The SAMPER project

(Semi-numerical AMPlitude EvaluatoR)

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(Phys.Rev.D72:054018,2005)

- Next-to-Leading Order: LHC requirements
- Issues in Loop calculus: the semi-numerical approach
- First proof of the method: Higgs + 4 partons at one loop
- Future directions: multiplicity and complexity

Next–to–Leading Order: LHC requirements

From the TEVATRON experiments we learned:

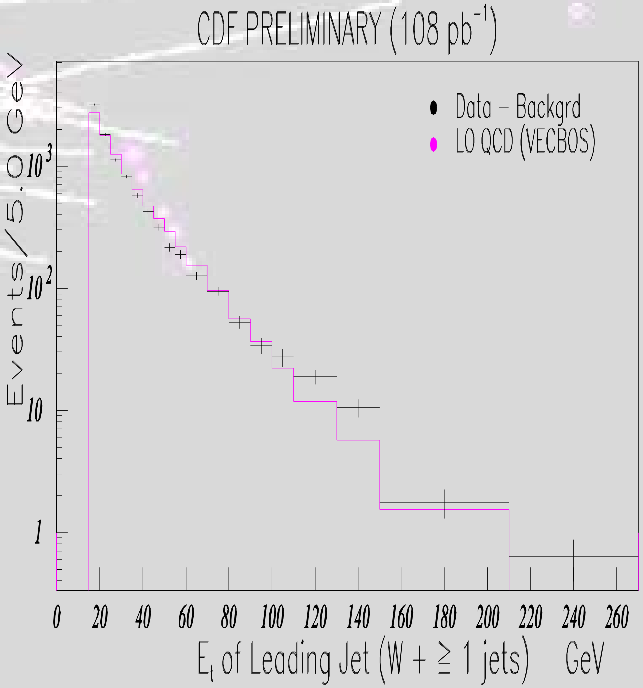
- Leading order works well for most *shape* predictions, but fail as far as cross section normalization goes.

- However, normalization fails.

At Next-to-Leading order:

- Understanding of the uncertainty on the shape of distribution.

- A first estimate of the cross section normalization. (i.e. a definition of the strong coupling constant)



Given the expected precision and types of searches at the LHC Next-to-Leading Order predictions are highly desirable.

Next–to–Leading Order: LHC requirements

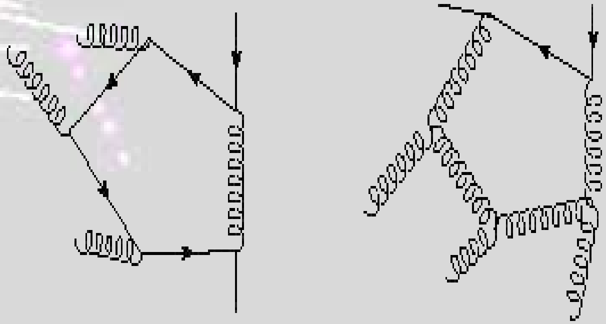
A start of the basic NLO needs for a serious phenomenology program at the LHC (from Les Houches 2005):

2. PP V V + jet (new physics and Higgs search background)
3. PP H + 2 jets (Higgs production through vector boson fusion background)
4. PP T Tbar + B Bbar (Higgs plus top quark background)
5. PP V V + B Bbar (new physics and Higgs search background)
6. PP V V + 2 jets (Higgs search background)
7. PP V + 3 jets (Generic background)
8. PP V V V (background to tri-lepton searchs)
9. Etcetera

$V \in \{ W, Z, \gamma \}$

Issues in Loop calculus

- At Leading order we have seen great progress through recursive techniques and algebraic computations.
- One loop calculations are still a “cottage industry”, much like Leading order ~20 years ago.
- To get anywhere near the priority list processes radical new techniques have to be introduced.
- Like at Leading order we want to reformulate the one loop calculation in an algorithm which can be numerically implemented.
- We will have to deal with analytical loop graphs: semi-numerical approach.



QGRAF-diagram 263

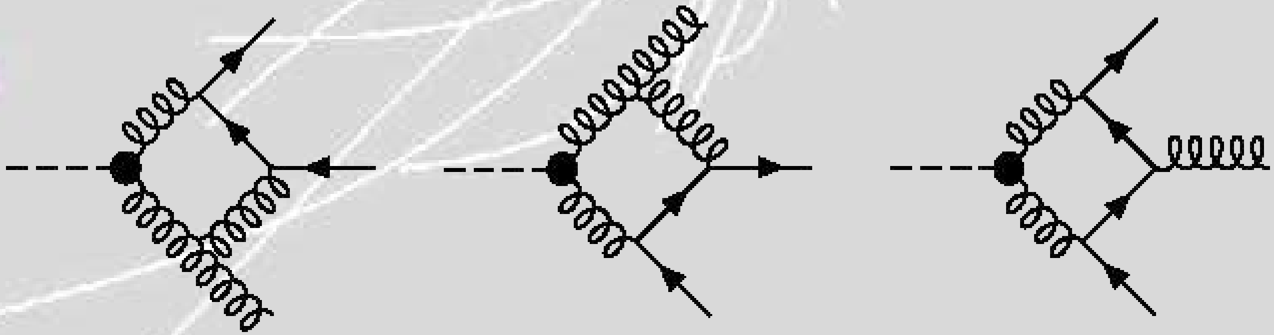
QGRAF-diagram 266

Figure 1. Sample Feynman diagrams contributing to the virtual corrections.

