From High Energy QCD to Statistical Physics
... and Back

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Based on:

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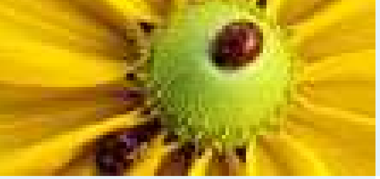
E.I., D. Triantafyllopoulos (hep-ph/0411405 & 0501193)



Introduction

- What is the high–energy limit of QCD scattering ?
- High energy QCD $\Longleftrightarrow$ High density gluonic matter
  - Weak coupling ! (by asymptotic freedom)
  - Elaborate resummations (high energy & many body)
- Theoretical progress leading to new tools
  - Color Dipole Picture (Al Mueller)
  - Color Glass Condensate (MV, JIMWLK)
- Conceptual & phenomenological consequences
  - Saturation of the parton densities
  - Unitarization of the scattering amplitudes
  - Geometric scaling ($\Longrightarrow$ DIS at small–x)
  - Cronin peak & High–p_T suppression in d+Au at RHIC
- Unexpected link to problems in statistical physics !

  High–energy QCD evolution: a classical stochastic process

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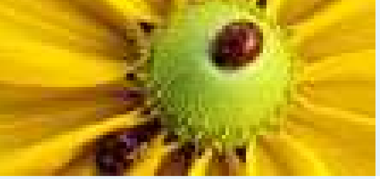
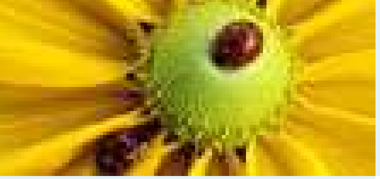
  High–energy QCD evolution: a classical stochastic process



Outline

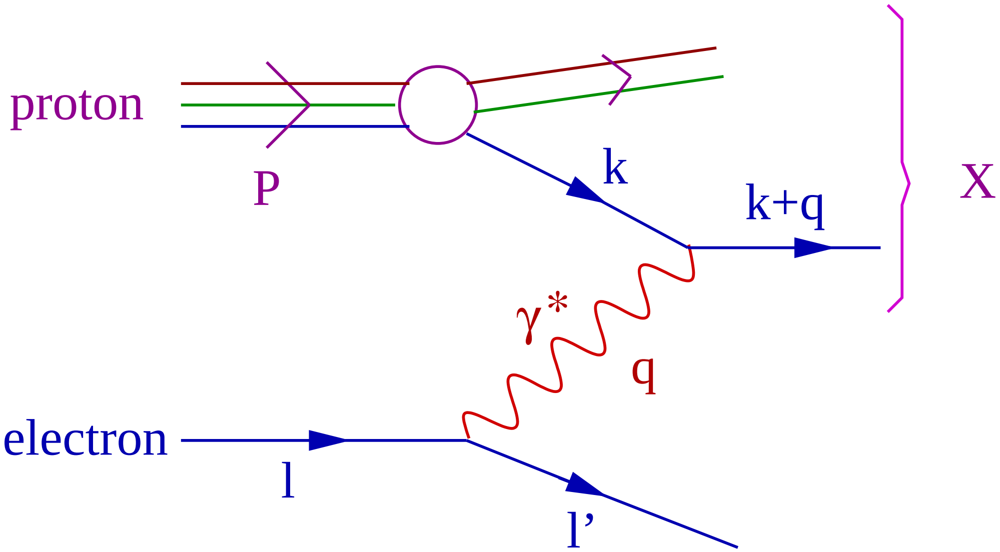
Introduction
Outline
High-energy QCD
Gluon Saturation
Effective theory for CGC
Unitarity & Geometric scaling
QCD vs. Statistical Physics
Pomeron loops
Conclusions

- Gluon evolution in QCD at high energy
  - Deep inelastic scattering at small x
  - Gluon distribution at HERA
  - BFKL evolution and its ‘small–x problem’
- Gluon Saturation: the general idea
- The QCD effective theory for gluon saturation: CGC
- From High–Energy QCD to Statistical physics
  - Relation to the “reaction–diffusion” process
  - The mean field approximation:
    Unitarization & Geometric scaling
  - Particle number fluctuations
- Evolution equations with ‘Pomeron loops’
- Conclusions & Open questions



Deep Inelastic Scattering at Small–x (1)

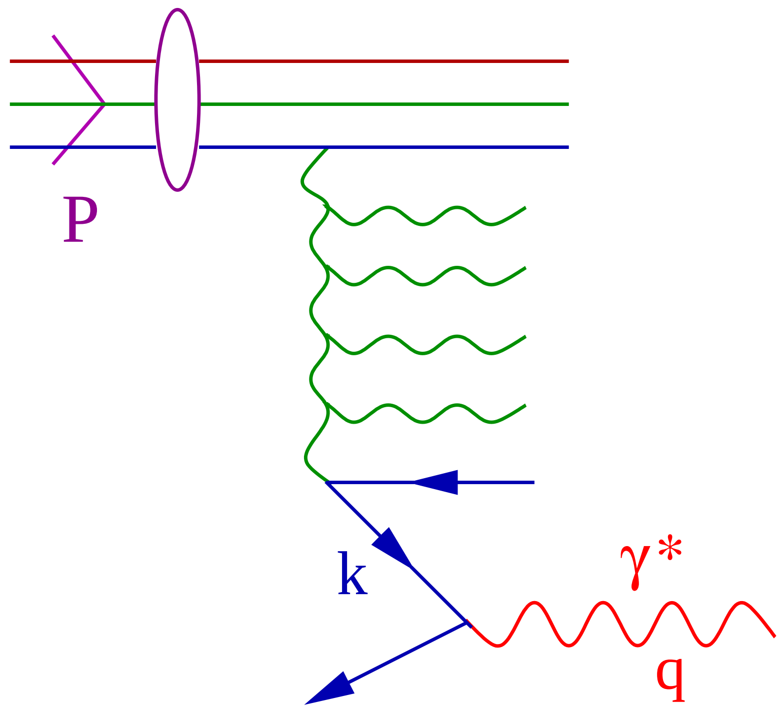
electron (l) + proton (P) $\longrightarrow$ electron (l') + X (P_X)



- Two independent kinematical invariants :
  - $Q^2 \equiv -q^\mu q_\mu \geq 0$
  - $x \simeq Q^2/s$ with $s \equiv (P+q)^2 \gg Q^2$
- Virtual photon absorbed by a quark excitation of the proton
  - with transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $k_z = xP$

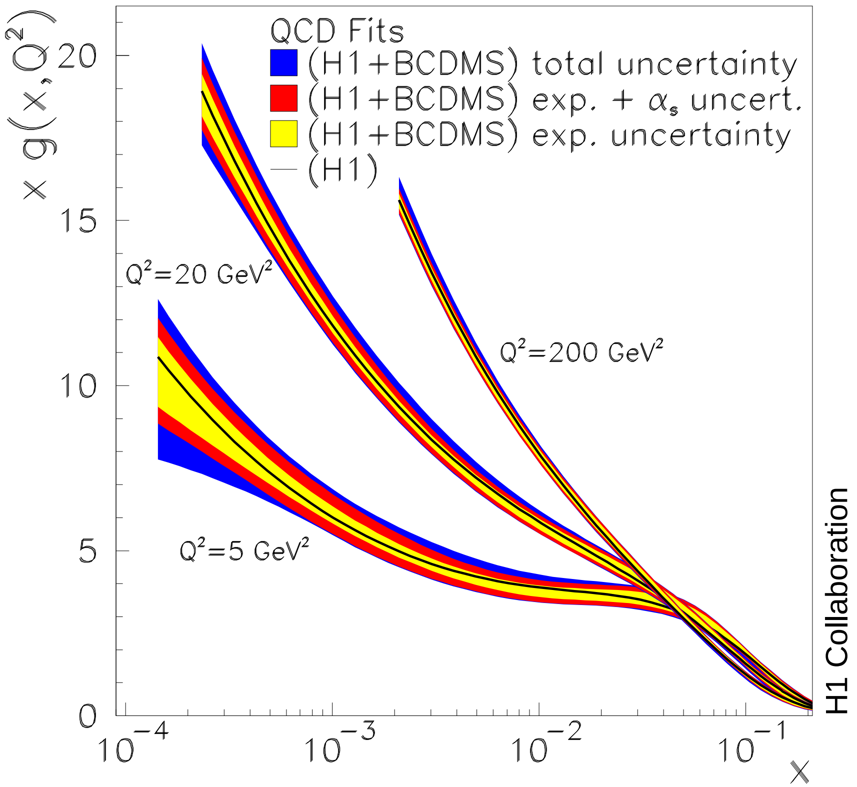
Deep Inelastic Scattering at Small–x (2)

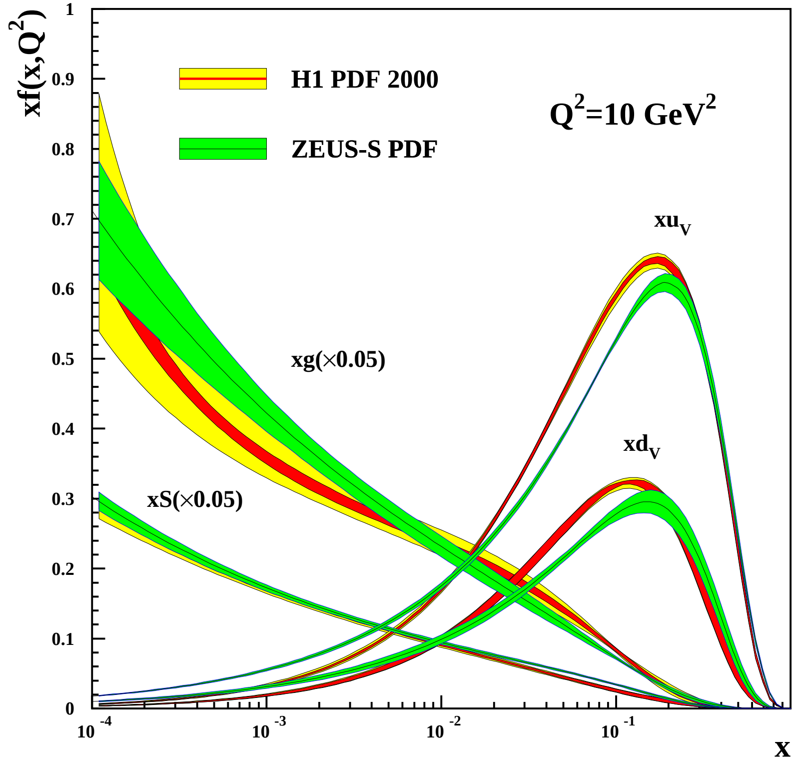
- High energy $(s \gg Q^2) \iff$ Small–x $(x \ll 1)$
- At small–x, the struck quark is typically radiated off the gluon distribution in the proton



