

# **Proton – Nucleus Potential at the LHC**

**Jianwei Qiu**  
*Iowa State University*

**QCD at Cosmic Energies - II**  
**The Highest Energy Cosmic Rays and QCD**  
**Skopelos, Greece, September 26 – 30, 2005**

# Outline of the Talk

- Introduction and Terminology
- Important role of  $d+Au$  at RHIC
- $pA$  collisions at the LHC
- Benchmark tests (universal  $A$ -dependence)
- Semihard processes  
(calculable  $A$ -dependence)
- Summary and Outlook

# Introduction and Terminology

□ Cosmic ray energy near GZK region:

$$S_{\text{GZK}} \sim 1000 S_{\text{LHC}}$$

□ typical momentum exchange ( $y \sim 0$ ):  $\langle x \rangle \sim \frac{m_T}{\sqrt{s}/2}$

$$\langle x \rangle_{\text{RHIC}} (y \sim 0) \sim 10^{-2}$$

$$\langle x \rangle_{\text{LHC}} (y \sim 0) \sim 10^{-3}$$

$$\langle x \rangle_{\text{GZK}} (y \sim 0) \sim 10^{-4}$$

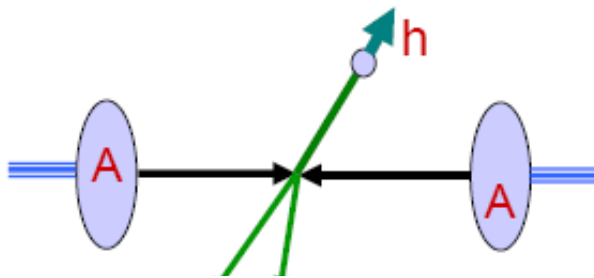
Unexplored region

$$\longrightarrow \langle x_2 \rangle_{\text{GZK}} (x_1 \sim 0.1) \sim 10^{-8}$$

□ Is leading power pQCD calculation valid?

❖ Processes  
❖  $\sqrt{s}$  (or  $x_1, x_2$ )

$$\frac{d\sigma}{dQ^2} = \int dx_1 dx_2 f^{(2)}(x_1, Q^2) f^{(2)}(x_2, Q^2) \frac{d\hat{\sigma}(x_1, x_2)}{dQ^2} \left[ 1 + O\left(\frac{1}{Q^\alpha}\right) \right]$$

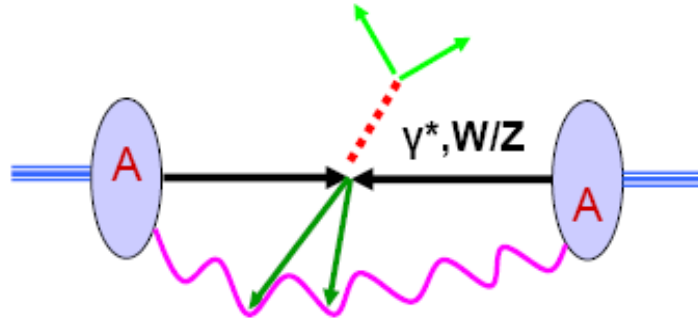


Neglect soft interaction between beams!

→ Universality of PDF's

# Factorization is an approximation

□ Drell-Yan cross section is **NOT** completely factorized!



$$\frac{d\sigma}{dQ^2} = f^{(2)} \otimes f^{(2)} \otimes \frac{d\hat{\sigma}^{(2)}}{dQ^2} + \frac{1}{Q^2} f^{(2)} \otimes f^{(4)} \otimes \frac{d\hat{\sigma}^{(4)}}{dQ^2} + \frac{1}{Q^4} F\left(\frac{Q^2}{S}\right) + \dots$$

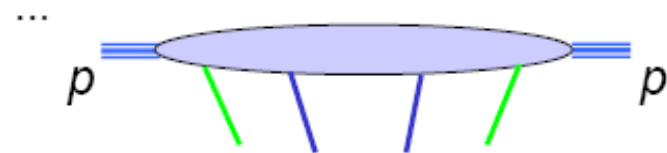
**Not factorized!**

- ❖ There is **always** soft gluon interaction between two hadrons!
- ❖ Gluon field strength is **one power** more Lorentz contracted than ruler

$$f^{(2)} \propto \langle p | \bar{\psi}(0) \gamma^+ \psi(y^-) | p \rangle, \\ \langle p | F^{+\alpha}(0) F_{\alpha}^+(y^-) | p \rangle$$



$$f^{(4)} \propto \langle p | \bar{\psi}(0) \gamma^+ F^{+\alpha}(y_1^-) F_{\alpha}^+(y_2^-) \psi(y^-) | p \rangle$$



# Sources of nuclear dependence

- ❑ **Universal** nuclear dependence:  
from **nuclear wave functions**

$$f_N^{(2)}(x, Q^2) \rightarrow f_A^{(2)}(x, Q^2) \propto \langle A | \bar{\psi}(0) \gamma^+ \psi(y^-) | A \rangle$$

- ❑ **Process-dependent** nuclear dependence  
(coherent power corrections)

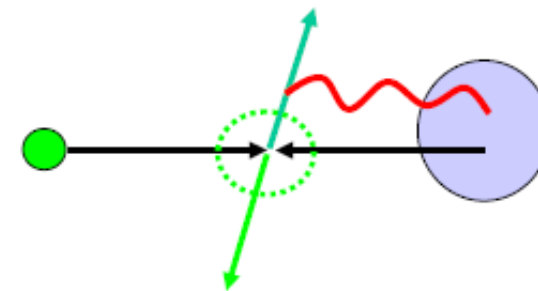
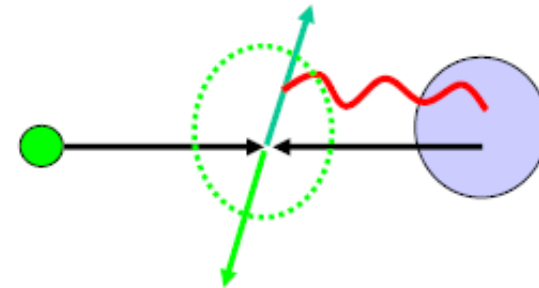
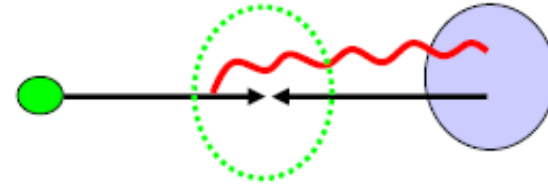
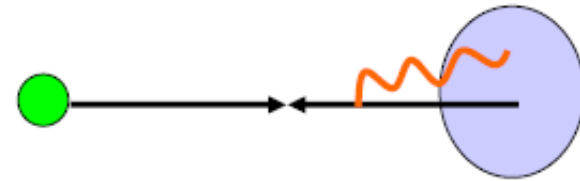
- **Initial-state:**
- **Final-state:**

$$f_N^{(2)}(x, Q^2) \rightarrow f_A^{(4)}(x_1, x_2, x_3, Q^2)$$

- ❖ **Change total cross section**

- ❑ Elastic scattering  
(incoherent multiple scattering)

- ❖ **Does not change total production rate**
- ❖ **Change the spectrum**

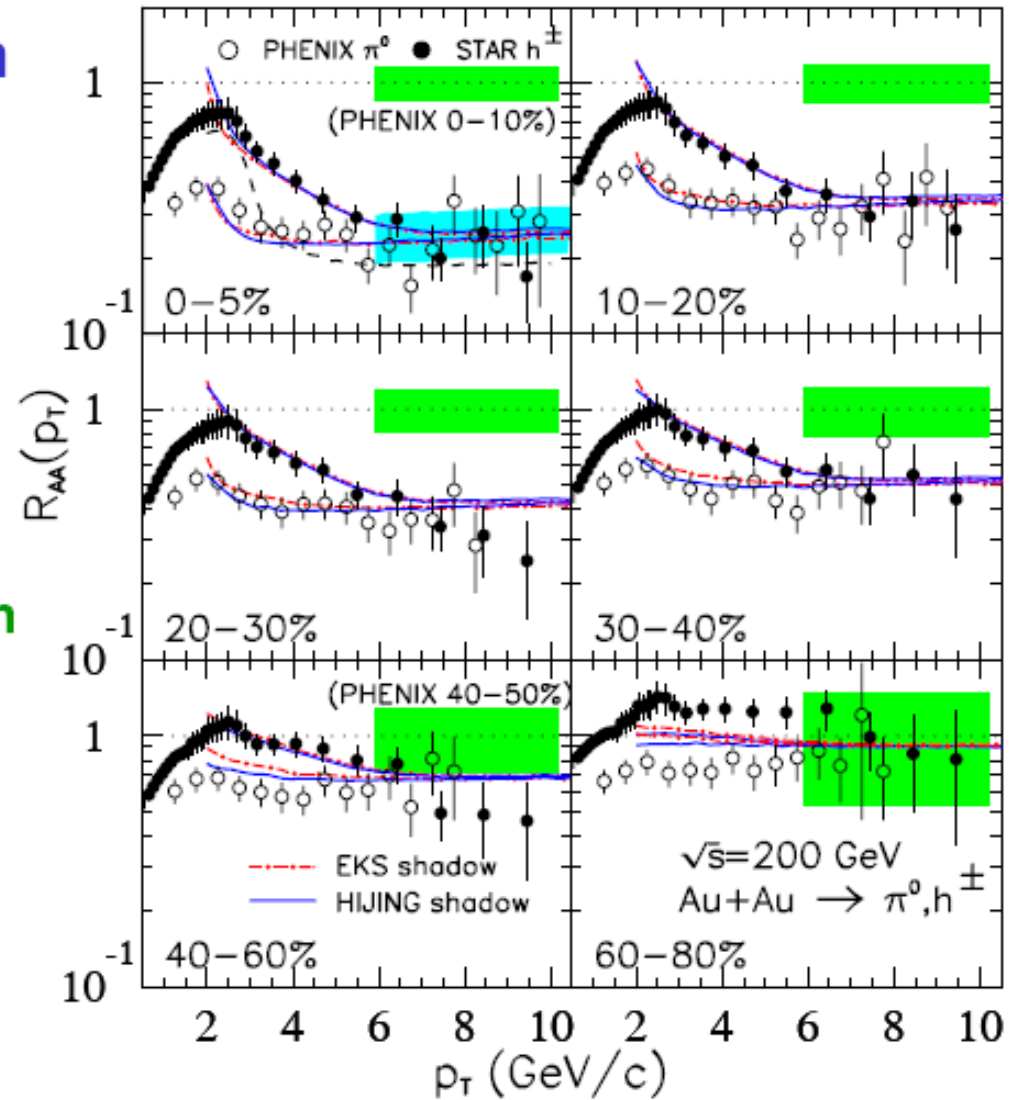


# Important role of $dA$ at RHIC

## Single inclusive hadron in AA collision

$$R^{AB} = \frac{\sigma_{AB}}{\langle N_{\text{binary}} \rangle_{AB} \langle \sigma_{NN} \rangle}$$

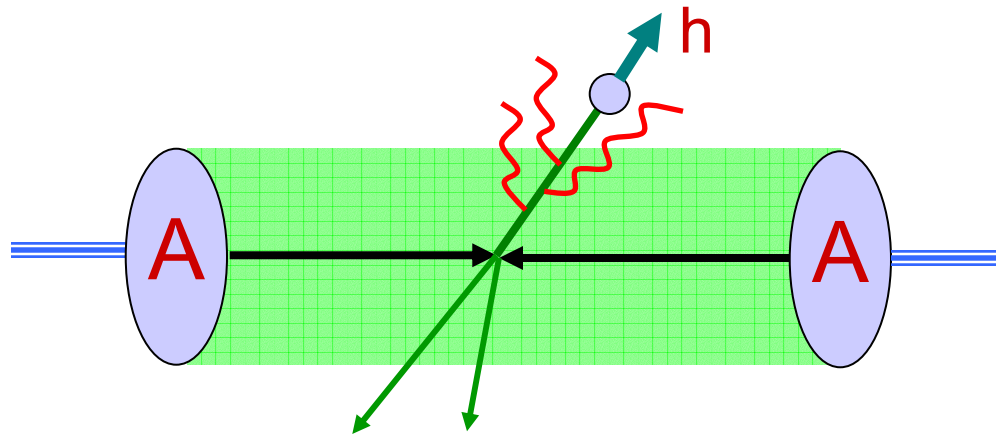
- ❖ What cause the suppression or enhancement?
- ❖ What should we expect for  $dA$ ?



# Jet quenching

## □ Assumptions:

- ❖ Soft interactions between the ions **does not** change the effective PDF's



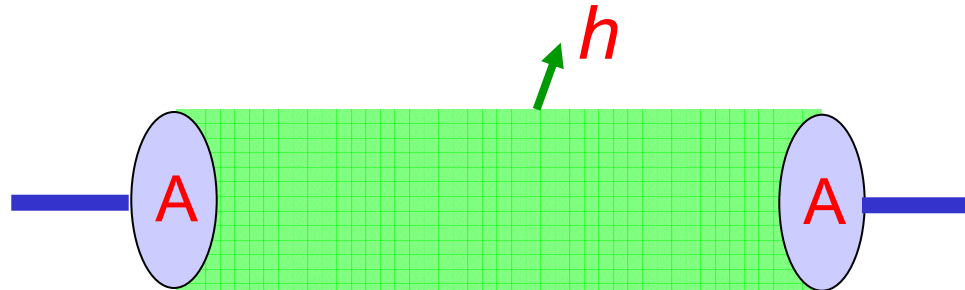
- ❖ Multiple scattering with the medium leads to energy loss
- ❖ Reduction of leading hadron momentum leads to suppression at high  $p_T$

**Suppression is a final-state effect**

**No suppression expected for dA**

# Saturation and CGC

- Soft interactions between the ions **alter** the PDF's



- ❖ Convolution of **two universal saturated distributions** at a saturation scale:  $Q_s \sim \text{GeV}$ ; or
- ❖ Solve classical Yang-Mills Equation
  - ⇒ **gluon density** from the AA collisions, then convert the gluons to the observed hadron
- Momentum of the GeV hadron  $h$  is balanced by many soft particles ⇒ **no back-to-back hadron correlation**

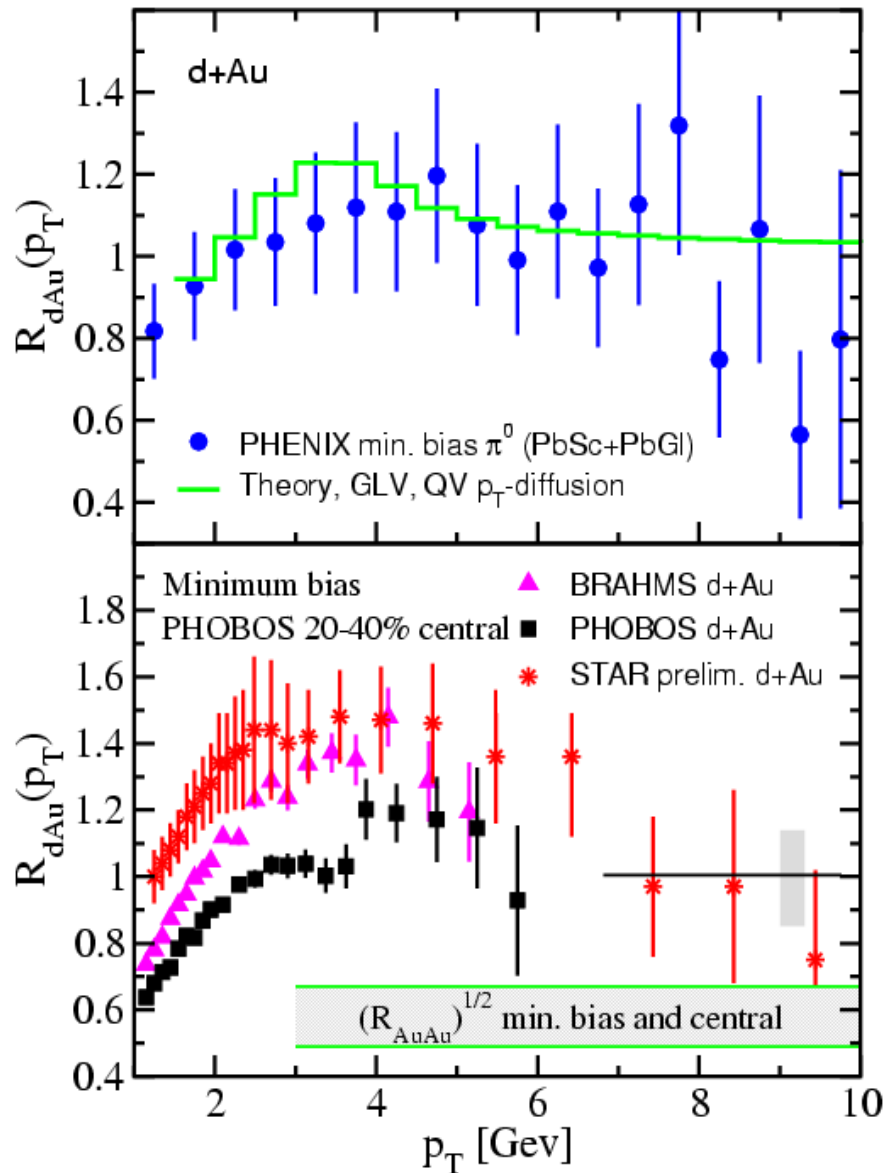
**Suppression is a initial-state effect**

