Multipactor effect in coaxial cables and dielectric-loaded waveguides.
Study of the electromagnetic spectrum radiated by a multipactor discharge

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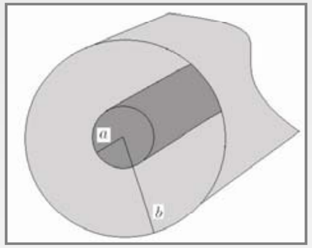
Theoretical model (1)



Objective: Study of the multipactor effect in coaxial waveguides for space telecom applications.

- A numerical algorithm **to predict multipactor breakdown voltage threshold** in coaxial guides has been implemented based on the **effective electron concept**.

- The TEM mode has been considered (electric and magnetic fields):

$$\vec{E}_{RF}(\vec{r},t) = \frac{(1-R)\ V_0}{r\ln\left(\frac{b}{a}\right)}\cos\left(\omega t - \beta z + \theta_1\right)\hat{r} + \frac{R\ V_0}{r\ln\left(\frac{b}{a}\right)}\cos\left(\omega t + \beta z + \theta_2\right)\hat{r}$$

$$\vec{B}_{RF}(\vec{r},t) = \frac{(1-R)\ V_0}{c\ r\ln\left(\frac{b}{a}\right)}\cos\left(\omega t - \beta z + \theta_1\right)\hat{\varphi} - \frac{R\ V_0}{c\ r\ln\left(\frac{b}{a}\right)}\cos\left(\omega t + \beta z + \theta_2\right)\hat{\varphi}$$

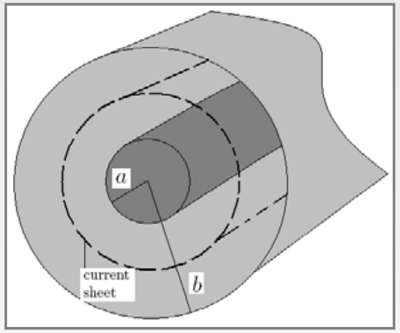
Forward and backward waves are considered:

R= Reflection coefficient

Travelling Wave (TW): R=0

Standing Wave (SW): R=0.5, θ_1=0, θ_2=π

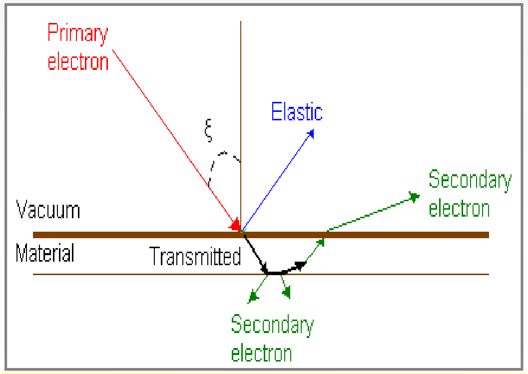
Theoretical model (2)

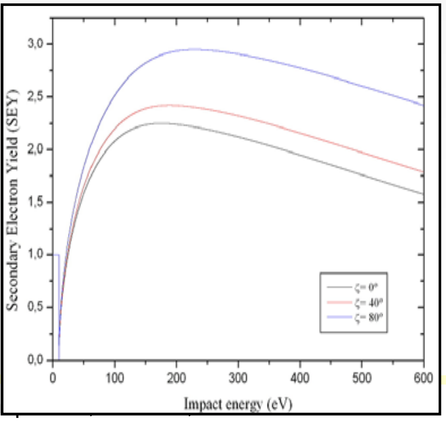


- **Model of space charge:** An electron sheet of surface density -σ is moving between electrodes.

$$\vec{E}_{sc} = \begin{cases} \frac{-\sigma r_{sc}}{\epsilon_0 r}\,\hat{r} & \text{if } r > r_{sc} \\ 0 & \text{if } r < r_{sc} \end{cases} \quad \text{where,} \quad -\sigma = \frac{-e\,N}{2\,\pi\,r_{sc}\,h}$$

- **Vaughan´s model for SEY:** Gaussian distribution of velocities and cosine law for secondary electrons

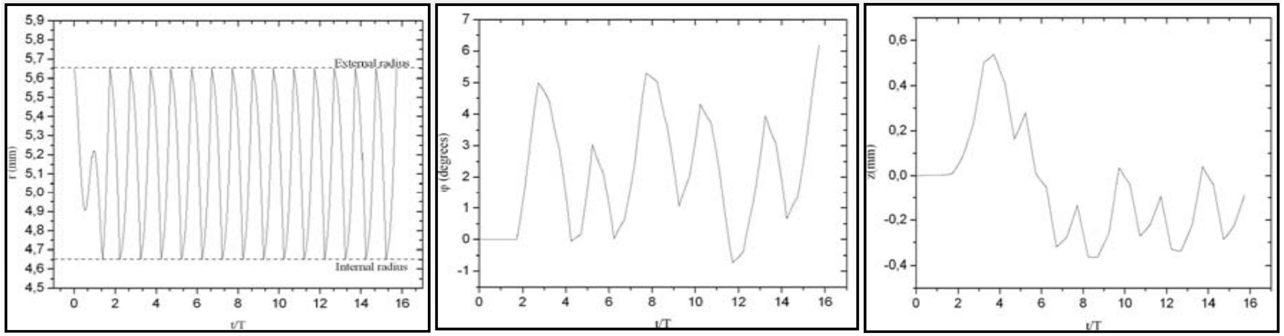




Theoretical model (3)