Issues in Loop calculus

- An one loop amplitude consists of a sum over Feyman diagrams (which can be generated using e.g. a fortran program like QGRAF):

$$\mathcal{M} = \sum_{i=1}^{N_D} A_i$$



```
c2(x1,x2)*
gg(a1,a2)*lp(k1+k2)*
c2(x3,x4)*
gg(a3,a4)*lp(-l1)*
c2(x5,x6)*
gg(a5,a6)*lp(l1-Q)*
c2(x7,x8)*
gg(a7,a8)*lp(-l1+k3)*
ggg(a-2,-k1,x-2,a-4,-k2,x-4,a1,k1+k2,x1)*
Hgg(a-1,Q,x-1,a3,-l1,x3,a5,l1-Q,x5)*
ggg(a-6,-k3,x-6,a4,l1,x4,a7,-l1+k3,x7)*
ggg(a2,-k1-k2,x2,a6,-l1+Q,x6,a8,l1-k3,x8);

G m47 = (+1)*
H(a-1)*g(a-2)*g(a-4)*g(a-6)*
c2(x1,x2)*
gg(a1,a2)*lp(k1+k3)*
c2(x3,x4)*
gg(a3,a4)*lp(-l1)*
c2(x5,x6)*
gg(a5,a6)*lp(l1-Q)*
c2(x7,x8)*
gg(a7,a8)*lp(-l1+k2)*
ggg(a-2,-k1,x-2,a-6,-k3,x-6,a1,k1+k3,x1)*
Hgg(a-1,Q,x-1,a3,-l1,x3,a5,l1-Q,x5)*
ggg(a-4,-k2,x-4,a4,l1,x4,a7,-l1+k2,x7)*
ggg(a2,-k1-k3,x2,a6,-l1+Q,x6,a8,l1-k2,x8);

G m48 = (+1)*
H(a-1)*g(a-2)*g(a-4)*g(a-6)*
c2(x1,x2)*
gg(a1,a2)*lp(k2+k3)*
c2(x3,x4)*
gg(a3,a4)*lp(-l1)*
c2(x5,x6)*
gg(a5,a6)*lp(l1-Q)*
c2(x7,x8)*
gg(a7,a8)*lp(-l1+k1)*
ggg(a-4,-k2,x-4,a-6,-k3,x-6,a1,k2+k3,x1)*
Hgg(a-1,Q,x-1,a3,-l1,x3,a5,l1-Q,x5)*
ggg(a-2,-k1,x-2,a4,l1,x4,a7,-l1+k1,x7)*
ggg(a2,-k2-k3,x2,a6,-l1+Q,x6,a8,l1-k1,x8);

G m49 = (+1)*
H(a-1)*g(a-2)*g(a-4)*g(a-6)*
c2(x1,x2)*
gg(a1,a2)*lp(-l1)*
c2(x3,x4)*
gg(a3,a4)*lp(l1-Q)*
c2(x5,x6)*
gg(a5,a6)*lp(-l1+k1)*
c2(x7,x8)*
gg(a7,a8)*lp(l1-Q+k2)*
Hgg(a-1,Q,x-1,a1,-l1,x1,a3,l1-Q,x3)*
ggg(a-2,-k1,x-2,a2,l1,x2,a5,-l1+k1,x5)*
ggg(a-4,-k2,x-4,a4,-l1+Q,x4,a7,l1-Q+k2,x7)*
```

Issues in Loop calculus

- Next each Feynman diagram is factorized in tensor integrals and kinematic factors using FORM

$$\mathcal{A}(p_1,\ldots,p_N;\varepsilon_1,\ldots,\varepsilon_N)=\sum_{M=0}^{N}K_{\mu_1\cdots\mu_M}(p_1,\ldots,p_N;\varepsilon_1,\ldots,\varepsilon_N)\,I^{\mu_1\cdots\mu_M}(D;q_1,\ldots,q_N)$$