**Large suppression expected for dA**

$$R_{dAu} = \sqrt{R_{AuAu}} \ll 1, \quad R_{AuAu} \simeq 0.2 - 0.4$$



# Comparison to the d+Au Data



## RHIC Data from:

B.Back *et al.* [PHOBOS],  
Phys.Rev.Lett. 91 (2003)

J.Adams *et al.* [STAR],  
Phys.Rev.Lett. 91 (2003)

S.Adler *et al.* [PHENIX],  
Phys.Rev.Lett. 91 (2003)

I.Arsene *et al.* [BRAHMS],  
Phys.Rev.Lett. 91 (2003)

## Theoretical predictions:

I.Vitev and M.Gyulassy,  
Phys.Rev.Lett. 89 (2002)

I.Vitev,  
Phys.Lett. B562 (2003)

D.Kharzeev, E.Levin, L.McLerran,  
Phys.Lett. B 561 (2003)

## Current Data from RHIC:

- support Cronin type effect in d+Au
- disfavor the saturation picture in d+Au

## Parton $x$ is not small enough:

- Increases collision energy – the LHC
- moves to the forward region – lower  $x$

## ***pA* program at the LHC**

- Calibrate the AA measurements  
(lesson from RHIC)**
- Test QCD dynamics that proton-proton  
cannot provide  
(differences between pA and AA)**
- Help extrapolate the hadronic collisions  
to cosmic ray energies**
- ...**

# Benchmark Tests

## □ Predictive Power of PQCD - Factorization

Scale of hadron wave function:  $\Lambda \sim 1/\text{fm} \sim 200 \text{ MeV}$  - nonperturbative

Scale of hard partonic collision:  $Q \gg \text{GeV}$  - perturbative

**Time dilation:**

dynamics at the scale of  $\Lambda$  is effectively frozen during the partonic hard collision at the scale  $Q$

**Parton model:**

$$\sigma_{\text{hadron}}(Q, \Lambda) \approx \hat{\sigma}_{\text{parton}}(Q) \otimes f(\Lambda)$$

**QCD Factorization:**

$$\sigma_{\text{hadron}}(Q, \Lambda) \approx \underbrace{\hat{\sigma}_{\text{parton}}(Q/\mu, \alpha_s(\mu))}_{\text{Infrared safe}} \otimes \underbrace{f^{(2)}(\mu, \Lambda)}_{\text{Universal upto power corrections}} + \mathcal{O}\left(\frac{1}{Q^\alpha}\right)$$

Infrared safe

Universal upto  
power corrections

□ Benchmark tests = “no” power corrections = hard probe

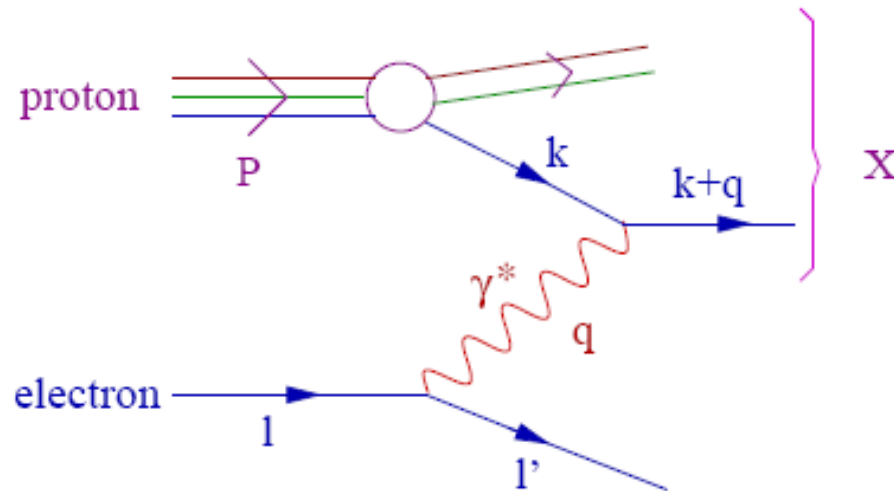
# Questions

- Where power corrections come from?
- How can we calculate or estimate the size of power corrections?
- Is power correction more (or less) important at small  $x$ ?
- Is power correction enhanced or suppressed in nuclear collisions?
- ...

# An example

Inclusive lepton – hadron deep inelastic scattering

$$\text{electron } (l) + \text{proton } (P) \longrightarrow \text{electron } (l') + X (P_X)$$



- Two independent kinematical invariants :
  - ◆  $Q^2 \equiv -q^\mu q_\mu \geq 0$
  - ◆  $x \simeq Q^2/s$  with  $s \equiv (P + q)^2 \gg Q^2$

lancu's talk

# Small-x and coherence length

- Hard probe – process with a large momentum transfer:

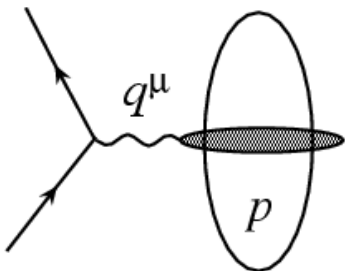
$$q^\mu \quad \text{with} \quad Q \equiv \sqrt{|q^2|} \gg \Lambda_{\text{QCD}}$$

- Size of a hard probe is very **localized** and much **smaller** than a typical hadron at rest:

$$\frac{1}{Q} \ll 2R \sim \text{fm}$$

- But, it might be **larger** than a **Lorentz contracted** hadron:

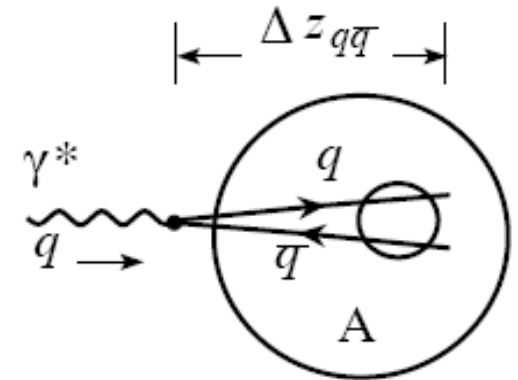
$$\frac{1}{Q} \sim \frac{1}{xp} \gg 2R \left( \frac{m}{p} \right) \quad \text{or equivalently} \quad x \ll x_c \equiv \frac{1}{2mR} \sim 0.1$$



If an active parton  $x$  is small enough  
the hard probe could cover several nucleons  
In a Lorentz contracted large nucleus!

# Coherence length in different frames

- Use DIS as an example – in target rest frame:  
virtual photon fluctuates into a q-qbar pair



- Lifetime of the  $q\bar{q}$  state:

$$\Delta E_{q\bar{q}} \sim \nu - E_{q\bar{q}} \sim \frac{Q^2}{2\nu} \left[ 1 + \mathcal{O}\left(\frac{m_{q\bar{q}}^2}{Q^2}\right) \right]$$

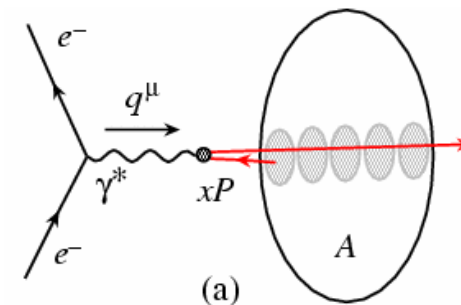
$$\Delta z_{q\bar{q}} \sim \frac{1}{\Delta E_{q\bar{q}}} \sim \frac{2\nu}{Q^2} = \frac{1}{m x_B}$$

- $\Delta z_{q\bar{q}} \gg 2$  fm, inter-nuclear distance, if  $x_B \ll 0.1$

- If  $x_B \ll 0.1$ , the probe – q-qbar state of the virtual can interact with who hadron/nucleus coherently.

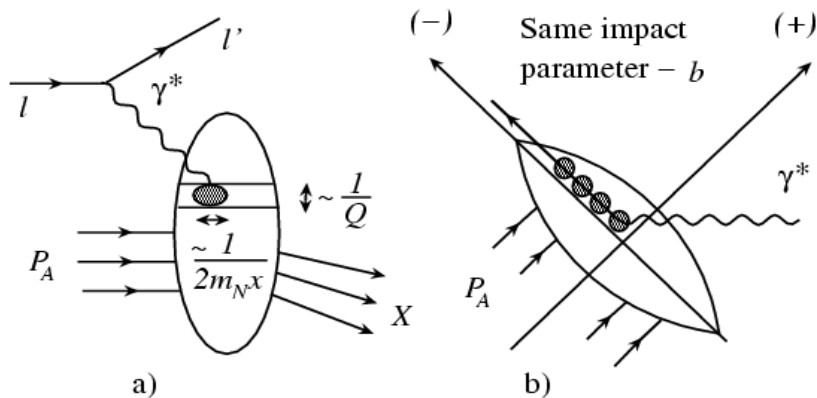
**The conclusion is frame independent**

- In Breit frame:  
coherent final-state rescattering



# Dynamical power corrections

- Coherent multiple scattering leads to dynamical power corrections:



$$d\sigma \approx d\sigma^{(S)} + d\sigma^{(D)} + \dots$$

Naïve power counting:

$$\frac{d\sigma^{(D)}}{d\sigma^{(S)}} \sim \alpha_s \frac{1/Q^2}{R^2} \langle F^{+\alpha} F_{\alpha}^+ \rangle A^{1/3}$$

2D lightcone dynamics

- Characteristic scale for the power corrections:  $\langle F^{+\alpha} F_{\alpha}^+ \rangle$

- For a hard probe:  $\frac{\alpha_s}{Q^2 R^2} \ll 1$

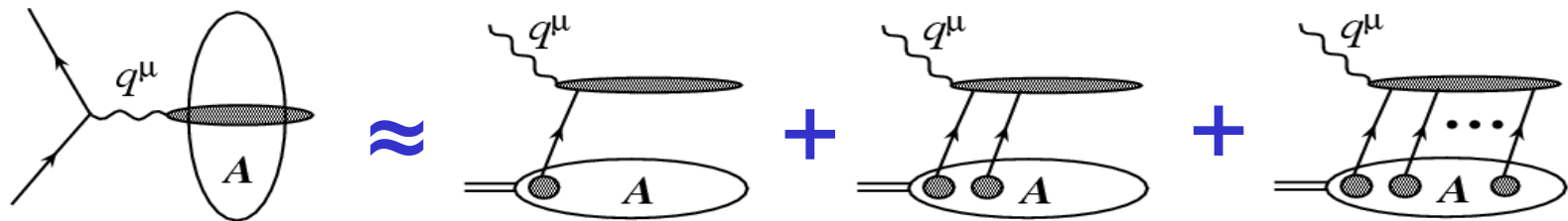
- Enhanced by nuclear radius:  $A^{1/3} \leq 6$

- Enhanced by the slope of small-x distribution:  $-\frac{\partial}{\partial x} \varphi(x)$

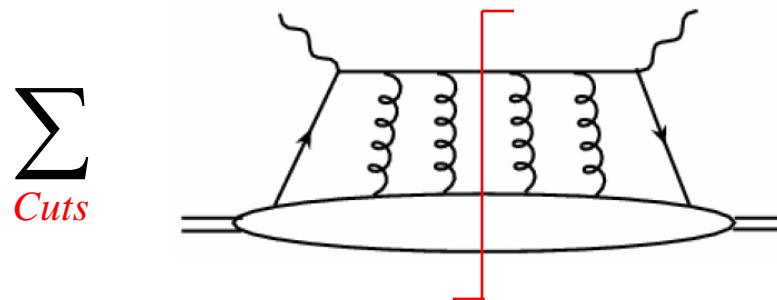


# Coherent multiparton interactions

At small  $x$ , the hard probe covers several nucleons, coherent multiple scattering could be equally important at relatively low  $Q$



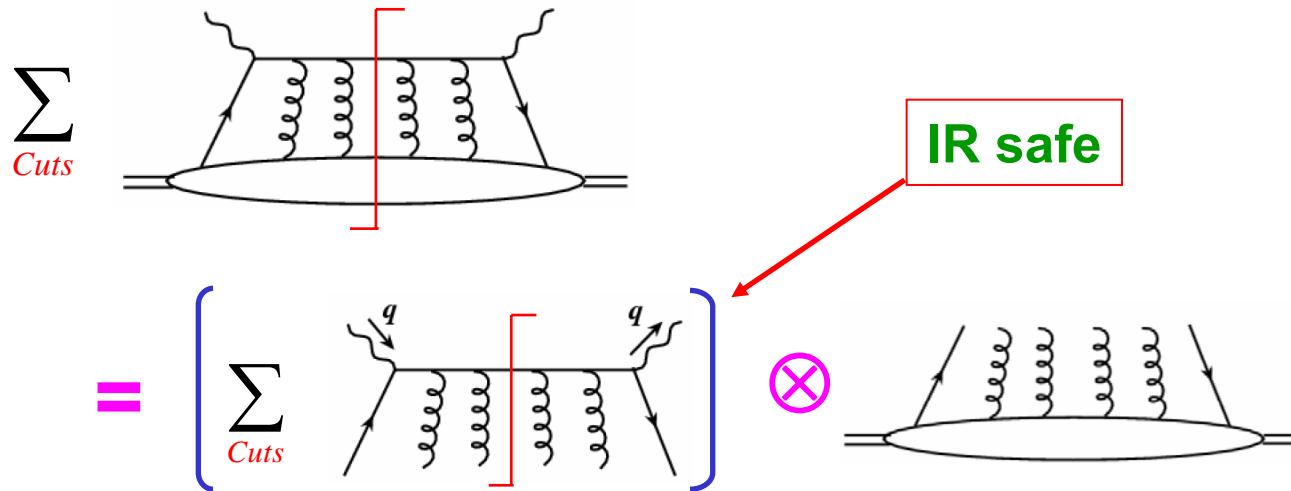
To take care of the coherence, we need to sum over all **cuts** for a given forward scattering amplitude



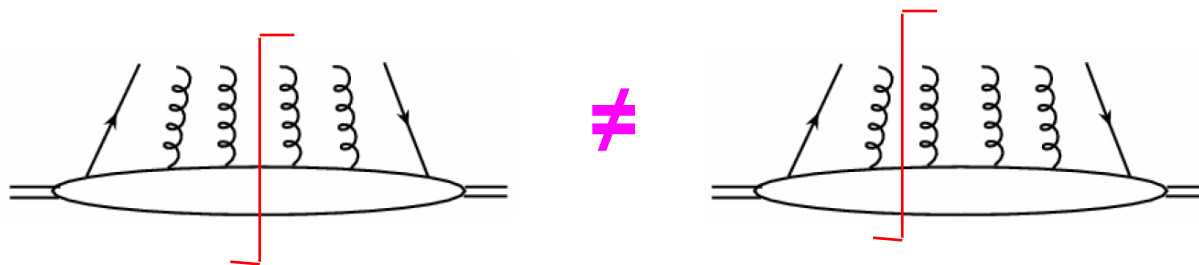
Summing over all **cuts** is also necessary for **IR** cancellation

# Collinear approximation is important

With collinear approximation:



Different cuts for matrix elements of partons with  $k_T$  are not equal:



# Factorization beyond leading power

## □ Consequence of OPE:

$$\begin{aligned}\sigma_{phys}^h &= \hat{\sigma}_2^i \otimes [1 + C^{(1,2)}\alpha_s + C^{(2,2)}\alpha_s^2 + \dots] \otimes T_2^{i/h}(x) \\ &+ \frac{\hat{\sigma}_4^i}{Q^2} \otimes [1 + C^{(1,4)}\alpha_s + C^{(2,4)}\alpha_s^2 + \dots] \otimes T_4^{i/h}(x) \\ &+ \frac{\hat{\sigma}_6^i}{Q^4} \otimes [1 + C^{(1,6)}\alpha_s + C^{(2,6)}\alpha_s^2 + \dots] \otimes T_6^{i/h}(x) \\ &+ \dots\end{aligned}$$

Leading twist

Power corrections

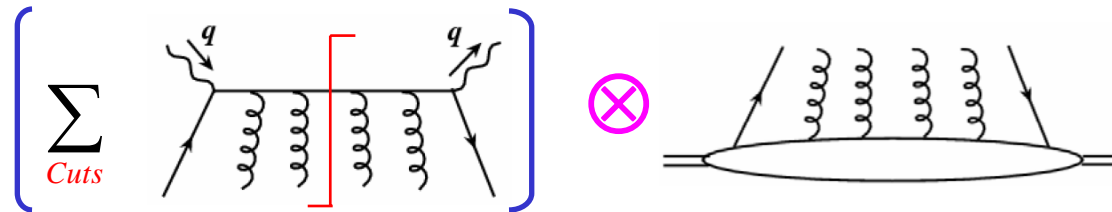
## □ Predictive power:

- ❖ Coefficient functions are IR safe
- ❖ Distributions/correlations/matrix elements are universal

## □ Distributions are defined to remove all collinear divergences of the partonic scattering

# Multiparton correlation functions

## □ Parton momentum convolution:



$$\propto \int \prod_i dy_i^- e^{ix_i p^+ y_i^-} \langle P_A | \prod_i F^{+\perp}(y_i^-) | P_A \rangle$$

All coordinate space integrals are **localized** if **x** is large

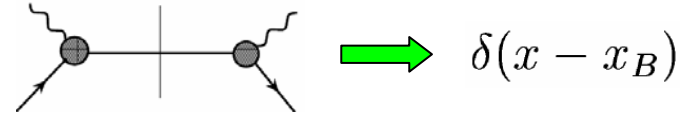
## □ Leading pole approximation for $dx_i$ integrals :

- $dx_i$  integrals are fixed by the poles (no pinched poles)
- $x_i=0$  removes the exponentials
- $dy$  integrals can be extended to the size of nuclear matter

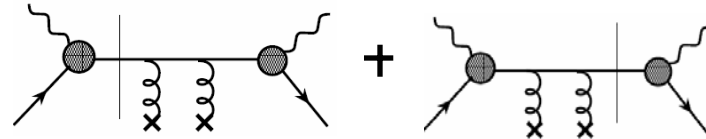
Leading pole leads to highest powers in medium length,  
a much small number of diagrams to worry about

# Multiple soft rescattering at tree-level

LO contribution to DIS cross section:



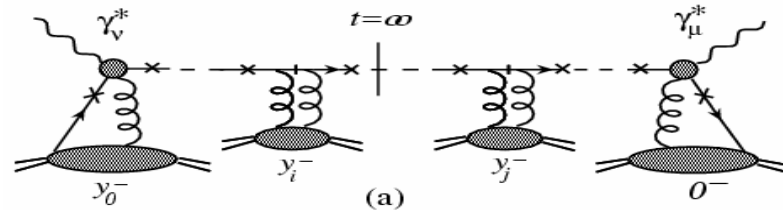
NLO contribution:



$$\rightarrow \frac{g^2}{Q^2} \left( \frac{1}{2N_c} \right) \left[ 2\pi^2 \tilde{F}^2(0) \right] x_B \lim_{x_1 \rightarrow x} \left[ \frac{1}{x - x_1} \delta(x_1 - x_B) + \frac{1}{x_1 - x} \delta(x - x_B) \right]$$

$$\int \frac{dy_2^- dy_1^-}{(2\pi)^2} \left[ F^{+\alpha}(y_2^-) F_{\alpha}^+(y_1^-) \right] \theta(y_2^-) \quad x_B \left[ -\frac{d}{dx} \delta(x - x_B) \right]$$

Nth order contribution:



$$\left[ \frac{g^2}{Q^2} \left( \frac{1}{2N_c} \right) \left[ 2\pi^2 \tilde{F}^2(0) \right] \right]^N x_B^N \lim_{x_i \rightarrow x} \sum_{m=0}^N \delta(x_m - x_B) \left[ \prod_{i=1}^m \left( \frac{1}{x_{i-1} - x_m} \right) \right] \left[ \prod_{j=1}^{N-m} \left( \frac{1}{x_{m+j} - x_m} \right) \right]$$

$$x_B^N \left[ (-1)^N \frac{1}{N!} \frac{d^N}{dx^N} \delta(x - x_B) \right]$$

**Infrared safe!**

# Model for the correlation functions

□ Matrix elements:

$$\left\langle P_A \left| \bar{\Psi}(0) \frac{\gamma^+}{2p^+} \Psi(y^-) \left[ \prod_{i=1}^N \int \tilde{F}^2(0) \right] \right| P_A \right\rangle$$

□ Approximation:

**Nucleus is made of a group of loosely bound nucleons**

$$|P_A\rangle \propto \prod_{i=1}^A |p\rangle \quad \text{with } p = \frac{P_A}{A}$$

$$\left\langle P_A \left| \hat{O}_0 \prod_{i=1}^N \hat{O}_i \right| P_A \right\rangle \propto A \langle p | \hat{O}_0 | p \rangle \prod_{i=1}^N \langle p | \hat{O}_i | p \rangle$$

□ Reduce the correlation functions to **one** unknown  
– a universal matrix element

$$\langle p | F^{+\alpha} F_{\alpha}^+ | p \rangle$$

# Contributions to DIS structure functions

## □ Transverse structure function:

Qiu and Vitev, PRL (2004)

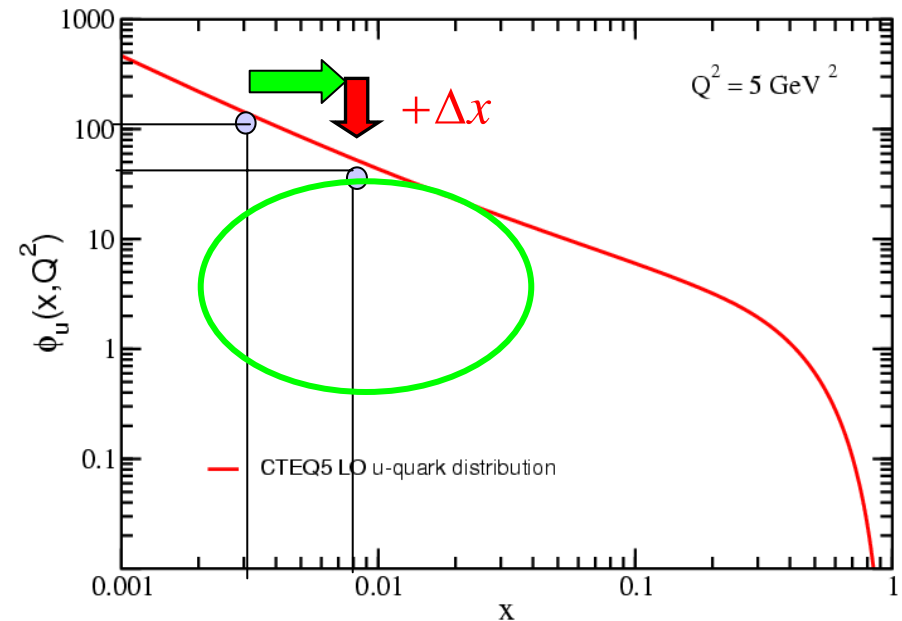
$$F_T(x_B, Q^2) = \sum_{n=0}^N \frac{1}{n!} \left[ \frac{\xi^2}{Q^2} (A^{1/3} - 1) \right]^n x_B^n \frac{d^n}{dx_B^n} F_T^{(0)}(x_B, Q^2)$$

$$\approx F_T^{(0)}(x_B(1 + \Delta), Q^2)$$

$$\Delta \equiv \frac{\xi^2}{Q^2} (A^{1/3} - 1)$$

$$\xi^2 = \frac{3\pi\alpha_s}{8R^2} \langle F^{+\alpha} F_{\alpha}^+ \rangle$$

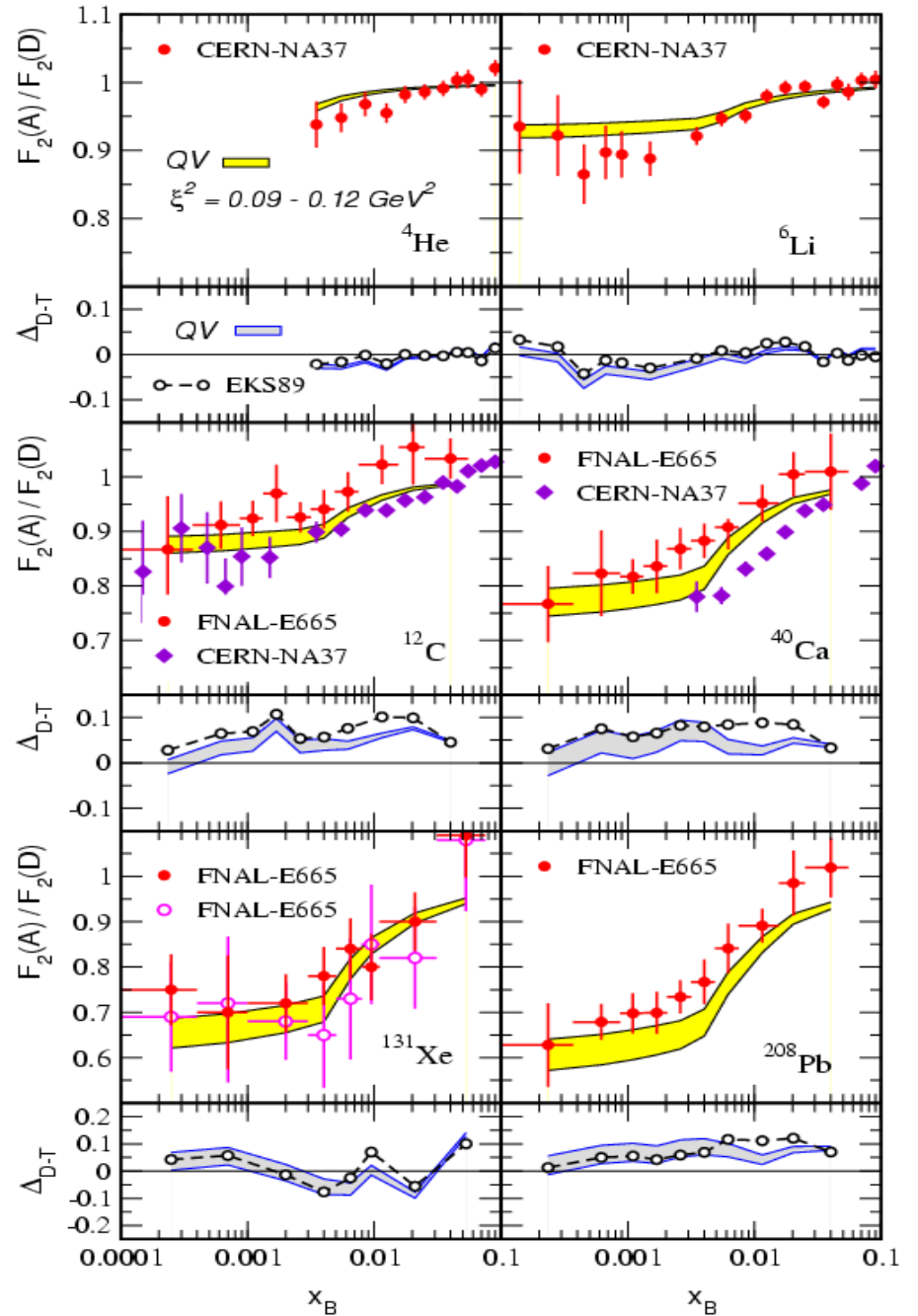
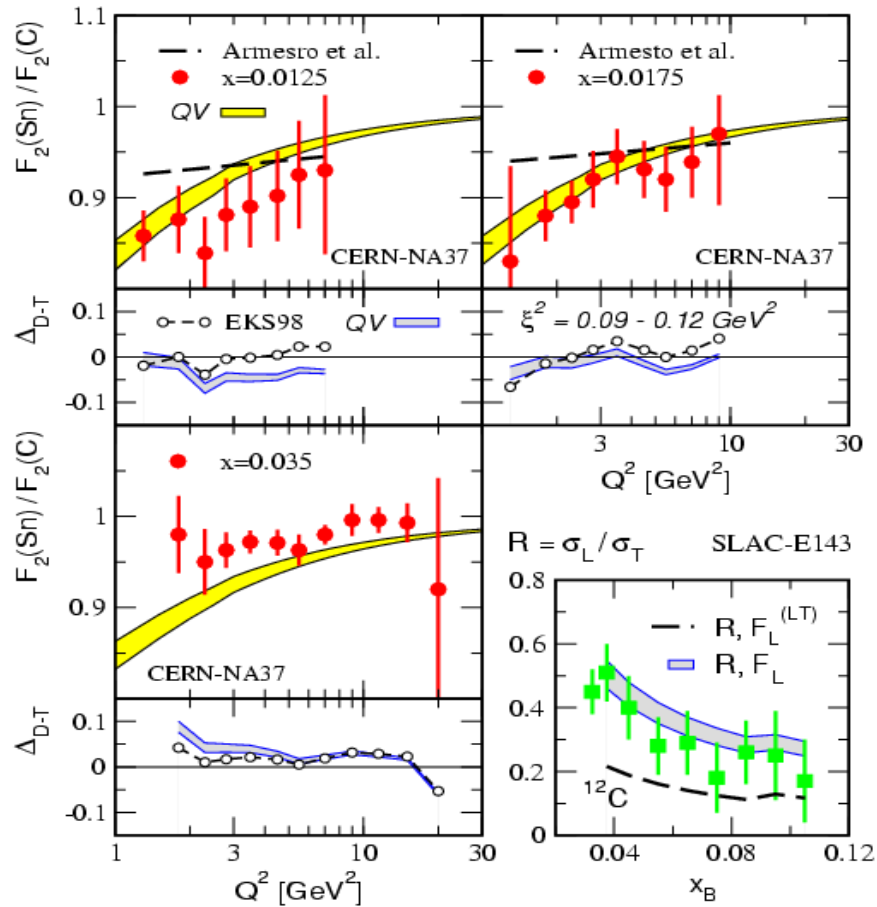
Single parameter for the power correction, and is proportional to the same characteristic scale



## □ Similar result for longitudinal structure function

# Neglect LT shadowing upper limit of $\xi^2$

$$\xi^2 \sim 0.09 - 0.12 \text{ GeV}^2$$





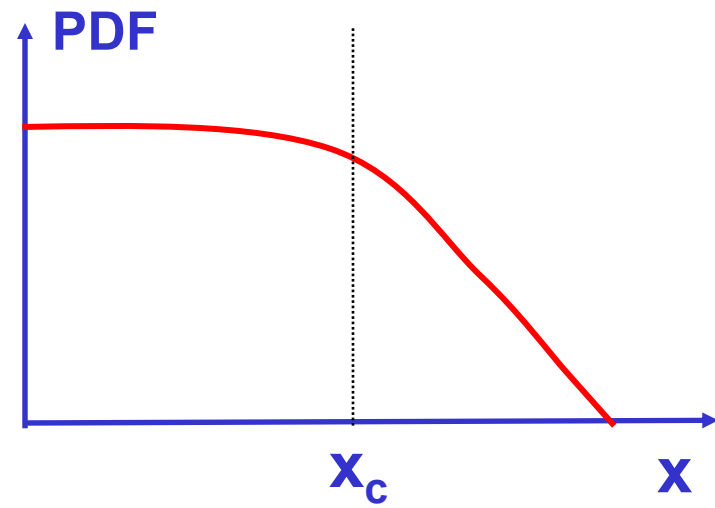
# Leading twist shadowing

## □ Power corrections complement to the leading twist shadowing:

- ❖ Leading twist shadowing changes the  $x$ - and  $Q$ -dependence of the **parton distributions**
- ❖ Power corrections to the **DIS structure functions** (or cross sections) are effectively equivalent to **a shift in  $x$**
- ❖ Power corrections **vanish** quickly as hard scale  $Q$  increases while the leading twist shadowing goes away **much slower**

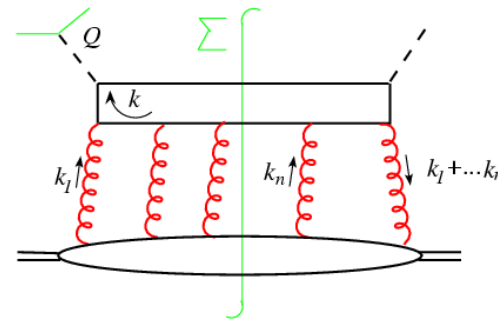
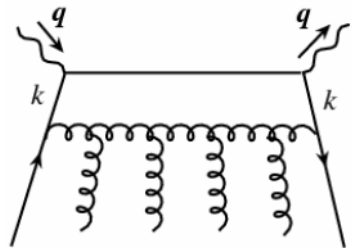
□ If leading twist shadowing is so strong that  **$x$ -dependence of parton distributions saturates** for  $x < x_c$ ,

additional power corrections, **the shift in  $x$** , should have **no effect to the cross section!**

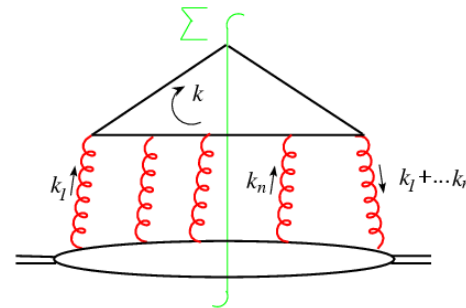
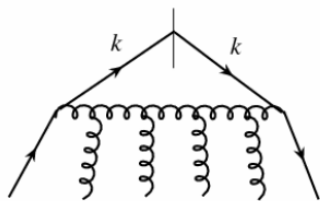