- **Electrons dynamics:** Velocity-Verlet algorithm

$$\vec{\mathbf{F}}_{\mathrm{L}} = q\,(\vec{\mathbf{E}}_{tot} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})\,;\ \vec{\mathbf{E}}_{tot} = \vec{\mathbf{E}}_{RF} + \vec{\mathbf{E}}_{sc}$$



Dynamics of an effective electron as a function of time is simulated

The developed software CAD tool is called **MULTICOAX**

Numerical and experimental results (1)

- Comparison with technical literature:

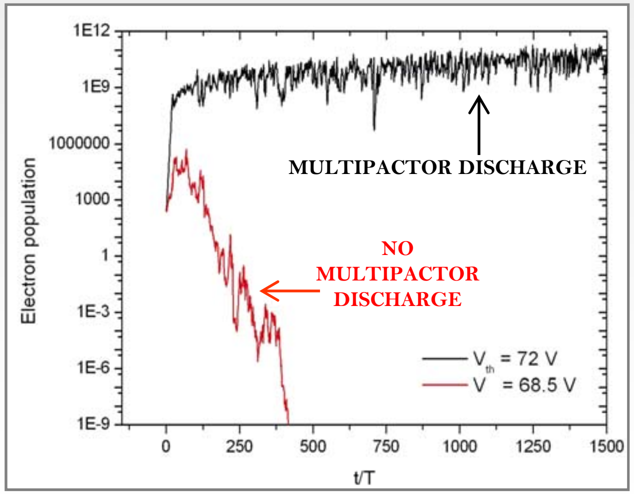
R. Woo, *J. Appl. Phys. , vol. 39, no. 13, pp. 1528-1533, 1968*

E. Somersalo, P. Ylä-Ojala, D. Porch, J. Sarvas, *Particle Accelerators , Vol. 59, pp. 107-141, 1998*

- **A coaxial sample has been designed, manufactured and measured at ESA/ESTEC Lab.:** A quarter-wave transformer at 1.35 GHz (return losses: ~20 dB)

Numerical and experimental results (2)

Criteria for multipactor breakdown voltage threshold in MULTICOAX: saturation of the electron population



Comparison between experiment and simulation of the multipactor breakdown power threshold:

COPPER

Experimental measurement	204.6 W
ESA/ESTEC Multipactor Calculator	316.2 W
MULTICOAX	209.9 W

SILVER

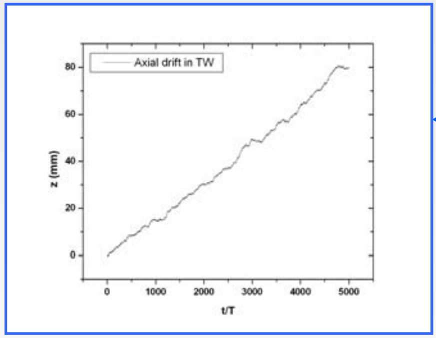
Experimental measurement	280.6 W
ESA/ESTEC Multipactor Calculator	289.4 W
MULTICOAX	286.1 W

Numerical and experimental results (3)

- Study of the axial drift in the TW and SW configurations (I):

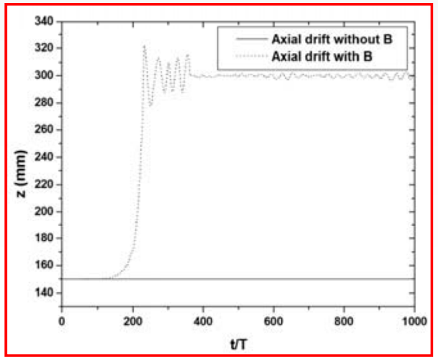
$$a_z = -\frac{q}{m} v_r B_\phi$$

TW



Electron moves in the z>0 axis (for a TW travelling in z>0 direction)