Gluon distribution at HERA

The gluon density rises very fast at small x ! (as a power of $1/x$)



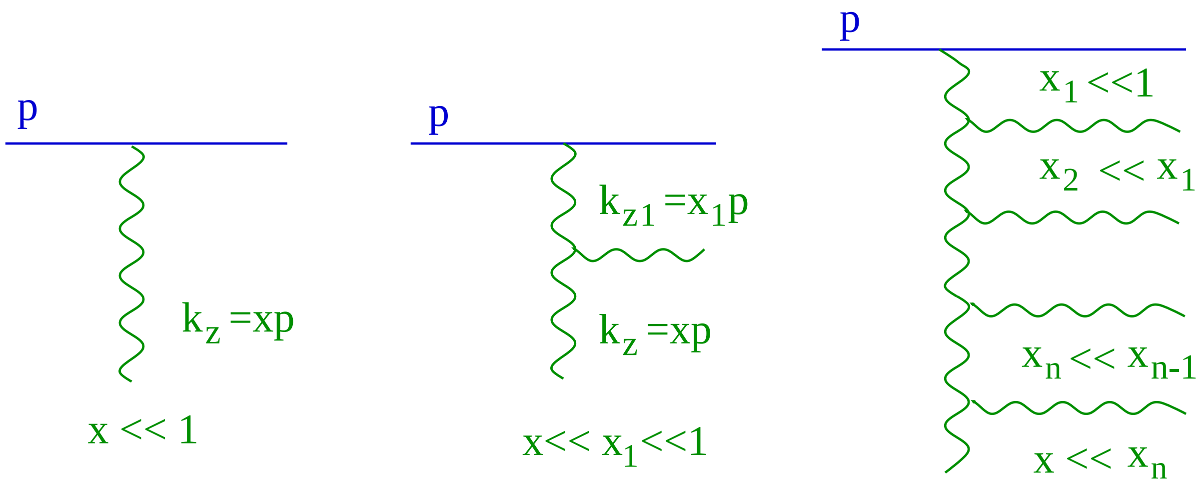


$xG(x, Q^2)$ = # of gluons with transverse size $\Delta x_\perp \sim 1/Q$ and $k_z = xP$

Can one understand this rise in QCD ?

BFKL evolution

‘Quantum evolution’ amplifies the gluon density at $x \ll 1$!



$\sim \alpha_s \ln \frac{1}{x}$ $\qquad$ $\sim \frac{1}{n!}\left(\alpha_s \ln \frac{1}{x}\right)^n$

- The differential probability for one gluon emission :

$$\mathrm{d}\mathcal{P} \simeq \alpha_s \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \frac{\mathrm{d}x}{x} \qquad \text{(bremsstrahlung)}$$

- The cost of one additional gluon :

$$\alpha_s \int_x^1 \frac{\mathrm{d}x_1}{x_1} = \alpha_s \ln \frac{1}{x}$$

The small–x problem of BFKL

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- DIS
- HERA data
- BFKL
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- When $\alpha_s \ln(1/x) \gtrsim 1 \Longrightarrow$ Need for resummation

$$N(x) \propto \sum_n \frac{1}{n!}\left(\omega\alpha_s \ln\frac{1}{x}\right)^n = \mathrm{e}^{\omega\alpha_s Y}$$

$Y \equiv \ln(1/x) \sim \ln s$: "rapidity"

- BFKL equation : $\omega = (12\ln 2)/\pi \simeq 2.7$

  (Balitsky, Fadin, Kuraev, Lipatov, 78)

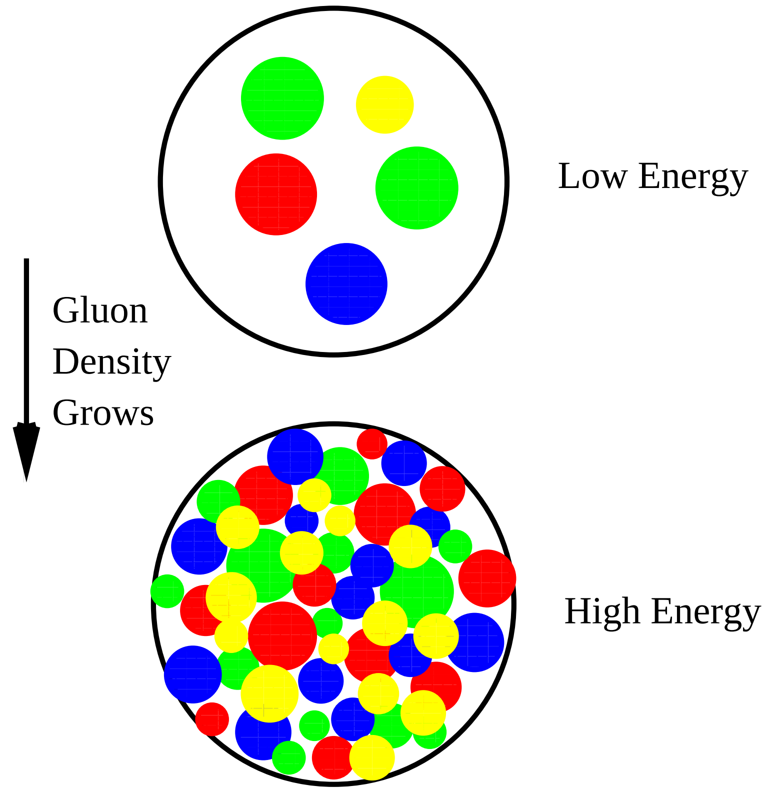
- Unstable growth of the gluon distribution !

- Conceptual difficulties in the high–energy limit
  - Violation of the unitarity condition for the S–matrix :

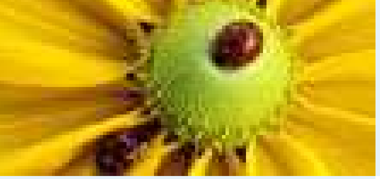
    $SS^\dagger = 1 \quad\Longrightarrow\quad T(s,b) \le 1$ ... but $T_{\mathrm{BFKL}}(s,b) \sim s^\omega$!
  - 'Infrared diffusion' : sensitivity to the non–perturbative domain at low momenta $Q^2 \lesssim \Lambda^2_{\mathrm{QCD}}$

High energy = High density

- QCD evolution with increasing energy:
  A rapid evolution towards increasing density !



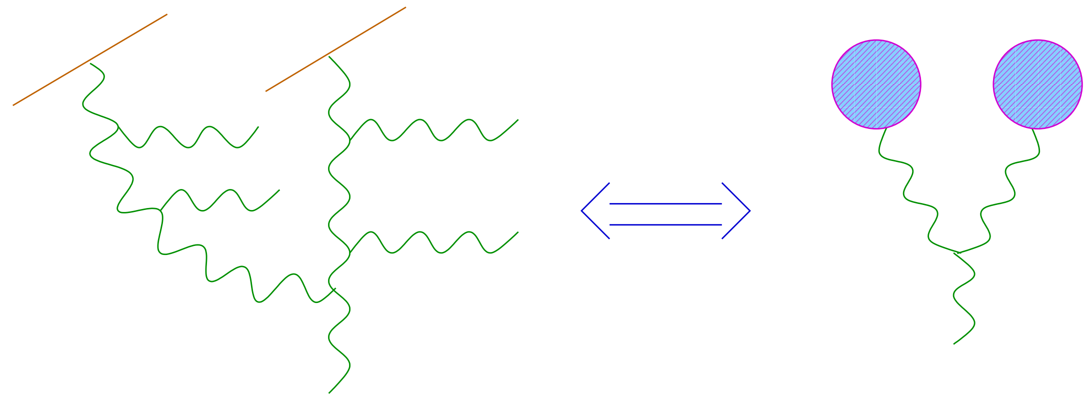
- But no feedback from the high density on the BFKL evolution



Gluon Saturation : The General Idea

(Gribov, Levin and Ryskin, 83 ; Mueller and Qiu, 86)

- The high density favorizes gluon recombination



- When RECOMBINATION $=$ RADIATION $\Longrightarrow$ SATURATION
- The gluons must be large enough to overlap with each other

$$n(Y,Q^2)\;\equiv\;\frac{xG(x,Q^2)}{Q^2\,\pi R^2}\;:\qquad \text{Occupation number}$$

- One expects a non–linear equation of the generic form:

$$\frac{\partial n}{\partial Y}\simeq\alpha_s n-\alpha_s^2\,n^2=0\qquad\text{when}\quad n(Y,Q^2)\sim\frac{1}{\alpha_s}\gg 1$$

The Saturation Momentum

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- High density
- The idea of saturation
- Saturation momentum
- From pQCD to saturation

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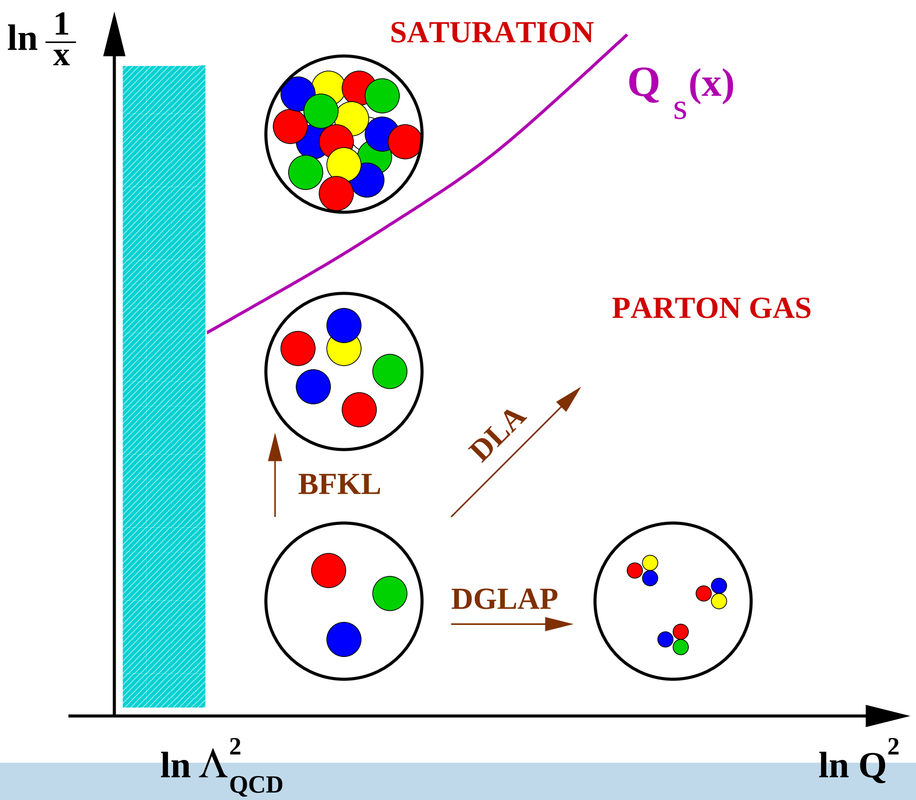
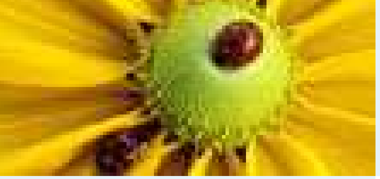
- For a given $Y = \ln 1/x$, this requires Q^2 to be smaller than

$$Q_s^2(x) \simeq \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2} \sim \frac{1}{x^\lambda}$$

$Q^2 \gg Q_s^2(x)$: Dilute regime (rapid growth: BFKL, DGLAP)

$Q^2 \lesssim Q_s^2(x)$: Saturation: $n \sim 1/\alpha_s$ (large but constant)





How to describe saturation within QCD ?