# Beyond the tree-level

- Correlation functions need to remove all collinear divergences in partonic scattering – factorization
- DGLAP evolved PDFs do not remove the collinear divergences beyond single scattering

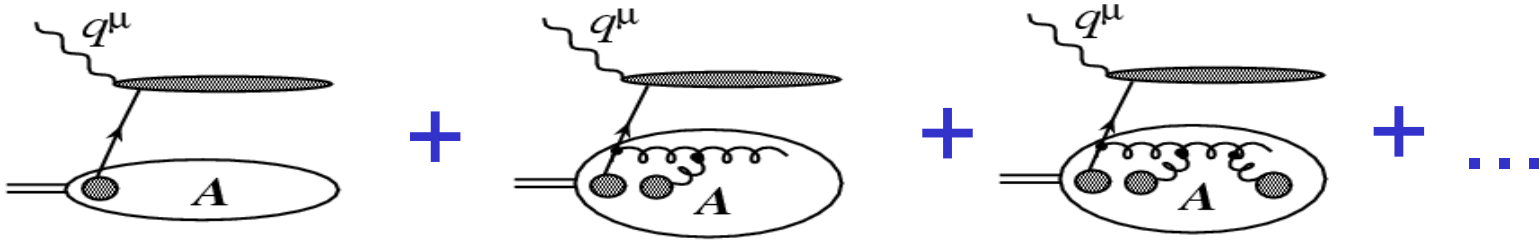


- Redefine PDFs to include all collinear divergences of partonic subprocesses → leading twist shadowing

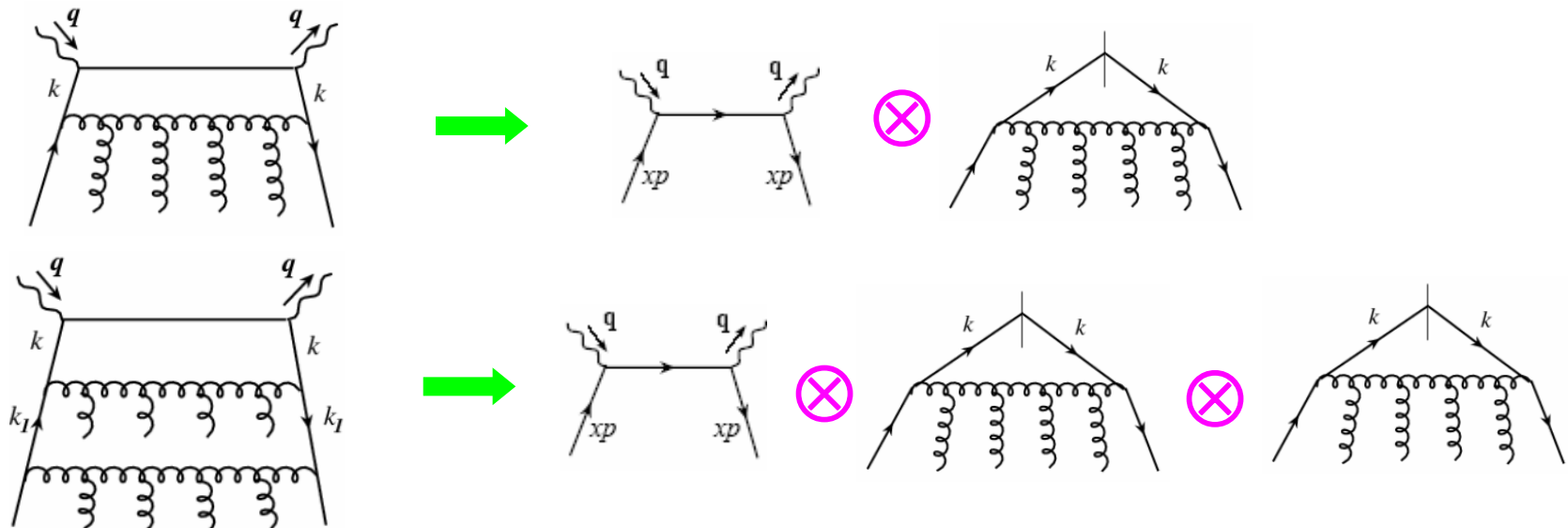


# Coherent power corrections to PDFs

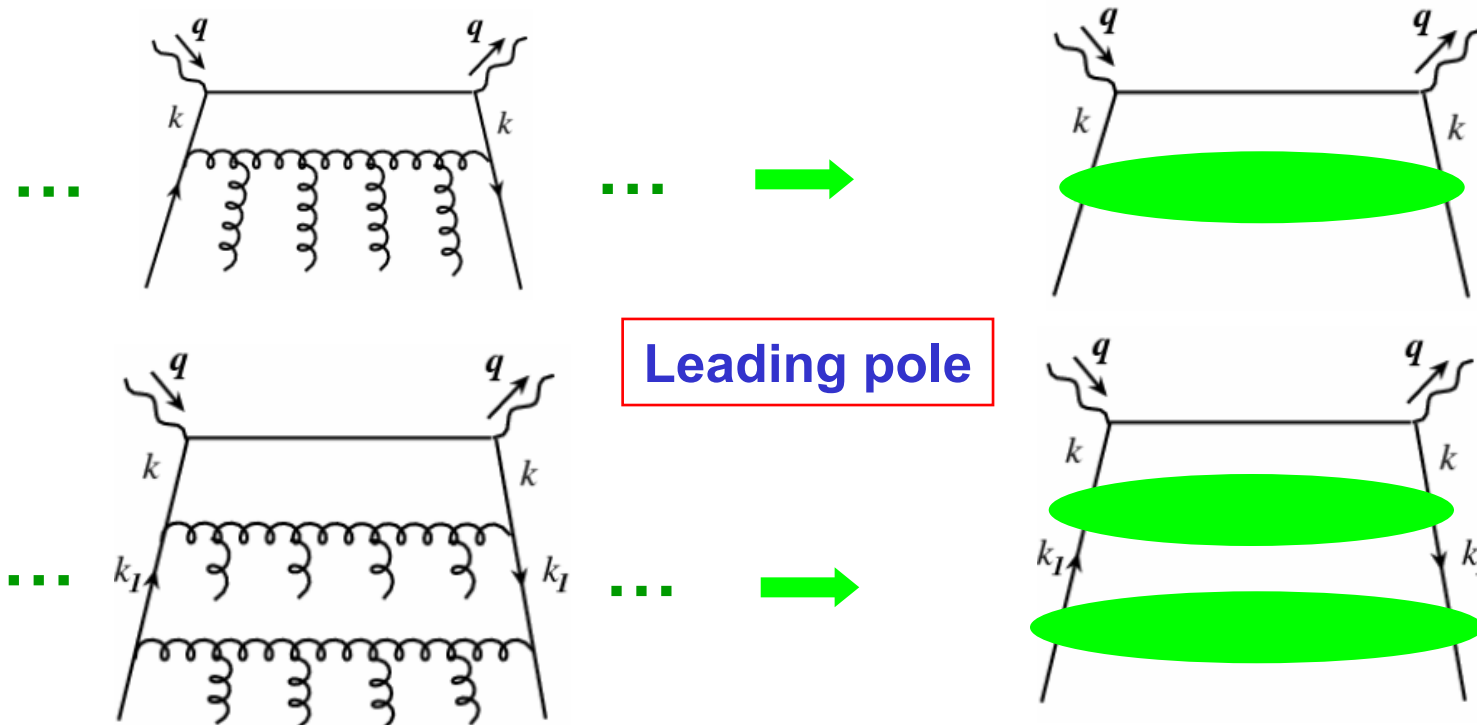
Hard probe sees only one effective parton:



Pinched poles in the ladder diagrams – corrections to evolution



# Modified ladder diagrams

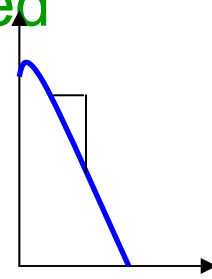


**Modified DGLAP evolution equations**

- ❑ Stay close to collinear factorization – PDF's
- ❑  $x$  is not too small
- ❑ Slow down the evolution in small- $x$  region

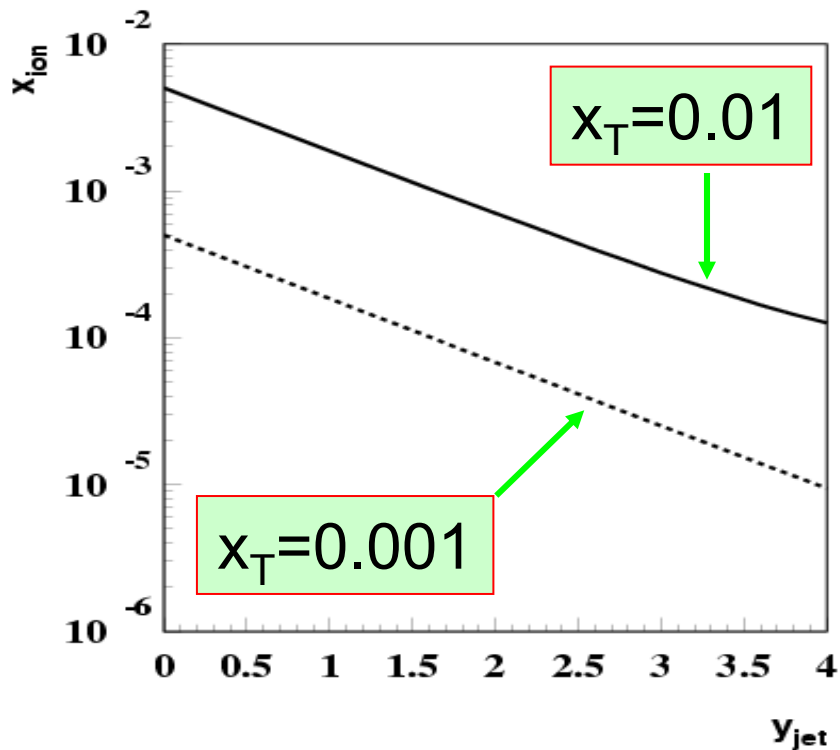
# A-dependence of benchmark tests

- ❑ Coherent multiple scattering is power suppressed
  - ❖ But, enhanced by nuclear size
  - ❖ Enhanced effect to steep falling distributions
- ❑ No power correction = Single hard scattering
- ❑ Leading power collinear factorized formula
- ❑ A-dependence of benchmark tests should only involve the universal nuclear dependence from PDF's
  - y-dependence of  $W$ ,  $Z$ , Higgs, Drell-Yan inclusive cross sections
  - $W$ ,  $Z$ , Higgs, Drell-Yan transverse momentum distributions
  - Low mass Drell-Yan at high  $p_T$ , and direct photon (isolation cut?)
  - Inclusive Jets at large  $E_T$
  - Heavy quarkonium transverse momentum distributions at large  $p_T$
  - *etc.*

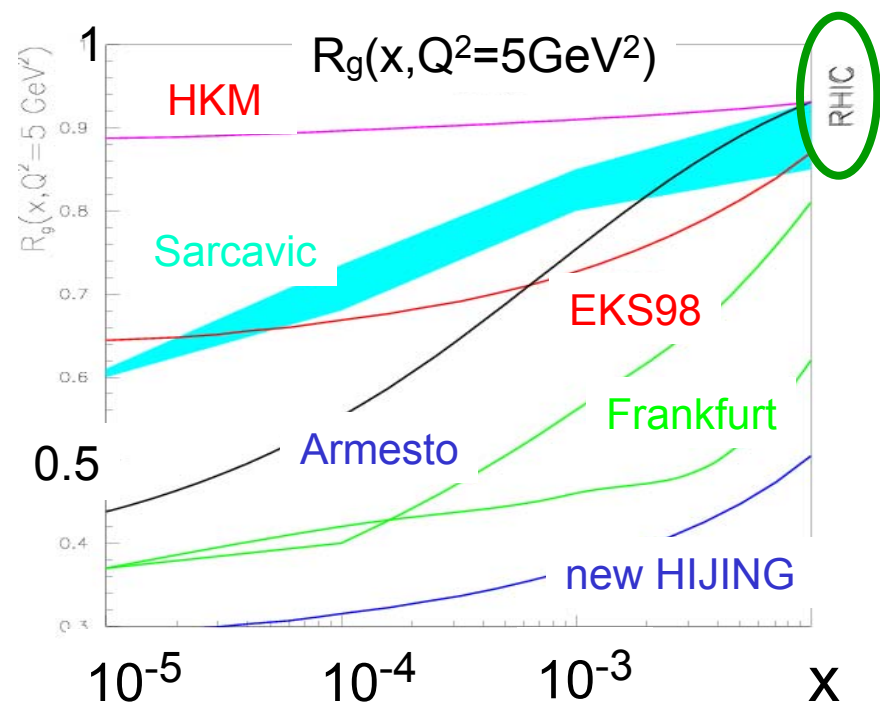


# Nuclear Parton Distribution Functions

- Probes small  $x$  region  
(for inclusive jet production)



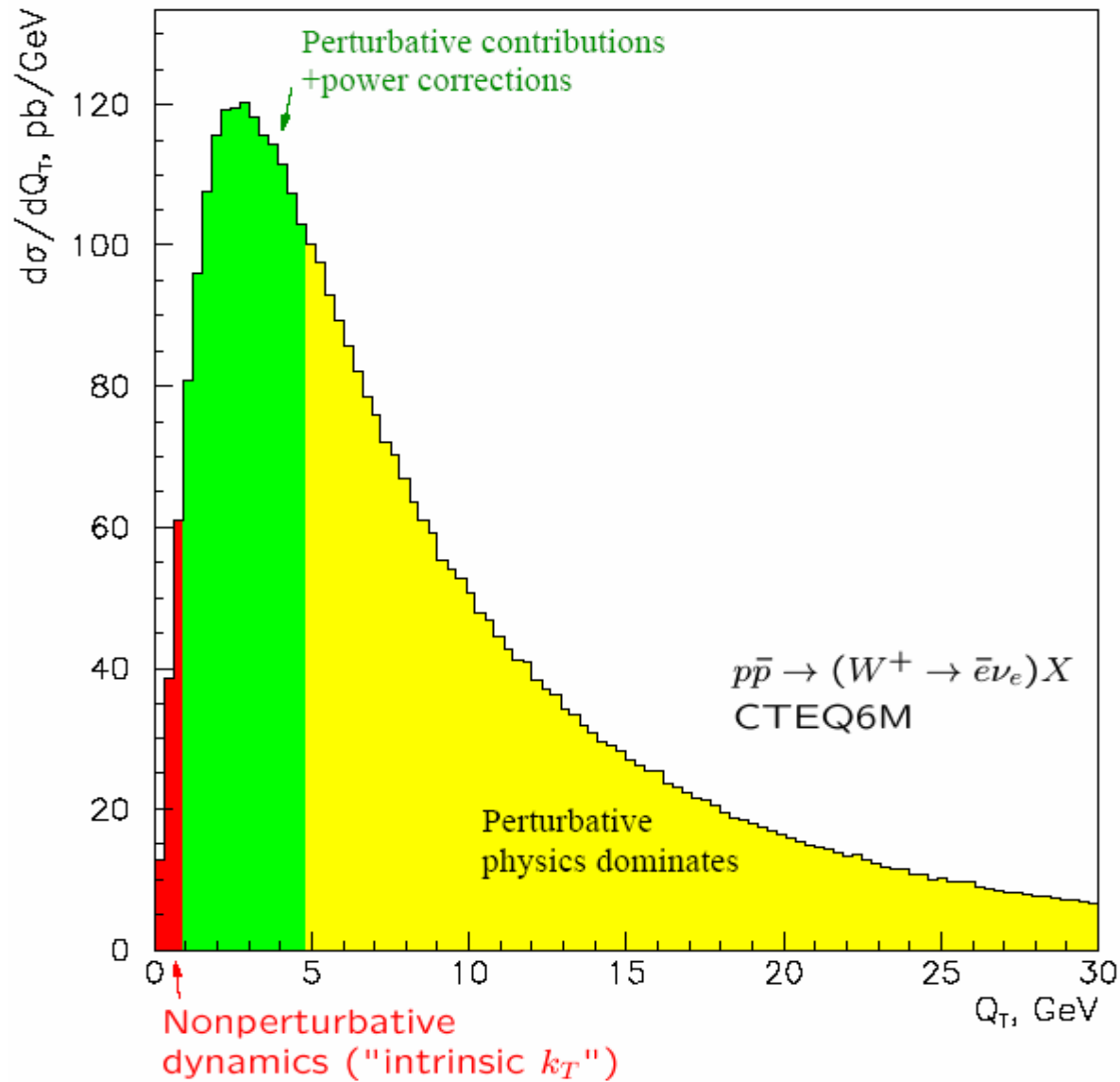
- Poor knowledge on nuclear parton distributions



Hard processes at the LHC can probe parton  $x$  as small as  $10^{-5}$ !

