The CKM Quark-Mixing Matrix

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Definitions

- The charged current $W^{\pm}$ interaction couples to the physical left-handed quarks as

$$V_{CKM} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- The CKM matrix can be parameterized by three mixing angles (s_{12},s_{23},s_{13}) angle and one CP-violating phase (δ)
- The hierarchical structure makes convenient to define:

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \qquad s_{23} = A\lambda^2 = \lambda \left| \frac{V_{cb}}{V_{us}} \right|, \qquad s_{13} e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar{\rho} + i\bar{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}\left[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})\right]}$$

- These relations ensure that the CKM matrix written in terms of A, λ, ρ and η is unitary to all orders in λ and that $(\rho + i\eta) = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$ is phase-convention independent

Magnitude of the CKM elements (I)

- $|V_{ud}|$
  - From: Superallowed $0^+ \rightarrow 0^+$ Nuclear transitions:
    $|V_{ud}| = 0.97418 \pm 0.00027$
  - Latest neutron lifetime measurement is off by 6.5 σ w.r.t. average
- $|V_{us}|$
  - Average of different semi-leptonic kaon decays:
    $|V_{us}| = 0.2255 \pm 0.0019$
  - Precision is limited by the accuracy on $f_+(0)$ (theory), calculations differ up to 2% (but lattice agrees with historical determination from Roos-Leutwyler)
- $|V_{cd}|$
  - Best precision still obtained from the ratio of double-muon to single-muon production in ν / $\overline{\nu}$ interactions
    $|V_{cd}| = 0.230 \pm 0.011$
- $|V_{cs}|$
  - Unquenched QCD calculations of the D_s meson decay constant and D form factor allow one to extract V_{cs} from semi-leptonic and leptonic decays. The average gives:
    $|V_{cs}|=1.04 \pm 0.06$

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Magnitude of the CKM elements (II)

- $|V_{cb}|$
  - From the average of inclusive and exclusive determination the V_{cb} V_{ub} mini review determines (with inflated errors):
    $|V_{cb}| = (41.2 \pm 1.1) \times 10^{-3}$
- $|V_{ub}|$
  - Determination is quite involved. Average from V_{cb} V_{ub} mini rev:
    $|V_{ub}| = (3.93 \pm 0.36) \times 10^{-3}$
  - To watch: determinations from inclusive and exclusive decays tend to move apart
- $|V_{tb}|$
  - From single top production (w/o unitarity assumption):
    $|V_{tb}| > 0.74$ (95% CL)
- $|V_{td}|$ and $|V_{ts}|$
  - $|V_{ts}|$ now measured from B_s-mixing
  - precise $|V_{td}/V_{ts}| = 0.209 \pm 0.001 \pm 0.006$
  - Error already dominated by theory:

$$\xi = \frac{f_{Bs}\sqrt{\widehat{B}_{Bs}}}{f_{Bd}\sqrt{\widehat{B}_{Bd}}} = 1.23 \pm 0.02 \pm 0.03$$

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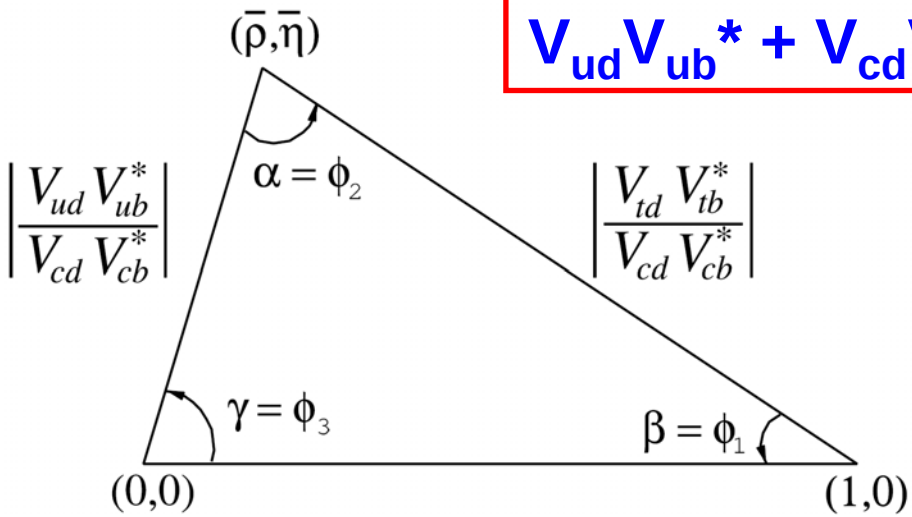
CKM Angles

From B $\to \pi\pi, \rho\pi, \rho\rho$ decays: $\alpha = (88\ ^{+6}_{-5})^\circ$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{tb}V_{tb}^* = 0$$



From tree decays (GLW, ADS, Dalitz):
$\gamma = (77^{+30}_{-32})^\circ$

Chamonium modes:
Sin 2 β = 0.681 ± 0.025

Unitarity and Global Fit in SM

- **Unitarity Checks:**

$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2= 0.9999 \pm 0.0011$ (1st row)

$|V_{cd}|^2+|V_{cs}|^2+|V_{cb}|^2= 1.136 \pm 0.125$ (2nd row)

$|V_{ud}|^2+|V_{cd}|^2+|V_{td}|^2= 1.002 \pm 0.005$ (1st column)

$|V_{us}|^2+|V_{cs}|^2+|V_{ts}|^2= 1.134 \pm 0.125$ (2nd column)