SW



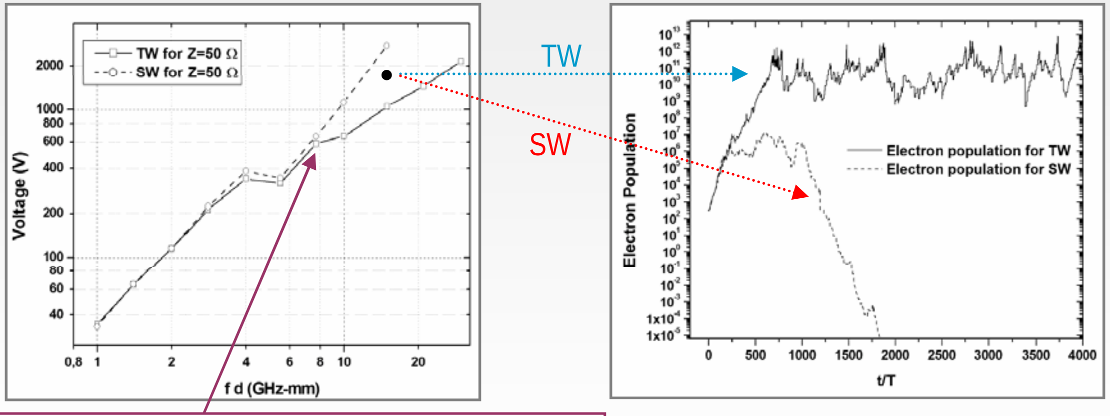
Electron moves to the ATTRACTOR (zero of the electric field SW pattern)

Multipactor discharge is extinguished !

Numerical and experimental results (4)

- Study of the axial drift in the TW and SW configurations (II):

Example: Coaxial waveguide, Z_0=50 Ω, d=20 mm



For (f d) > ~7 GHz mm TW and SW voltage threshold are splitted

Multipactor breakdown voltage threshold is higher for SW than for TW configurations for higher values of (f d)

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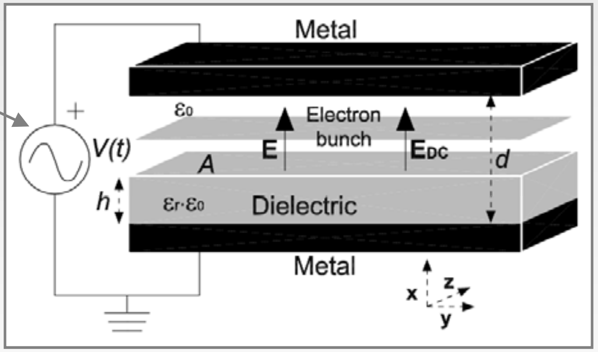
Theory

$$\boxed{V(t) = V_0 \cos(\omega t + \alpha)}$$

$$\boxed{\mathbf{E} = E_0 \cos(\omega t + \alpha)\hat{\mathbf{x}}.}$$

$$\boxed{E_0 = \frac{V_0 \varepsilon_r}{h + \varepsilon_r (d - h)}}$$

$$\boxed{E_{dc}(t + \Delta t) = E_{dc}(t) + \frac{e N_i(t)(\delta - 1)}{2 A \varepsilon_0}}$$



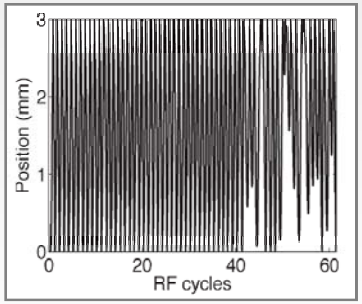
<u>Objective:</u> To study two-surface multipactor regime in a parallel-plate waveguide dielectric-loaded with a uniform slab

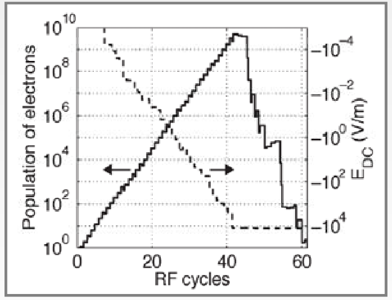
- Simultaneous tracking of multiple effecttive electrons
- Dielectric surface static charge (positive or negative) is accounted: DC electric field
- Space-charge effects are included in the simulation (a dynamic current sheet produces the electron repulsion)
- Total electric field: RF + DC + SC
- Different SEY curves for metal and dielectric; gaussian velocity distribution has been used for secondary electrons.

Simulations (1)

Mitigation of the multipactor discharge: Case study I

Material	W_{max}(eV)	W_1(eV)	W_2(eV)	W_0(eV)	δ_{max}
Silver	165.0	30.0	5000.0	16.0	2.2
Alumina1	350.0	85.5	1414.0	12.9	6.5





An electron is in resonance (first order of multipactor)

30 eV< KINETIC IMPACT ENERGY < 85.5 eV

Electrons: emitted on the metal, absorbed on the dielectric

DC electric field becames **negative**

Electrons are **repelled** on the dielectric surface

Resonance is **lost**

Electrons collide in the metal with low energy

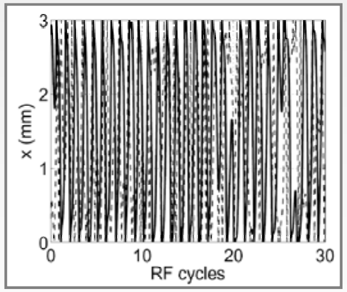
Electrons population diminish !

