Charm physics from the lattice

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Outline

1. Introduction
   - Why Charm physics?
   - Why charm and LQCD?
2. Lattice QCD
   - Light quark methods (for sea and valence)
   - Heavy quarks
3. Results
   - Charm quark mass
   - Spectroscopy
   - Decays

Why the (renewed) interest in charm physics?

- An exciting time ... lots of new resonances in $c\bar{c}$ and D systems.
- A benchmark for B-physics calculations and a good test of theoretical tools.
- A theoretical challenge. Is $m_c \sim 1.3\text{GeV}$ too light for HQET? Certainly much heavier than m_u and m_d so light quark methods don't work.
- Possible new physics to be found? Search for exotica, CP violation etc.

What can be learned from the lattice?

- Test lattice calculations of stable ground state masses.
- Test lattice calculations of amplitudes eg the CKM-independent ratio $\frac{\mathcal{B}(D \to \ell\nu)}{\mathcal{B}(D \to \pi\ell\nu)}$.
- With good lattice calculations can measure CKM matrix elements, V_{cd} & V_{cs}.
- Lattice calculations can explore the spectrum of radial and orbital excitations, hybrids and exotics. Needed to understand new states.

Quenched approximation removed

In new unquenched ($N_f \geq 2$) calculations

- Many quantities agree at $\sim$ % level with experiment.
- Precision calculations of "gold-plated" quantities.
  - $\longrightarrow$ stable particles (usually mesons), one-hadron processes
- Wealth of v. interesting, v. challenging "solid gold" quantities await.
  - $\longrightarrow$ unstable particles, multihadron final states, strong decays, baryons, ... Lots of progress recently.

A number of different fermion formulations

Goal: reduce m_q $\Rightarrow$ chiral extrapolations $\Rightarrow$ better precision.

Light quark formulations

- Overlap/domain wall: Very good chiral behaviour, expensive, theoretically sound.
- Staggered: Good chiral behaviour, cheap but theoretical questions unresolved.
- Wilson and Clover: Poor chiral behaviour, reasonable cost, theoretical sound.

With some caveats ...

Cost of Staggered < Cost of Wilson < Cost of Overlap

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Example

Typical pion masses in lattice simulations

- MILC: $m_\pi \sim 230$ MeV
- Others: $m_\pi \sim 330 - 490$ MeV
- Physical: $m_\pi = 140$ MeV

Methods for heavy quarks - m_b only

HQET

Many calculations in the static limit and now with $1/m_b$ corrections (ALPHA Collab.)

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An expansion in HQ velocity. Improvable but nonrenormalisable. $\mathcal{O}(1/(am_q))^n$ errors.

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Methods for heavy quarks - reaching m_c

Fermilab (El-Khadra, Kronfeld & Mackenzie (1996))

Mass-dependent operator renormalisation. Works at all quark masses. Many operators for $\mathcal{O}(a^2)$ improvement.

- substantial work in quenched QCD.
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Relativistic improved

Must control $\mathcal{O}(am_q)$ errors. Recent results:

- Dudek *et al* (overlap)
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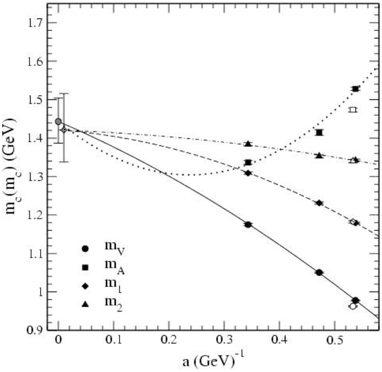
The charm quark mass



UKQCD (2004)

Observations

$\Leftarrow$ • quenched; ○ unquenched.

- Different mass defs agree at $a = 0$.
- Little difference quenched vs unquenched.
- For heavy light quarks.

$$\text{UKQCD} \begin{cases} N_f = 0 & m_c^{\overline{MS}}(m_c) = 1.29(7)(13)\ \text{GeV} \\ N_f = 2 & m_c^{\overline{MS}}(m_c) = 1.247(3)\binom{+20}{-4}\ \text{GeV} \end{cases}$$

Precision spectroscopy and LQCD

Renewed interest with lots of new states discovered.

New Charm States

$D^*_{sJ}(2317), D_{sJ}(2460), D_{sJ}(2630), D^*_0(2308), D'_1(2440)$

New Charmonium States

$X(3872), X(3943), Y(3943), Z(3931), Y(4260)$

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Precise theoretical calculations of S, P, D, F waves and radial excitations, hybrids and exotics would help identify these states. In principle, LQCD offers the best theoretical hope ...