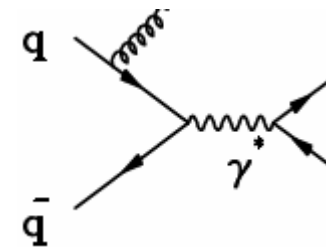
But, nuclear PDF's (in particular, gluon) are poorly constrained!

# W/Z, Higgs, Drell-Yan $Q_T$ distribution



Showing the different theoretical regions in momentum space

Drell-Yan type subprocess



Photon can be replaced by W, Z, Higgs, etc.

# QCD resummation

□ For processes with two large observed scales,

$$Q_1^2 \gg Q_2^2 \gg \Lambda_{\text{QCD}}^2 \quad \text{e.g. } p_T\text{-distribution of } Z^0$$

we could choose:  $\mu = Q_1$  or  $Q_2$ , or somewhere between

→  $\alpha_s(Q_1^2)$  is small,  $\alpha_s(Q_1^2) \ln(Q_1^2 / Q_2^2)$  is not necessary small

Cannot remove the logarithms by choosing a proper  $\mu$

→ **Resummation of the logarithms is needed**  
– the virtual photon fragmentation functions

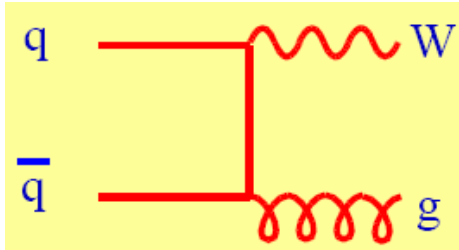
□ For a massless theory, we can get two powers of the logarithms at each order in perturbation theory:

$$\alpha_s(Q_1^2) \ln^2(Q_1^2 / Q_2^2)$$

**because of an overlap region of IR and CO divergences**



# Double log resummation



**LO Differential  $Q_T$ -distribution as  $Q_T \rightarrow 0$  :**

$$\frac{d\sigma}{dydQ_T^2} \approx \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \times 2C_F \left( \frac{\alpha_s}{\pi} \right) \frac{\ln(Q^2/Q_T^2)}{Q_T^2} \Rightarrow \infty$$

**□ Resum the double leading logarithms – DDT formula:**

$$\frac{d\sigma}{dydQ_T^2} \approx \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \times 2C_F \left( \frac{\alpha_s}{\pi} \right) \frac{\ln(Q^2/Q_T^2)}{Q_T^2} \times \exp \left[ -C_F \left( \frac{\alpha_s}{\pi} \right) \ln^2(Q^2/Q_T^2) \right] \Rightarrow 0$$

**as  $Q_T \rightarrow 0$**

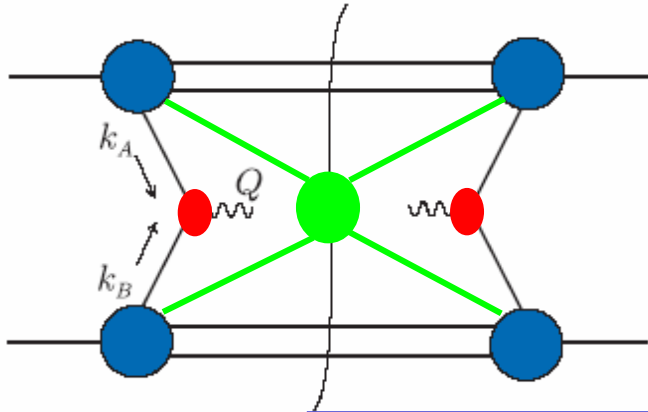
**□ Experimental fact:**  $\frac{d\sigma}{dydQ_T^2} \Rightarrow$  finite [neither  $\infty$  nor 0!] as  $Q_T \rightarrow 0$

Double leading logarithm approximation (DLLA) over constrains phase space of radiated gluons (strong ordering in transverse momenta)

ignore overall transverse momentum conservation

# CSS b-space resummation formalism

□ Leading order  $K_T$ -factorized cross section:



$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} = \sum_f \int d\xi_a d\xi_b \int \frac{d^2k_{A_T} d^2k_{B_T} d^2k_{s,T}}{(2\pi)^6}$$

$$\times P_{f/A}(\xi_a, k_{A_T}) P_{\bar{f}/B}(\xi_b, k_{B_T}) H_{\bar{f}\bar{f}}(Q^2) S(k_{s,T})$$

$$\times \delta^2(\vec{Q}_T - \vec{k}_{A_T} - \vec{k}_{B_T} - \vec{k}_{s,T})$$

$$\delta^2(\vec{Q}_T - \prod_i \vec{k}_{i,T}) = \frac{1}{(2\pi)^2} \int d^2b e^{i\vec{b}\cdot\vec{Q}_T} \prod_i e^{-i\vec{b}\cdot\vec{k}_{i,T}}$$

$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} \equiv \frac{1}{(2\pi)^2} \int d^2b e^{i\vec{b}\cdot\vec{Q}_T} \tilde{W}_{AB}(b, Q) + Y_{AB}(Q_T^2, Q^2)$$

resummed

No large log's

$$= \frac{1}{(2\pi)^2} \int_0^\infty db J_0(bQ_T) b \tilde{W}_{AB}(b, Q) + \left[ \frac{d\sigma_{AB}^{(\text{Pert})}}{dQ^2 dQ_T^2} - \frac{d\sigma_{AB}^{(\text{Asym})}}{dQ^2 dQ_T^2} \right]$$

The  $Q_T$ -distribution is determined by the b-space function:  $b\tilde{W}_{AB}(b, Q)$

# The b-space resummation

- **The b-space distribution:**  $\tilde{W}_{AB}(b, Q) \equiv \sum_{i,j} \tilde{W}_{ij}(b, Q) \hat{\sigma}_{ij}(Q)$

- The  $\tilde{W}_{ij}(b, Q)$  obeys the evolution equation

$$\frac{\partial}{\partial \ln Q^2} \tilde{W}_{ij}(b, Q) = [K(b\mu, \alpha_s) + G(Q/\mu, \alpha_s)] \tilde{W}_{ij}(b, Q) \quad (1)$$

- Evolution kernels satisfy RG equations

$$\frac{\partial}{\partial \ln \mu^2} K(b\mu, \alpha_s) = -\frac{1}{2} \gamma_K(\alpha_s(\mu)) \quad (2)$$

$$\frac{\partial}{\partial \ln \mu^2} G(Q/\mu, \alpha_s) = \frac{1}{2} \gamma_K(\alpha_s(\mu)) \quad (3)$$

- CSS Resummation of the large logarithms  $\iff$

- Integrate  $\ln \mu^2$  in Eq.(2) from  $\ln \frac{c^2}{b^2}$  to  $\ln \mu^2$
- Integrate  $\ln \mu^2$  in Eq.(3) from  $\ln Q^2$  to  $\ln \mu^2$
- Integrate  $\ln Q^2$  in Eq.(1) from  $\ln \frac{c^2}{b^2}$  to  $\ln Q^2$
- $c = 2e^{-\gamma_E} \sim 1$

**Leading  
power in  $1/Q^2$**

- homogeneous evolution equation  
 $\Rightarrow$  solution proportional to boundary condition

$$W_{ij}(b, Q) = W_{ij}(b, \frac{1}{b}) e^{-S_{ij}(b, Q)}$$

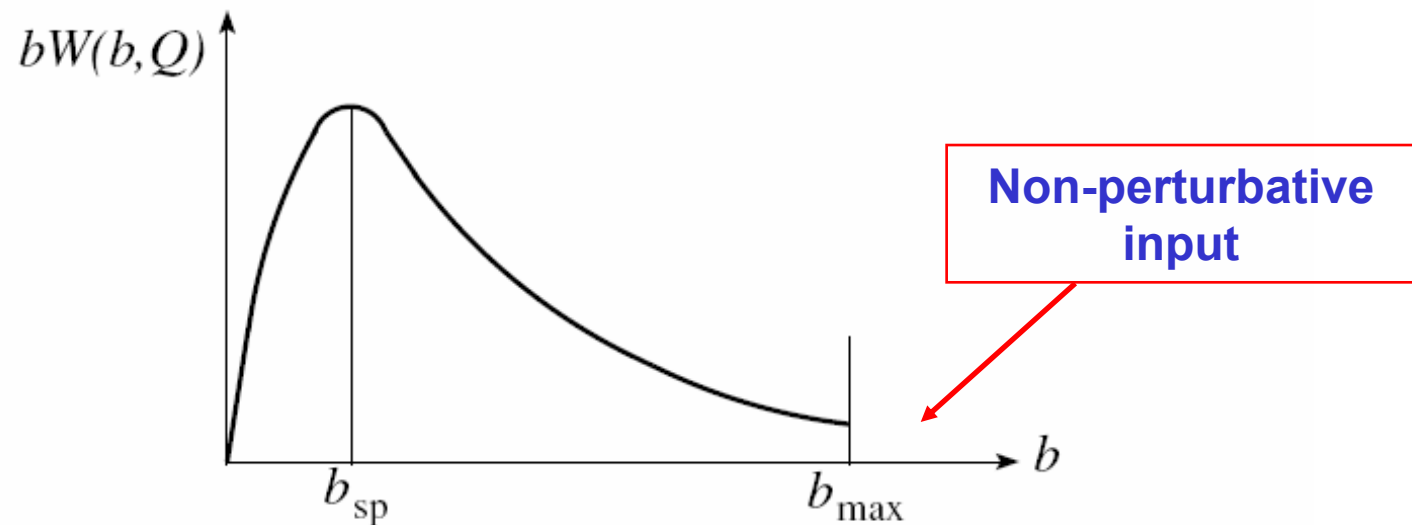
- if  $b \ll 1/\Lambda_{\text{QCD}}$ , boundary condition  $W_{ij}(b, 1/b)$ 
  - depends only on one perturbative scale  $\sim 1/b$
  - should be fully perturbative, and
  - have no large logarithms $\Rightarrow$  perturbative  $b$ -distribution

$$W^{\text{pert}}(b, Q) = \sum_{a, b, i, j} \sigma_{ij \rightarrow C}^{(LO)} \left[ \phi_{a/A} \otimes C_{a \rightarrow i} \right] \\ \otimes \left[ \phi_{b/B} \otimes C_{b \rightarrow j} \right] \times e^{-S(b, Q)}$$

## □ Sudakov form factor:

$$S(b, Q) = \int_{c^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ A(\alpha_s(\mu^2)) \ln\left(\frac{Q^2}{\mu^2}\right) + B(\alpha_s(\mu^2)) \right]$$

- all large logarithms are summed into  $S(b, Q)$ , and  $S(b, Q)$  is perturbative for  $b$  not too large
- functions:  $C_{a \rightarrow i}$  and  $C_{b \rightarrow j}$  are perturbative



- Need non-perturbative input at large  $b$ :

# Predictive power of the formalism

- $b$ -space distribution:

$$\int_0^\infty db J_0(q_T b) b e^{-S(b, Q)} \left[ \phi_{a/A} \otimes C_{a \rightarrow j} \right] \otimes \left[ \phi_{b/B} \otimes C_{b \rightarrow \bar{j}} \right]$$

- pQCD dominates if  $\int_0^{b_{max}} db(\dots) \gg \int_{b_{max}}^\infty db(\dots)$

- or saddle point  $b_{sp} \ll b_{max}$ :

- $b$ -dep of  $b e^{-S(b, Q)} \rightarrow b_{sp} \propto \left( \frac{\Lambda_{\text{QCD}}}{Q} \right)^\lambda, \lambda \sim 0.4$

- $b$ -dep of  $\phi_{a/A}(x, \frac{1}{b})$  and  $\phi_{b/B}(x', \frac{1}{b})$

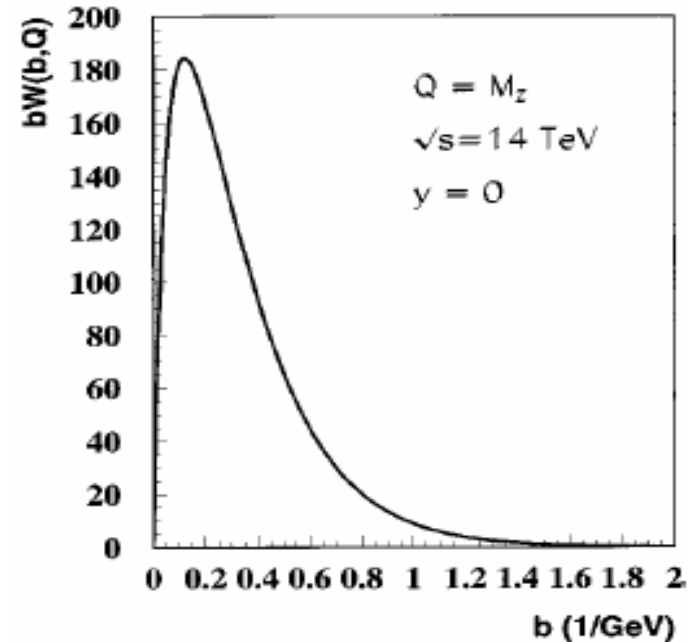
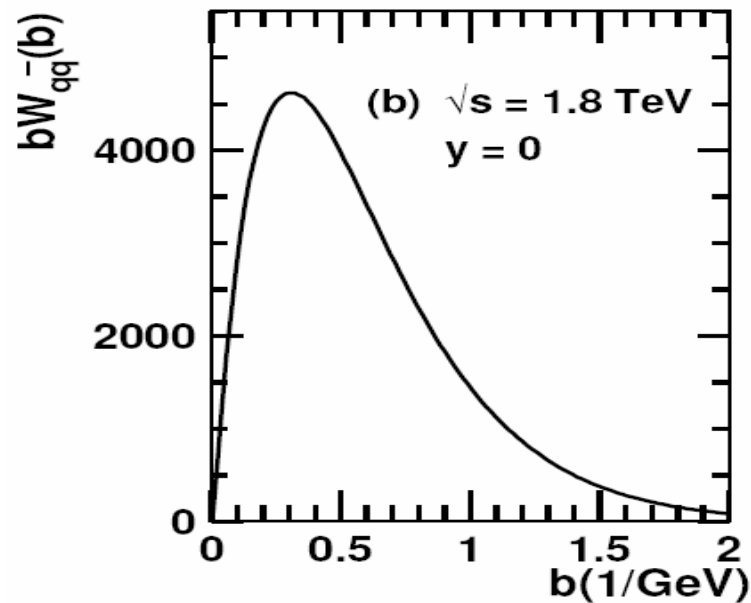
$\Leftrightarrow$  DGLAP evolution

$$\frac{d}{db} \phi(x, \frac{1}{b}) = -\frac{1}{b} \frac{d}{d \ln \frac{1}{b}} \phi(x, \frac{1}{b}) < 0 \quad \text{for } x < x_c \sim 0.1$$

$\Rightarrow$  larger  $\sqrt{S}$ , smaller  $x$ , and smaller  $b_{sp}$

# Location of the saddle point

## □ Z production (collision energy dependence):

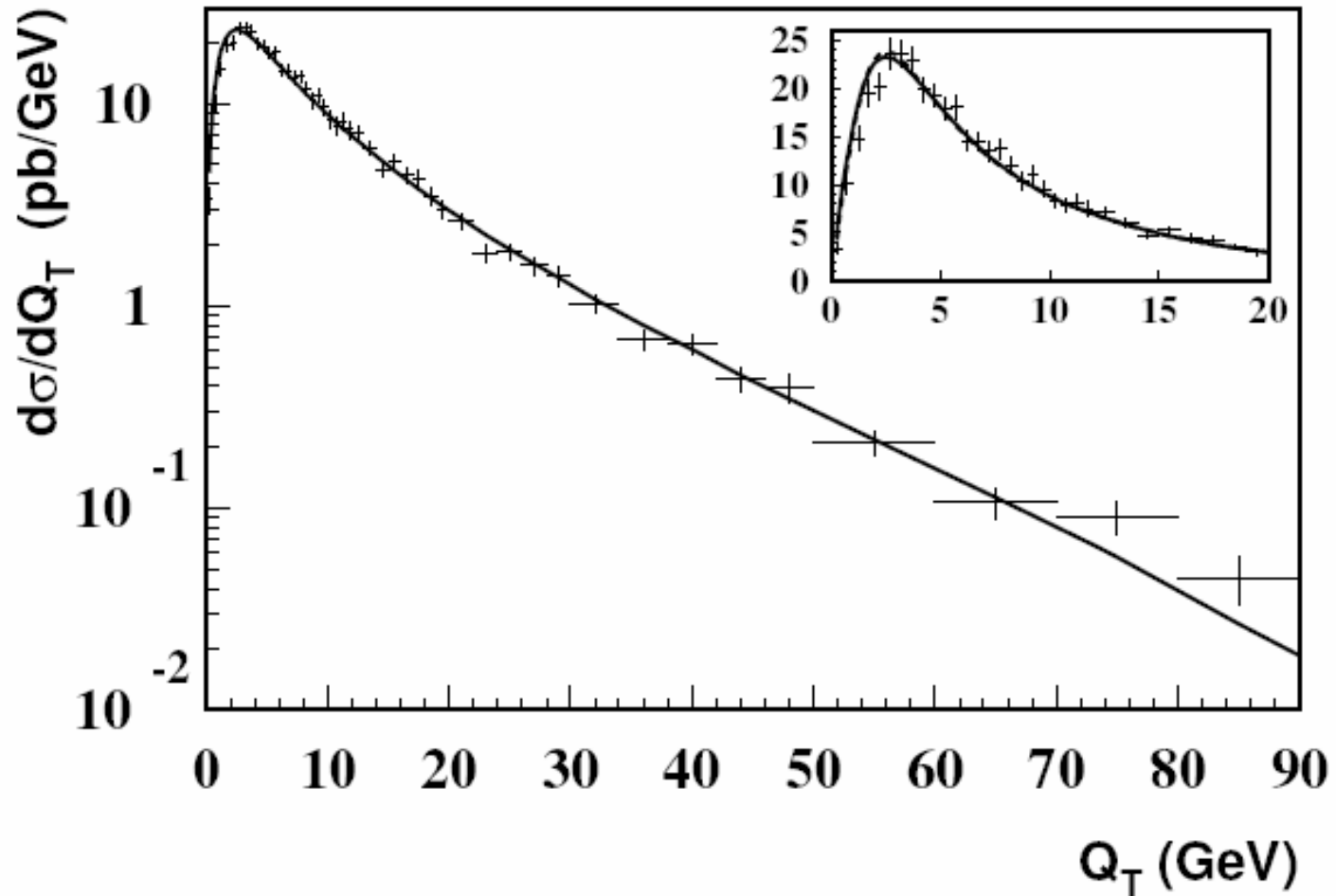


Higher collision energy = larger phase space  
= more gluon shower  
= larger parton  $k_T$

Shift of the peak is calculated perturbatively!