G. Torregrosa, Á. Coves, C. P. Vicente, A. M. Pérez, B. Gimeno, V. E. Boria, IEEE Electron Device Letters, vol. 27, no. 7, pp. 619-621, July 2006

Simulations (2)

Mitigation of the multipactor discharge: Case study II

Material	W_{max}(eV)	W_1(eV)	W_2(eV)	W_0(eV)	δ_{max}
Silver	165.0	30.0	5000.0	16.0	2.2
Alumina2	1300.0	23.8	14135.0	7.5	6.5



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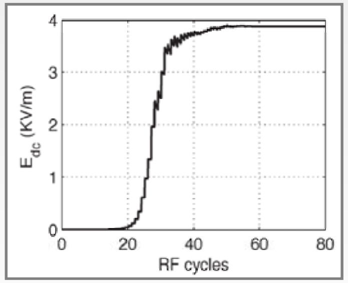

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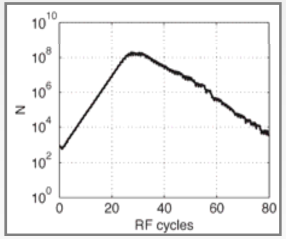

An electron is in resonance (first order) → KINETIC IMPACT ENERGY > 30.0 eV → Electrons are released on both metal and dielectric surfaces →

→ DC electric field becames positive → Electrons are attracted to the dielectric surface, and resonance is lost → Electrons are absorbed on metal → More e^- are absorbed on metal than emitted on diel. → Electrons population diminish !

G. Torregrosa, Á. Coves, C. P. Vicente, A. M. Pérez, B. Gimeno, V. E. Boria, IEEE Trans. Electron Device, vol. 55, no. 9, pp. 2505-2511, Sept. 2008

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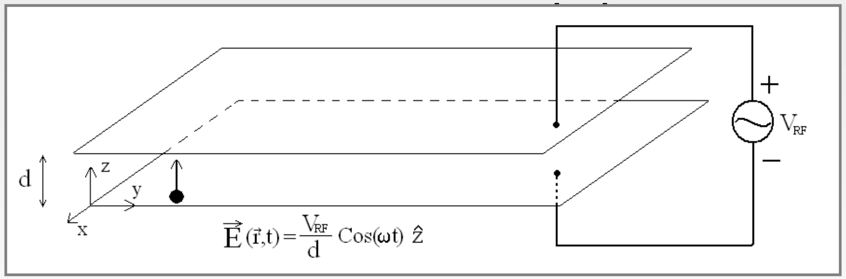
Theory_I (1)

Objective: Evaluation of the **full spectrum radiated by a multipactor discharge** occuring within a parallel-plate waveguide under an **harmonic voltage**

Application on measurements test set-up



$$\vec{E}(\vec{r},t)=\frac{V_{RF}}{d}\,\mathrm{Cos}(\omega t)\ \hat{z}$$

- We consider the 1D electron movement (along the z-direction)
- **Sombrin's model** allows to calculate in a simple way the electron kinematics and the perfect resonance conditions:

$$F_z=-e\,E=m_e\,a_e \qquad a_e(t)=-\frac{1}{m_e}\frac{e\,V_{RF}}{d}\mathrm{Cos}\,(\omega t)$$

$$v_e(t)=v_0+\frac{eV_{RF}}{m_e\omega\,d}\left[\mathrm{Sin}(\alpha)-\mathrm{Sin}(\omega t)\right]$$

$$z_e(t)=\frac{v_0}{\omega}(\omega t-\alpha)+\frac{eV_{RF}}{m_e\omega^2\,d}\left[\mathrm{Cos}(\omega t)-\mathrm{Cos}(\alpha)+(\omega t-\alpha)\mathrm{Sin}(\alpha)\right]$$

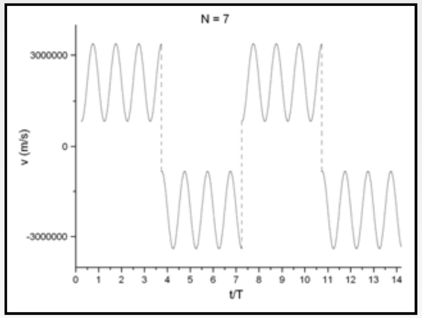
RESONANCE CONDITION

$$\underbrace{\omega t_i=\alpha+N\pi \qquad N=1,3,5,\ldots}$$

$$\boxed{\tilde{V}_{RF}=\frac{m_e\omega d\left[\omega d-N\pi v_0\right]}{e\left[N\pi\,\mathrm{Sin}(\alpha)-2\,\mathrm{Cos}(\alpha)\right]}}$$

Theory_I (2)

Velocity of an effective electron in perfect resonance conditions:

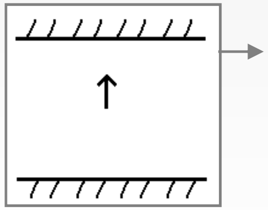


$$v_e(t)=\begin{cases} v_0+\frac{eV_{rf}}{m_e\omega d}\left(\mathrm{Sin}(\alpha)-\mathrm{Sin}(\omega t)\right), & t\in\left]t_\alpha, t_\alpha+N\frac{T}{2}\right] \\ -v_0-\frac{eV_{rf}}{m_e\omega d}\left(\mathrm{Sin}(\alpha)+\mathrm{Sin}(\omega t)\right), & t\in\left]t_\alpha+N\frac{T}{2}, t_\alpha+NT\right] \end{cases}$$