$$I^{\mu_1\cdots\mu_M}(D;q_1,\ldots,q_N)\equiv\int\frac{d^Dl}{i\pi^{D/2}}\,\frac{l^{\mu_1}\ldots l^{\mu_M}}{d_1d_2\cdots d_N},\qquad d_i\equiv(l+q_i)^2,\qquad q_i\equiv\sum_{j=1}^{i}p_j$$

```
if (count(L,1)==5);
Id L(a0?)*L(a1?)*L(a2?)*L(a3?)*L(a4?)*I'N'(D?,?q)=
  1/4*((d_(a0,a1)*d_(a2,a3)+d_(a0,a2)*d_(a1,a3)+d_(a0,a3)*d_(a1,a2))*P
(a4,?q)
      +(d_(a0,a1)*d_(a2,a4)+d_(a0,a2)*d_(a1,a4)+d_(a0,a4)*d_(a1,a2))*P
(a3,?q)
      +(d_(a0,a1)*d_(a3,a4)+d_(a0,a3)*d_(a1,a4)+d_(a0,a4)*d_(a1,a3))*P
(a2,?q)
      +(d_(a0,a2)*d_(a3,a4)+d_(a0,a3)*d_(a2,a4)+d_(a0,a4)*d_(a2,a3))*P
(a1,?q)
      +(d_(a1,a2)*d_(a3,a4)+d_(a1,a3)*d_(a2,a4)+d_(a1,a4)*d_(a2,a3))*P
(a0,?q))*I'N'(D+4,?q)
 -1/2*(d_(a0,a1)*P(a2,?q)*P(a3,?q)*P(a4,?q)+d_(a0,a2)*P(a1,?q)*P(a3,?q)*P
```

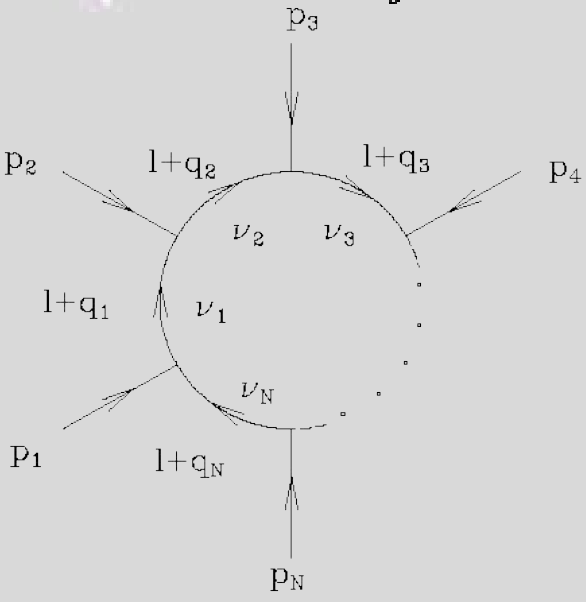


FIG. 1: The generic N-point loop graph.

Issues in Loop calculus

- We can evaluate the kinematic factors numerically
- The tensor integrals have to be further processed using a decomposition

$$
\begin{aligned}
I^{\mu_1\cdots\mu_M}(D;q_1,\ldots,q_N) = & \sum_{i_1 i_2\cdots i_M}^{N} a_{i_1 i_2\cdots i_M}\, q_{i_1}^{\mu_1} q_{i_2}^{\mu_2}\cdots q_{i_M}^{\mu_M} \\
& + \sum_{i_3 i_4\cdots i_M}^{N} a_{00 i_3 i_4\cdots i_M}\, g^{\mu_1\mu_2} q_{i_3}^{\mu_3} q_{i_4}^{\mu_4}\cdots q_{i_M}^{\mu_M} \\
& + \sum_{i_5 i_6\cdots i_M}^{N} a_{0000 i_5\cdots i_M}\, g^{\mu_1\mu_2} g^{\mu_3\mu_4} q_{i_5}^{\mu_5} q_{i_6}^{\mu_6}\cdots q_{i_M}^{\mu_M} \\
& + \cdots,
\end{aligned}
$$

- The coefficients are proportional to generalized scalar integrals (Davydychev, 1991)

$$
I(D;\{q_1,\nu_1\},\ldots,\{q_N,\nu_N\}) \equiv I(D;\nu_1,\nu_2,\ldots,\nu_N) \equiv \int \frac{d^D l}{i\pi^{D/2}} \frac{1}{d_1^{\nu_1} d_2^{\nu_2}\cdots d_N^{\nu_N}}
$$

Issues in Loop calculus

- Example:

$$\begin{aligned}
I^{\mu_1\mu_2}(D;q_1,\dots,q_N) &= -\frac{1}{2}\, I(D+2;\{1\}_{k=1}^{N})\; g^{\mu_1\mu_2} \\
&\quad + \sum_{P\{\mu_1,\mu_2\}}^{2!} \sum_{i_1\le i_2}^{N} I(D+4;\{1+\delta_{i_1 k}+\delta_{i_2 k}\}_{k=1}^{N})\; q_{i_1}^{\mu_1} q_{i_2}^{\mu_2} \\
&= -\frac{1}{2} I(D+2;1,1,1,\dots,1)\; g^{\mu_1\mu_2} \\
&\quad +2\, I(D+4;3,1,1,\dots,1)\; q_1^{\mu_1} q_1^{\mu_2} \\
&\quad +\; I(D+4;2,2,1,\dots,1)\; (q_1^{\mu_1} q_2^{\mu_2} + q_1^{\mu_2} q_2^{\mu_1}) \\
&\quad +\cdots ,
\end{aligned}$$

