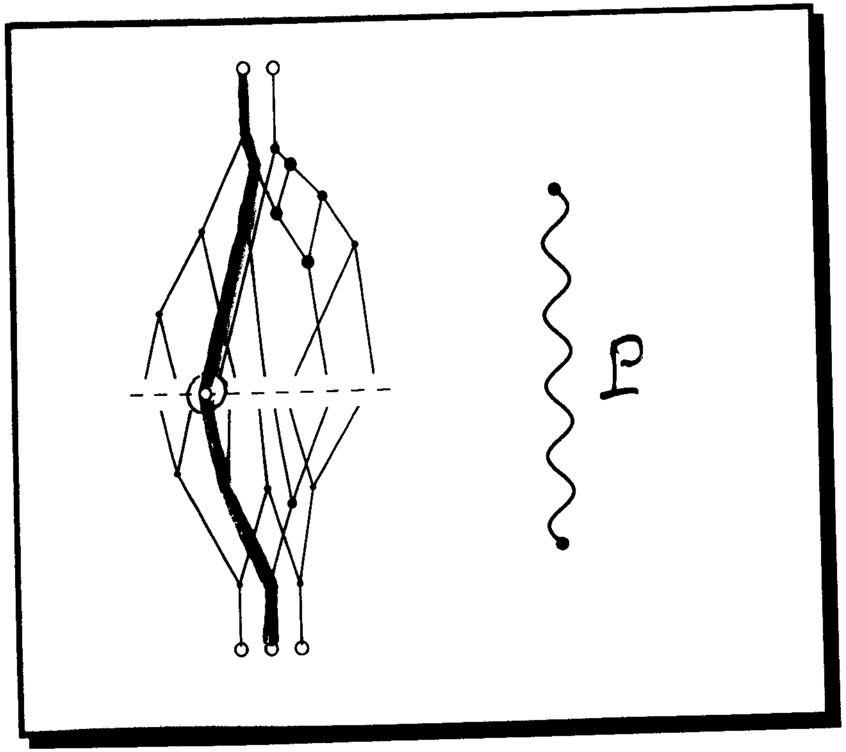
$$\Downarrow \qquad = \langle \mathcal{G}(0;b)|\hat{T}e^{\hat{H}(y+\tilde{y})}|G(0;0)\rangle = \langle \mathcal{G}(0;b)|\hat{T}|G(y+\tilde{y};0)\rangle$$

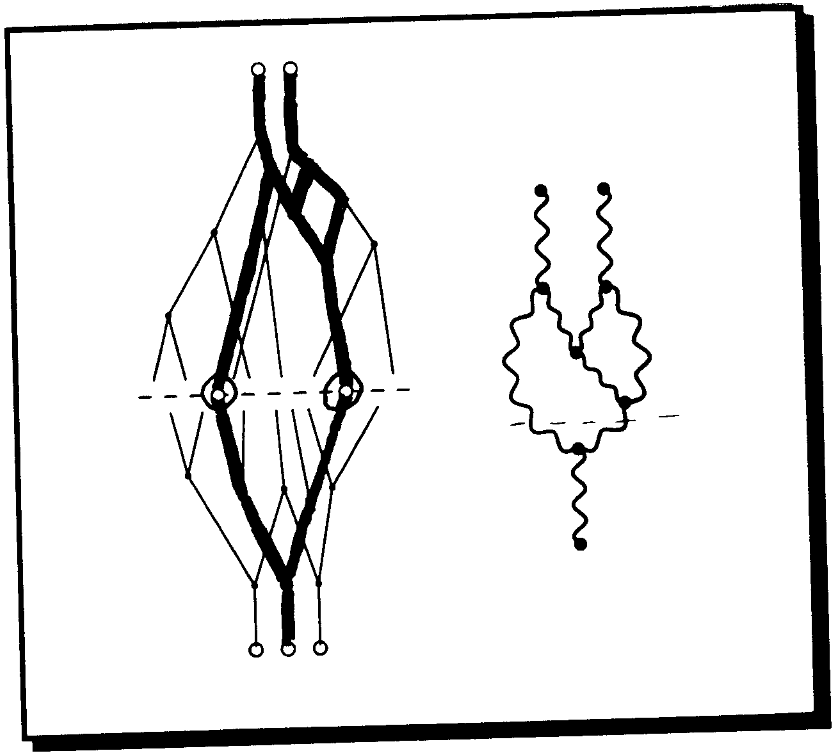
holds only if $\boxed{2m_2+\nu = \epsilon\lambda}$ $\Rightarrow$ relations among regge vertices

Looks rather natural because splitting in one frame is fusion in another frame.