↑

Electron velocity is a periodic function of period NT

$$t_\alpha=\frac{\alpha}{\omega}$$



Electron movement is a singular point-current density:

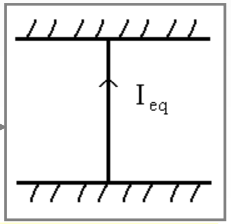
$$J_z(z,t) = -ev_e\delta\left(z-z_e(t)\right)$$

Spatial average of the electron point-current:

Observation Point >> $\frac{2d^2}{\lambda}$

Equivalent multipactor wire current

$$I(t)\equiv\langle J_z(z,t)\rangle=\frac{1}{d}\int_0^d -e\,v_e\delta\left(z-z_e(t)\right)dz=\frac{-e}{d}v_e(t)$$



Theory_I (3)

Equivalent wire current is expanded into a Fourier series:

$$I(t) = \frac{-e}{d}\sum_{m=1}^{+\infty} c_m \cos\left(m\frac{\omega}{N}t + \psi_m\right)$$

where expansion coefficients are obtained in a simple closed form:

$$a_m = \frac{-2\,(1-(-1)^m)\,(v_{e_0} + G\sin\alpha)\,\sin(m\alpha/N)}{m\pi}$$

$$b_m = -G\delta_{m,N} + \frac{2\,(1-(-1)^m)\,(v_{e_0} + G\sin\alpha)\,\cos(m\alpha/N)}{m\pi}$$

$$m = 1, 3, 5, \ldots$$

$$c_m = \sqrt{a_m^2 + b_m^2} \text{ and } \psi_m = -\arctan(b_m/a_m)$$

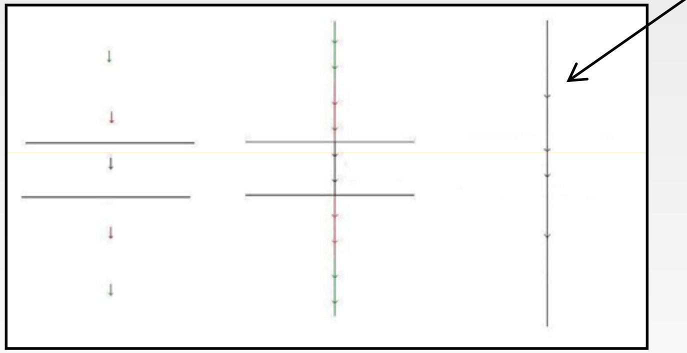
The equivalent wire current is expressed in phasor complex currents radiating at discrete frequencies:

$$I(t) = \Re\left(\sum_{m=1}^{+\infty} i_m e^{jm\frac{\omega}{N}t}\right)$$

$$i_m \equiv \frac{-e}{d}\, c_m\, e^{j\psi_m}, \qquad m = 1, 3, 5, \ldots$$

Theory_I (4)

Radiation of the equivalent wire current in the parallel-plate scenario is considered using image theory:



$$\phi_m = j\frac{c}{k_m}\nabla\cdot(A_{z_m}\hat{z}) = j\frac{c}{k_m}\frac{\partial A_{z_m}}{\partial z} = 0$$

This infinite wire current radiates in free-space

Free-space Green´s function is used to calculate the electromagnetic radiated field pattern in terms of scalar electric and vector magnetic potentials:

$$g_0(\vec{r},\vec{r}\,') = \frac{e^{-jkR}}{4\pi R}$$

$$A_{z_m}(\vec{r}) = \mu_0 i_m \int_{-\infty}^{+\infty} g_0(\vec{r},\vec{r}\,')dz'$$

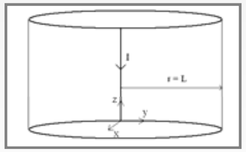
$$\begin{aligned} A_{z_m}(\vec{r}) &= \frac{\mu_0}{4\pi} i_m \int_{-\infty}^{+\infty} \frac{\exp\left(-jk_m\sqrt{r^2+(z-z')^2}\right)}{\sqrt{r^2+(z-z')^2}}dz' \\ &= \frac{\mu_0}{4\pi} i_m(-j)\pi H_0^{(2)}(k_m r) \\ &= \mu_0 i_m\left(\frac{H_0^{(2)}(k_m r)}{4j}\right) \end{aligned}$$

Theory_I (5)

Electric and magnetic are analytically evaluated in the far-field region:

$$\boxed{H_0^{(2)}(x) \approx \sqrt{\frac{2}{\pi x}}\, e^{-j(x-\pi/4)}} \longrightarrow \boxed{\begin{aligned} \vec{E}_m &= -jk_m c A_{z_m}\hat{z} \approx -\eta i_m \sqrt{\frac{k_m}{8\pi r}}\, e^{-j(k_m r-\pi/4)}\hat{z} \\ \vec{H}_m &= \frac{1}{\mu_0}\nabla\times(A_{z_m}\hat{z}) \approx i_m \sqrt{\frac{k_m}{8\pi r}}\, e^{-j(k_m r-\pi/4)}\hat{\varphi}. \end{aligned}}$$

