Charm Theory Review (Non-lattice)

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In this talk, I review aspects of D^0-$\bar{D}^0$ mixing, rare decays of D mesons and CP violations in D meson decays, in a manner which emphasizes the interplay between experiment and theory.

1. INTRODUCTION

The organizers of this conference recommended three subjects to choose amongst – D^0 mixing, rare decays and CP-violations. Since there is currently activity in all three areas, this report will touch on each of them.

2. MIXING

The most newsworthy topic in the area of meson-antimeson oscillations is the recent detection of B_s-$\bar{B}_s$ mixing at Fermilab. This leaves D^0-$\bar{D}^0$ as the only undetected candidate for mixing. As such it will (I suspect) become an area of heightened experimental and theoretical scrutiny.

On a quantitative level, the current PDG data collection lists [1]

$$x_{\rm D} = \frac{\Delta M_{\rm D}}{\Gamma_{\rm D}} < 0.029 \quad (95\% \text{ C.L.})$$
$$y_{\rm D} \simeq \frac{\Delta \Gamma_{\rm D}}{2\Gamma_{\rm D}} < 0.008 \pm 0.005 \quad .$$

D^0-$\bar{D}^0$ mixing is indeed small and we know why – there is Cabibbo angle suppression $[\sin^2\theta_{\rm C}]$ and quark mass suppression $[(m_s^2/m_c^2)^n]$. We cannot, however, yet precisely predict the size of mixing. This frustration is partly a reflection of the quark mass, too large to justify chiral methods and too small to yield a rapidly convergent heavy quark expansion.

2.1. Mixing in the Standard Model

Our discussion will emphasize $\Delta\Gamma_{\rm D}$,

$$\Delta\Gamma_{\rm D} = -\frac{\mathcal{I}m\ \langle \bar{D}^0 | i\int d^4x T\left(H_{\rm w}(x) H_{\rm w}(0)\right) | D^0\rangle}{M_{\rm D}} \ .$$

To utilize this relation, we insert intermediate states between the $|\Delta C| = 1$ weak hamiltonians $H_{\rm w}$. This can be done using either quark or hadron degrees of freedom. Let us consider each of these possibilities in turn.

2.1.1. Quark-level Analysis

Using the heavy quark methodology to expand the mixing in terms of operator dimension, we find two four-fermion operators of dimension six, fifteen six-fermion operators of dimension nine, *etc.* [2–4] Consider the familiar $D = 6$ (box diagram) case. If the b-quark contribution is ignored, there are $s\bar{s}$, $s\bar{s}$ and $s\bar{d} + d\bar{s}$ intermediate states. Taking $m_d = 0$, the mixing functions will depend on $z \equiv m_s^2/m_c^2 \simeq 0.006$. Table 1 uses one of the mixing functions for $\Delta\Gamma_{\rm D}$ to show what happens when an expansion in powers of z is carried out. The individual intermediate states are *not* intrinsically small – they begin to contribute at $\mathcal{O}(z^0)$. However, flavor cancellations remove all contributions through $\mathcal{O}(z^2)$, so the net result is $\mathcal{O}(z^3)$, which is tiny. The corresponding result for $\Delta M_{\rm D}$ turns out to be $\mathcal{O}(z^2)$ (the extra factor of z for $\Delta\Gamma_{\rm D}$ is due to 'helicity suppression', the same mechanism that occurs in pion leptonic decay).

For any given operator dimension, one must expand in powers of $\mathcal{O}(\alpha_s)$. Thus the work just described gives the leading-order (LO) term in an expansion in QCD, with $x_{\rm D}^{\rm (LO)} \gg y_{\rm D}^{\rm (LO)}$.

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Table 1
Flavor Cancellations in $\Delta\Gamma$.

Int. State	$\mathcal{O}(z^0)$	$\mathcal{O}(z^1)$	$\mathcal{O}(z^2)$
$s\bar{s}$	$1/2$	$-3z$	$3z^2$
$d\bar{d}$	$1/2$	0	0
$s\bar{d} + d\bar{s}$	-1	$3z$	$-3z^2$
Total	0	0	0

The NLO calculation has recently been done in Ref. [5], and the effects of QCD are found to be significant. Summing the LO and NLO terms, the net result is $x_{\rm D} \simeq y_{\rm D} \sim 10^{-6}$. No higher order QCD contributions have yet been computed.