- This can be implemented numerically *provided* we can evaluate the generalized scalar integrals numerically.

```
template <class T1,class T2,typename Y> Y Contract(Tensor<T1> v1,Tensor<T2> v2) {
  int rank=v1.rank;
  assert(v2.rank==rank);
  Y result=0.0;
  double g[4]={1.0,-1.0,-1.0,-1.0};
  if (rank==0) result=v1.t0*v2.t0;
  else { for (int i1=0;i1<4;i1++) {
```

Issues in Loop calculus

- The generalized scalar integrals can be calculated recursively, e.g. (red line):

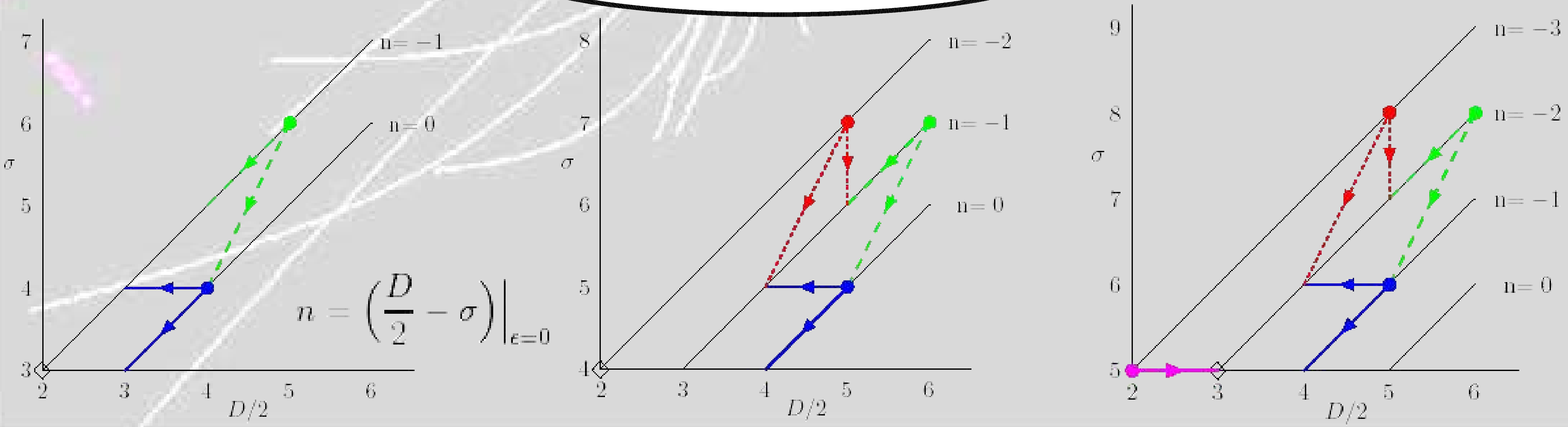
Based on integration by part techniques: Tkachov & Chetyrkin, 1981

$$(\nu_l - 1) I(D; \{\nu_k\}_{k=1}^{N})$$

$$= -\frac{b_l}{B} I(D-2; \{\nu_k - \delta_{lk}\}_{k=1}^{N}) + \sum_{i=1}^{N} \left(\frac{b_l b_i}{B} - S_{li}^{-1} \right) I(D-2; \{\nu_k - \delta_{ik} - \delta_{lk}\}_{k=1}^{N}).$$

$$S_{ij} \equiv (q_i - q_j)^2 \qquad \sigma \equiv \sum_{i=1}^{N} \nu_i; \qquad b_i \equiv \sum_{j=1}^{N} S_{ij}^{-1}; \qquad B \equiv \sum_{i=1}^{N} b_i = \sum_{i,j=1}^{N} S_{ij}^{-1}$$

Numerically calculable



$$n = \left(\frac{D}{2} - \sigma \right) \Big|_{\epsilon=0}$$

Issues in Loop calculus

- We can setup a recursion algorithm which reduces all generalized scalar integrals to a few standard scalar integrals (which are known analytical)

Glover & Giele, 2004

- Example:

$$I_4(D=4-2\epsilon;1,1,1,1,\{s_{12},s_{23},0,p_2^2,p_3^2,p_4^2\}) = \frac{c_\Gamma}{(s_{23}s_{12}-p_2^2p_4^2)}$$

$$\times \Big[\frac{2}{\epsilon^2}\Big((-s_{12})^{-\epsilon}+(-s_{23})^{-\epsilon}-(-p_2^2)^{-\epsilon}-(-p_3^2)^{-\epsilon}-(-p_4^2)^{-\epsilon}\Big)$$

$$+\frac{1}{\epsilon^2}\Big((-p_2^2)^{-\epsilon}(-p_3^2)^{-\epsilon}\Big)/(-s_{23})^{-\epsilon}+\frac{1}{\epsilon^2}\Big((-p_3^2)^{-\epsilon}(-p_4^2)^{-\epsilon}\Big)/(-s_{12})^{-\epsilon}$$

$$-2\,\mathrm{Li}_2\left(1-\frac{p_2^2}{s_{12}}\right)-2\,\mathrm{Li}_2\left(1-\frac{p_4^2}{s_{23}}\right)+2\,\mathrm{Li}_2\left(1-\frac{p_2^2p_4^2}{s_{12}s_{23}}\right)\Big]+\mathcal{O}(\epsilon)$$