Integration of the complex Poynting´s vector in a cylindrical surface allows to evaluate the **total radiated power by the multipactor discharge in a closed analytical expression**:



$$\vec{N}_n^{rad}(r) = \vec{E}_n^{rad} \times \vec{H}_n^{*rad} \longrightarrow P_n^{rad} = \frac{1}{2}\Re\left[\int_{S(r=L)} \vec{N}_n^{rad}\cdot d\vec{S}\right] \longrightarrow$$

$$\boxed{P_n^{rad} = \frac{\mu_0\,\omega\,n\,e^2}{8\,N\,d}\left[\delta_{N,n}\left(G^2 - \frac{8UQ(WS-2)}{n\pi}\right) + \left(\frac{4U}{n\pi}\right)^2\right]}$$

(n = 1, 3, 5, 7,….)

$$Q = \frac{\omega d - N\pi v_0}{4 + (N\pi)^2}$$

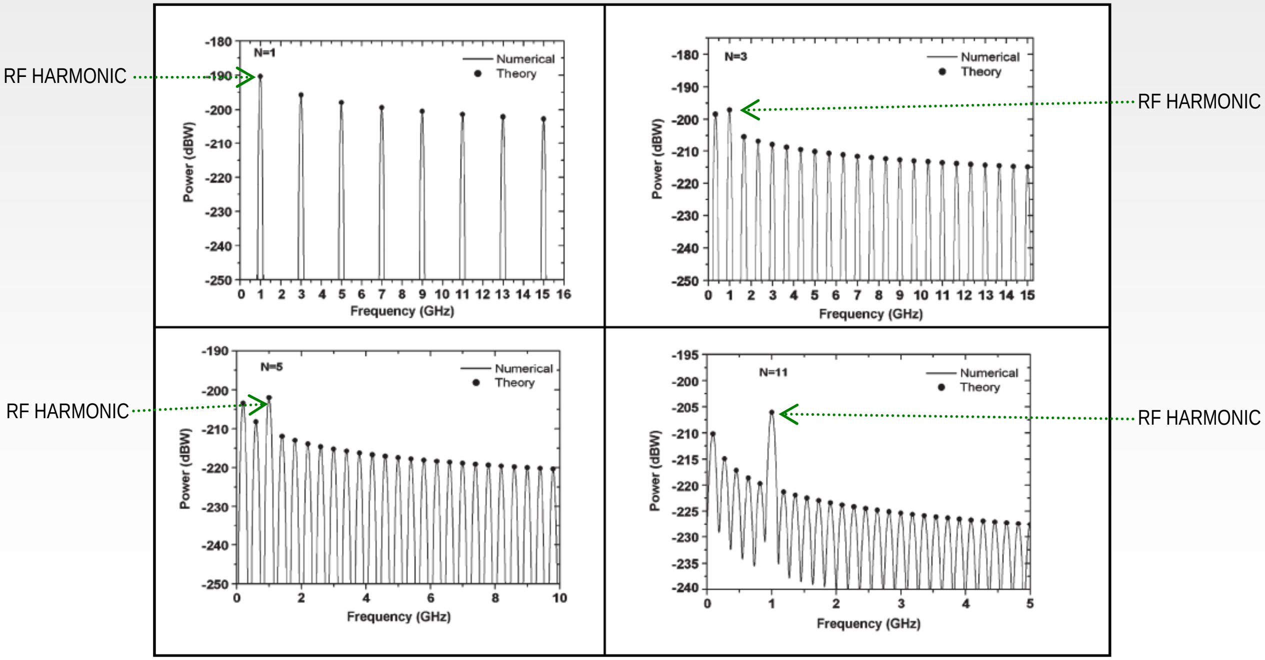
$$U = v_0 + Q\,C$$

$$S = \sqrt{\frac{e^2 V_{RF}^2\left(4+(N\pi)^2\right)}{(\omega d - N\pi v_0)^2} - (m_e \omega d)^2} \qquad G \equiv \frac{eV_{RF}}{m_e \omega d} \qquad W = \frac{N\pi}{\omega\, d\, m_e} \qquad C = N\pi + \frac{2S}{\omega\, d\, m_e}$$

SPECTRUM RADIATED BY A MULTIPACTOR DISCHARGE IN A PARALLEL-PLATE WAVEGUIDE

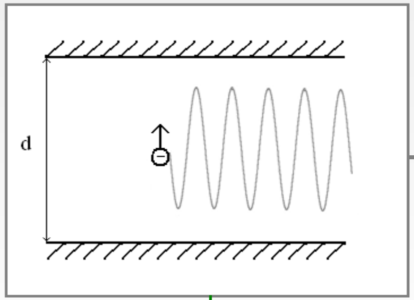
Results_I (1)