- The high–density gluons are **weakly coupled** :

$$Q_s^2(x) \gg \Lambda_{\rm QCD} \simeq 200\ {\rm MeV} \quad \Longrightarrow \quad \alpha_s(Q_s^2) \ll 1$$

- ... yet, their dynamics is **fully non–linear** !

$$n \sim 1/\alpha_s \quad \Longrightarrow \quad \alpha_s\, n \sim 1$$

- Weak coupling + Large occupation numbers

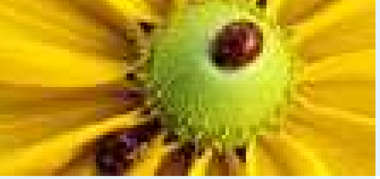
$\Longrightarrow$ Strong classical 'color' fields

- Color Glass Condensate

A classical effective theory for the small–x gluons as obtained after integrating out the gluons at large x in perturbation theory

L. McLerran, R. Venugopalan (94) : a model for a large nucleus

E.I., A. Leonidov & L. McLerran (00) : quantum theory



The Color Glass Condensate

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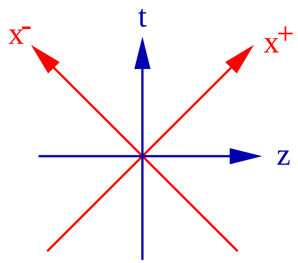
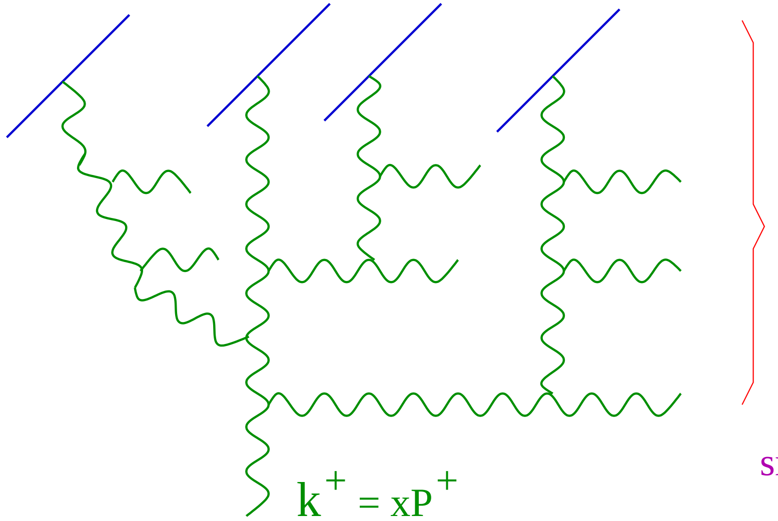
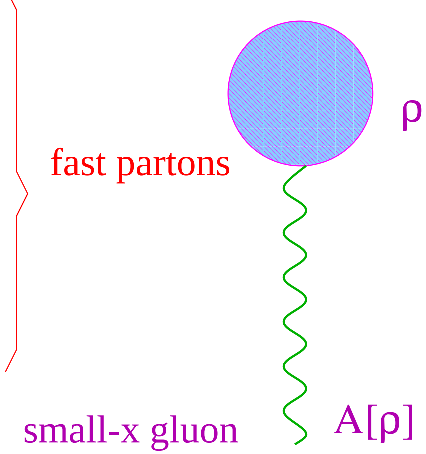


$$x^{\pm} = (t \pm z)/\sqrt{2}$$

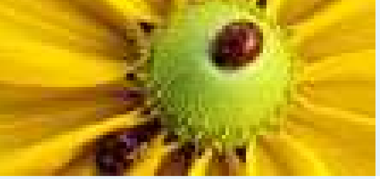




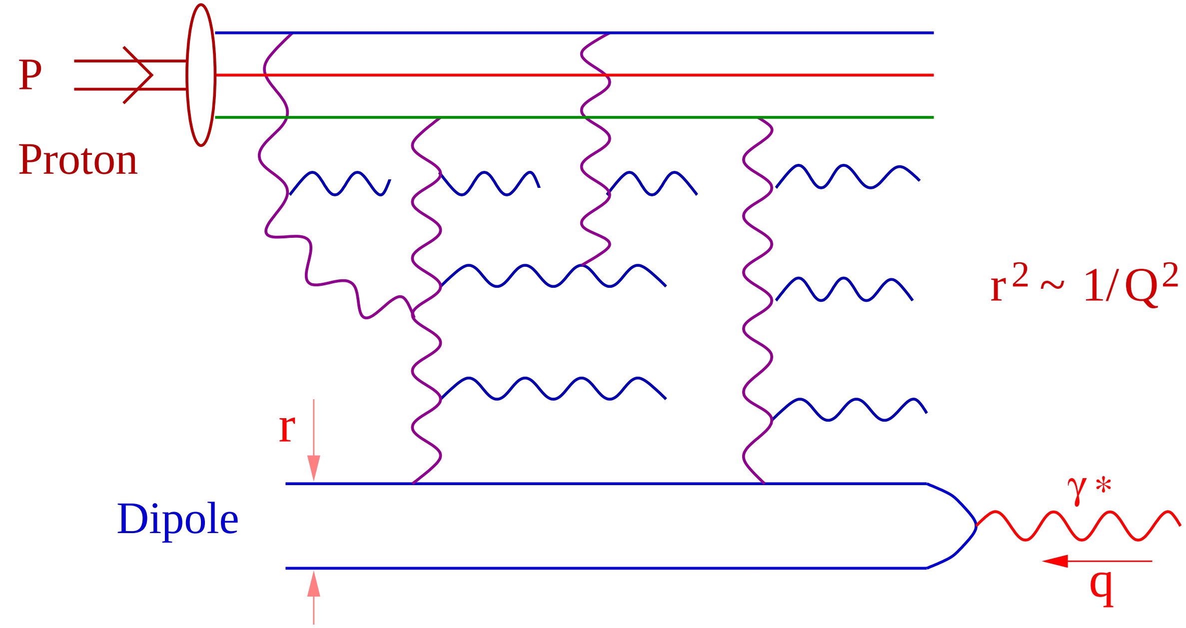
- Small–x gluons : The classical field $A[\rho]$ radiated by fast $(x' > x)$ partons having a color charge density ρ_a
- The fast partons are 'frozen' (by Lorentz time dilation) in some random configuration

  $\Longrightarrow$ Probability distribution $W_Y[\rho]$ for the color charge
- With increasing Y, new quanta are included in ρ (evolution)

$$\frac{\partial W_Y[\rho]}{\partial Y} = -H\left[\rho, \frac{\delta}{\delta\rho}\right] W_Y[\rho] \qquad \text{(JIMWLK)}$$



Deep Inelastic Scattering off the CGC

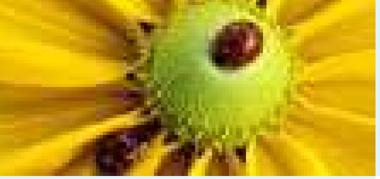


- γ^* fluctuates into a quark–antiquark pair ('color dipole')
- The dipole scatters (multiply) off the CGC in the proton

$$S_Y = \frac{1}{N_c}\left\langle \mathrm{tr}\left(V_{\boldsymbol{x}}^{\dagger}\, V_{\boldsymbol{y}}\right)\right\rangle_Y = \int \mathrm{D}[A^+]\, W_Y[A^+]\, \frac{1}{N_c}\,\mathrm{tr}\Big(V_{\boldsymbol{x}}^{\dagger}[A^+]\, V_{\boldsymbol{y}}[A^+]\Big)$$

$$V(\boldsymbol{x}) \equiv \mathrm{P}\exp\left(\mathrm{i}g\int dx^- A_a^+(x^-,\boldsymbol{x})T^a\right) \qquad \text{Wilson line (eikonal)}$$

- Quenched average (e.g., spin glass) $\Longrightarrow$ "Color Glass"

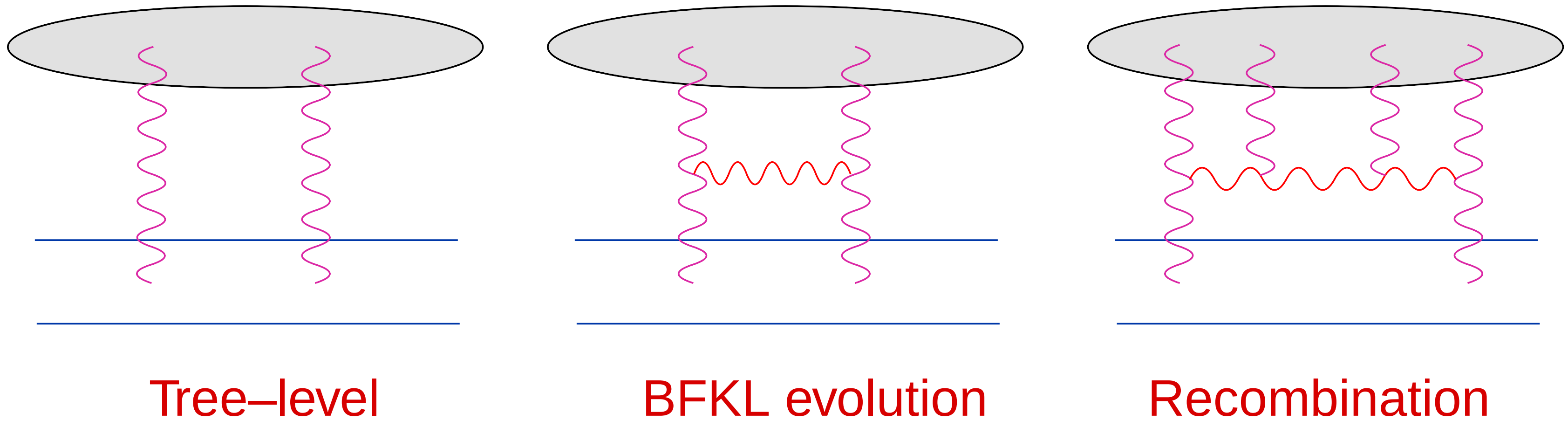


Evolution equation for the dipole S–matrix

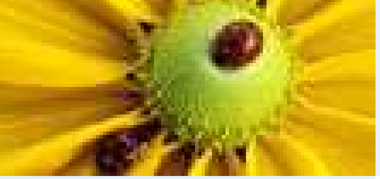
$$\partial_Y S(\boldsymbol{x},\boldsymbol{y}) = \int \mathrm{D}[A]\,(\partial_Y W_Y)\,\frac{1}{N_c}\mathrm{tr}(V_{\boldsymbol{x}}^\dagger V_{\boldsymbol{y}})$$

$\Longrightarrow$ a relatively simple equation !