Numerically calculable

Issues in Loop calculus

- This means we can completely evaluate the tensor loop integral numerical

$$
\begin{aligned}
I^{\mu_1\cdots\mu_M}(D;q_1,\ldots,q_N) &= \sum_{i_1 i_2\cdots i_M}^{N} a_{i_1 i_2\cdots i_M}\, q_{i_1}^{\mu_1} q_{i_2}^{\mu_2}\cdots q_{i_M}^{\mu_M} \\
&+ \sum_{i_3 i_4\cdots i_M}^{N} a_{00 i_3 i_4\cdots i_M}\, g^{\mu_1\mu_2} q_{i_3}^{\mu_3} q_{i_4}^{\mu_4}\cdots q_{i_M}^{\mu_M} \\
&+ \sum_{i_5 i_6\cdots i_M}^{N} a_{0000 i_5\cdots i_M}\, g^{\mu_1\mu_2} g^{\mu_3\mu_4} q_{i_5}^{\mu_5} q_{i_6}^{\mu_6}\cdots q_{i_M}^{\mu_M} \\
&+ \cdots,
\end{aligned}
$$

- and consequently each Feynman diagram numerical

$$
\mathcal{A}(p_1,\ldots,p_N;\varepsilon_1,\ldots,\varepsilon_N) = \sum_{M=0}^{N} K_{\mu_1\cdots\mu_M}(p_1,\ldots,p_N;\varepsilon_1,\ldots,\varepsilon_N)\, I^{\mu_1\cdots\mu_M}(D;q_1,\ldots,q_N)
$$

Issues in Loop calculus

- The final issue is instabilities in the recursion relation due to the fact that the kinematic matrix *S* can be (almost) singular in certain regions of phase space (for example the Higgs momentum is small).
- Solution is to rewrite the recursion relations as expansions in the small determinant of the matrix *S*

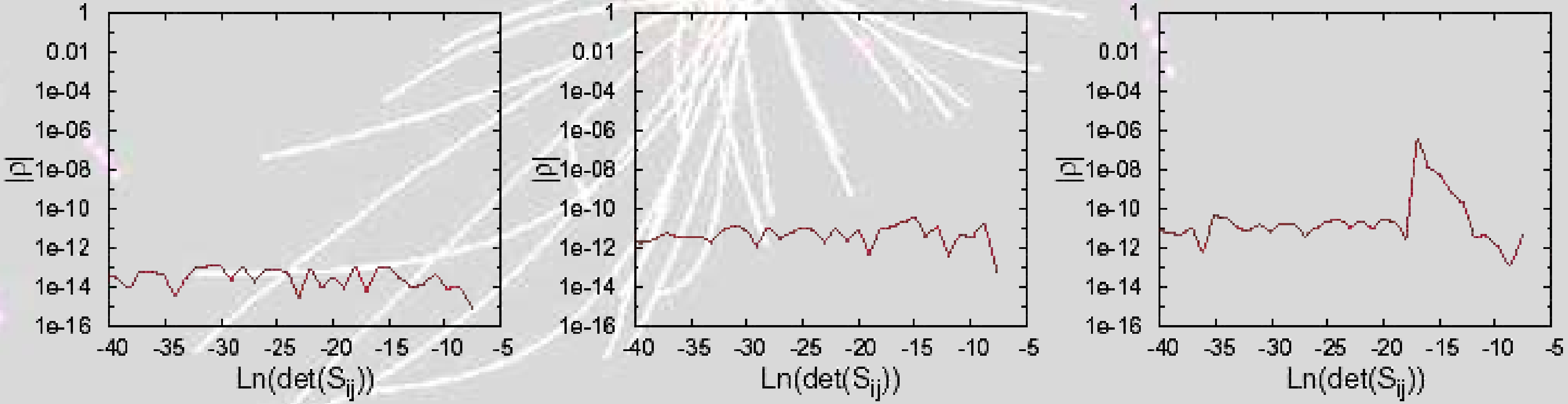


FIG. 6: Relative accuracy $|\rho|$ for the $1/\epsilon^2$ pole (left), the $1/\epsilon$ pole (center) and the constant part (right) of the one-loop amplitude squared for $H \to q\bar{q}q'\bar{q}'$ as a function of the determinant of the kinematics matrix S_{ij}.