Correspondence to reggeon graphs

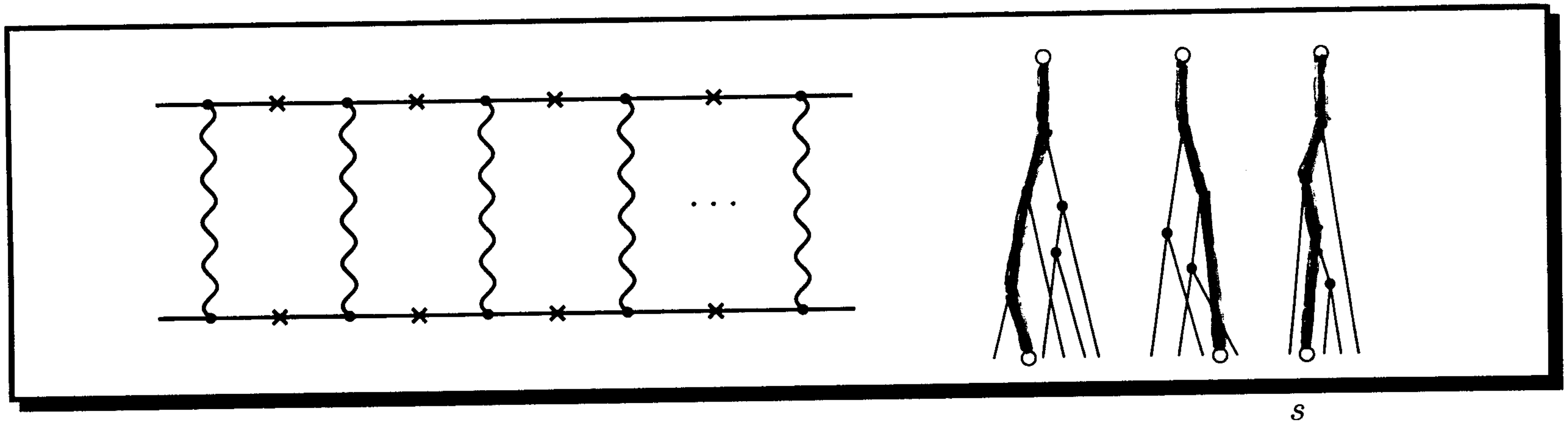




One-pomeron exchange – one-parton density ($s = 1$)

Double-pomeron exchange – two-particle density ($s = 2$), etc

The eikonal approximation



- Independent partons in initial set

$$f_s = \prod_{a=1}^{s} f_1(y; z_a)$$

- Independent evolution of each parton

$$\tilde{f}_s = \prod_{a=1}^{s} f_1(\tilde{y}; \tilde{z}_a)$$

$$T^{eik}(Y,b) = \sum_{s=1}^{\infty} \frac{(-1)^{s-1}}{s!} [\chi(Y,b)]^s = 1 - e^{-\chi(Y,b)}$$

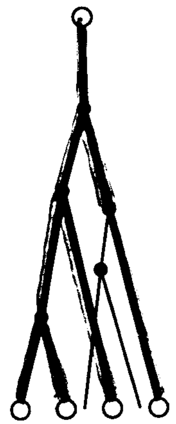
$$\chi(Y,b) = \epsilon \int dz f_1(y,z) \tilde{f}_1(\tilde{y}, z-b) \quad \rightarrow \quad \frac{\epsilon}{4\pi DY} \exp(-b^2/4DY)$$

$e^{\Delta Y}$

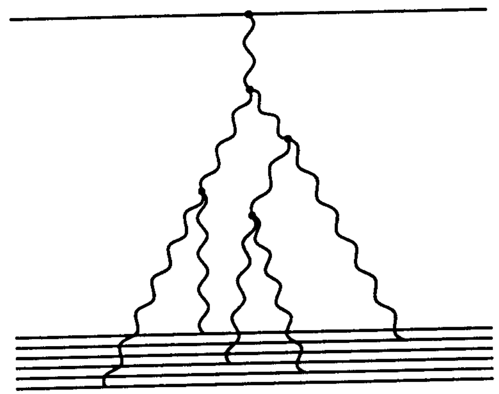
Note that for $\Delta > 0$ the eikonal approximation looks inconsistent

The Schwimmer approximation

Splitting without fusion ($\lambda \neq 0, \nu = 0$)



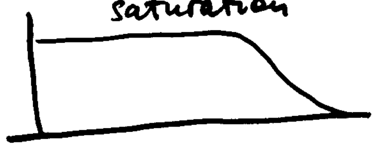
$$T_{sch}(Y) = \frac{g_1 g_2 e^{\Delta Y}}{1 + g_2 \frac{\lambda}{\Delta}(e^{\Delta Y} - 1)}$$



The Schwimmer approximation is still inconsistent for different frames
splitting → fusion

Splitting + fusion give the Froissart regime at $Y \to \infty$

$$\boxed{R^2(Y) = 4\lambda DY}$$



Advantages of the Model

- Fock wave function of fast hadron
- Lorentz-invariant partonic interpretation of high-energy interactions
- Reveals inconsistencies of commonly used RFT approximations
- New analytic methods for RFT analysis:
  - thermodynamical methods
  - stochastic diff eqs
- Simple MC algorithm
  allowing numerical summation of the RFT diagrams

Problems

(some are inherent to RFT itself)

- ✦ Matching with perturbative calculations
- ✦ Identification of soft partons
- ✦ Role of quantum numbers of partons
- ✦ Calculation of cross sections for particular inelastic processes
- ✦ Account for energy-momentum conservation
- ✦ ...

First steps ?

- Identification of reggeon diagrams from parton set history
- Estimate of role of enhanced diagrams at RHIC and LHC energies
- Generalization to several types (and scales) of partons
- Numerical analysis of the Froissart regime onset
- Calculation of diffractive dissociation cross section (gap events)
- ???