Precision spectroscopy and LQCD

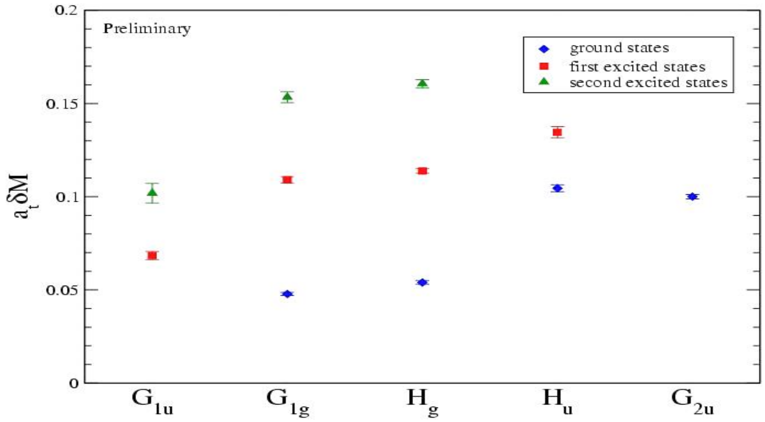
In practice, this is very difficult! Not gold-plated.

Stumbling blocks

- Understand states created by lattice ops $\Leftrightarrow$ continuum counterparts.
- Disentangle degeneracies, sometimes across the lattice irreps.
- Resolve spin-splittings and spin-orbit effects.
- All this for states at/near threshold.

But, it can be done

S,P,D wave splittings with the B_s meson in the static limit

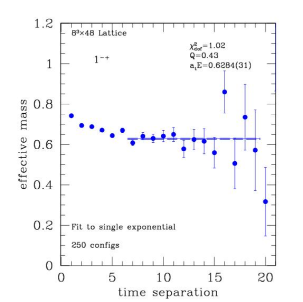


J.Foley *et al* (Lattice 2005)

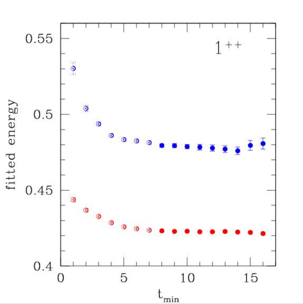
New methods in lattice spectroscopy

$\Rightarrow$ variational basis of operators with all-to-all propagators.

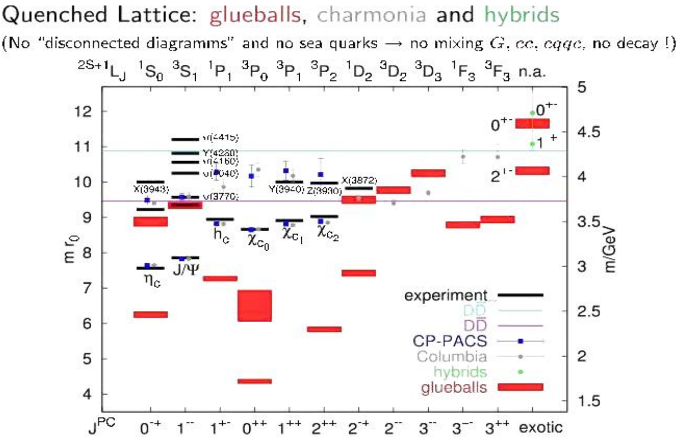
$c\bar{c}$ hybrid



Radial excitations



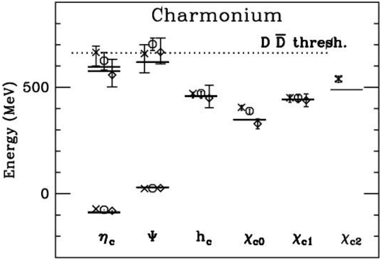
Quenched $c\bar{c}$ spectrum of S,P and D waves



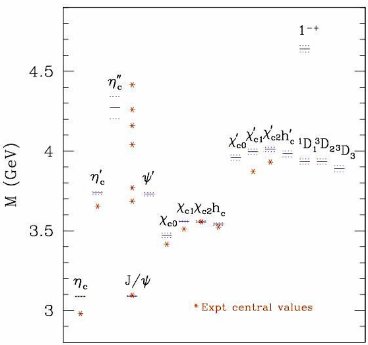
G.Bali, Charm2006, Beijing

Unquenched $c\bar{c}$ spectrum of S,P and D waves

Fnal+MILC
($N_f = 2 + 1$)



TrinLat (preliminary)
($N_f = 2, L < 2$fm)



The hyperfine splitting

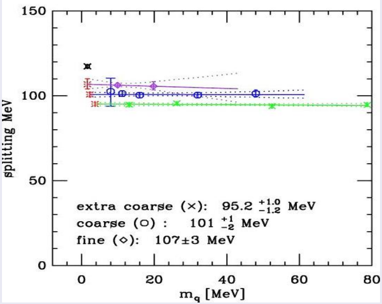
- Consistently underestimated in lattice calculations.
- Improvements seem to effect HFS in same direction: continuum extrapolation (↑), unquenching (↑), improved actions (↑), disconnected contributions (↑ ?).