- One–step evolution of the gluons in the target (CGC)

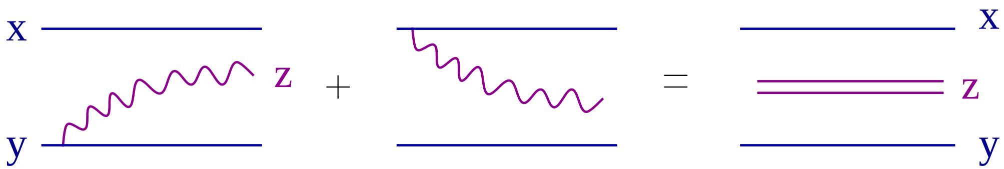


- BFKL evolution : $2 \rightarrow 2$ vertex
- Gluon recombination: $n \rightarrow 2$ vertex, with $n > 2$ $\Longrightarrow$ Unitarity
- All this is encoded in the JIMWLK equation

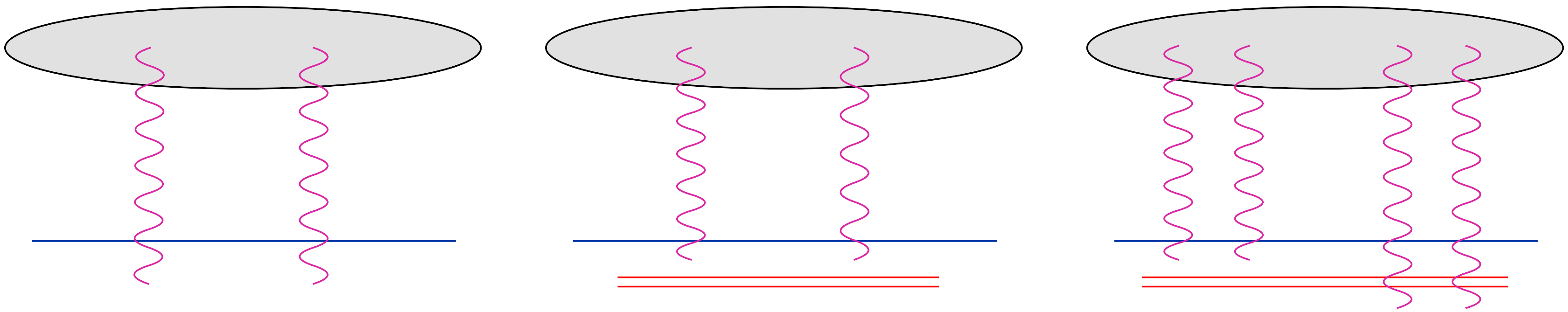


Projectile (Dipole) Evolution

- Projectile evolution $\Longleftrightarrow$ Dipole splitting (at large N_c)

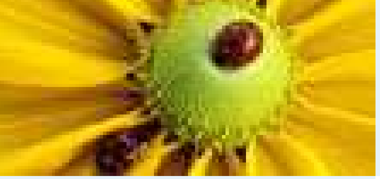


$$p(\boldsymbol{x},\boldsymbol{y}|\boldsymbol{z})\,d^2\boldsymbol{z} = \frac{\alpha_s N_c}{2\pi^2}\,\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\,d^2\boldsymbol{z}$$



Tree–level | Single scattering | Double scattering

- BFKL evolution: A single child dipole scatters off the target
- Unitarity corrections: Both child dipoles scatter off the target



The Balitsky–Kovchegov equation

$$\frac{\partial}{\partial Y}\langle T(\boldsymbol{x},\boldsymbol{y})\rangle_Y = \bar{\alpha}_s \int_{\boldsymbol{z}} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \Big\langle \underbrace{-T(\boldsymbol{x},\boldsymbol{y}) + T(\boldsymbol{x},\boldsymbol{z}) + T(\boldsymbol{z},\boldsymbol{y})}_{\text{BFKL (linear)}} - \underbrace{T(\boldsymbol{x},\boldsymbol{z})T(\boldsymbol{z},\boldsymbol{y})}_{\text{non–linear}} \Big\rangle_Y$$

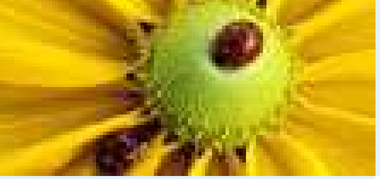
- The first equation in an infinite hierarchy ! (Balitsky, 96)
- Mean field approximation : $\langle T\,T\rangle \approx \langle T\rangle\langle T\rangle$

  $\Longrightarrow$ A closed, non–linear, equation for $\langle T\rangle$: BK equation (99)
- $T = 1$: Fixed point at high energy $\Longrightarrow$ “Black Disk Limit”
- The same universality class as the F–KPP equation

  (Munier & Peschanski, 03)

  “F–KPP” : Fischer, Kolmogorov, Petrov, Piskounov ($\sim$ 1940)

  $\triangleright$ Familiar in statistical physics, chemistry, biology, ...



Reaction–diffusion process $A \rightleftharpoons 2A$

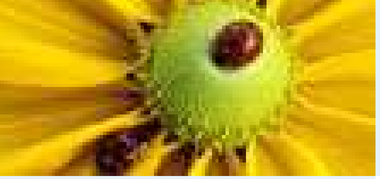
- Particles of type A distributed on an one-dimensional lattice
  - Particle splitting (rate α) : $A \xrightarrow{\alpha} A + A$
  - Particle merging (rate β) : $A + A \xrightarrow{\beta} A$
  - A particle can diffuse to a neighboring site
- $n(x,t)$: number of particles on site x at time t

  At large t, $n(x,t)$ saturates at a value $N \equiv \alpha/\beta \gg 1$
- $N \to \infty$: $h(x,t) = n(x,t)/N$ obeys the F–KPP equation :

$$\partial_t h(x,t) = \underbrace{\partial_x^2 h(x,t)}_{\text{diffusion}} + \underbrace{h(x,t)}_{\text{growth}} \underbrace{- \; h^2(x,t)}_{\text{recombination}}$$

- Two fixed points: $h = 0$ (unstable) and $h = 1$ (stable)
- "Traveling wave" : a front propagating into the unstable state

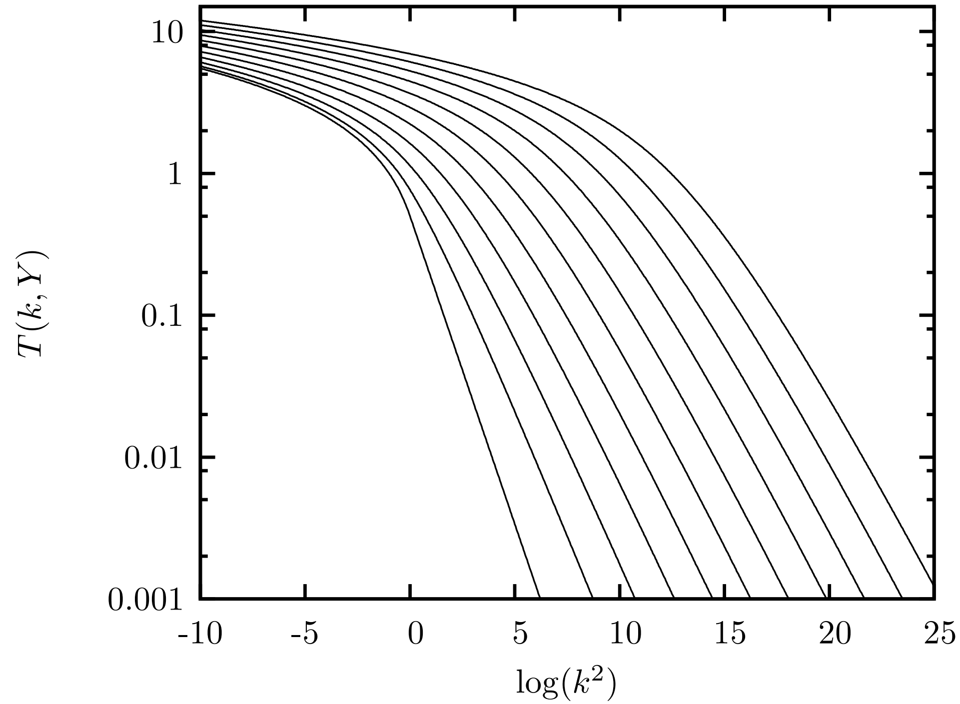
$$h(x,t) \simeq F(x - vt)\,, \quad F(z \to -\infty) \to 1\,, \quad F(z \gg 1) \sim \mathrm{e}^{-\gamma z}$$



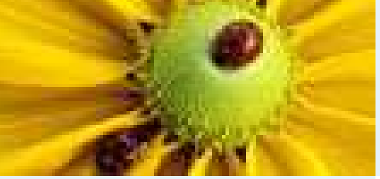
BK Equation: The Traveling Wave

$$\partial_Y T(\rho,Y) = \underbrace{\partial_\rho^2 T(\rho,Y) + T(\rho,Y)}_{\text{BFKL evolution}} \quad \underbrace{- \; T^2(\rho,Y)}_{\text{recombination}}$$

- $Y \sim \ln s$ ('time') & $\rho \equiv \ln 1/r^2 \sim \ln k^2$ (dipole inverse size)



- Large $\rho \Longrightarrow$ Small dipole $\Longrightarrow$ Weak interaction
- Small $\rho \Longrightarrow$ Strong scattering $\Longrightarrow$ Unitarization