Qiu, Zhang

- Fermilab CDF data on  $Z$  at  $\sqrt{S} = 1.8$  TeV

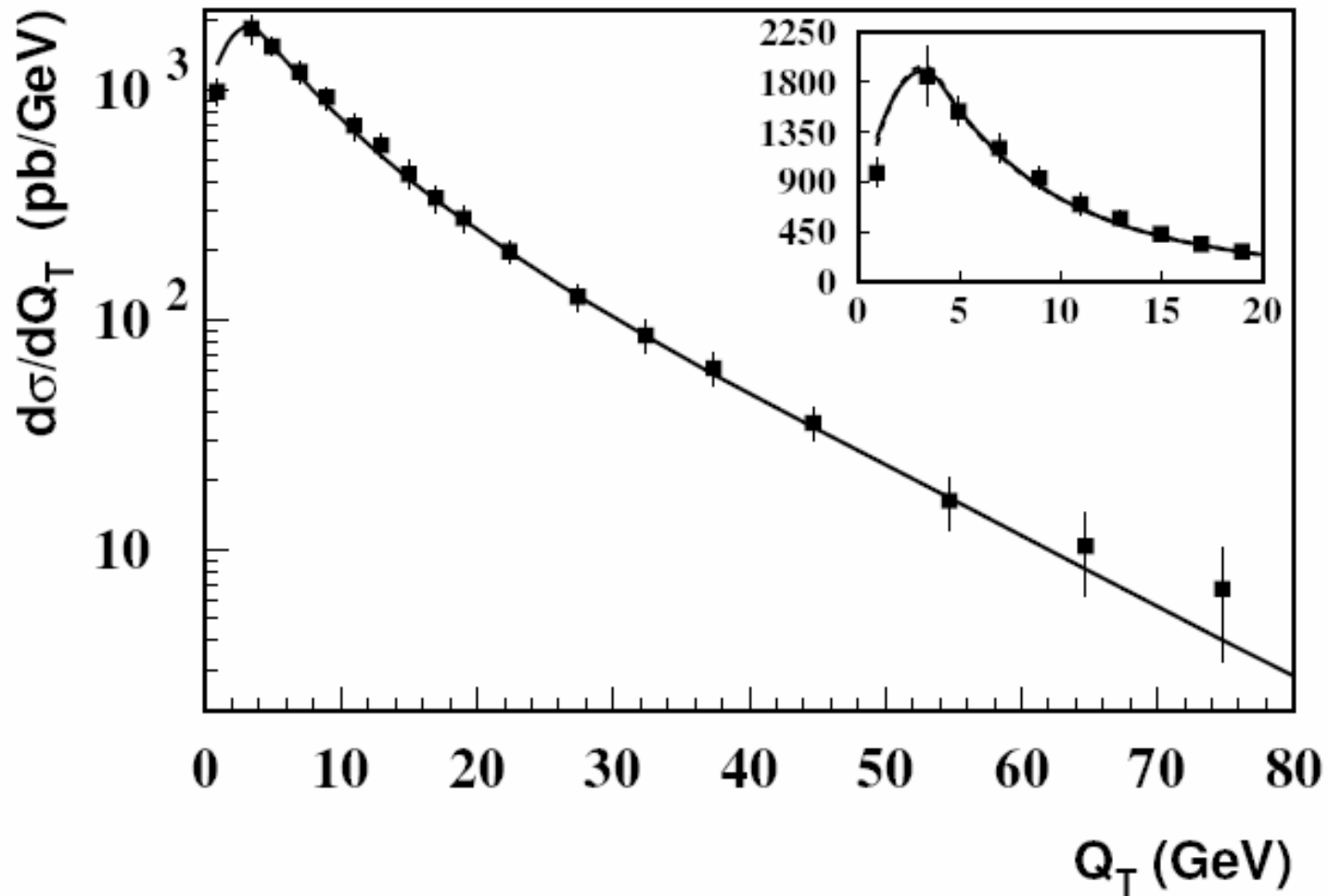


**Power correction is very small, excellent prediction!**

Qiu, Zhang

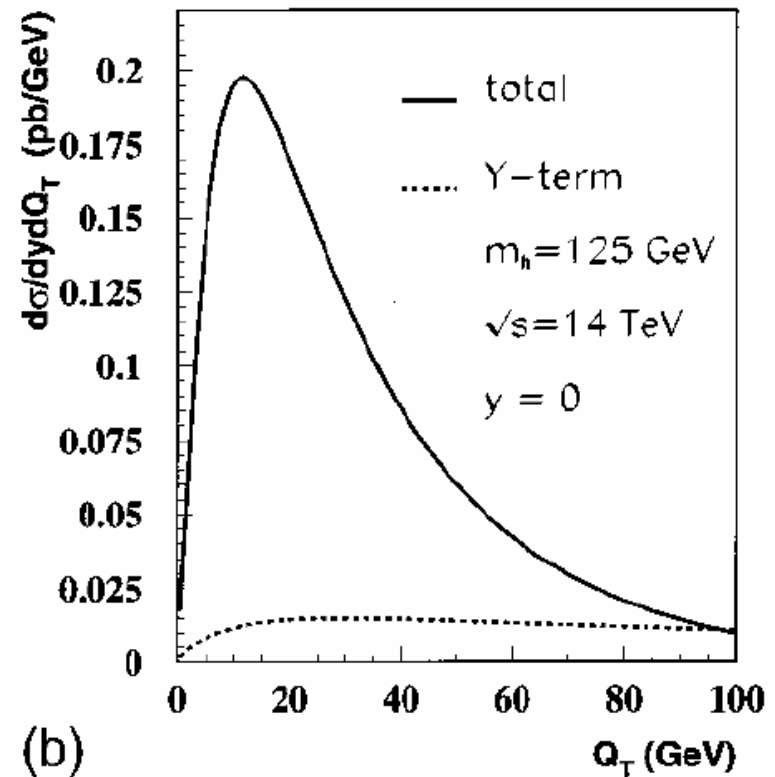
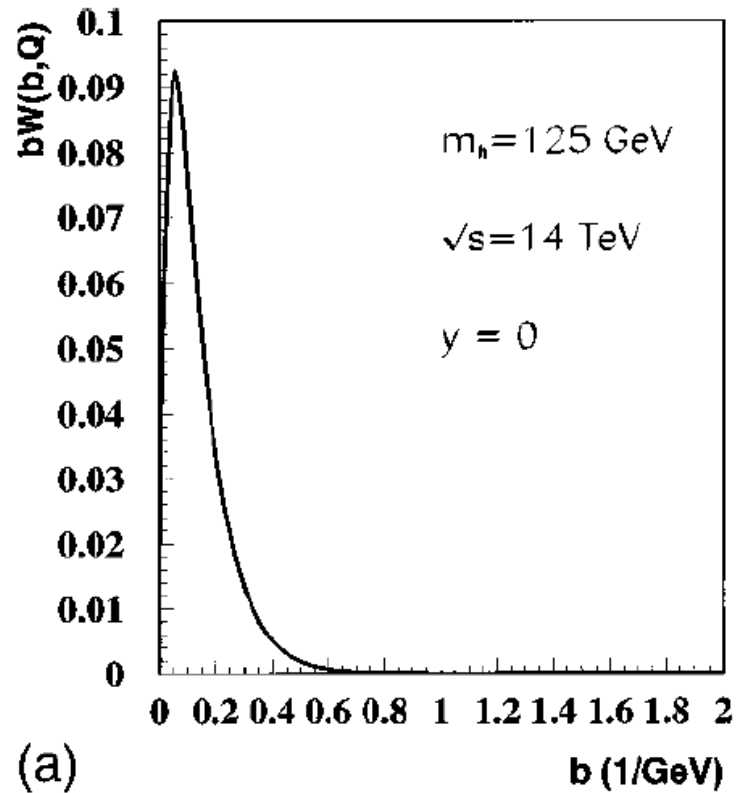


- Fermilab D0 data on  $W$  at  $\sqrt{S} = 1.8$  TeV



**No free fitting parameter!**

# Higgs production

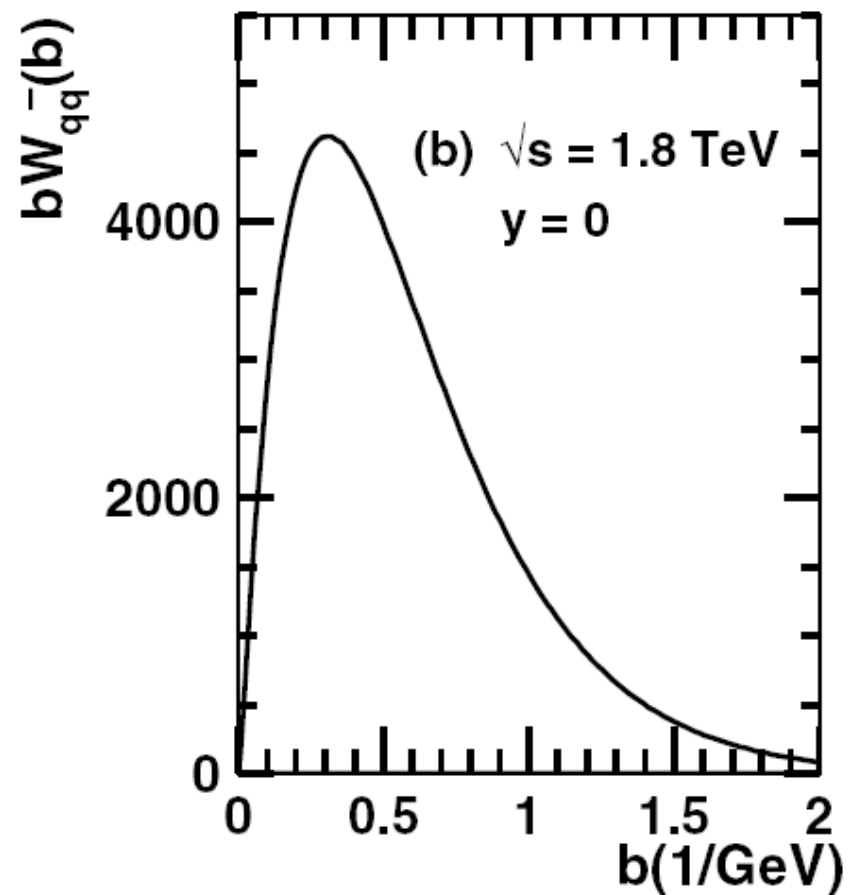
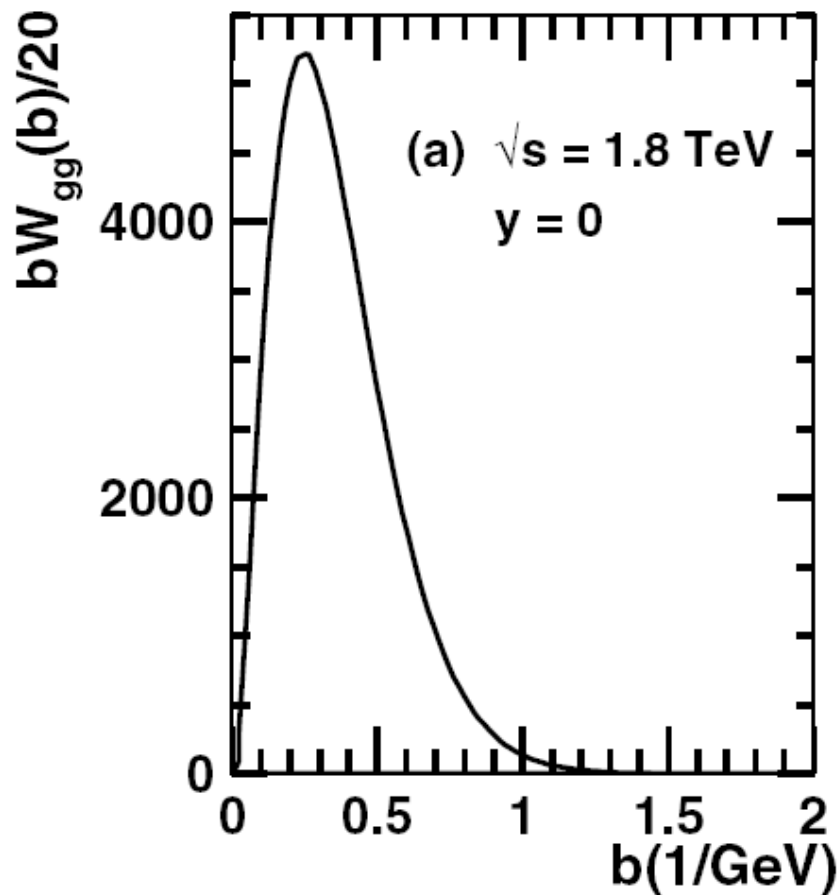