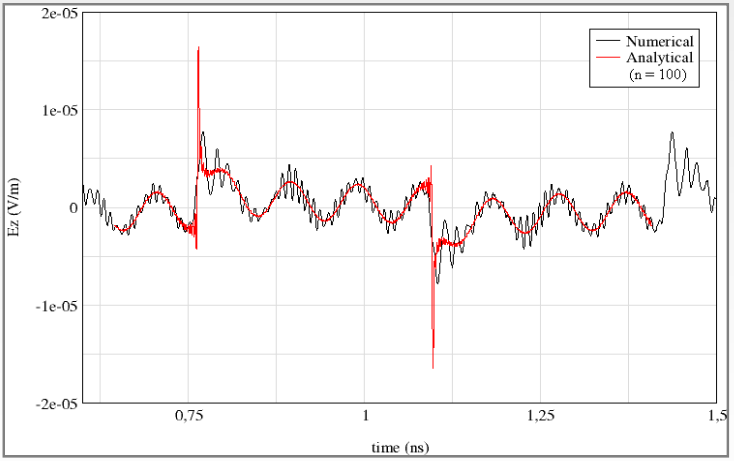
Power spectrum radiated by a multipactor discharge at different multipactor orders (N) in comparison with numerical results obtained with a PIC code:



Results_I (2)

Electric field radiated by an electron as a function of time in comparison with a PIC code for multipactor order N=7:



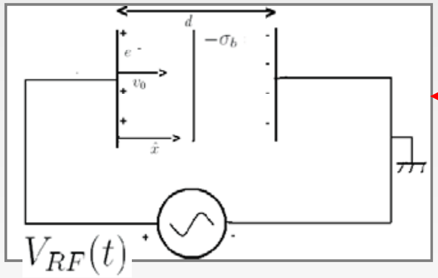


It might be used for wake-fields calculations

E. Sorolla, S. Anza, B. Gimeno, A. M. Pérez, C. Vicente, J. Gil, F. J. Pérez-Soler, F. D. Quesada, A. Álvarez, V. E. Boria,
IEEE Trans. Electron Devices, vol. 55, no. 8, pp. 2252-2258, Aug. 2008

Theory_II (1)

Objective: Evaluation of the **full spectrum radiated by a multipactor discharge** occuring within a parallel-plate waveguide under **an arbitrary voltage**



The voltage $V_{RF}(t)$ is an **arbitrary function of time**

- We consider the 1D effective electron movement (along the x-direction)
- **Space-charge effects** are considered with a planar current-sheet moving between electrodes:

$$-\sigma_b = \frac{-e n_e(t)}{A} \longrightarrow \vec{E}_b = \begin{cases} \frac{-\sigma_b}{2\epsilon_0}(-\hat{x}) & x\epsilon[0, x_b[\\ \frac{-\sigma_b}{2\epsilon_0}(\hat{x}) & x\epsilon[x_b, d] \end{cases}$$

- The effective electron dynamics is numerically calculated by means of the Velocity-Verlet algorithm considering Lorentz's force and the total electric field:

$$\vec{E}_{total} = \vec{E}_{RF} + \vec{E}_b = E_{total}\hat{x}$$

Theory_II (2)

- The equivalent wire current is given by the following expression:

$$i_{eq}(t) = \frac{-e}{d} \; n(t) \; v(t)$$

where the current electron population generated by the considered effective electron is accounted. The Fourier transform is numerically evaluated by means of the FFT algorithm:

$$I(\omega) = \int_{t_1}^{t_2} i_{eq}(t) \; e^{-j \; \omega \; t} \; dt$$

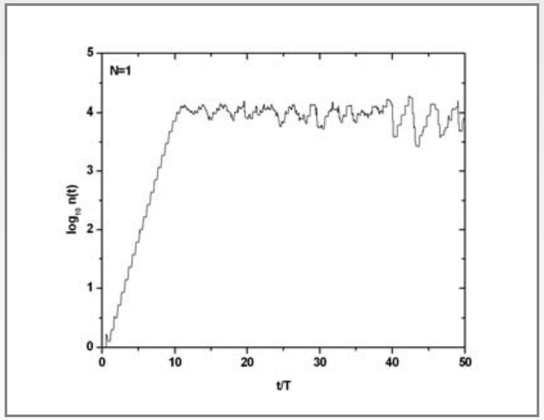
Finally, the total electromagnetic radiated power by the multipactor discharge is calculated:

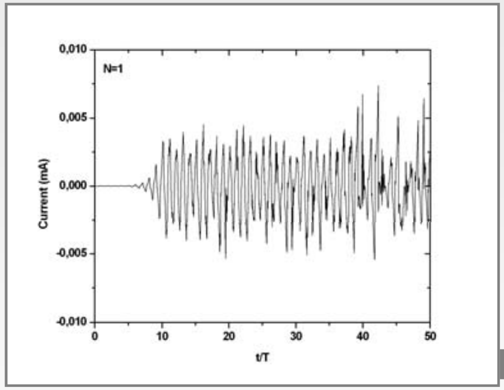
$$P(\omega) = \frac{\omega \; \mu_0 \; d}{8} \; |I(\omega)|^2$$