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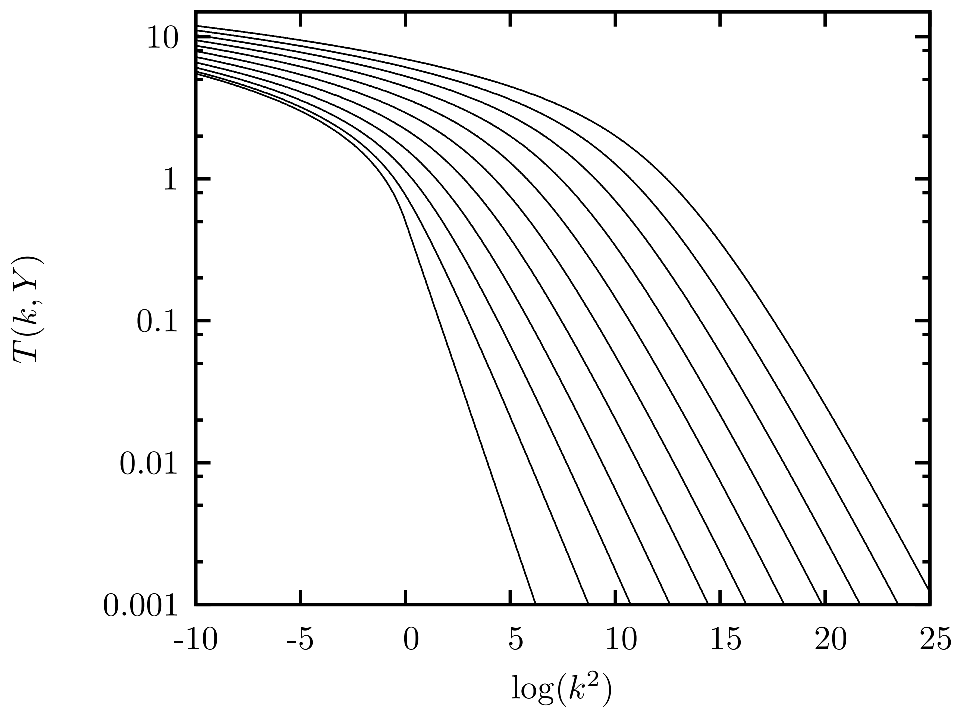
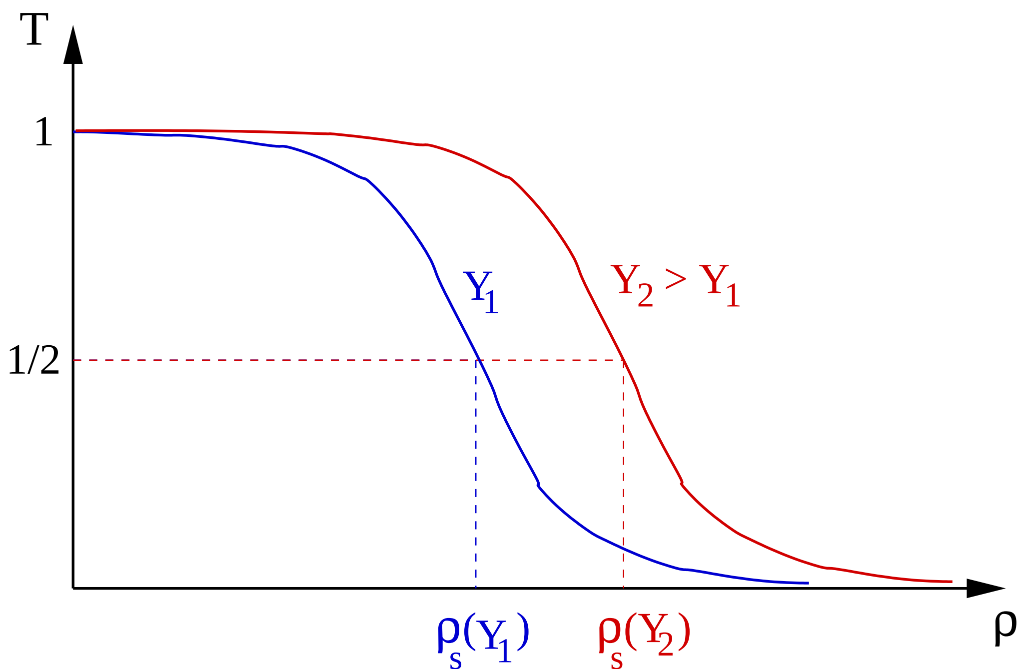


- $T \sim$ (unintegrated) gluon distribution
- High–$k_\perp$: power spectrum. Low–$k_\perp$: flat spectrum

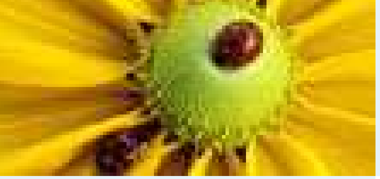
- The position of the front: $T(\rho, Y) = 1/2$ for $\rho = \rho_s(Y)$
  - $\rho > \rho_s(Y) \Longrightarrow$ Color transparency & BFKL growth

    $$T(\rho, Y) \sim r^{2\gamma}\, s^{\bar{\alpha}_s \omega} \sim \mathrm{e}^{-\gamma\rho}\, \mathrm{e}^{\bar{\alpha}_s \omega Y}$$

  - $\rho < \rho_s(Y) \Longrightarrow$ Black disk limit



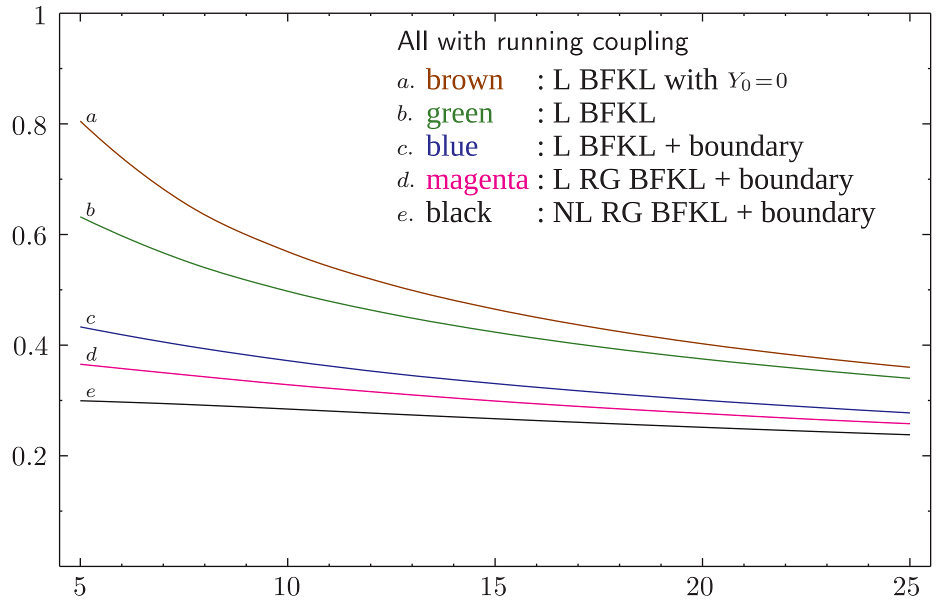
- $T(\rho_s, Y) = 1/2 \Longrightarrow \rho_s(Y) \simeq \lambda_0 \bar{\alpha}_s Y\,,\ \lambda_0 \equiv \omega/\gamma = 4.883...$



The saturation momentum at NLO

$$\rho_s(Y) \equiv \ln Q_s^2(Y) \simeq \lambda_0 \bar{\alpha}_s Y \quad \Longrightarrow \quad Q_s^2(Y) \propto \mathrm{e}^{\lambda_0 \bar{\alpha}_s Y}$$

- N.B. : λ_0 is fully determined by the BFKL dynamics !
- The saturation exponent: $\lambda(Y) \equiv \mathrm{d} \ln Q_s^2(Y)/\mathrm{d}Y$



- Next–to–leading order BFKL + Collinear resummation (a la Salam et al) : Triantafyllopoulos, 2002
- $\lambda(Y) \approx 0.3$ as opposed to $\lambda_0 \bar{\alpha}_s \approx 1$!!

Geometric Scaling

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- Dipole evolution
- BK equation
- Reaction–diffusion
- Traveling wave
- Saturation momentum
- QSAT at NLO
- Geometric Scaling

QCD vs. Statistical Physics

Pomeron loops

Conclusions

- Traveling wave: The front propagates without distorsion

$$T(\rho, Y) \simeq F(\rho - \rho_s(Y)) \equiv \mathcal{F}\big(r^2 Q_s^2(Y)\big)$$

$\Longrightarrow$ "Geometric scaling" (a function of a single variable)

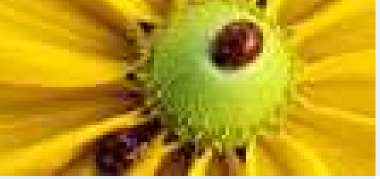
(E.I., Itakura, McLerran, 02 ; Mueller, Triantafyllopoulos, 02)

- A natural explanation for a new scaling law identified in the HERA data for DIS at small–x

(Staśto, Golec-Biernat, and Kwieciński, 2000)