**Gloun-gluon domination = narrow  $b$ -distribution  
= large  $\langle Q_T \rangle$**

**Large  $\langle Q_T \rangle$  here is generated by gluon shower,  
but, is perturbatively calculated!**

Berger, Qiu

# Upsilon production

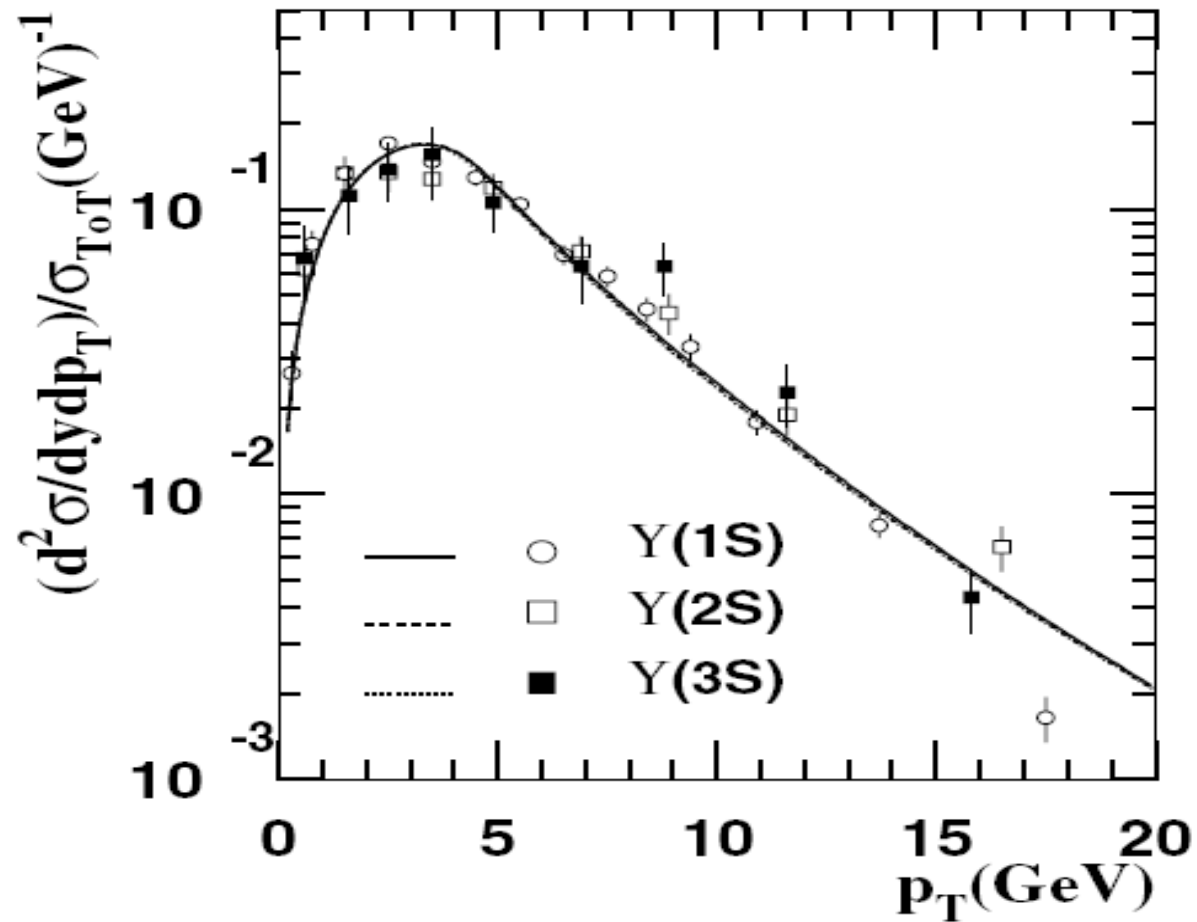


Dominated by gluon-gluon fusion

Narrow  $b$ -distribution = reliable perturbative calculation

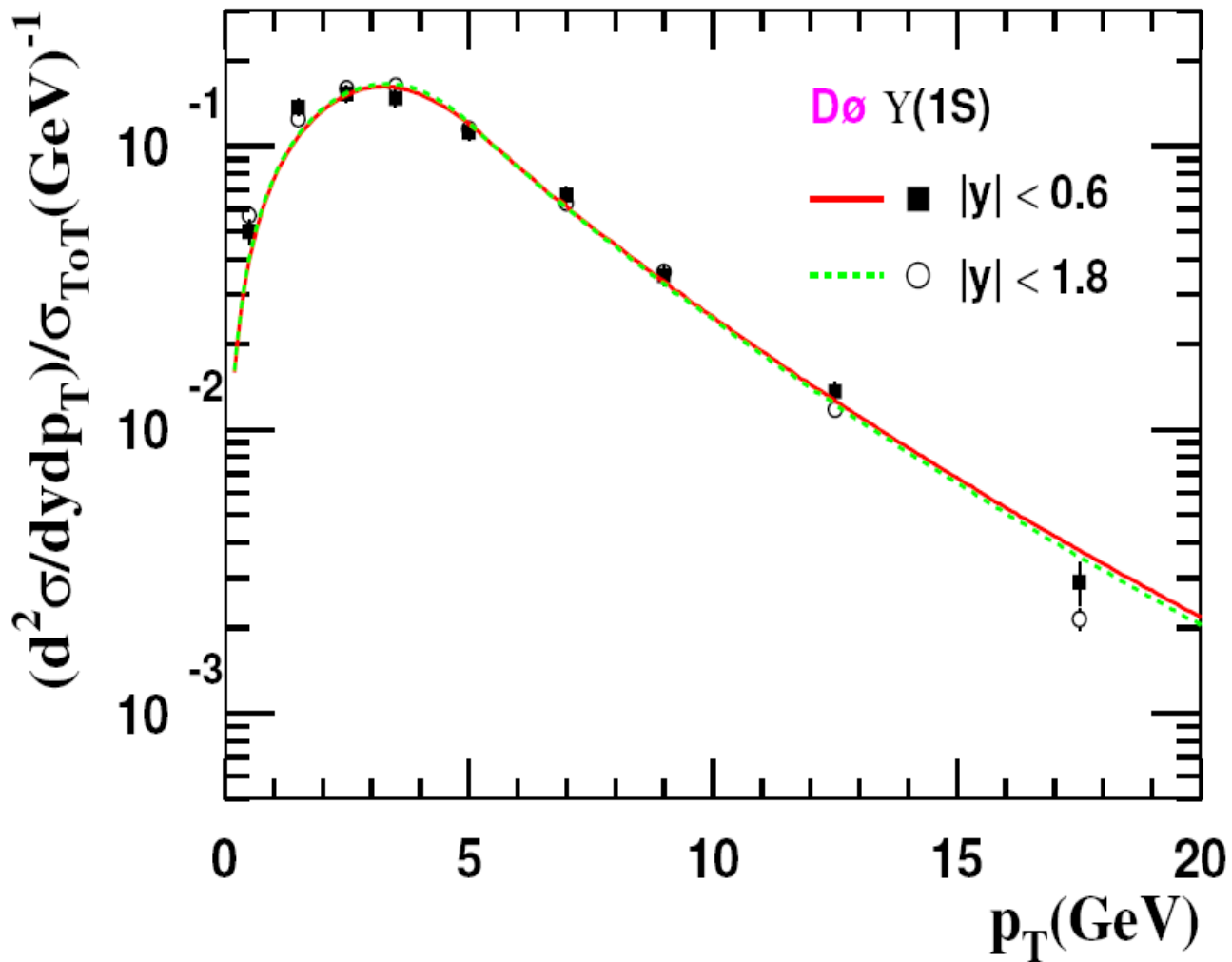
Berger, Qiu, Wang

# CDF Run – I Upsilon data



Berger, Qiu, Wang

# D0 Run – II Upsilon data



Berger, Qiu, Wang

# A good probe of gluon distribution

- Resummed Drell-Yan type process is a good probe of gluon distribution at small-x

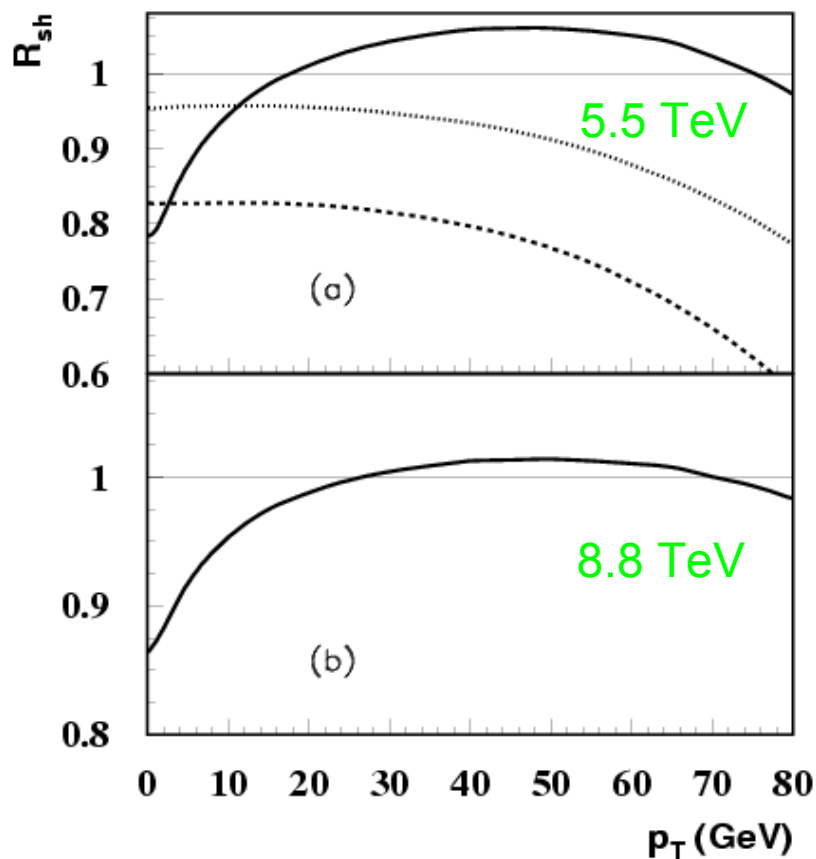
Resummed  $p_T$ -distribution is determined by b-space distribution

$$x_A = \frac{Q}{\sqrt{S}} e^y, \quad x_A = \frac{Q}{\sqrt{S}} e^{-y}, \quad \text{at } \mu \sim \frac{1}{b_{sp}}$$

- Although infinite soft gluon radiation involved, the broadening of  $p_T$  distribution is **perturbatively** calculable
- Since these particle does not interact much with hadronic matter, this process is a good probe of nuclear gluon distribution in pA collision

# Shadowing can lead to enhancement in $p_T$ distributions of W and Z production

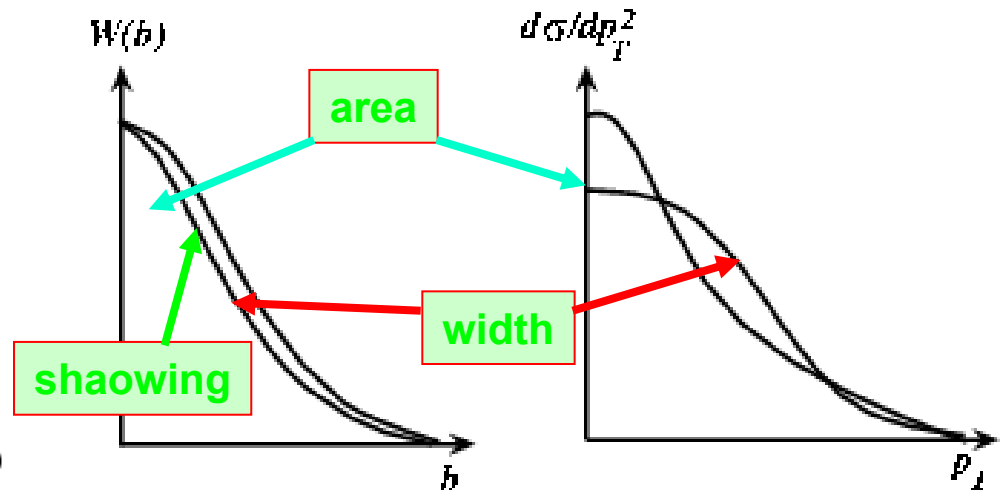
- W/Z production is dominated by low  $p_T$  region
- the shape is controlled by the gluon shower



$$R_{sh} = \frac{d\sigma^{(sh)}(p_T, A, B)}{dp_T^2} / \frac{d\sigma(p_T, A, B)}{dp_T^2}$$

Fixed order pQCD:  $x_{parton} < 0.05$ ,  $R_{sh} < 1$

Resummed pQCD: shadowing in  $b$ -space



Fai, Qiu, Zhang

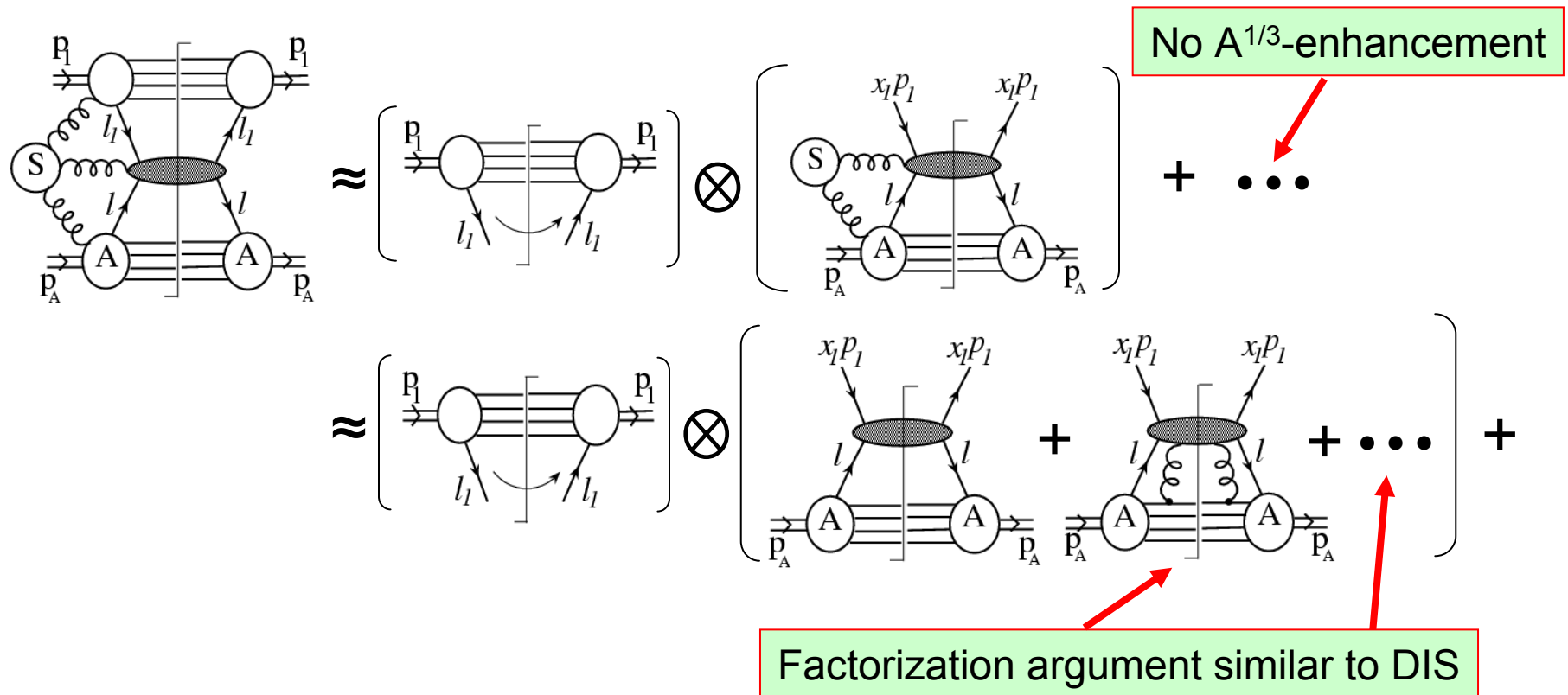
# Semihard processes

- ❑ Momentum exchange in the hard collisions,  $Q$ , is much larger than non-perturbative hadronic scale:  $1/fm \sim \Lambda_{\text{QCD}}$
- ❑ But, the scale,  $Q$ , is not large enough that the medium size enhanced power corrections are important
  - ➔  $(A^{1/3}-1)\xi^2/Q^2$  is not too much less than 1
  - $\xi^2$  is a medium sensitive scale  $\propto \langle F^{+\alpha} F^+_{\alpha} \rangle$
- ❑ Like the leading power, predictive power of pQCD for the power corrections also relies on the factorization
- ❑ Without the factorization, calculations and predictions are model dependent
- ❑ Factorization holds for A-enhanced power corrections in  $pA$
- ❑ Factorization fails for AA beyond  $1/Q^2$



# Factorization in p-nucleus collisions

- **A-enhanced power corrections,  $A^{1/3}/Q^2$ , are factorizable:**



- **But, power corrections to hard parts are process-dependent, and they are different from DIS**

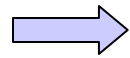
# Multiple Scattering

- ❑ **Single hard scattering** of momentum exchange,  $Q$ , is **localized** in space-time of  $1/Q$ , which is much less than nucleon size  $\sim$  fm
  - ➡ the scattering is only sensitive to the local parton densities (or distributions)
- ❑ Need **multiple scattering** to probe the medium properties (or structure)
  - **Coherent** multiple scattering is suppressed by the powers of the hard momentum scale,  $1/Q^n$ 
    - ➡ Need **semihard processes** to probe the coherent medium effect
  - **Incoherent** multiple scattering ➡ Glauber formalism

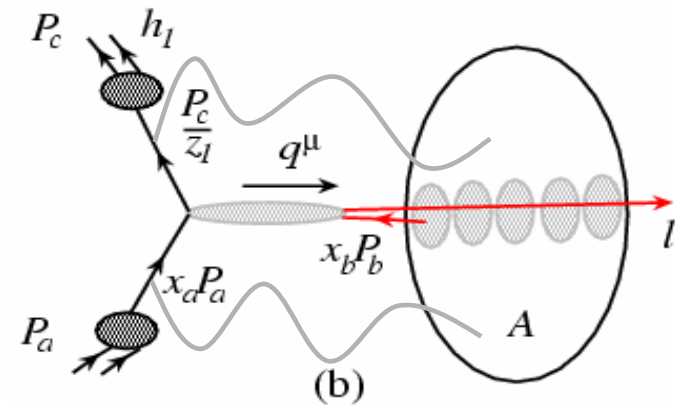
Qiu, Sterman, 2003

# Power Corrections in **p+A** Collisions

- Hadronic factorization fails for power corrections of the order of  $1/Q^4$  and beyond
- Medium size enhanced dynamical power corrections in p+A could be factorized