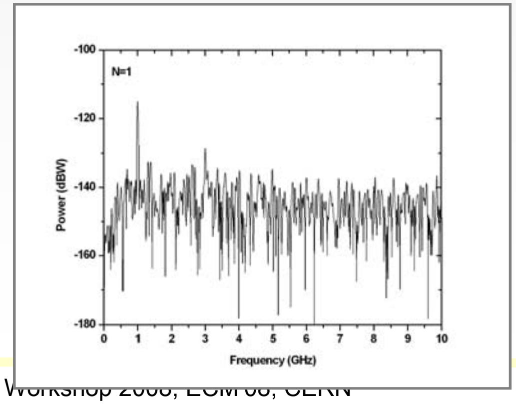
SPECTRUM RADIATED BY A MULTIPACTOR DISCHARGE IN A PARALLEL-PLATE WAVEGUIDE

Results_II (1)

- Example: Single-carrier analysis







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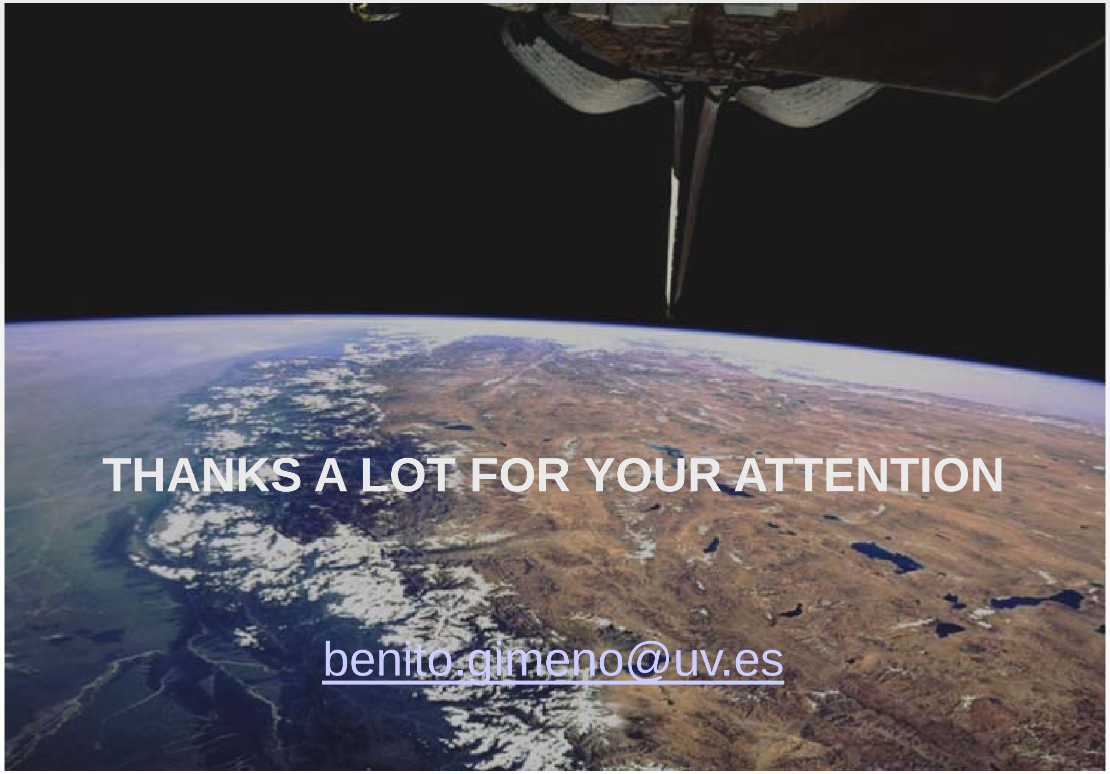
Conclusions and future lines

- Multipactor in coaxial waveguides:
  - Simulation tool: MULTICOAX
  - Tested and validated with technical literature and experimental data
  - Attractors in SW configuration migth produce a partial mitigation of the discharge
  - Influence of attractors has been studied: voltage threshold is splitted for TW and SW
  - Numerical evaluation of susceptibility charts including multipactor higher-order modes
  - To study multipactor effect in circular waveguide

- Multipactor in dielectric-loaded waveguides:
  - Simulation tool has been developed
  - Two cases-study have been analyzed: mitigation of the multipactor discharge is possible in the presence of dielectric materials
  - Design of an experiment to measure a multipactor discharge including a dielectric slab

- Analysis of the electromagnetic power spectrum radiated by a multipactor discharge:
  - Sombrin´s model provides a simple descripction of the multipactor phenomena ->
    Analytical evaluation of the electromagnetic fields radiated by the discharge ->
    Closed expression for the total radiated power
  - Simulation tool for the calculation of the radiated power of a discharge under arbitrary time regime
  - Calculation of the electromagnetic fields radiated by a charged particle within a waveguide ->
    Wake fields calculation



THANKS A LOT FOR YOUR ATTENTION

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