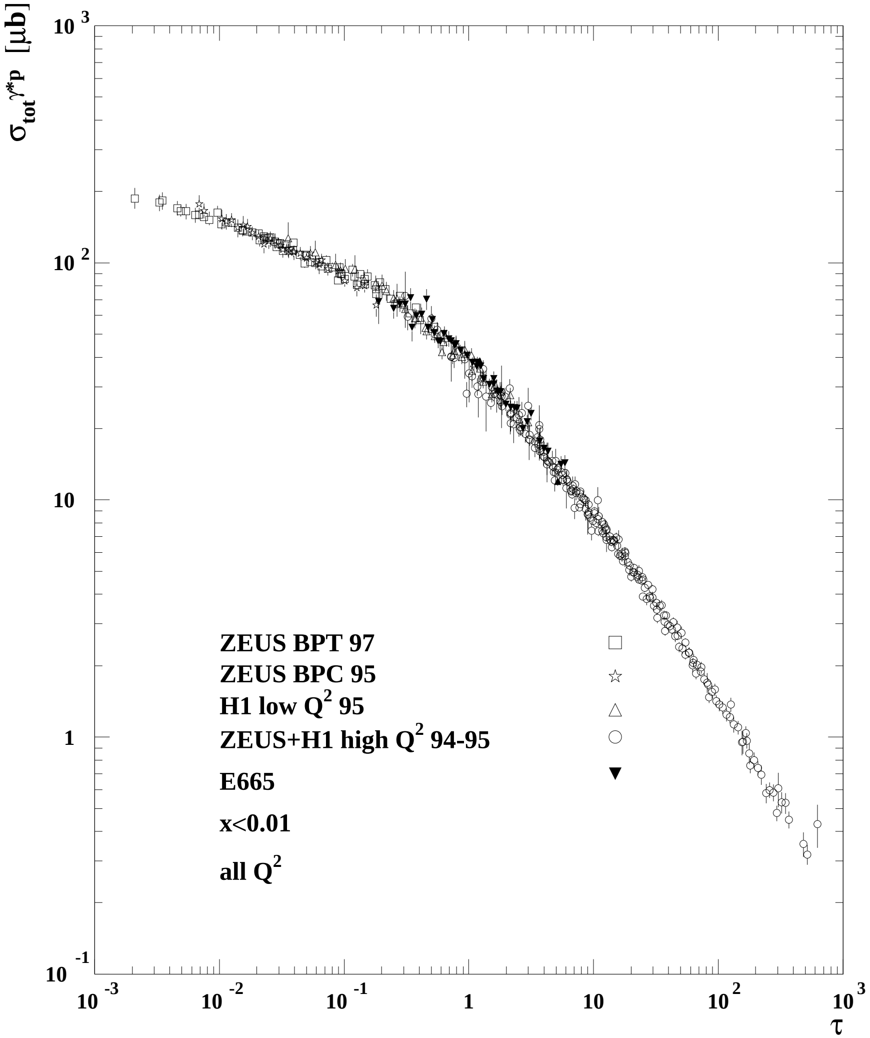
- Relevant for the high–p_T suppression observed in deuteron-gold collisions at RHIC

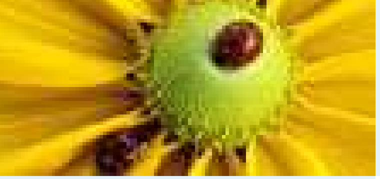
(Kharzeev, Levin, McLerran, 02 ; E.I., Itakura, Triantafyllopoulos, 04)



Geometric Scaling at HERA

$$\sigma_{\gamma^* p}(x, Q^2) \approx \sigma(\tau)\,, \quad \text{with} \ \ \tau \equiv \frac{Q^2}{Q_s^2(x)}\,, \quad Q_s^2(x) \sim 1/x^{\lambda}\,, \quad \lambda \simeq 0.3$$





Beyond the mean field approximation

Introduction
Outline
High-energy QCD
Gluon Saturation
Effective theory for CGC
Unitarity & Geometric scaling
QCD vs. Statistical Physics
- Fluctuations
- Discreteness
- Pulled fronts
- Saturation exponent
- Front diffusion
- Breakdown of BFKL
Pomeron loops
Conclusions

1. The correspondence statistical physics extends also beyond the mean field approximation
2. This is important, and also useful, since the physics of saturation is very sensitive to fluctuations !

   (E.I., A. Mueller, S. Munier, 04)

- The importance of fluctuations for the high–energy evolution in QCD has been anticipated in previous work by

  A. Mueller (94), G. Salam (95), E.I. and A. Mueller (03), A. Mueller and A. Shoshi (04)

- What is the origin of the fluctuations in the case of QCD ?

  Recall: Reaction–diffusion process $A \rightleftharpoons 2A$:

  Fluctuations in the number of particles

- Why are the effects of the fluctuations so important ?

Gluon number fluctuations

- In a given event, the dipole–hadron scattering amplitude counts the number of gluons in the target wavefunction:

$$T(r,Y) \approx \alpha_s^2\, n(r,Y) \quad \text{with} \quad n(r,Y) = 0, 1, 2, \ldots$$

  - $n(r,Y)$ = number of ‘equivalent dipoles’ with size r
  - α_s^2 : amplitude for dipole–dipole scattering

$\Longrightarrow$ In an event–by–event description, T is discrete !

$\Longrightarrow$ Gluon number fluctuations entail fluctuations in T :

$$\delta T \sim \alpha_s^2 \sqrt{n} \sim \sqrt{\alpha_s^2\, T} \Longrightarrow \delta T \sim T \quad \text{when} \quad T \lesssim \alpha_s^2$$

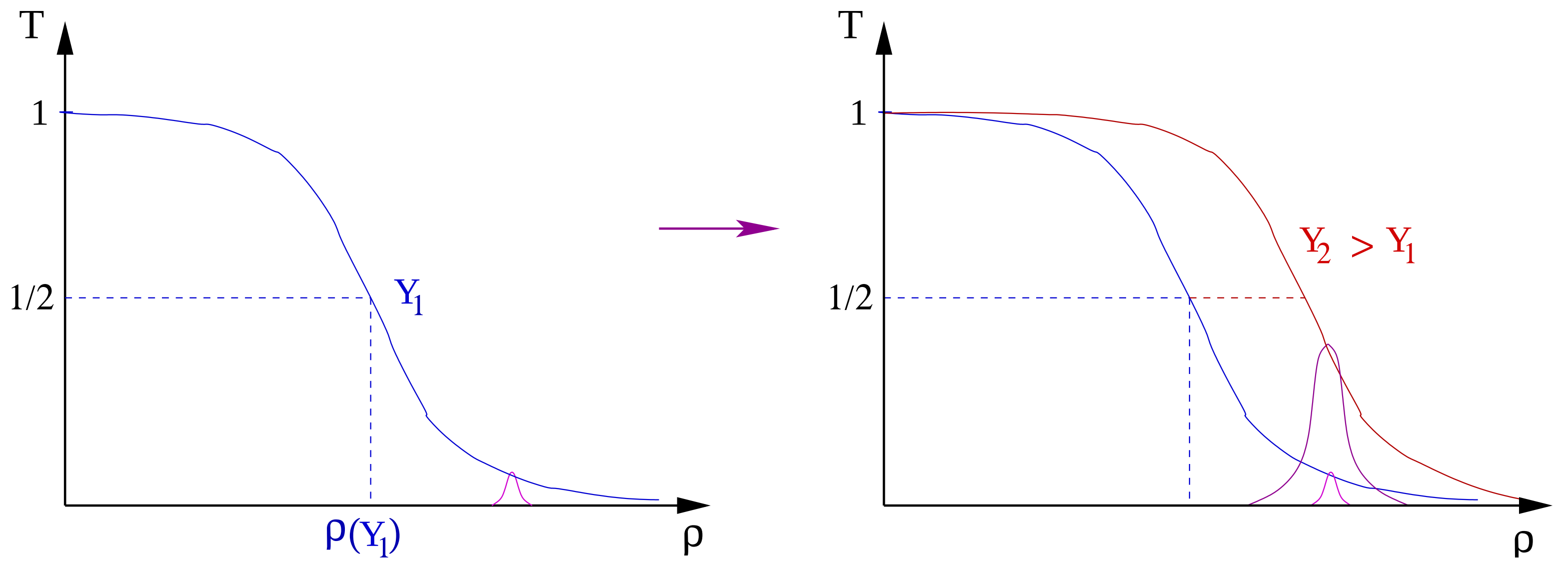
- Mean field approximation is not reliable in the tail of the front!

And so what ?!

Pulled Fronts & Fluctuations

- The propagation of the front is driven by the growth and spreading of the small perturbations about the unstable state
- The 'pulled front' dynamics is very sensitive to fluctuations !

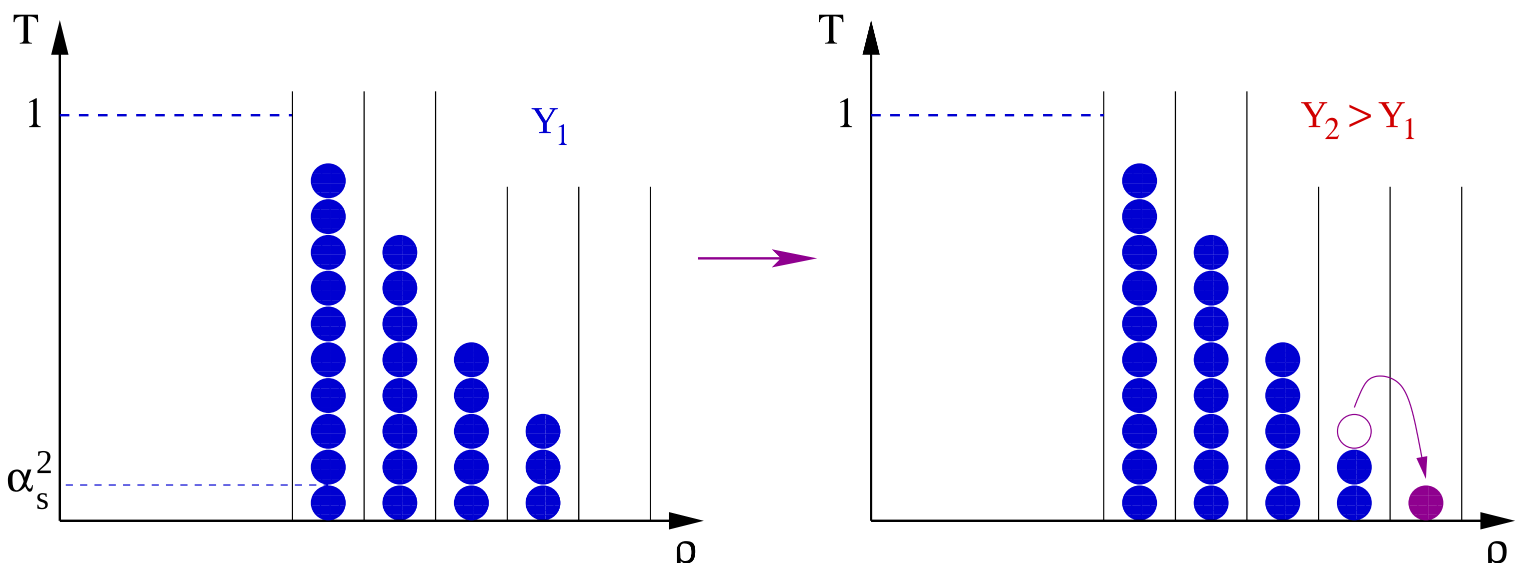
  "Fluctuating pulled fronts" (see arXiv:cond–mat after 97)
- Discreteness modifies the mechanism for front propagation
  - MFA : BFKL growth in the tail



Pulled Fronts & Fluctuations

- The propagation of the front is driven by the growth and spreading of the small perturbations about the unstable state
- The 'pulled front' dynamics is very sensitive to fluctuations !