Current best estimates of $(m_{J/\psi} - m_{\eta_c})$

	N_f	ΔM (MeV)
QCD-TAR0	0	77(2)(6)
Columbia & CP-PACS	0	89(8)
Tamhanker *et al*	0	88(4)
Dudek *et al*	0	97(6)
Fnal+MILC	2+1	$107(3)_{a^{-1} \sim 2.3 GeV}$

getting the right answer ...

Continuum & chiral extrapolations Fnal+MILC



$a \to 0 \Rightarrow \Delta M = 117\text{MeV}?$

getting the right answer ...

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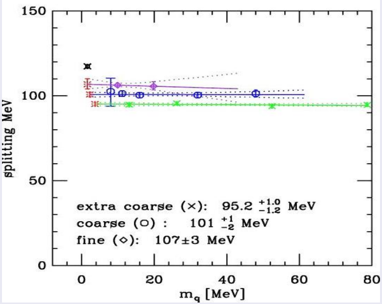

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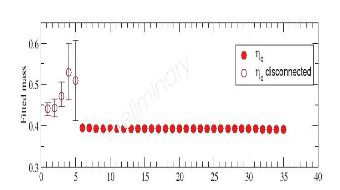
Disconnected diagrams

UKQCD, QCD-TARO: 20MeV?

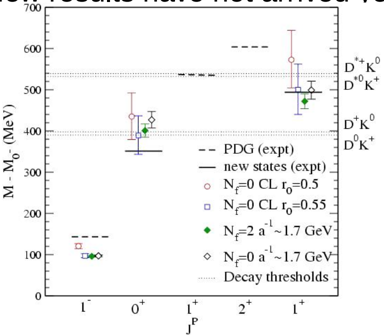
TrinLat - direct measurement



Needs all-to-all propagators with noise reduction

D_s spectroscopy

New results have not arrived yet. From UKQCD (2003)



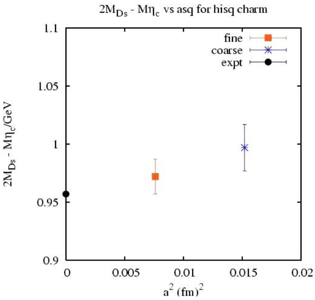
No evidence exotics are required to explain new states in $c\bar{s}$ spectrum.

Large systematics remain (need to control m_c and m_l dependent errors.)

Should be recalculated with better statistics and systematics

Controlling both $c\bar{c}$ and D physics?

Can be studied by looking at quantities like $(2M_{D_s} - M_{\eta_c})$
From Davies et al with staggered heavy and light quarks.



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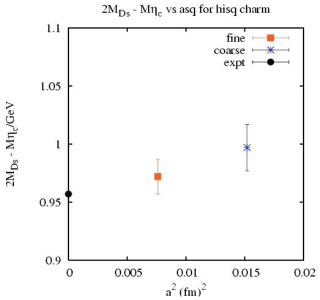


Should also be demonstrated for other formulations

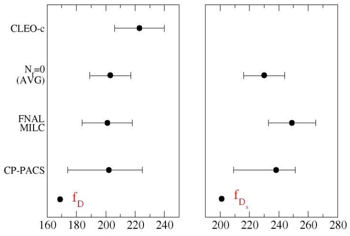
LQCD and the CKM matrix

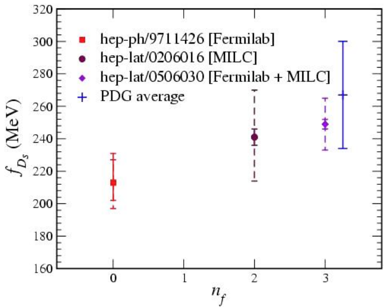
Determining the CKM elements

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \ell\nu & K \to \pi\ell\nu & B \to \pi\ell\nu \\ V_{cd} & V_{cs} & V_{cb} \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^{(*)}\ell\nu \\ V_{td} & V_{ts} & \\ \langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & V_{tb} \end{pmatrix}$$

- All CKM elements except V_{tb} can be determined from LQCD
- LQCD offers the only way for precision determinations of some eg V_{td}, V_{ts}.