- **Global Fit:**

$\lambda=0.2257^{+0.0009}_{-0.0010}$ $\quad A= 0.814^{+0.021}_{-0.022}$

$\bar{\rho}=0.135^{+0.031}_{-0.016}$ $\quad \bar{\eta}=0.349^{+0.015}_{-0.017}$

$J=(3.05^{+0.19}_{-0.20}) \times 10^{-5}$

Constraints on the $\bar{\rho}$ $\bar{\eta}$ plane

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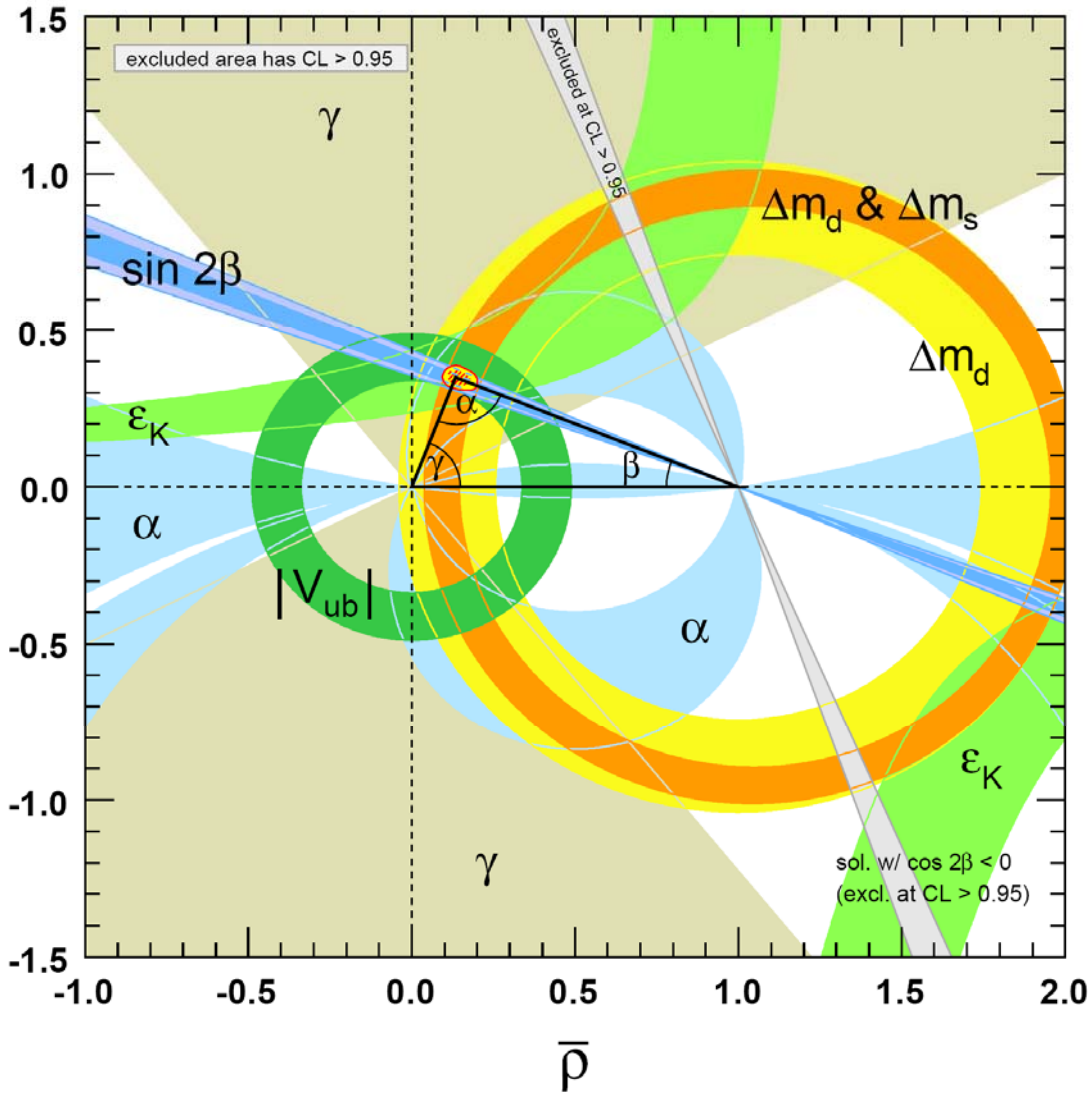

Puzzles

- Hints of large B_s mixing phase
- $\Delta A = \mathcal{A}(K^{+/-}\pi^0) - \mathcal{A}(K^{+/-}\pi^{-/+}) = 0.148 \pm 0.028$
- Sin $(2\beta)_{eff}$ (from penguin dominated modes) is moving closer to Sin (2β) measured from the Charmonium modes

Issues (some)

- V_{ub} exclusive vs. inclusive determinations
- Frequentistic vs. Bayesian Statistics
- Treatment of theoretical "systematics"
- Use of input hadronic parameters (e.g. pre-dictions vs. post-dictions)
- Consensus on lattice results (e.g. f_{Ds} from HPQCD and tension in $D^+_s \to \ell^+ \nu$ decays)

Outlook

- **Present / Near Future**
  - **B factories**
  - **Tevatron**
  - **LHC**
- **Future**
  - **Rare K Decays ($K \to \pi \nu\bar{\nu}$)?**
  - **Super Flavour Factories?**