Proof of method

- It is easy to come up with calculational schemes, but its only worth something if one can calculate new processes
- We chose *H + 4 partons* (one of the processes on the Les Houches list)
- We calculated *H + 4 quarks* both analytic and numerical
- We calculated *H + 2 quarks + 2 gluons* and *H + 4 gluons* numerical (with gauge invariance tests etc…)

$$
\begin{aligned}
Y^A(k_1,k_2,k_3,k_4) = &-N_c \frac{c_\Gamma \mu^{2\epsilon}}{\epsilon^2}\Big[(-s_{14})^{-\epsilon}+(-s_{23})^{-\epsilon}\Big] \\
&+ \frac{1}{N_c}\frac{c_\Gamma \mu^{2\epsilon}}{\epsilon^2}\Big[(-s_{12})^{-\epsilon}+(-s_{34})^{-\epsilon}-2(-s_{13})^{-\epsilon}+2(-s_{14})^{-\epsilon}+2(-s_{23})^{-\epsilon}-2(-s_{24})^{-\epsilon})\Big] \\
&- \frac{c_\Gamma \mu^{2\epsilon}}{\epsilon}\Big[3C_f-b_0\Big]\Big[(-s_{12})^{-\epsilon}+(-s_{34})^{-\epsilon}\Big]-\frac{20}{9}n_f+\frac{152}{9}N_c-16C_f \\
&+ \frac{1}{N_c}\Big[\mathrm{Ls}^{2\mathrm{me}}_{-1}(s_{134},s_{234};s_{34},M_H^2)+\mathrm{Ls}^{2\mathrm{me}}_{-1}(s_{123},s_{124};s_{12},M_H^2)\Big] \\
&- \frac{2}{N_c}\Big[\mathrm{Ls}^{2\mathrm{me}}_{-1}(s_{123},s_{134};s_{13},M_H^2)+\mathrm{Ls}^{2\mathrm{me}}_{-1}(s_{124},s_{234};s_{24},M_H^2)\Big] \\
&- (N_c-\frac{2}{N_c})\Big[\mathrm{Ls}^{2\mathrm{me}}_{-1}(s_{124},s_{134};s_{14},M_H^2)+\mathrm{Ls}^{2\mathrm{me}}_{-1}(s_{123},s_{234};s_{23},M_H^2)\Big], \qquad (29)
\end{aligned}
$$

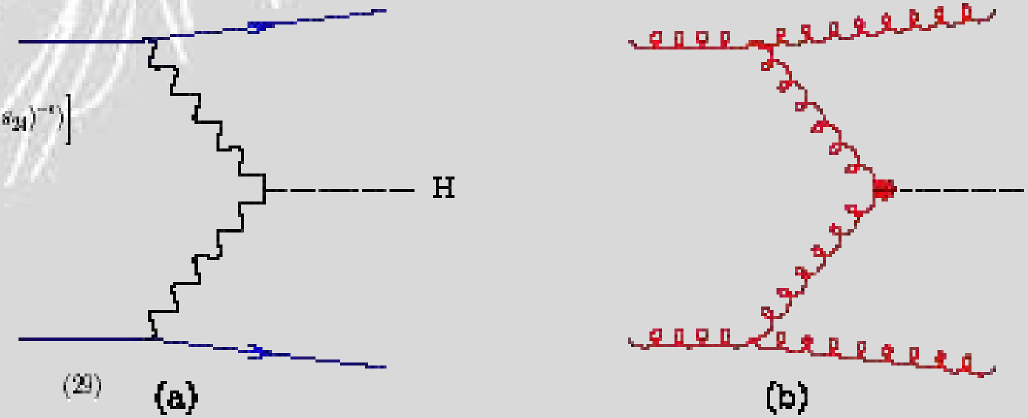
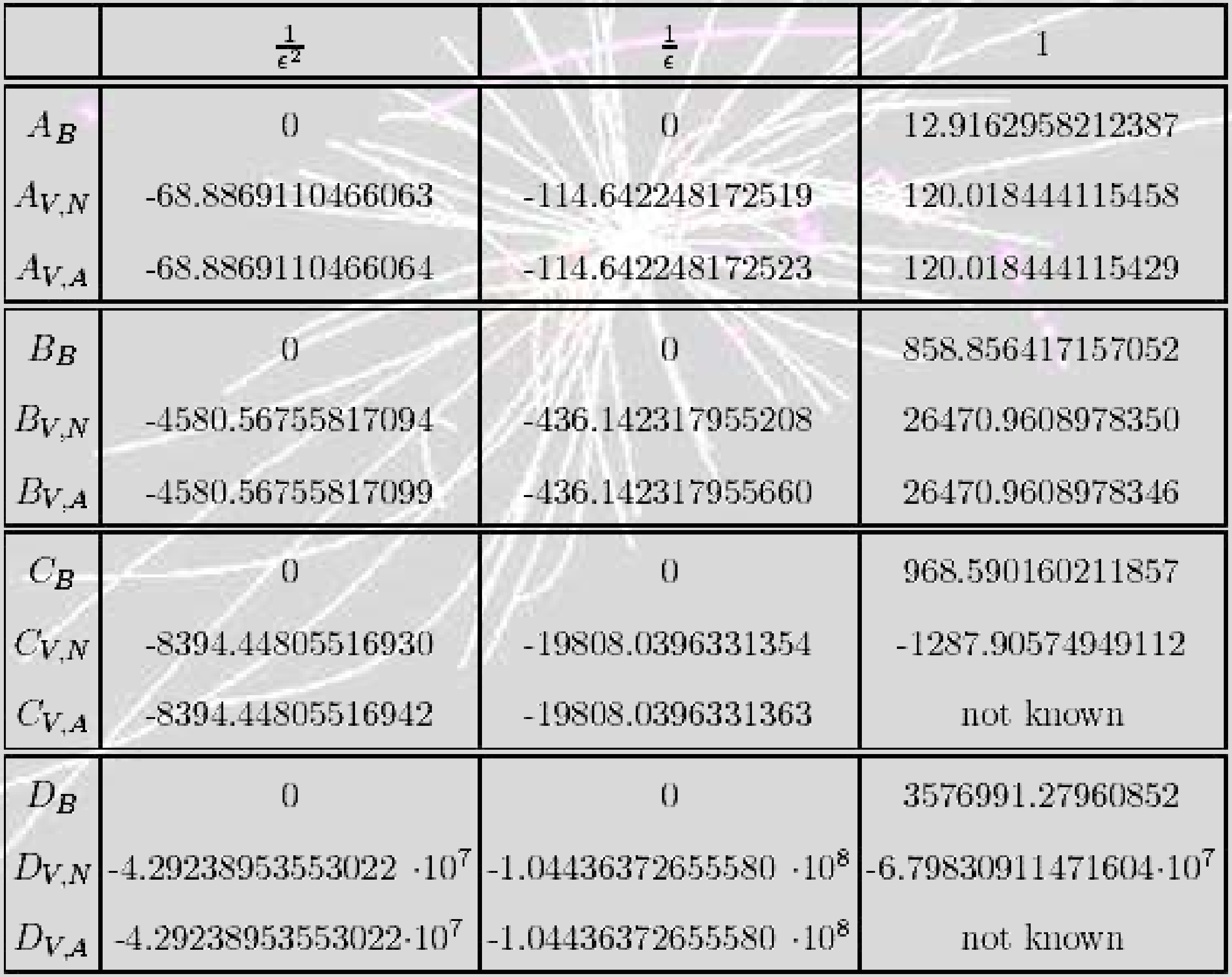