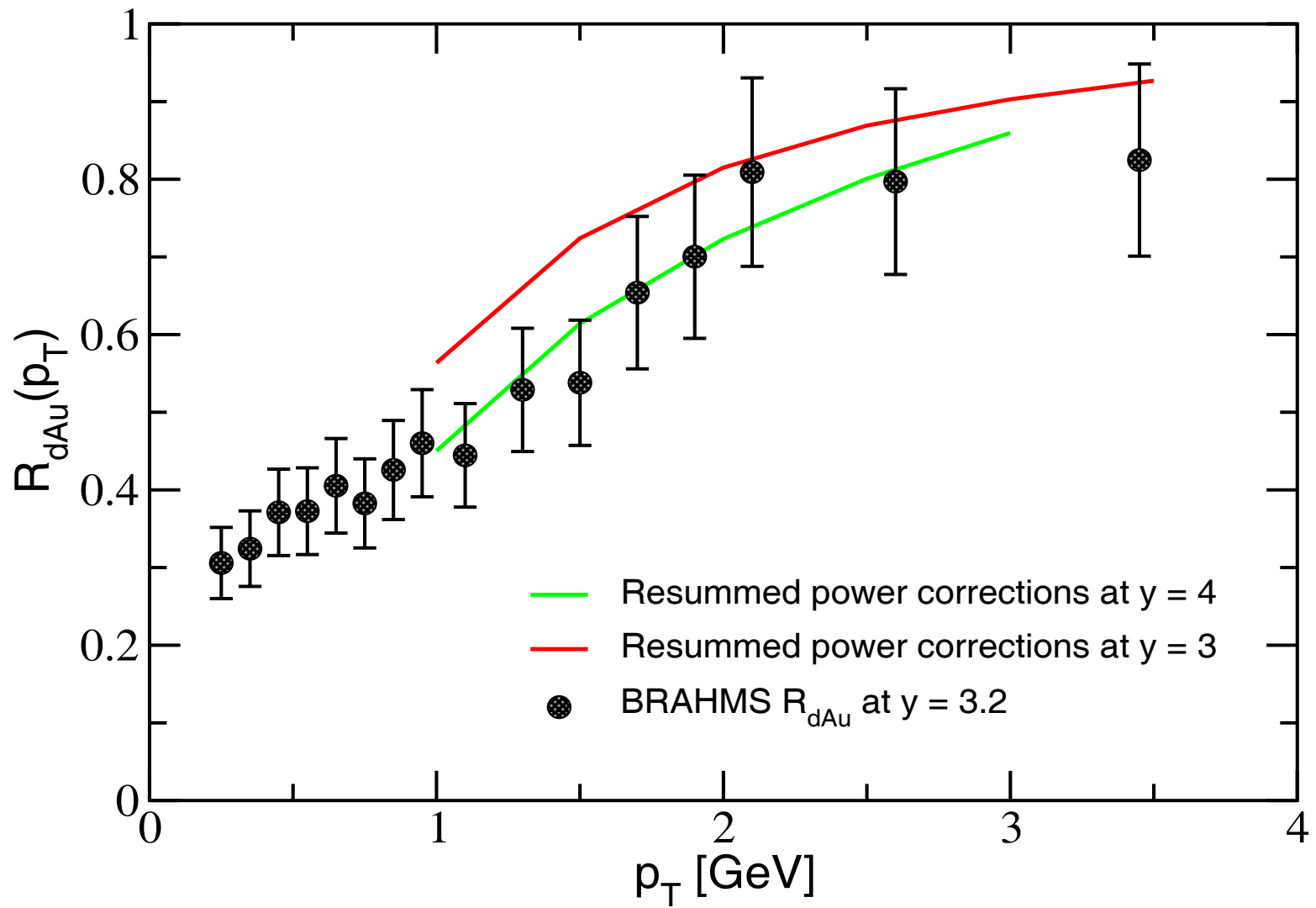
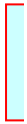
to make predictions for p+A collisions



- Single hadron inclusive production:

Once we fix the incoming parton momentum from the beam and outgoing fragmentation parton, we uniquely fix the momentum exchange,  $q^\mu$ , and the probe size

↔ **coherence** along the direction of  $q^\mu - p^\mu$



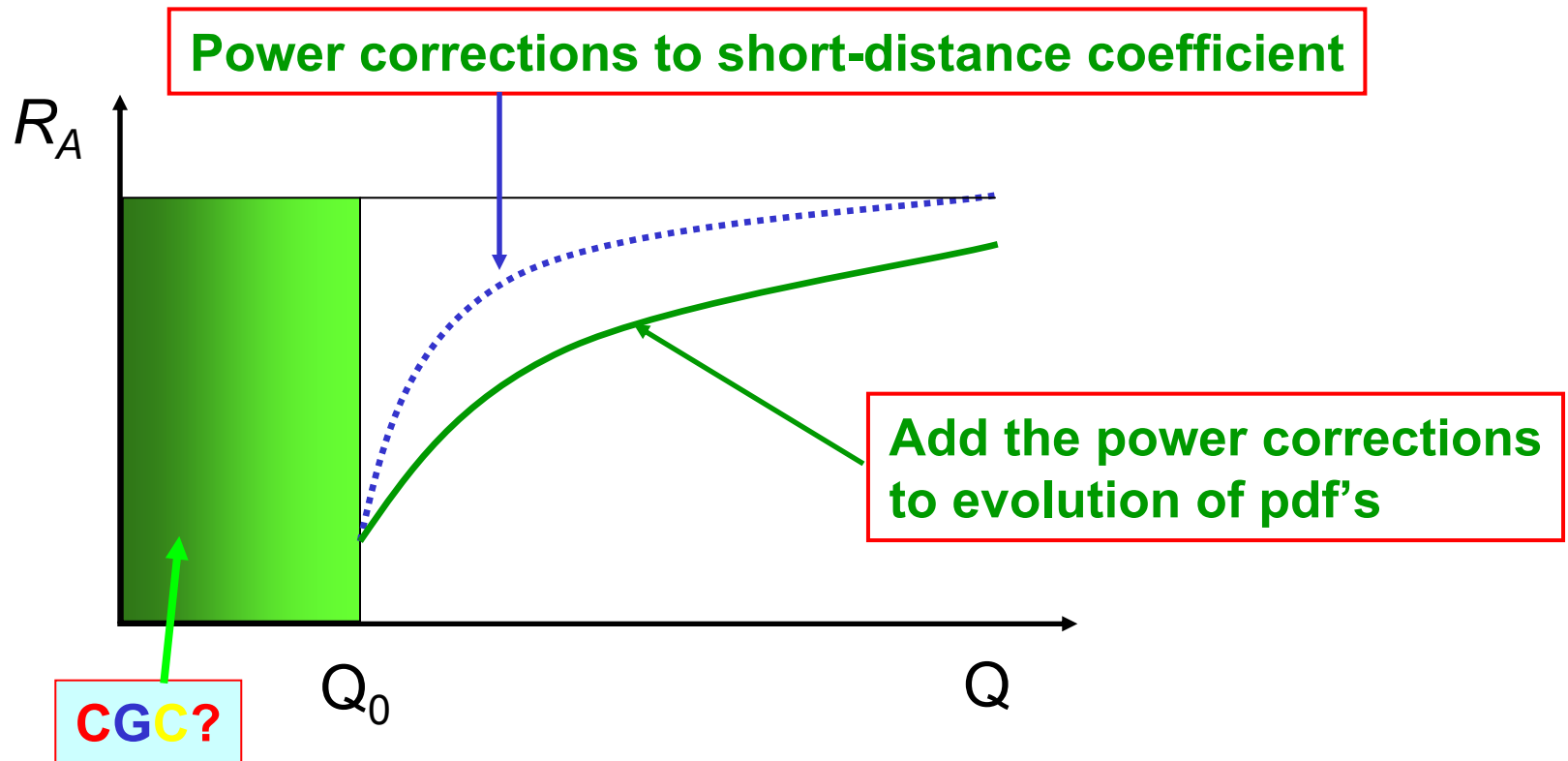
Vitev

# Role of coherent power corrections

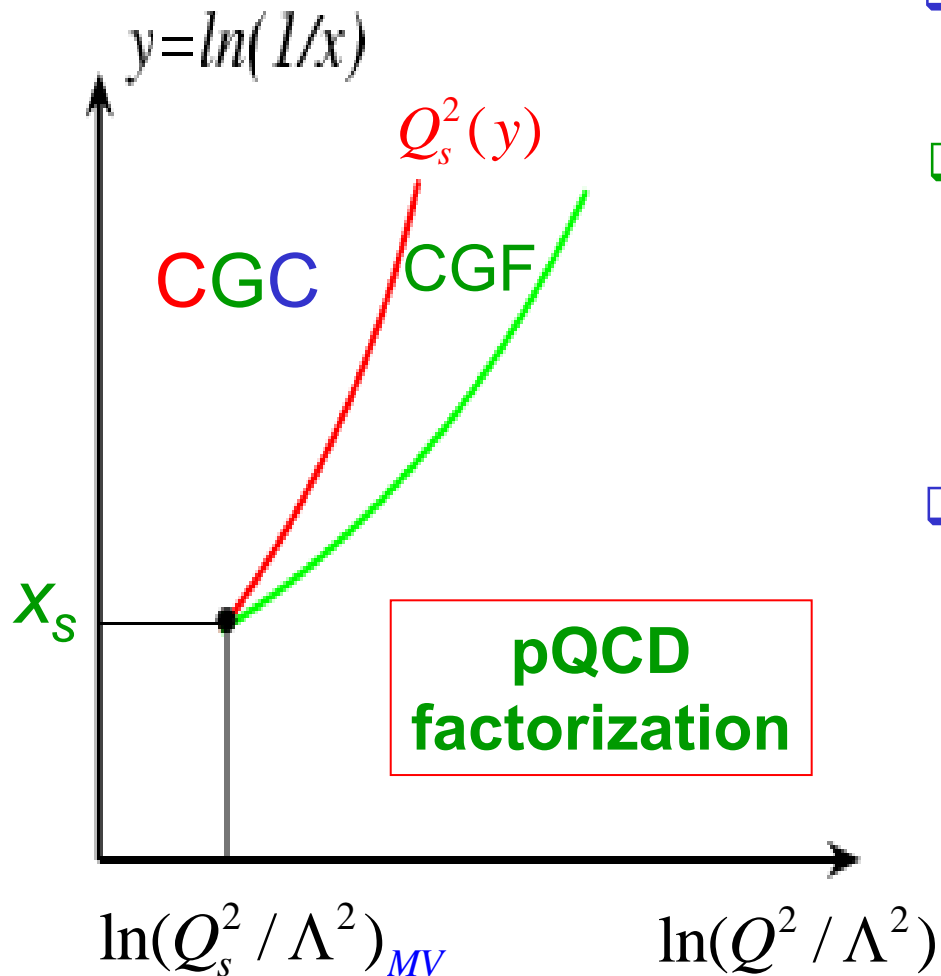
□ Ratio of **physical observables**:  $R_A$

$$R_A \equiv \frac{F_2^A/A}{F_2^D/2}, \quad \frac{\sigma^{dA}}{\langle N_{coll} \rangle \sigma^{NN}}, \quad \text{etc.}$$

- ❖ power correction to cross section
- ❖ power correction to evolution equation of pdf's



# Phase diagram of parton densities



❑ Experiments measure **cross sections**, not **PDFs**

❑ **PDFs are extracted based on**

- factorization
- truncation of perturbative expansion

❑ How to probe the boundary between different regions?

❑ **Look for where pQCD factorization fails**

❑ **Power corrections**  
– improve predictive power of factorization approach

# Conclusions

- ❑ Test the **predictive power** of pQCD in nuclear collisions by verifying the **universality of nPDF's** through the **hardest probes**, only available at the LHC
- ❑ Measure the **nPDF's** over an **unprecedented range of  $x$ ,  $Q^2$** 
  - find out the true nuclear modification to the PDF's by probing ultra-soft gluons through the  $y$ -dependence
  - QCD resummation significantly improve the predictive power, including the low  $p_T$  region, which is sensitive to soft gluon shower.
- ❑ Study the **multiple parton correlations** in nuclear medium by probing the **semihard subprocesses**
  - Heavy quarkonium, low mass Drell-Yan pair, dijet or di-hadron correlations, ...
- ❑ pA at the LHC can certainly provide a much needed help for extrapolating the hadronic collision to the GZK energy

# A new approach to the large b-region

$$\tilde{W}_{QZ}(b, Q) = \begin{cases} \tilde{W}(b, Q) & b \leq b_{max} \\ \tilde{W}(b_{max}, Q) \tilde{F}_{QZ}^{NP}(b, Q; b_{max}) & b > b_{max} \end{cases}$$

- ❖ solution of the CSS evolution equation in small-b region
- ❖ preserve the perturbative small b-region unchanged
- ❖ solution of the modified CSS evolution equation, including leading power corrections, in large b-region

$$F_{QZ}^{NP}(b, Q; b_{max}) = \exp \left\{ - \ln \left( \frac{Q^2 b_{max}^2}{c^2} \right) \left[ g_1 \left( (b^2)^\alpha - (b_{max}^2)^\alpha \right) + g_2 \left( b^2 - b_{max}^2 \right) \right] - \bar{g}_2 \left( b^2 - b_{max}^2 \right) \right\}$$

Leading twist

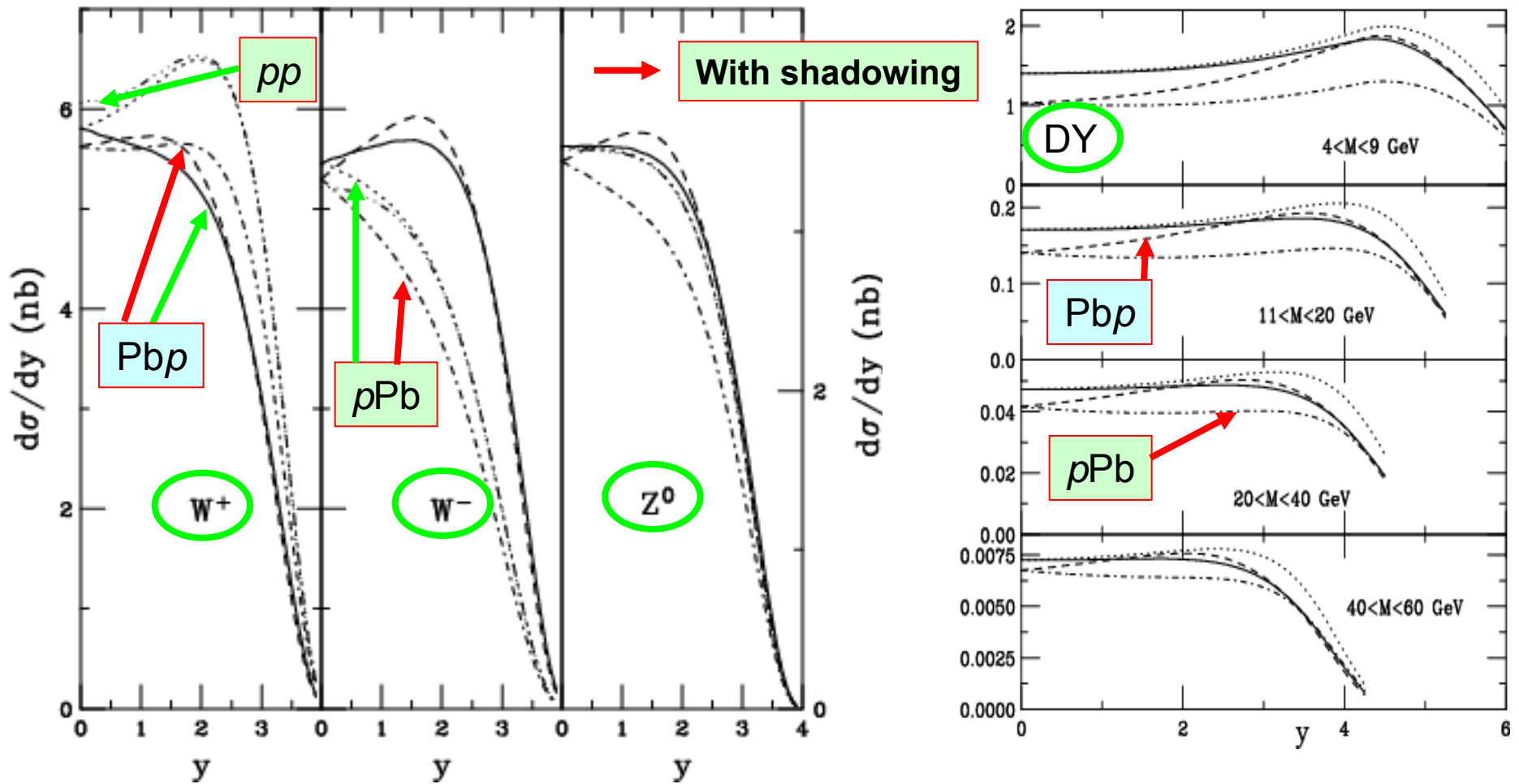
Intrinsic power corrections

Dynamical power corrections

- ❖  $g_1$  and  $\alpha$  are fixed by the continuity of  $W(b, Q)$  at  $b_{max}$
- ❖  $\sqrt{S}$  is built in the value of  $g_1$  and  $\alpha$



# Rapidity $y$ -dependence for W, Z and Drell-Yan cross sections



For  $\sqrt{s} = 5.5$  TeV  $pp$ ,  $pPb$ ,  $Pbp$  collisions

R. Vogt