  "Fluctuating pulled fronts" (see arXiv:cond–mat after 97)
- Discreteness modifies the mechanism for front propagation
  - Discrete system : Diffusion of gluons in the foremost bin



- As compared to the MFA, the front should slow down !

Saturation exponent with fluctuations

- Brunet–Derrida (97, for the reaction-diffusion process) :

  “There should be at least one gluon per bin for the BFKL growth to get started” : $n \geq 1$, or $T \gtrsim \alpha_s^2$.

$$\partial_Y T(\rho, Y) \,=\, \partial_\rho^2 T(\rho, Y) \,+\, \Theta(T - \alpha_s^2)\big(T - T^2\big)$$

- The speed of the front (saturation exponent) for $\alpha_s \to 0$:

$$\lambda_s \,\equiv\, \frac{1}{\bar{\alpha}_s}\,\frac{d\rho_s(Y)}{dY} \,\approx\, \lambda_0 \,-\, \frac{\mathcal{C}}{\ln^2(1/\alpha_s^2)}\,, \qquad \lambda_0 \approx 4.88, \quad \mathcal{C} \approx 150\,(!)$$

  Mueller, Shoshi (04); E.I., Mueller, Munier (04)

- An exact result in QCD in the limit $\alpha_s \to 0$

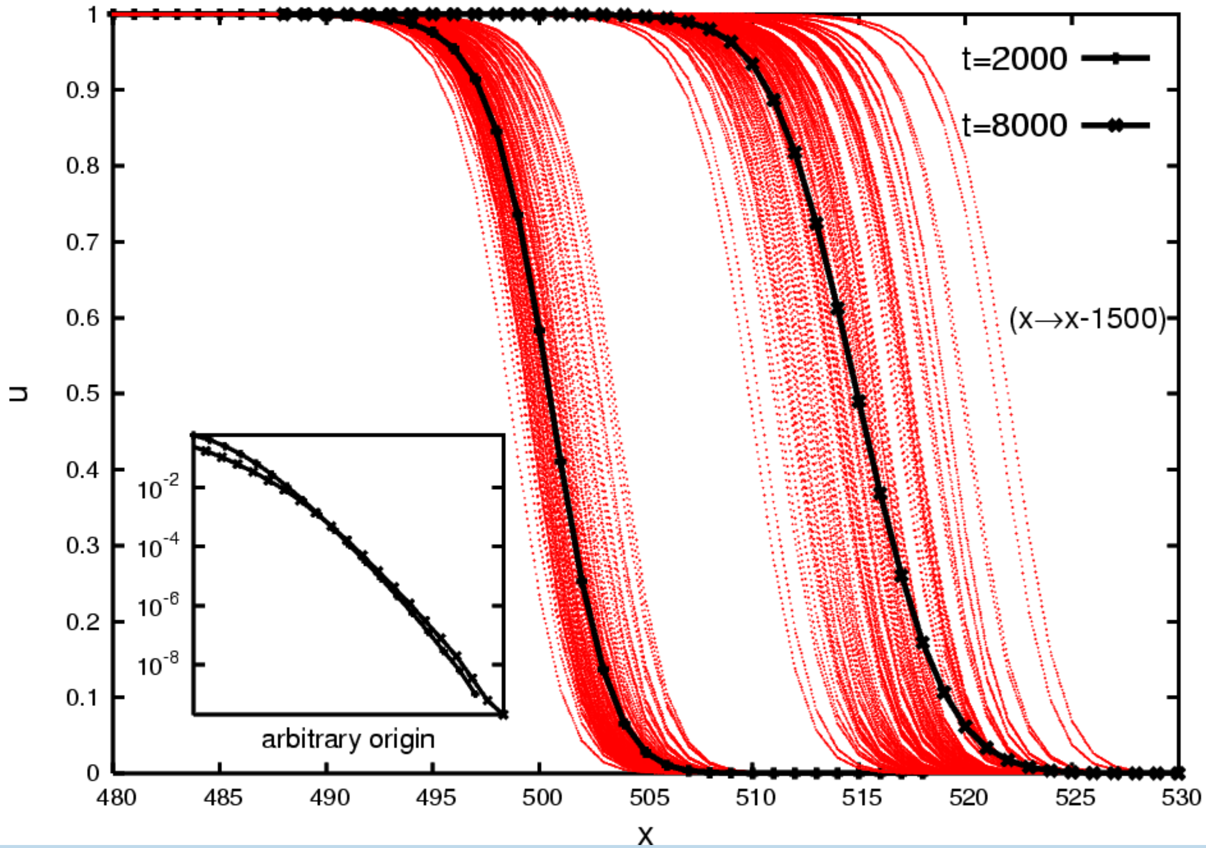
  ... but pretty useless for practical applications !

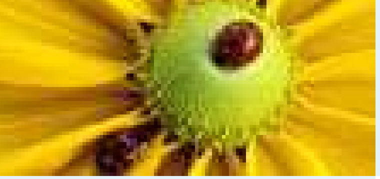
- Fluctuations are parametrically more important than the NLO BFKL corrections

Front diffusion violates geometric scaling

- A stochastic evolution generates un ensemble of fronts.
- The position $\rho_s(Y)$ of the front shows a diffusive wandering around its average value :

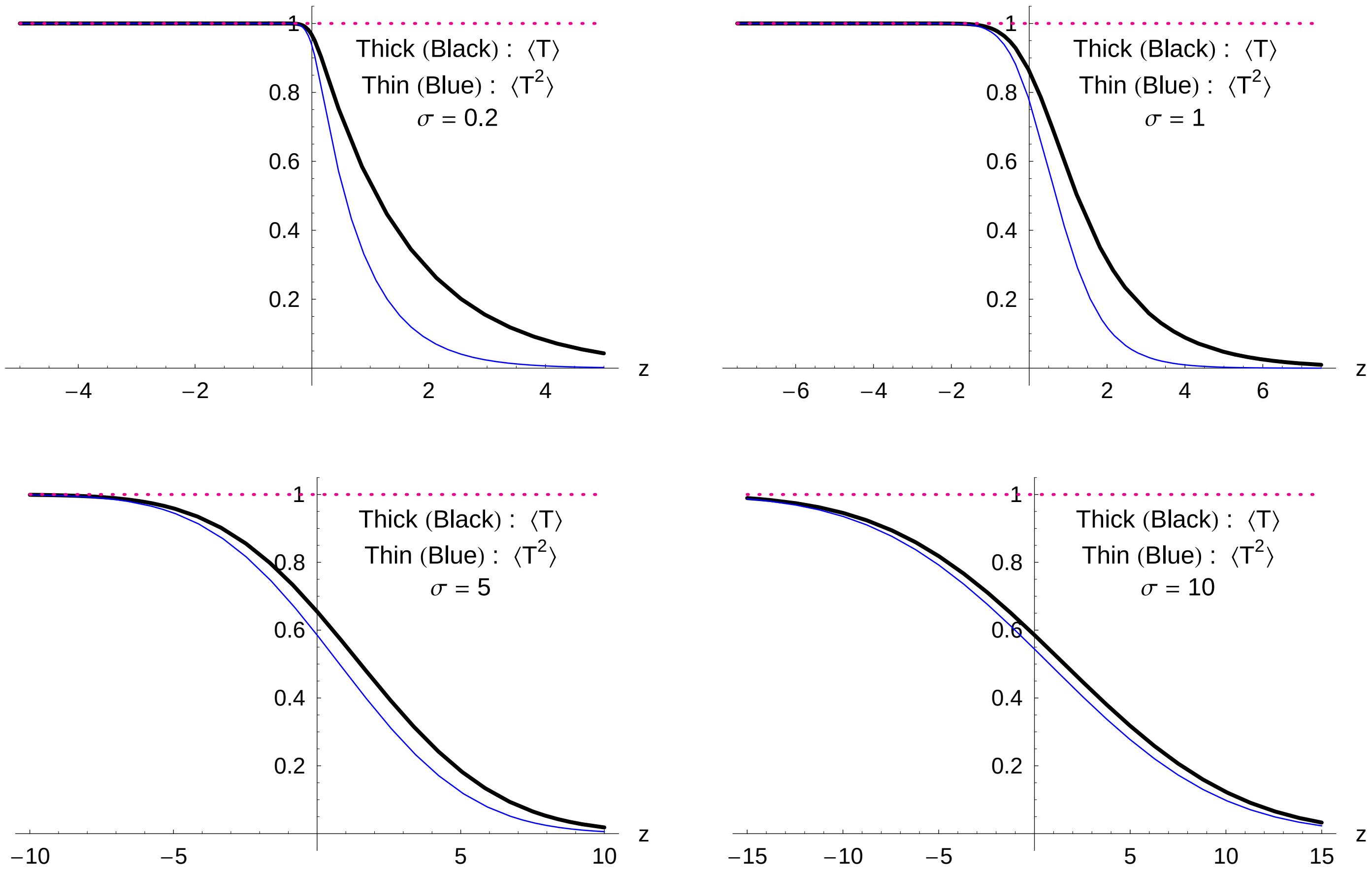
$$\langle \rho_s(Y) \rangle = \lambda_s \bar{\alpha}_s Y, \quad \langle \rho_s^2 \rangle - \langle \rho_s \rangle^2 = D \bar{\alpha}_s Y, \quad D \sim \frac{1}{\ln^3(1/\alpha_s^2)}$$





Breakdown of BFKL approx. in the tail

- In the MFA (BFKL) : $\langle T^2 \rangle \approx \langle T \rangle \langle T \rangle$ in the tail
- In the presence of fluctuations : $\langle T \rangle \approx \langle T^2 \rangle \cdots \approx \langle T^n \rangle$



- Averages in the tail are dominated by rare fluctuations

Beyond JIMWLK: Pomeron loops

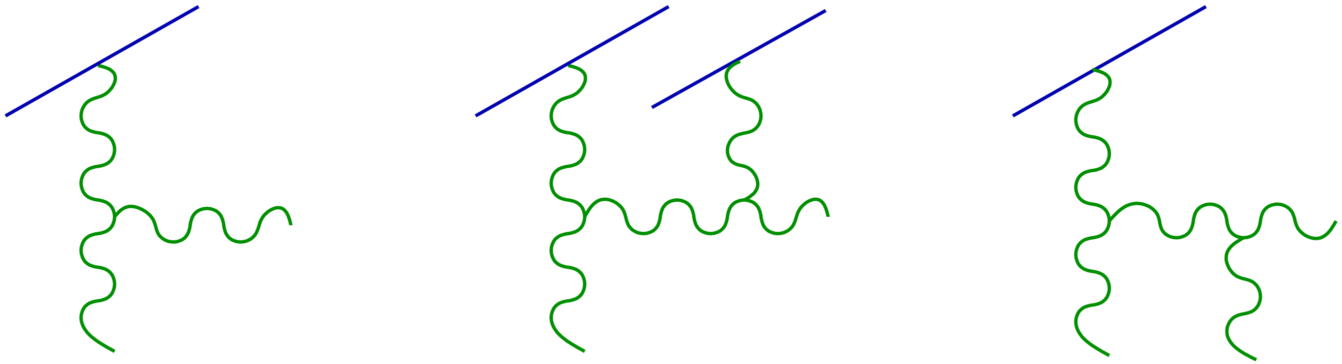
- How to go beyond asymptotic results ?
- The subasymptotic behaviour is not universal !