Leptonic decays





- Fnal+MILC: $N_f = 2 + 1$, staggered light, Fnal heavy.
- CP-PACS: $N_f = 2$, clover light, RHQ heavy.

f_D and f_{D_s}

Decay constants

	f_D (MeV)	f_{D_s} (MeV)
Fnal/MILC	$201 \pm 3 \pm 17$	$249 \pm 3 \pm 16$
CP-PACS	$202 \pm 12^{+20}_{-25}$	$238 \pm 11^{+07}_{-27}$
CLEO-c	$223 \pm 17 \pm 3$	
BABAR		$279 \pm 17 \pm 6 \pm 19$

Errors

- LQCD dominated by discretisation error - need multi-a.
- CLEO-c dominated by statistics - reduced by full data set.

Exclusive semileptonic decays

Calculating $|V_{cx}|$

Hadronic matrix elements parameterised by form factors ($\leftarrow$ LQCD)

$$\langle P|V^\mu|D\rangle = \sqrt{2m_D}\left[v^\mu f_{||}(E) + p_\perp^\mu f_\perp(E)\right]$$

with $(v = p_D/m_D,\ p_\perp = p_P - Ev)$. $f_{||}, f_\perp$ related to f_0, f_+.

$\rightarrow$ Decay rate, $\Gamma/|V_{cx}|^2$ from $\int_{q^2}$ (phase space) $\times\ |f_+(q^2)|$

$\rightarrow$ $|V_{cx}|$ using measured lifetime and branching ratios.

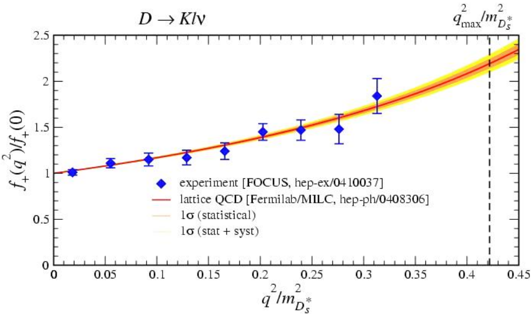
Precision form factors?

Requires

- Control of heavy quark discretisation effects $\rightarrow$ improved actions.
- Control of light quark effects $\rightarrow$ < 300 MeV pions in χ-extrapolation.
- $a \longrightarrow 0 \Rightarrow$ multiple a.
- Nonperturbative or $\geq$ 1-loop perturbative renormalisation.
- High statistics.
- $N_f = 2, 2+1$.

A prediction: $D \longrightarrow K\ell\nu$

The shape of $D \to K\ell\nu$ form factor from Fnal+MILC



Note: LQCD covers full kinematic range (unlike $B \to \pi$). Similar results for $D \to \pi\ell\nu$.
CLEO-c "threatens" dramatic improvement . . .

$D \longrightarrow \pi\ell\nu$ and $D \longrightarrow K\ell\nu$

FNAL+MILC results

$$f_+^{D\to\pi}(0) = 0.64 \pm 3 \pm 6$$
$$f_+^{D\to K}(0) = 0.73 \pm 3 \pm 7$$
$$\frac{f_+^{D\to\pi}(0)}{f_+^{D\to K}(0)} = 0.87 \pm 3 \pm 9$$

Error budget for $f_{+,0}$

time fits 3%
chiral fits 3%
BK param 2%
P.Th 1%
Discretisation 2%, 5%, 7%

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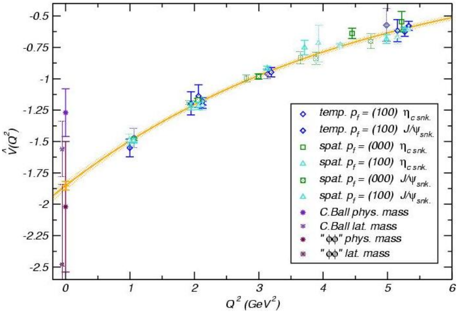
	LQCD	PDG
$\lvert V_{cd}\rvert$	0.239(10)(24)(2)	0.224(12)
$\lvert V_{cs}\rvert$	0.969(39)(94)(24)	0.9745(8)

First lattice determination of radiative transitions in $c\bar{c}$

Dudek *et al* (2006) $\Gamma(J/\Psi \to \eta_c \gamma_{M1}) =$ kinematics $\times\ |\hat{V}(0)|^2$



Also calculated: $h_1 \to \eta_c \gamma_{E1,C1}$, $\chi_{c0,c1} \to J/\Psi \gamma_{E1,(M2),C1}$, $f_{(J/\Psi,\eta_c,\Psi',\eta_c')}$.

Needs tuned m_c, unquench, . . .

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- Little to say from lattice about possible molecular states. I believe it is better to determine $q\bar{q}$ spectrum first.
- The leptonic and semileptonic decays are well-determined by the FNAL/MILC collaboration. It would be nice to see confirmation by other groups and methods.