FIG. 1: (a) Lowest order process for vector boson fusion. (b) Example of a diagram contributing to the gluon fusion process in association with two jets.

Proof of method

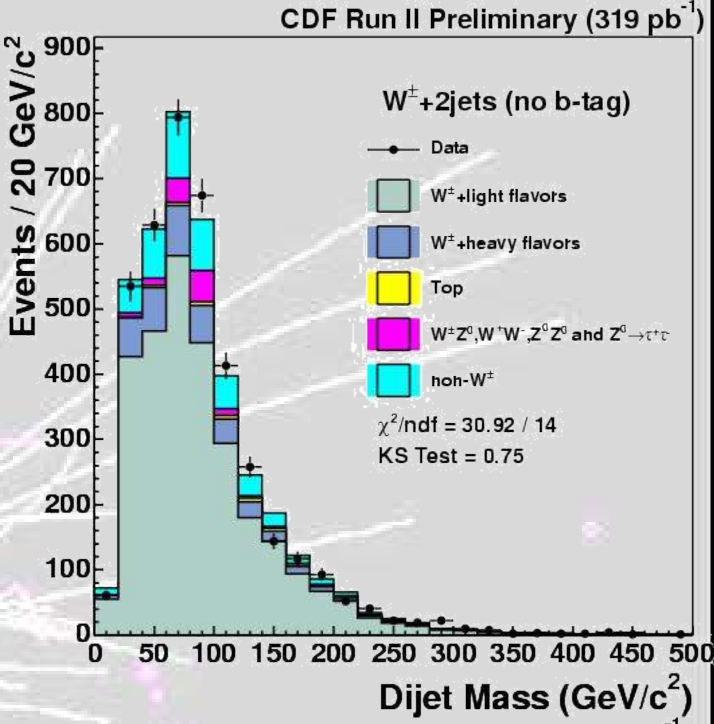
- Results for a phase space point:

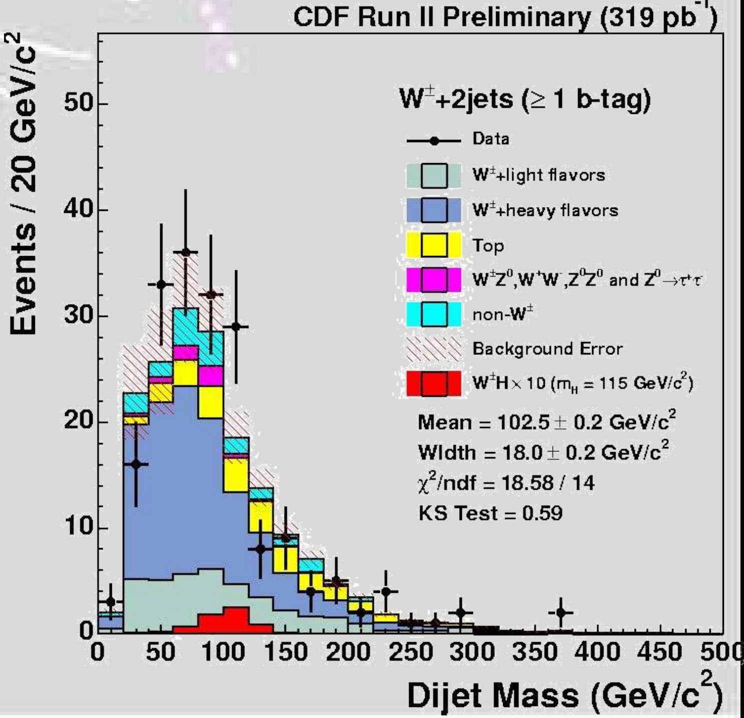


		$\frac{1}{\epsilon^2}$	$\frac{1}{\epsilon}$	1	
	A_B	0	0	12.9162958212387	Born
H+qq+qq	$A_{V,N}$	-68.8869110466063	-114.642248172519	120.018444115458	Numerical
	$A_{V,A}$	-68.8869110466064	-114.642248172523	120.018444115429	Analytic
	B_B	0	0	858.856417157052	
H+qq+q'q'	$B_{V,N}$	-4580.56755817094	-436.142317955208	26470.9608978350	
	$B_{V,A}$	-4580.56755817099	-436.142317955660	26470.9608978346	
	C_B	0	0	968.590160211857	
H+qq+gg	$C_{V,N}$	-8394.44805516930	-19808.0396331354	-1287.90574949112	
	$C_{V,A}$	-8394.44805516942	-19808.0396331363	not known	
	D_B	0	0	3576991.27960852	
H+gggg	$D_{V,N}$	$-4.29238953553022 \cdot 10^7$	$-1.04436372655580 \cdot 10^8$	$-6.79830911471604 \cdot 10^7$	
	$D_{V,A}$	$-4.29238953553022 \cdot 10^7$	$-1.04436372655580 \cdot 10^8$	not known	

Future Directions

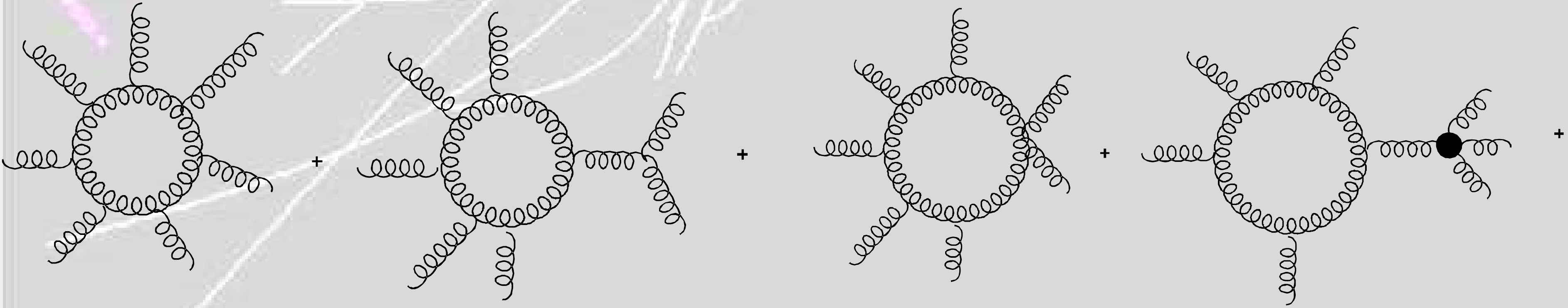
- At this point we are sure we can calculate *2   3* one loop amplitudes efficiently and maintain numerical stability all over phase space.
- This leaves us with implementing the virtual corrections in a monte carlo and add the unresolved Leading order *2   4* contributions. Currently this is in progress for the process *PP   H + 2 jets* through gluon fusion.
- When successful we can start implementing *all 2   3* processes for the LHC at Next-to-Leading order.





Future Directions

- How about going beyond *2   3* processes (e.g. *PP   TTbar+BBbar* or even *PP   W+4 jets*) ?
- We have started an “exploration project” to develop methods (simply running QGRAF will not do) and gauge required computer resources for running such a monte carlo.
- Following the Leading order developments the perfect process to do all of this is *gluon gluon   n gluons* pushing *n* as high a possible.



Conclusions

- We have a method which will give us the ability to calculate the relevant *2 3* processes at Next-to-Leading order for the LHC.
- Given enough manpower and computer resources this can be done in a relatively short time span.
- We are exploring the feasibility of going beyond *2 3* processes. The limitations will in the end be computer resources (just as it is for Leading order processes).