$\Longrightarrow$ One needs the actual evolution equations of QCD

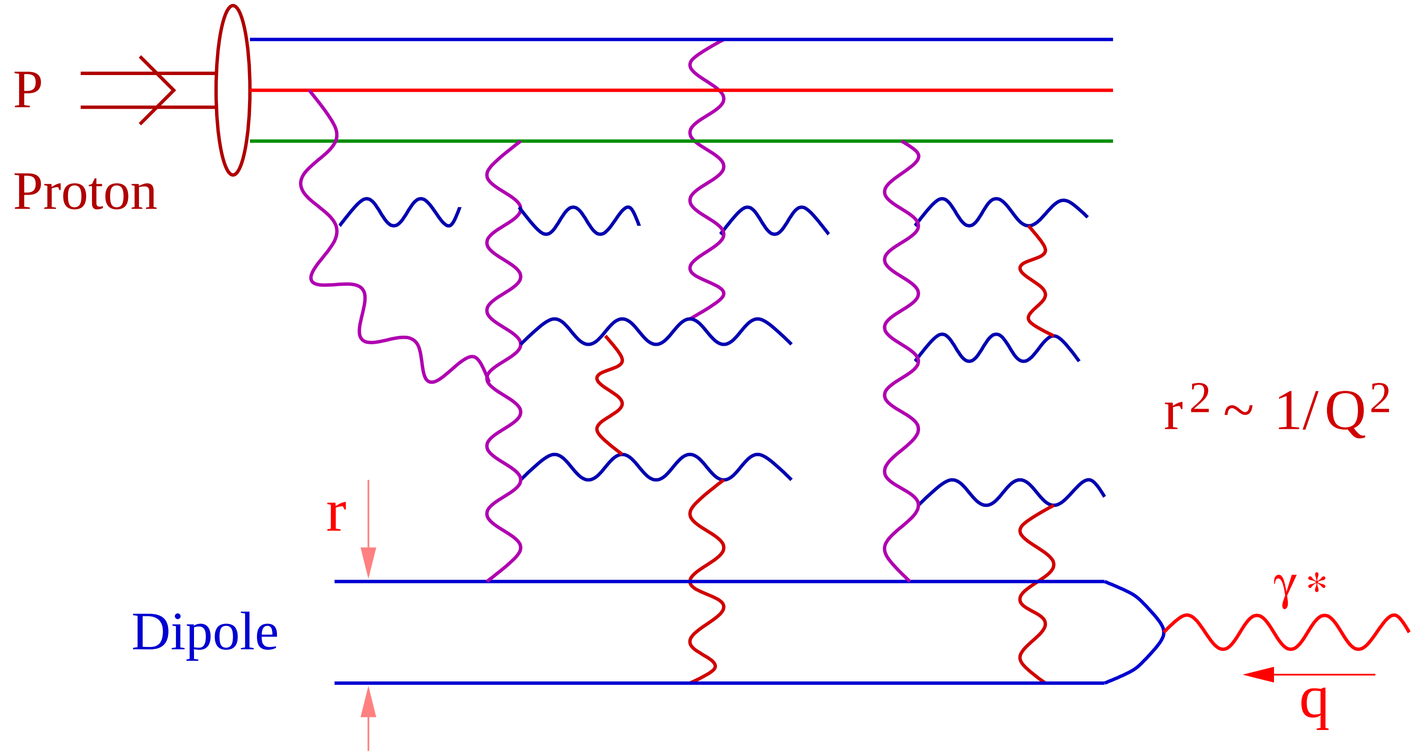
  - BFKL evolution
  - Gluon recombination (saturation) : JIMWLK
  - Particle number fluctuations

- The fluctuations are induced by radiation processes (bremsstrahlung) which are not included in JIMWLK !

(E.I., D. Triantafyllopoulos, 04)

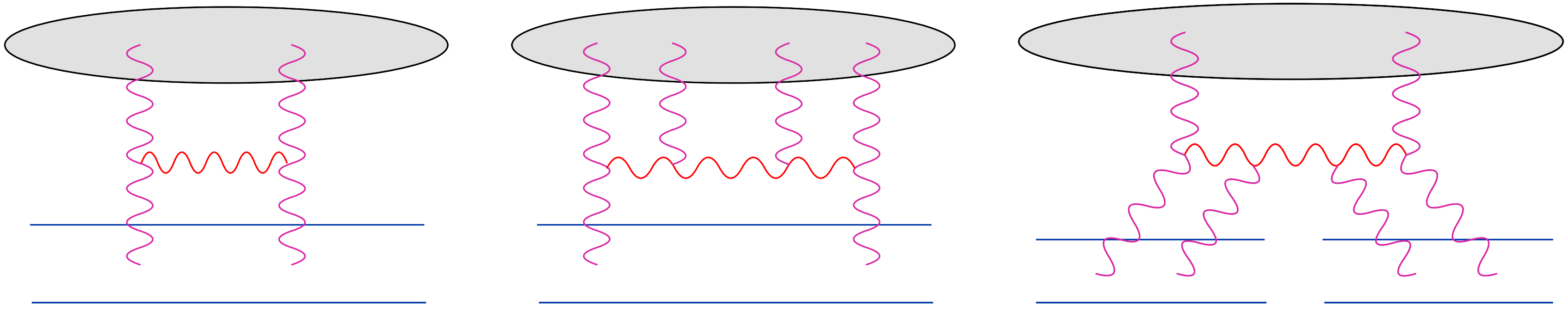


Update on DIS at small–x



- A summary of the QCD evolution required for computing deep inelastic scattering at (very !) small–x
- Gluon splitting + merging $\Longrightarrow$ ‘Pomeron loops’

E.I., D. Triantafyllopoulos (04, 05)



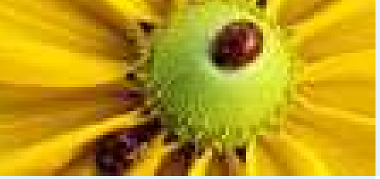
- Approximation: SU(3) $\longrightarrow$ SU(N_c) with $N_c \gg 1$

$$\frac{\mathrm{d}T}{\mathrm{d}Y} = T - T^{(2)}$$

$$\frac{\mathrm{d}T^{(2)}}{\mathrm{d}Y} = T^{(2)} - T^{(3)} + T$$

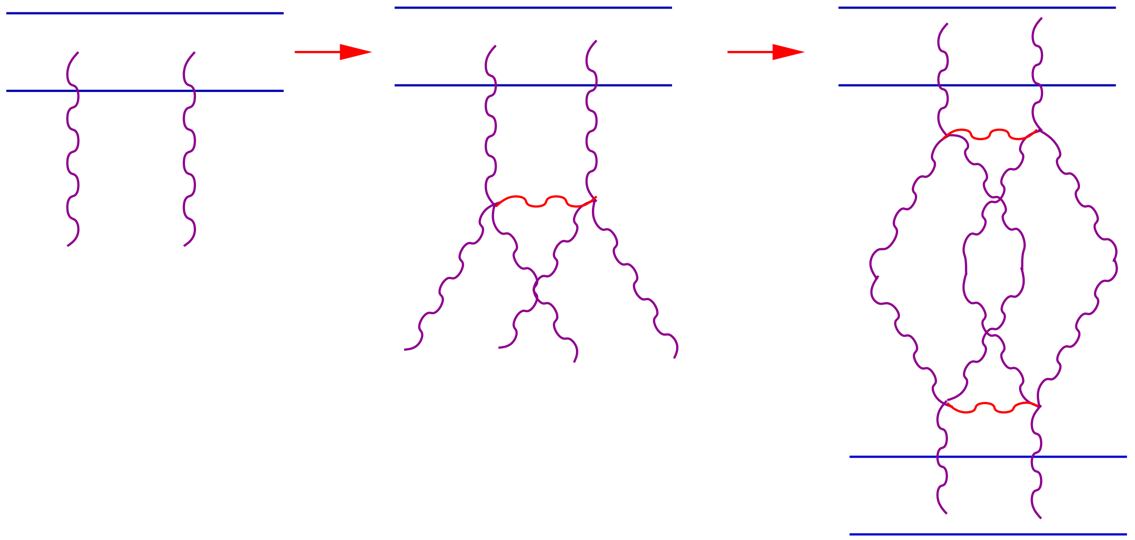
$$\cdots$$

$$\frac{\mathrm{d}T^{(n)}}{\mathrm{d}Y} = \underbrace{T^{(n)}}_{\text{BFKL}} - \underbrace{T^{(n+1)}}_{\text{merging}} + \underbrace{T^{(n-1)}}_{\text{splitting}}$$



Evolution equations with Pomeron loops (2)

- Splitting + Recombination $\Longrightarrow$ Pomeron loops



- Evolution Hierarchy $\approx$ A Langevin Equation ($\sim$ sFKPP)

$$\frac{\mathrm{d}T}{\mathrm{d}Y} = T - T^2 + \sqrt{\alpha_s^2\, T}\,\nu \quad \text{with} \quad \langle \nu(Y)\nu(Y')\rangle = \delta(Y - Y')$$

N.B. $T(r,Y) \approx \alpha_s^2\, n(r,Y) \quad \Longrightarrow \quad \delta T \sim \alpha_s^2 \sqrt{n} \sim \sqrt{\alpha_s^2\, T}$

- Well suited for numerical simulations

G. Soyez, hep-ph/0504129; Enberg, Golec–Biernat, Munier, 0505101

Conclusions & Open questions

- Color Glass Condensate
  High–density, but weakly coupled, form of gluonic matter
  which controls hadron interactions at high energies
- QCD evolution at high energy: Classical stochastic process
  - mean field aspects & fluctuations
  - non–linear effects $\Longrightarrow$ gluon saturation
  - unitarization of scattering amplitudes
  - geometric scaling and its violations
- Interesting new developments
  - relation to problems in statistical physics, chemistry, ...
  - evolution equations with Pomeron Loops
- Intense activity & Many open problems
  - urgent need for better estimates & numerics
  - evolution equations for $N_c = 3$
  - next–to–leading order corrections