Higher dimensional sectors in the OPE are expected to be important, in part because the flavor cancellations will not universally be so severe. [3,4] For example, it is known that in the $D = 9$ sector the contributions are $\mathcal{O}(z^{3/2})$. A thorough study of z dependence is underway. [6]

To summarize, a proper analysis of D^0 mixing at the quark level involves a *triple* expansion – in dimension D, in powers of α_s and in powers of m_s^2/m_c^2. It is possible that as-yet uncalculated terms can lift $x_{\rm D}$ and $y_{\rm D}$ well above the LO + NLO $\mathcal{O}(10^{-6})$ scale for dimension six cited above.

2.1.2. Hadron-level Analysis

Using hadronic intermediate states in the formula for $\Delta\Gamma_{\rm D}$ has been a well-studied approach to the problem.

One possibility is to construct a theoretical model of $|\Delta C| = 1$ decays and to fit any undetermined model parameters via experiment. In 1995, a rather extensive model/fit of D^0 decays was applied to the determination of $\Delta\Gamma_{\rm D}$ and the result $y_{\rm D} \simeq 10^{-3}$ was found. [7] This is almost an order of magnitude smaller than the current experimental bound that I cited earlier.

Even earlier (in 1985), a more phenomenological method had been proposed in which one divides out phase space from experimental branching fractions and takes the square root to reconstruct the magnitudes of individual decay amplitudes. This can be used to then determine $y_{\rm D}$. [8] The two-body sector of charged pseudoscalar branching fractions was explored as a trial horse,

$$y_{\rm D}^{(P^+P^-)} = \mathcal{B}_{[D^0\to K^+K^-]} + \mathcal{B}_{[D^0\to \pi^+\pi^-]}$$
$$-2\cos\delta \left[\mathcal{B}_{[D^0\to K^-\pi^+]} \cdot \mathcal{B}_{[D^0\to K^+\pi^-]}\right]^{1/2}$$

where δ is the relative phase between the $K^-\pi^+$ and $K^+\pi^-$ modes. This contribution vanishes in the limit of exact SU(3) flavor symmetry, and the same must be true for all other decay sectors. However, since the real world exhibits large SU(3) symmetry breaking it was conjectured that this constraint might be invalidated. Unfortunately, little was known in 1985 about the $D^0 \to K^+\pi^-$ mode, and Ref. [8] could only reach the qualitative conclusion that SU(3) breaking might induce an unexpectedly large value for $y_{\rm D}$. A more recent and much more complete analysis indicates that each decay sector experiences only muted SU(3) breaking. [9] If substantiated over time, this presumably reflects the fact that in D^0-$\bar{D}^0$ mixing SU(3) breaking is of second order. [10] There must be exceptions, however, since certain multi-particle final states (*e.g.* four-kaon modes) are disallowed due to phase space restrictions. It has been stated that such multi-particle sectors might lead to a value as large as $y_{\rm D} \sim 0.01$. As one of the originators of the 'phenomenological' method, I would very much like for it to give the best prediction for $y_{\rm D}$. Unfortunately, sector-by-sector it is the least precisely known branching fraction that controls the predictive power of the analysis. Let us hope that this approach is not doomed to supply only 'fuzzy' estimates.

Finally, there is the possibility of exploiting the presence of resonances nearby the D^0 in mass, for which an enhancement of the mixing amplitude occurs. [11] This promising approach is currently hindered by an inadequate database regarding such nearby resonances.

2.2. Mixing Beyond the Standard Model

It has long been appreciated that meson-antimeson oscillations may be sensitive to heavy degrees of freedom which propagate in the mixing loop-amplitudes. Detection of mixing in K^0 and B_d systems led to predictions about charm and top quarks before these particles were actually discovered. In like manner, by comparing observed mixing with predictions of the Standard Model we constrain models of New Physics, impressively so for B_s-$\bar{B}_s$ mixing.

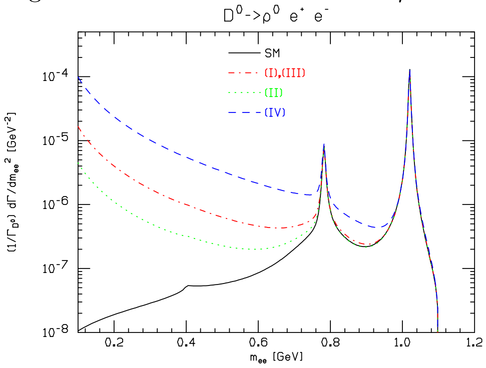
Admittedly, this has not yet led to the discovery of New Physics and we are all becoming increasingly frustrated. But it is important not to give up, so we keep trying. D^0 mixing has the intriguing property that the SM signal is very small and vanishes in the not-so-distant world of exact SU(3) symmetry. A group of us is currently involved in providing an up-to-date summary of New Physics in D^0 mixing. [12] Among other things, any such work should include a complete menu of NP possibilities (*e.g.* extra gauge bosons, *etc*), a proper treatment of NP amplitudes at the charm scale (basis operators and their QCD corrections) and an up-to-date knowledge of allowed NP parameter space.

A somewhat related topic involves the realization that NP *can* affect $\Delta\Gamma_{\rm D}$. There is often confusion about this point. Although it is true that only light quarks and gluons are allowed as intermediate states in the mixing amplitude for $\Delta\Gamma_{\rm D}$, it is likewise true that one of the interaction verticies can be NP. This line of reasoning would lead to a relation between NP contributions to $\Delta\Gamma_{\rm D}$ and to $|\Delta C| = 1$ decays. [13]

3. RARE DECAYS

The era of positive experimental results for rare charm decays is underway. Happily, there is already a sizable and modern theoretical literature on this subject (*e.g.* [14–16]). There has, however, been a lag in observations because the experimental signals are so small. This is about to change. One noteworthy, if somewhat distressing, theoretical result is that not all rare decay modes are of equal interest for detecting New Physics interactions at the quark level. A prime example is the sequence of flavor changing radiative decays $D \to V\gamma$, where $V = \rho, K^*, etc$ is a neutral spin-one particle. Here, one would hope to be probing the underlying quark transition $c \to u\gamma$. This mode is, of course, absent at tree level in the SM but can occur at higher orders. Even though the QCD corrections are relatively large, the $c \to u\gamma$ signal is still unobservably tiny.

Figure 1. Contributions to $D^0 \to \rho^0 e^+ e^-$.



[17] Such is not the case for the so-called long-distance contributions. These are calculated in terms of hadronic degrees of freedom and are estimated to give branching fractions as large as $\mathcal{O}(10^{-6} \to 10^{-5})$, [14] consistent with the scale that the PDG data compilation registers for the branching fraction $\mathcal{B}_{D^0\to\phi+\gamma} = (2.5^{+0.7}_{-0.6}) \cdot 10^{-5}$. This result is welcome but it informs us only about details of hadronic dynamics and nothing more.

Of greater potential for probing New Physics are the decays $D \to M\ell^+\ell^-$ where $M = \pi, K^*, etc.$ The output of observing such modes is not just a branching fraction, but also a distribution (most usually in dilepton invariant mass $q^2 \equiv (p_{\ell^+} + p_{\ell^-})^2$). Such a distribution could be of real interest, but not all parts of it. Consider the situation depicted in Fig. 1 for $D^0 \to \rho^0\ell^+\ell^-$, where the SM contribution is shown in black and the predictions of a version of supersymmetry with nonuniversal soft breaking effects are shown in color. [15] This figure has two striking features:

[1] The first is the presence of sharp peaks for intermediate values of q^2. These are induced by the weak transition $D^0 \to R^0\rho^0$ (R^0 is a neutral noncharmed resonance) followed by the electromagnetic conversion $R^0 \to \ell^+\ell^-$. This part of the graph is clearly a manifestation of SM physics, and reflects comments made above regarding the flavor-changing radiative decays.

[2] More interesting from the viewpoint of New

Table 2
Current Status of Some Decay Asymmetries ($A_{\rm CP}$)

Neutral	PDG Value	Charged	PDG Value
$D^0 \to K^+K^-$	0.005 ± 0.016	$D^\pm \to \phi\pi^\pm$	-0.014 ± 0.033
$D^0 \to \pi^+\pi^-$	0.021 ± 0.026	$D^\pm \to \pi^\pm K^+K^-$	0.002 ± 0.011
$D^0 \to K_s\pi^0$	0.001 ± 0.013	$D^\pm \to \phi\pi^\pm$	-0.02 ± 0.05
$D^0 \to \pi^0\pi^+K^-$	-0.031 ± 0.086	$D^\pm \to \pi^\pm\pi^+\pi^-$	-0.02 ± 0.04

Physics is the region of small q^2. The SM signal is relatively tiny here, giving the NP signal a chance to stand out. Several possible supersymmetric scenarios are shown and we refer the reader to Ref. [15] for details.

Recent experimental work at FermiLab has begun to make inroads on this subject via the decays $D^+, D_s^+ \to \pi^+\mu^+\mu^-$. Details of this impressive work is presented by Paul Karchin in the following talk. [18] It suffices to point out that the ϕ peak in the intermediate transition $D^+, D_s^+ \to \phi\pi^+$ has been observed and that for the region sensitive to New Physics the data is about 500 times above the SM signal, just 10 times above the signal expected from the Little Higgs model [16], but already constrains a version of R-parity violating supersymmetry [15].

4. CP-VIOLATIONS

To this date, no effects involving CP violation (CPV) have been observed for charmed hadrons. We comment below on two potentially interesting areas, involving respectively two-body decay modes and three-body decay modes.

4.1. Direct CPV in Two-body Modes

On the experimental side, direct CPV for two-body decays of D mesons involves measurement of the asymmetry

$$A_{\rm CP} = \frac{\Gamma_{D\to f} - \Gamma_{\bar{D}\to\bar{f}}}{\Gamma_{D\to f} + \Gamma_{\bar{D}\to\bar{f}}} \quad .$$

Direct CPV has been observed for kaons (ϵ'/ϵ) and B$_d$-mesons by studying $K \to \pi\pi$ and $B_d \to K\pi$ decays respectively. Despite years of effort and the presence of sizeable branching ratios, attempts to measure CP violating asymmetries in D decays have yielded only null results. The present level of experimental sensitivity is indicated by the entries in Table 2.

What does theory have to say about direct CPV? In all cases, it is QCD which inhibits a complete treatment. For ϵ'/ϵ in kaon physics, chiral symmetry provides a successful means for predicting the electroweak penguin contribution [19], but the QCD penguin component remains an enigma. For heavy quark systems, the strong phase shifts generally depend on a mix of perturbative and nonperturbative QCD dynamics. The nonperturbative component (surely dominant at the charm scale) has thus far precluded a model-independent description.

4.2. Dalitz Plots of Three Body Modes

There have been a number of interesting Dalitz studies of charm decays over the years and they are listed in Ref. [20]. Some of these are sensitive to both D^0 mixing and CPV. A recent example, $D^0 \to K_{\rm S}^0\pi^+\pi^-$, yields the mixing limits [21]

$$-0.045 < x_{\rm D} < 0.093$$
$$-0.064 < y_{\rm D} < 0.036 \quad .$$

Although not yet at the level of the experimental bounds listed earlier, they are not that far off. It is reassuring that the literature is continuing to attract interesting suggestions for further work. [22,23]

5. SUMMARY

Taking experiment and theory together, it is clear that the study of charm electroweak phenomena is quite an active area. But there is much left to do, as I summarize in the following.

5.1. Mixing

Experiment: A number of experimental studies have been underway, which is good. But nothing which could be called a discovery has yet been seen, so more sensitivity is needed. I especially feel that the range $0.001 < y_{\rm D} < 0.01$ should be probed in ongoing and forthcoming experiments. Are recent results from Belle and BaBar 'hints' or just fluctuations?

Theory: Crisp Standard Model predictions for $x_{\rm D}$ and $y_{\rm D}$ remain frustratingly elusive, but at least we know why. At the quark level, what is involved is a triple expansion in dimension D, QCD coupling α_s and the mass ratio $z \equiv m_s^2/m_c^2$. An NLO analysis with $D = 6$ gives $x_{\rm D} \simeq y_{\rm D} \simeq 10^{-6}$. One anticipates progress in attacking the problem, but not without difficulty. Performing the theoretical study of $y_{\rm D}$ at the hadron level has yielded some interesting insights. A specific model of $|\Delta C| = 1$ decays has $y_{\rm D} \simeq 10^{-3}$ whereas an up-to-date phenomenological analysis appears to allow $y_{\rm D} \simeq 10^{-2}$. Improved $|\Delta C| = 1$ decay data should impact this latter approach.

For reasons already explained, D^0-$\bar{D}^0$ remains a possibly fertile area for New Physics studies. Because the field of New Physics is in a state of constant flux (new models, constrained parameter space, *etc*), there is a need to update or redo the calculations. A modern treatment of this problem is underway.

5.2. Rare Decays and CP-violations

Experimental studies of charm rare decays and CPV have been performed and as a result we now know what levels of sensitivity are attainable. The work thus far has laid the foundations for real progress in the field, but the next round of experiments will face the challenge of doing even better. We must bridge the gap between current experimental bounds and theoretical predictions. For theorists, some studies of rare decays are already in the literature, but we need more input regarding expectations of various CPV signals.

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